THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical & Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves

2025 W1

A Supplement to

Chapter 9 – Time-Varying Fields and Maxwell's Equations

in W. H. Hayt, Jr. and J. A. Buck, *Engineering Electromagnetics*, 9th ed., McGraw-Hill, 2019, pp. 279-302.

The purposes of this supplement are:

- to assist the reader in identifying key points to be recognized as they apply the SQ3R (Survey, Question, Read, Recite, Review) process to reading and reviewing the chapter and
- to provide comments and supplemental information that fill in apparent gaps in the textbook.

Introduction

For the case of *static* fields, Maxwell's equations in point form reduce to

$$\nabla \cdot \boldsymbol{D} = \rho_v$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{E} = 0$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

and, when observed from a fixed point in space, *i.e.,* a static observer, the electric and magnetic fields exist independently of each other.

For the case of time-varying fields, the electric and magnetic fields are coupled and the above description needs to be revised. Such coupling between the electric and magnetic fields allows for the possibility of propagating electromagnetic waves which travel at the speed of light in a medium.

The observations by: 1) Oersted in 1820 that a fixed current (associated, of course, with a static electric field) gives rise to a magnetic field and 2) Faraday in 1831 that a time-varying magnetic field gives rise to an electric field established the link between electric and magnetic fields and the notion that electromagnetism is a single unified phenomenon. However, the descriptions of the curl of the magnetic and electric fields given by Ampère's Law and Faraday's Law, respectively, did not completely capture the full relationship between time-varying electric and magnetic fields.

In 1860, Maxwell resolved the continuity-of-current paradox by identifying displacement current as the missing component on the right-hand side of Ampere's Law. With a complete description of the electromagnetics in hand, Maxwell was able to predict the existence of electromagnetic waves that propagate at the speed of light and thereby establish light as an electromagnetic wave and thereby confirm Faraday's speculation of decades earlier.

We quickly note, however, Maxwell expressed his work in the form of quaternions, a mathematical formulation originally devised by Hamilton in 1843 for use in classical mechanics. The modern vector calculus notation that you became familiar with in ELEC 211 and which you will use here is due to the work of Oliver Heaviside, Josiah Gibbs, and others (the Maxwellians) in the late 1800's.

The remainder of this *chapter* is concerned with the complete description of the electromagnetic field that was first described by James Clerk Maxwell and rendered in their modern form by Oliver Heaviside and Josiah Gibbs.

The remainder of this *course* is concerned with the practical manifestation of propagating electromagnetic waves along guided structures (transmission lines and waveguides) and through unbounded media.

9.1 Faraday's Law

The section:

- provides the historical context for the revelation of Faraday's law
- defines emf and magnetic flux and gives Faraday's law in customary, point and integral form
- considers how emf may result from:
  - a time-varying magnetic field
  - a time-constant flux and a moving closed path
  - a time-varying magnetic field *and* a moving closed path
  - contrived examples where application of Faraday's law may be difficult

Comments

The descriptions are very complete but much more could be said regarding the application of special relativity to electromagnetics and the consistency of Maxwell's equations with special relativity. Indeed, when Einstein was once asked if he stood on the shoulders of Newton, he replied "No, on the shoulders of Maxwell."[1]

Suppose a fixed observer sees only a static electric field. An observer moving with respect to the charges or potentials that give rise to the electric field will see both a static electric field and a static magnetic field! Similarly, where a fixed observer sees only a static magnetic field, an observer moving with respect to the current that gives rise to the magnetic field will also see both a static electric field and a static magnetic field!

The details of the *Lorentz transformation of the electric and magnetic fields between different rest frames* are a consequence (or predictor?) of special relativity but are generally not considered in engineering electromagnetic courses. However, engineering students should at least be aware of this phenomenon which can have important engineering consequences beyond the terse comments in the last paragraph of p. 285.

A case in point is the TRIUMF cyclotron on the UBC south campus, http://www.triumf.ca. The 520 MeV cyclotron accelerates negative hydrogen ions to about 75% of the speed of light. In their rest frame, the fast-moving negative hydrogen ions perceive the static magnetic field that guides their spiral trajectory through the cyclotron as having both magnetic and electric field components. If the electric field component is too strong, the negative hydrogen ion will lose its extra electron and become a neutral atom that would no longer be guided by the magnetic field.

[1] https://www.theguardian.com/science/2015/dec/08/einstein-inspired-by-james-clerk-maxwell

As a result, the designers of the 520 MeV cyclotron were forced to use a fairly weak magnetic field in order to reduce the strength of the static electric field experienced by negative hydrogen ions travelling at $0.75c$. This results in the ions having a fairly large orbital radius and is the reason why the TRIUMF 520 MeV cyclotron is so physically large compared to cyclotrons with similar energies that accelerate protons instead, *e.g.,* the 590 MeV cyclotron at the Paul Scherrer Institut in Switzerland.

9.2 Displacement Current

This section:

- considers the limitations of Ampère's law when fields are time-varying
- the manner in which the introduction of displacement current resolves the issue
- expressions for the time-variation version of Ampère's law in both point and integral form
- an illustration of displacement current within a parallel-plate capacitor

Comments

The descriptions are very complete.

9.3 Maxwell's Equation in Point Form

This section:

- gives the four Maxwell's equations in point form
- considers the physical implications of the equations as stated
- gives the auxiliary equations, also known as the constitutive relations
- gives the relationships between field strength and flux density involving the polarization and magnetization fields
- gives the Lorentz force equation

Comments

The descriptions are very complete. However, the notion that the electric and magnetic fields are each completely described by their divergence and curl is not peculiar to electromagnetics but is instead a consequence of Helmholtz's theorem which states that any vector field $\boldsymbol{A}$ can be expressed as the sum of irrotational and rotational components where:

· the irrotational component is completely described by the divergence of $\boldsymbol{A}$, and,

· the rotational component is completely described by the curl of $\boldsymbol{A}$.

Accordingly, there must be four equations in Maxwell's set as shown above to completely describe the electric and magnetic fields.

Having said that, the expressions on the right-hand side of each of the divergence and curl equations cannot be derived from first principles and are simply experimental observations. For example, there is no reason that $\nabla \cdot \boldsymbol{B}$ must be equal to zero except that magnetic charges have never been observed. (The first person to experimentally demonstrate the existence of free magnetic charges will rewrite electromagnetics textbooks and almost certainly win the Nobel Prize.)

9.4 Maxwell's Equations in Integral Form

This section:

- gives the four Maxwell's equations in integral form
- notes that Maxwell's equations in integral form can be used to derive the boundary conditions for electromagnetic field quantities at interfaces between material media
- explains why it is practically imperative to know the boundary conditions for electromagnetic field quantities at interfaces between material media

Comments

The descriptions are fairly complete but the details of the derivation of the boundary conditions for flux density and field strength at material boundaries are strangely absent. We will resolve this in the problem set solutions.

9.5 The Retarded Potentials

This section:

- gives the scalar electric and magnetic vector potentials for the static/dc case
- shows how they can be used to obtain the fundamental fields
- shows how the expression for $\mathbf{E}$ in terms of V in the static case is inadequate in the time-varying case and shows how it can be resolved by adding a new term, $-\partial\mathbf{A}/\partial t$
- introduces the retarded scalar electric and magnetic vector potentials

Comments

The descriptions are very complete and will play a key role in solving the problem of radiation from a current element in ELEC 411 – Antennas and Propagation.

References

In addition to the seven references given at the end of the chapter, ELEC 311 students may also wish to consult the following:

[1] R. F. Harrington, *Introduction to Electromagnetic Engineering*, McGraw-Hill, 1958; Dover, 2003.

[2] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*, McGraw-Hill, 1961; Wiley – IEEE Press, 2001.

These two books are written in a similar style with the former aimed at undergraduate students and the latter aimed at graduate students. Both books are renowned for their clarity and style. *Time-Harmonic Electromagnetic Fields* is widely regarded as a classic work in this field and was re-issued by IEEE Press in 2001. *Introduction to Electromagnetic Engineering* was re-issued by Dover in 2003.

[3] R. Feynmann, *The Feynman Lectures on Physics, Vol. II: Mainly Electromagnetism and Matter,"* Addison Wesley, 1964; Basic Books, 2011.

Told from a physics perspective, Feynmann's lectures have attracted considerable praise for their unique combination of style and substance. Originally published in 1964, the three-volume set was reissued as a Millennium Edition by Basic Books in 2011.

[4] D. Fleisch, *A Student's Guide to Maxwell's Equations,* Cambridge University Press, 2008.

This book and its companion website at http://www.danfleisch.com/maxwell/ have attracted an enormous following and very strong reviews from readers.

On the book's website, readers will find:

Complete solutions to every problem in the book

You can get a series of hints to help you solve the problem, or you can see the full solution straight away. Just use the menu on the left to click on one of the Chapters, select "Problems," and pick the problem you want to work on.

Audio podcasts

For every module in the chapters covering the four Maxwell's Equations, Fleisch will walk you through the major concepts contained within that module. These audio files can be streamed to your computer so you can hear them immediately, or you can use your favorite podcast-catching software to grab and store them.

Three-dimensional models of electric and magnetic fields

The twelve sketches of electric and magnetic fields shown in Figures 1.1 and 2.1 in the book are available on this site as VRML files so you can see the fields in three dimensions. You'll need a VRML plug-in for your browser to view these files – you can find several free 3D viewers on the Web (Fleisch has been using Cortona® with good success).

Review of rectangular, cylindrical, and spherical coordinates

To keep the book short (and inexpensive), Fleisch didn't include a review of coordinate systems in the text – so it's here on the website. If you're a bit fuzzy on how to get from from (x,y,z) to (r, theta, phi), or from (A_x, A_y, A_z) to (A_r, A_{theta}, A_{phi}), then you should take a look at this review, which is in .pdf format.