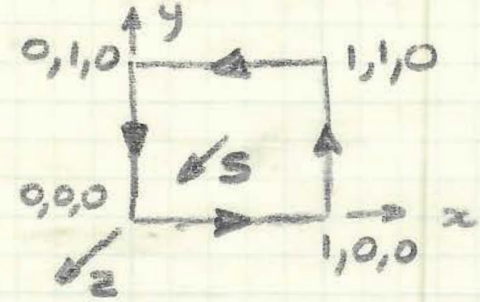
Given $\mathbf{H} = 300\mathbf{a}_z \cos(3 \times 10^8 t - y)$ A/m in free space, find the emf developed in the general $\mathbf{a}_\phi$ direction about the closed path having corners at (*a*) (0, 0, 0), (1, 0, 0), (1, 1, 0), and (0, 1, 0); (*b*) (0, 0, 0) ($2\pi$, 0, 0), ($2\pi$, $2\pi$, 0), and (0, $2\pi$, 0).

a.

- THE PATH LIES WITHIN THE $x$-$y$ PLANE




- THE MAGNETIC FLUX DENSITY IS ALIGNED WITH THE NORMAL TO THE PATH

$$B = \mu_o H = 300\mu_o \cos(3\times10^8 t - y)\,\hat{z} \text{ A/m}$$

$$\text{emf} = \frac{d\Phi}{dt} \qquad \Phi = \iint_S \bar{B}\cdot d\bar{S}$$

$$\Phi = \int_0^1\int_0^1 300\mu_o \cos(3\times10^8 t - y)\,dx\,dy$$

$$= 300\mu_o \sin(3\times10^8 t - y)\Big|_0^1$$

$$= 300\mu_o\left[\sin(3\times10^8 t - 1) - \sin(3\times10^8 t)\right] \text{Wb}$$

$$\text{emf} = -\frac{d\Phi}{dt}$$

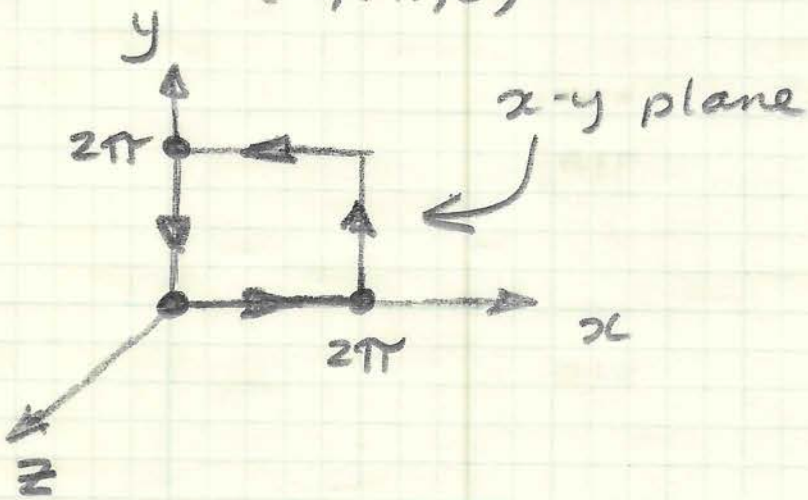
$$= 300(3\times10^8)(4\pi\times10^{-7})\left[\cos(3\times10^8 t - 1) - \cos(3\times10^8 t)\right]$$

$$= -1.13\times10^5\left[\cos(3\times10^8 t - 1) - \cos(3\times10^8 t)\right] \text{V}$$

b. CORNERS AT

(0,0,0) $(2\pi, 0, 0)$ $(2\pi, 2\pi, 0)$

$(0, 2\pi, 0)$




$$\Phi(t) = 2\pi \, 300 \mu_0 \sin(3\times10^8 t - y)\Big|_0^{2\pi}$$

$$= 0 \text{ wb.}$$

emf IS THEREFORE 0 AS WELL.

9.5

9.5 The location of the sliding bar in Figure 9.5 is given by $x = 5t + 2t^3$, and the separation of the two rails is 20 cm. Let $\mathbf{B} = 0.8x^2\mathbf{a}_z$ T. Find the voltmeter reading at (*a*) $t = 0.4$ s; (*b*) $x = 0.6$ m.

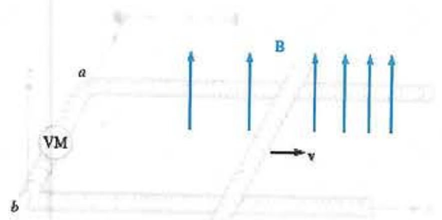



Fig. 9.5

a) $t = 0.4\,s$

① $\Phi = \int_0^{0.2}\int_0^{x} 0.8\, x'^2 dx'\, dy \Leftarrow \iint B \cdot dS$

$= \frac{0.16}{3} x^3 = \frac{0.16}{3}(5t + 2t^3)^3$ wb

② $emf = -\frac{d\Phi}{dt} = -\frac{0.16}{\cancel{3}}\cancel{3}(5t+2t^3)^2(5+6t^2)$

$= -0.16\,[5 \times .4 + 2 \times .4^3]^2\,[5+6.4^2]$

$= -4.32$ V

b) $x = 0.6$ m $\qquad 0.6 = 5t + 2t^3 \Rightarrow$

$t = 0.1193$

② $emf = -\frac{d\Phi}{dt}$

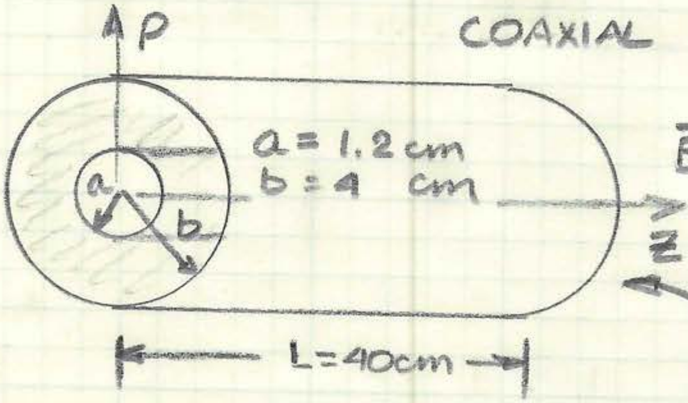
$= -0.16\,[5(.1193) + 2(.1193)^3]^2\,[5+6(.1193)^2]$

$= -0.293$ V

9.11

9.11 Let the internal dimensions of a coaxial capacitor be $a = 1.2$ cm, $b = 4$ cm, and $l = 40$ cm. The homogeneous material inside the capacitor has the parameters $\epsilon = 10^{-11}$ F/m, $\mu = 10^{-5}$ H/m, and $\sigma = 10^{-5}$ S/m. If the electric field intensity is $\mathbf{E} = (10^6/\rho)\cos 10^5 t\mathbf{a}_\rho$ V/m, find (*a*) $\mathbf{J}$; (*b*) the total conduction current $I_c$ through the capacitor; (*c*) the total displacement current $I_d$ through the capacitor; (*d*) the ratio of the amplitude of $I_d$ to that of $I_c$, the quality factor of the capacitor.

COAXIAL CAPACITOR




$$\bar{E} = \frac{10^6}{\rho}\cos 10^5 t\,\hat{\rho}\ \frac{V}{m}$$

$\epsilon = 10^{-11}$ F/m

$\mu = 10^{-5}$ H/m

$\sigma = 10^{-5}$ S/m

a. $\bar{J} = \sigma\bar{E} = \frac{10}{\rho}\cos 10^5 t\,\hat{\rho}\ A/m^2$

b. $I_c = \iint \bar{J}\cdot d\bar{S}$ $\qquad ds = d\varphi\, dL$

$= \frac{10}{\cancel{\rho}}\cos 10^5 t\ \ 2\pi\cancel{\rho} L$

$= 8\pi\cos 10^5 t$ A $=$ TOTAL CONDUCTION CURRENT

c. $\bar{J}_d = \frac{\partial\bar{D}}{\partial t} = \frac{\partial\epsilon\bar{E}}{\partial t} = \epsilon\frac{\partial\bar{E}}{\partial t}$

$= -\frac{\cancel{10^{-11}\,10^{6}\,10^{5}}^{\,1}}{\rho}\sin 10^5 t\,\hat{\rho}\ \frac{A}{m}$

$I_d = 2\pi\rho L J_d = -2\pi L\sin 10^5 t$

$= -0.8\pi\sin 10^5 t$ A $=$ TOTAL DISPLACEMENT CURRENT

d. $|I_d|/|I_c| = 0.8/8 = 0.1$ $=$ QUALITY FACTOR OF THE CAPACITOR

TOPS

9.12 The magnetic flux density $\mathbf{B} = B_0 \cos(\omega t)\cos(k_0 z)\,\mathbf{a}_y$ Wb/m$^2$ exists in free space. $B_0$ and $k_0$ are constants. Find (*a*) the displacement current density; (*b*) the electric field intensity; (*c*) $k_0$.

a. DISPLACEMENT CURRENT DENSITY

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} = \epsilon_0 \frac{\partial \bar{E}}{\partial t} = \nabla \times \frac{\bar{B}}{\mu_0} = \bar{J}_d$$

- THE FIELD IS y-DIRECTED, ONLY VARIES WITH z

$$\nabla \times \bar{H} = \nabla \times \frac{\bar{B}}{\mu_0} = -\frac{1}{\mu_0}\frac{\partial B_y}{\partial z}\hat{x} = \frac{\partial \bar{D}}{\partial t}$$

$$\bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \frac{k_0 B_0}{\mu_0}\cos(\omega t)\cos(k_0 z)\,\hat{x} \ \text{A/m}^2$$

b. ELECTRIC FIELD INTENSITY

$$\bar{E} = \frac{1}{\epsilon_0}\int \frac{\partial \bar{D}}{\partial t}\,dt + C$$

$$= \frac{k_0 B_0}{\mu_0 \epsilon_0}\sin k_0 z\,\hat{x}\int \cos \omega t\,dt$$

$$= \frac{k_0 B_0}{\omega \mu_0 \epsilon_0}\sin k_0 z \sin \omega t\,\hat{x}$$

$C = 0$ because there is no steady electric field component when H is time-varying

c. $k_0$

FIND THIS BY ENSURING CONSISTENCY BETWEEN THE TWO MAXWELL CURL EQUATIONS.

FARADAY'S LAW

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow \bar{B} = -\int \nabla \times \bar{E}\, dt$$

$C = 0$ because there is no steady magnetic field when E is time-varying.

FROM b), $\bar{E}$ IS $x$-DIRECTED AND VARIES WITH $z$ ONLY.

THUS, THE CURL OF $\bar{E}$ SIMPLIFIES TO A SINGLE TERM.

$$\bar{B} = -\int \nabla \times E\, dt = -\int \frac{\partial E_z}{\partial z}\, \hat{y}\, dt$$

$$= \int -\frac{k_0^2 B_0}{\omega \mu_0 \epsilon_0} \cos k_0 z \sin \omega t\, \hat{y}\, dt$$

$$= \frac{k_0^2 B_0}{\omega^2 \mu_0 \epsilon_0} \cos k_0 z \cos \omega t\, \hat{y}$$

$$\frac{k_0^2}{\omega^2 \mu_0 \epsilon_0} = 1 \Rightarrow k_0 = \omega \sqrt{\mu_0 \epsilon_0} \ \ m^{-1}$$

We'll see this again in Chapter 11...

9.15

9.15 Use each of Maxwell's equations in point form to obtain as much information as possible about (*a*) **H**, if **E** = 0; (*b*) **E**, if **H** = 0.

THE TWO CASES CORRESPOND TO MAGNETOSTATIC AND ELECTROSTATIC SCENARIOS, RESPECTIVELY.

$\mathbb{H}$ IF $\mathbb{E} = 0$

$\nabla \times \mathbb{H} = \mathbb{J}_c$ — $\mathbb{H}$ IS SOLELY DUE TO CONDUCTION CURRENT.

$\frac{\partial \mathbb{B}}{\partial t} = 0$ — $\mathbb{B}$ IS CONSTANT

$\rho_v = 0$ — NO CHARGE IS PRESENT

$\nabla \cdot \mathbb{B} = 0$ — ALWAYS TRUE

BECAUSE $\mathbb{J}$ IS NON-ZERO BUT $\mathbb{E} = 0$, THE CURRENT CARRYING MEDIUM IS A PERFECT CONDUCTOR.

$\mathbb{E}$ IF $\mathbb{H} = 0$

$\nabla \times \mathbb{E} = 0$ — $\mathbb{E}$ IS CONSTANT IN TIME & CONSERVATIVE

$\nabla \times \mathbb{H} = \mathbb{J}_c + \frac{\partial \mathbb{D}}{\partial t} = 0 \Rightarrow \mathbb{J} = \frac{\partial \mathbb{D}}{\partial t} = 0$

$\nabla \cdot \mathbb{D} = \rho_v$ — $\mathbb{E}$ ARISES FROM FREE CHARGE ONLY

$\nabla \cdot B = 0$ — $\mathbb{B} = 0$ ANYWAY.

TOPS

9.18

9.18 The parallel-plate transmission line shown in Figure 9.7 has dimensions $b = 4$ cm and $d = 8$ mm, while the medium between the plates is characterized by $\mu_r = 1$, $\epsilon_r = 20$, and $\sigma = 0$. Neglect fields outside the dielectric. Given the field $\mathbf{H} = 5\cos(10^9 t - \beta z)\mathbf{a}_y$ A/m, use Maxwell's equations to help find (*a*) $\beta$, if $\beta > 0$; (*b*) the displacement current density at $z = 0$; (*c*) the total displacement current crossing the surface $x = 0.5d$, $0 < y < b$, $0 < z < 0.1$ m in the $\mathbf{a}_x$ direction.

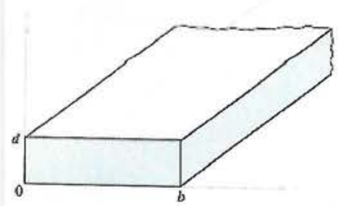



Fig. 9.7

a. $\nabla \times \bar{H} = -\frac{\partial H_y}{\partial z}\hat{x} = -5\beta \sin(10^9 t - \beta z)\hat{x}$

$= 20\epsilon_0 \frac{\partial E}{\partial t}$

$\bar{E} = \int -\frac{5\beta}{20\epsilon_0} \sin(10^9 t - \beta z)\hat{x}\, dt$

$= \frac{\beta}{4\times10^9 \epsilon_0} \cos(10^9 t - \beta z)\hat{x}$

$\nabla \times \bar{E} = \frac{\partial E_x}{\partial z}\hat{y} = \frac{\beta^2}{4\times10^9 \epsilon_0} \sin(10^9 t - \beta z)\hat{y}$

$= -\mu_0 \frac{\partial \bar{H}}{\partial t}$

$\bar{H} = \int \frac{-\beta^2}{4\times10^9 \mu_0\epsilon_0} \sin(10^9 t - \beta z)\hat{x}\, dt$

$= \frac{\beta^2}{4\times10^{18} \mu_0\epsilon_0} \cos(10^9 t - \beta z)$

$= 5\cos(10^9 t - \beta z)\hat{y}$

TOPS

$$\frac{\beta^2}{4\times 10^{18}\mu_0\epsilon_0} = 5$$

$$\beta^2 = \frac{4\times 10^{18}\times 5}{(3\times 10^8)^2} \qquad \beta = \frac{\sqrt{2\times 10^{19}}}{3\times 10^8}$$

$$= 14.9 \ m^{-1}$$

b. DISPLACEMENT CURRENT DENSITY @ $z=0$

$$\sigma = 0 \ ; \quad \nabla\times\bar{H} = \bar{J}_d = -5\beta \sin(10^9 t - \beta z)\hat{x}$$

$$= -74.5 \sin(10^9 t - \beta z)\hat{x}$$

$$= -74.5 \sin 10^9 t \ \hat{x} \ \frac{A}{m}$$

at $z=0$

c. TOTAL DISPLACEMENT CURRENT CROSSING THE SURFACE S

$x = 0.5d \quad 0 < y < b \quad 0 < z < 0.1\,m$

$= 4mm \quad 0 < y < 4cm$

$$I_d = -74.5\,b\int_0^{0.1} \sin(10^9 t - 14.9z)\,\hat{x}\cdot\hat{x}\,dz$$

$$= \frac{-74.5\times 0.04}{-14.9}\cos(10^9 t - 14.9z)\Big|_0^{0.1}$$

$$= 0.20\left[\cos(10^9 t - 1.49) - \cos(10^9 t)\right] A$$

TOPS

9.22

9.22 In a sourceless medium in which $\mathbf{J} = 0$ and $\rho_v = 0$, assume a rectangular coordinate system in which $\mathbf{E}$ and $\mathbf{H}$ are functions only of $z$ and $t$. The medium has permittivity $\epsilon$ and permeability $\mu$. (*a*) If $\mathbf{E} = E_x\mathbf{a}_x$ and $\mathbf{H} = H_y\mathbf{a}_y$, begin with Maxwell's equations and determine the second-order partial differential equation that $E_x$ must satisfy. (*b*) Show that $E_x = E_0 \cos(\omega t - \beta z)$ is a solution of that equation for a particular value of $\beta$. (*c*) Find $\beta$ as a function of given parameters.

a.

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow \frac{\partial E_x}{\partial z}\hat{y} = -\mu \frac{\partial H_y}{\partial t}\hat{y}$$

$\partial/\partial z$

$$\nabla \times \bar{H} = -\frac{\partial \bar{D}}{\partial t} \Rightarrow -\frac{\partial H_y}{\partial z}\hat{x} = \epsilon \frac{\partial E_x}{\partial t}\hat{x}$$

$\partial/\partial t$

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu \frac{\partial^2 H_y}{\partial t \partial z}$$

$$\frac{\partial^2 H_y}{\partial z \partial t} = -\epsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$$

THE 2ND ORDER PDE THAT $E_x$ MUST SATISFY.

b. SHOW THAT $E_x = E_0 \cos(\omega t - \beta z)$ IS A SOLUTION

$$\frac{\partial^2 E_x}{\partial z^2} = -\beta^2 E_0 \cos(\omega t - \beta z)$$

$$\mu\epsilon \frac{\partial^2 E_x}{\partial t^2} = -\omega^2 \mu\epsilon E_0 \cos(\omega t - \beta z)$$

MUST BE EQUAL

c. $\beta^2 = \omega^2 \mu\epsilon$, $\beta = \omega\sqrt{\mu\epsilon}$

TOPS

9.26 Write Maxwell's equations in point form in terms of **E** and **H** as they apply to a sourceless medium, where **J** and $\rho_v$ are both zero. Replace $\epsilon$ with $\mu$, $\mu$ with $\epsilon$, **E** with **H**, and **H** with −**E**, and show that the equations are unchanged. This is a more general expression of the *duality principle* in circuit theory.

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad (1)$$

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \quad (2)$$

MAXWELL'S EQUATIONS

- POINT FORM
- SOURCELESS MEDIUM

$$\nabla \cdot \epsilon \bar{E} = 0 \quad (3)$$

$$\nabla \cdot \mu \bar{H} = 0 \quad (4)$$

$\epsilon \rightarrow \mu \qquad \mu \rightarrow \epsilon \qquad \bar{E} \rightarrow \bar{H} \qquad \bar{H} \rightarrow -\bar{E}$

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \quad (2)$$

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad (1)$$

$$\nabla \cdot \mu \bar{H} = 0 \quad (4)$$

$$\nabla \cdot \epsilon \bar{E} = 0 \quad (3)$$

(1) → (2)
(2) → (1)
(3) → (4)
(4) → (3)

TOPS