ELEC 311
Electromagnetic Fields and Waves

Prof. David Michelson

David.Michelson@ubc.ca

W. H. Hayt, Jr. and J. A. Buck, *Engineering Electromagnetics*, 9th ed.,
McGraw-Hill, Chapters 9, 2019.

Annotation of these lecture notes is recommended and encouraged.

Introduction

- In previous courses, you were introduced to the manner in which time-varying electric and magnetic fields are coupled to each other, as described by Maxwell's equations.

- This course is concerned with propagating waves on transmission lines and waveguide waves and through unbounded media.

- Historically, this work ushered in the age of electronic communication and electrical power systems.

- Much of this material resulted from the work of three men:

  o James Clerk Maxwell (1831-1879)

  o Oliver Heaviside (1850-1925)
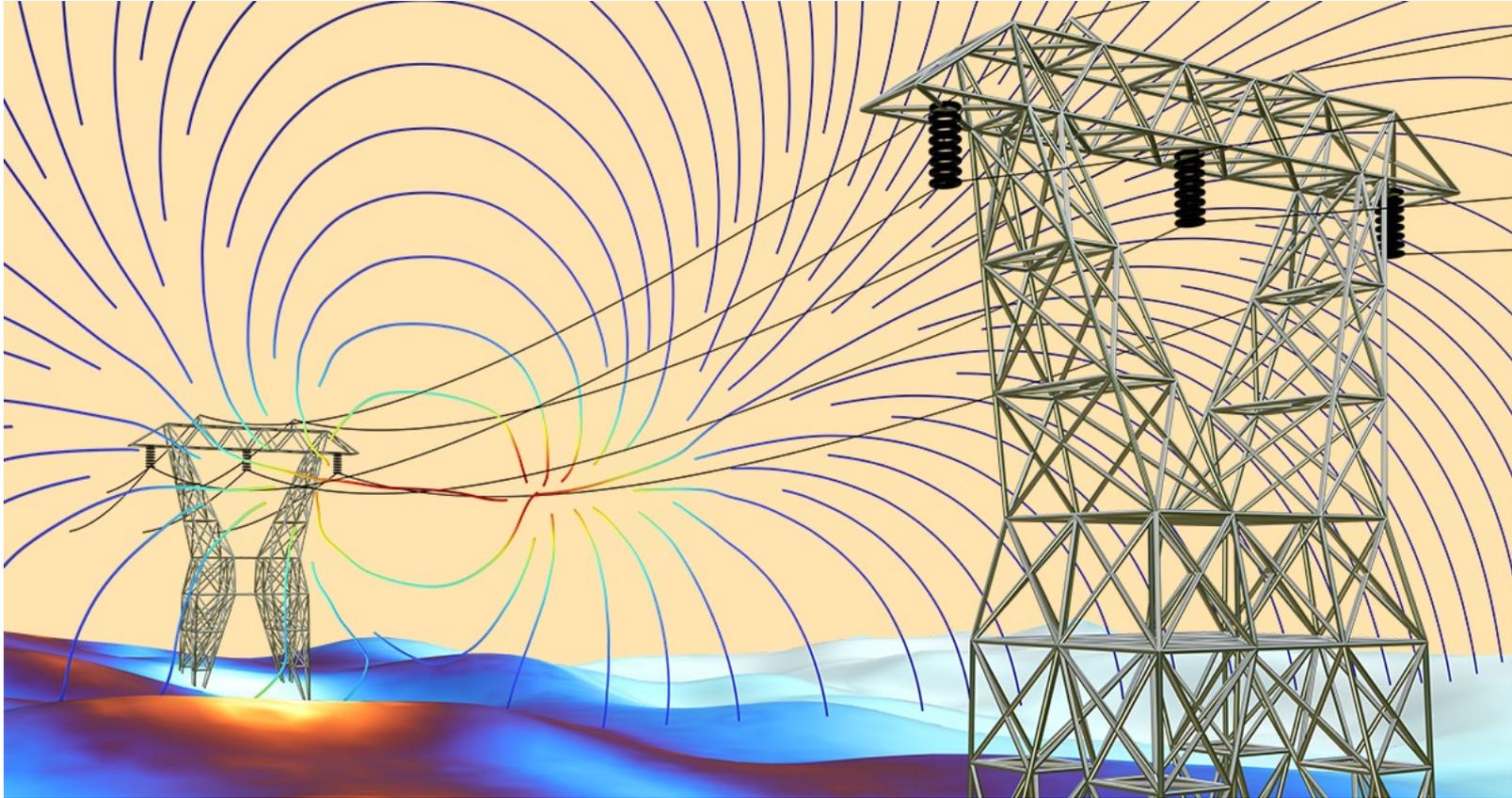
  o Philip Smith (1905-1987)

A circuit is a system designed to transmit electrical signals from one point to another, typically over short distances, at low voltages or frequencies where the wave nature of the signal need not be considered.
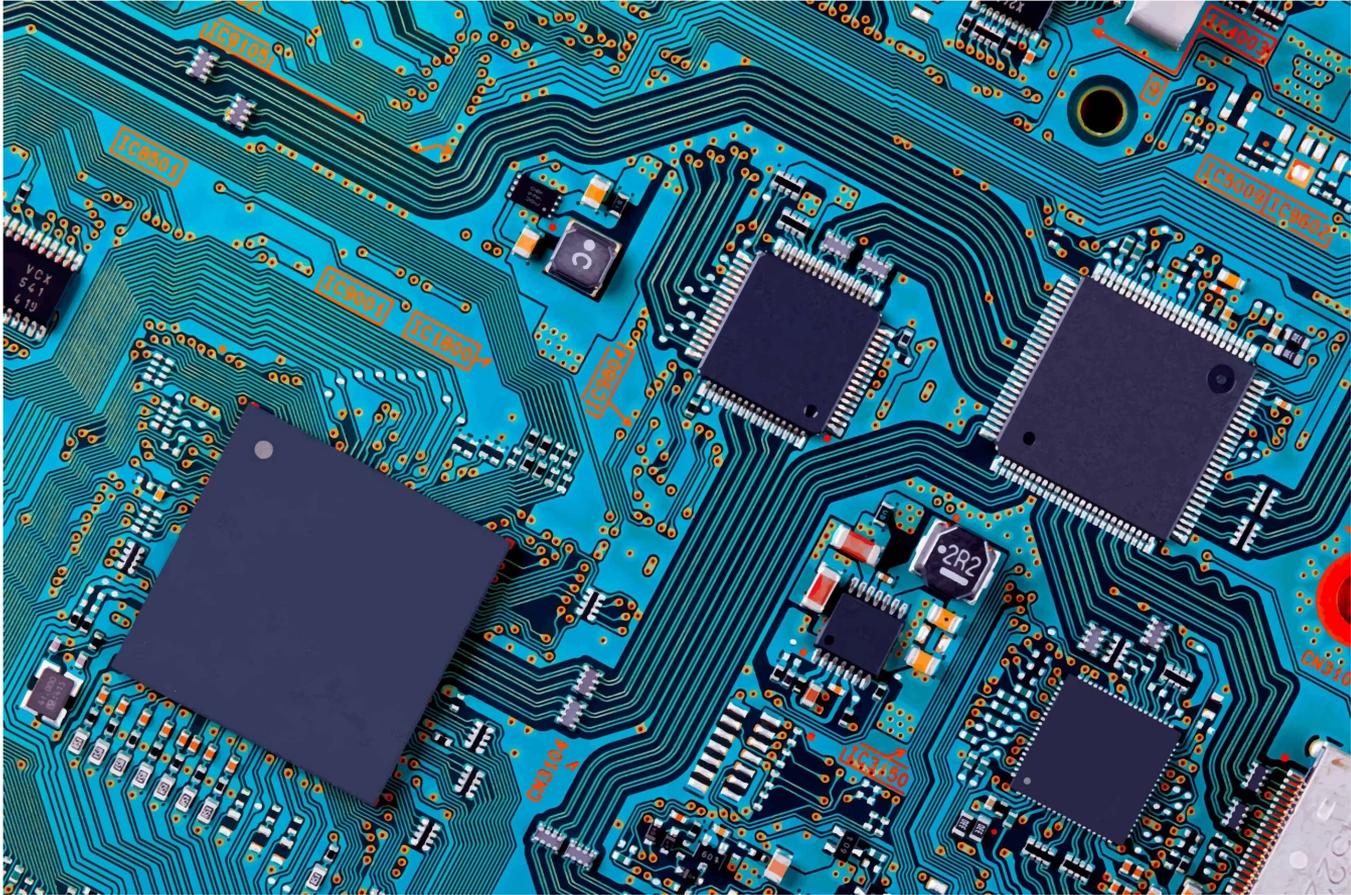
A circuit is a system designed to transmit electrical signals from one point to another, typically over short distances, at low voltages or frequencies where the wave nature of the signal need not be considered.

A transmission line is a structure designed to transmit electrical signals from one point to another, typically over a considerable distance, at high voltages or frequencies, where the wave nature of the signal must be considered.
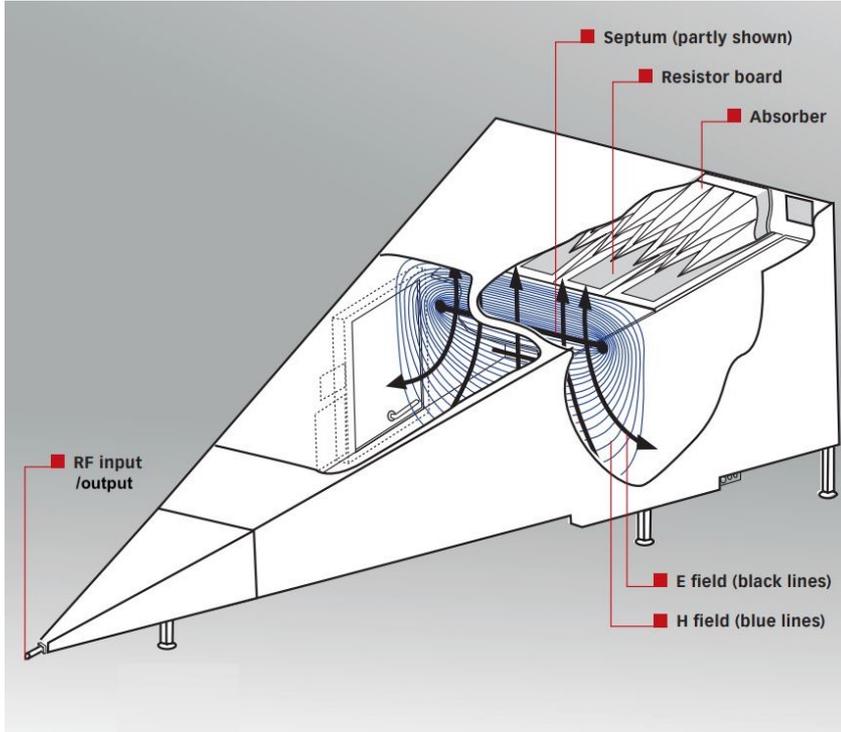
A transmission line is a structure designed to transmit electrical signals from one point to another, typically over a considerable distance, at high voltages or frequencies, where the wave nature of the signal must be considered.
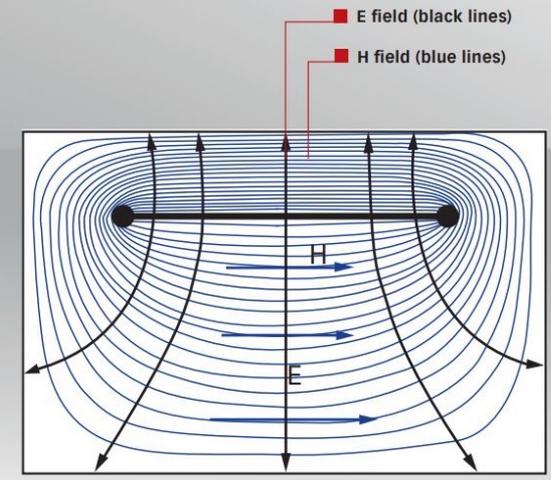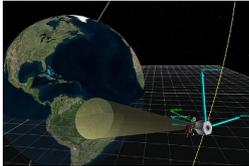
Septum (partly shown)
Resistor board
Absorber
RF input /output
E field (black lines)
H field (blue lines)

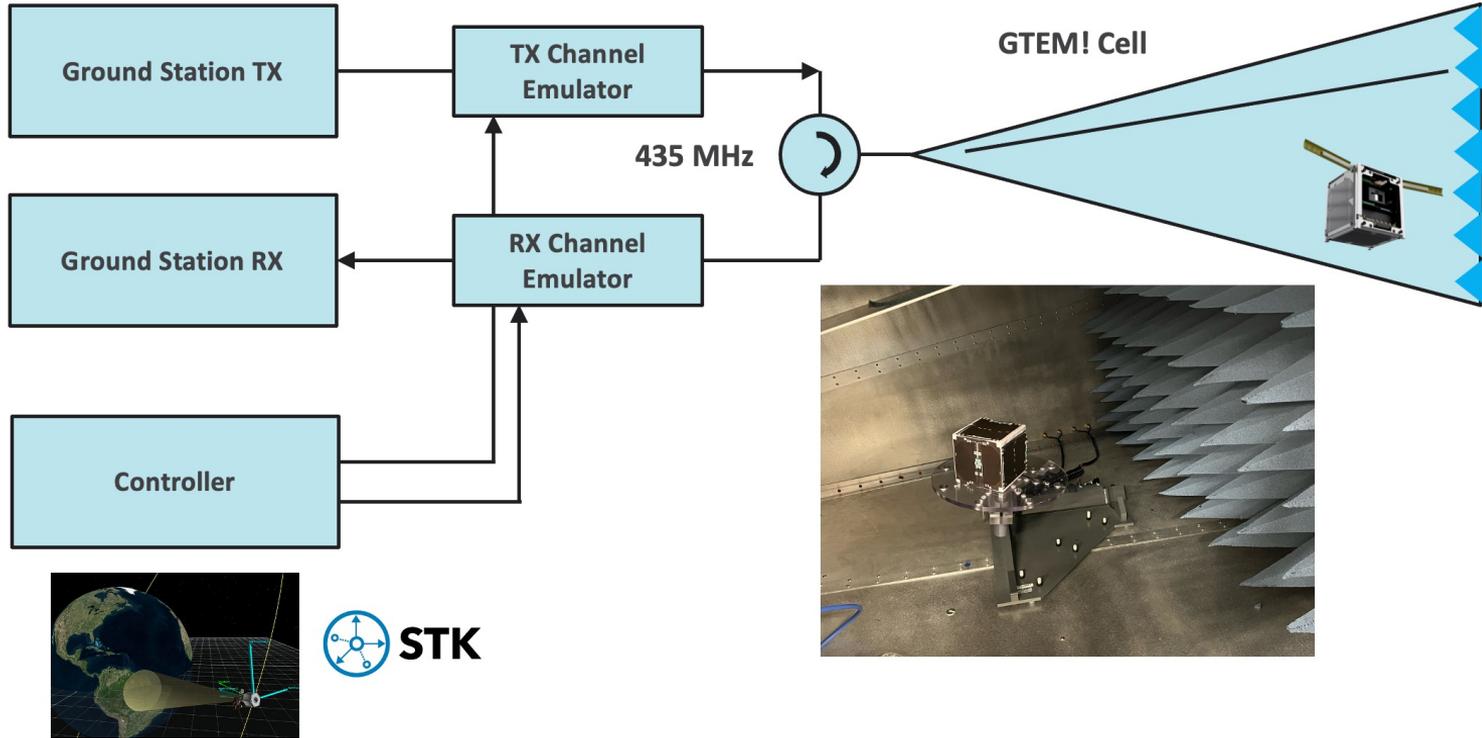**Electromagnetic fields** can be generated by the connection of an RF generator and power amplifier to the feeding point of the GTEM. TEM waves start to propagate between the septum and outer conductor (cell body).

The electromagnetic field intensity is directly related to both the supplied input power and to the distance between septum and outer conductor.

E field (black lines)
H field (blue lines)

Ground Station TX

TX Channel Emulator

GTEM! Cell

435 MHz

Ground Station RX

RX Channel Emulator

Controller

STK

Equivalent Circuit Model of a Transmission Line
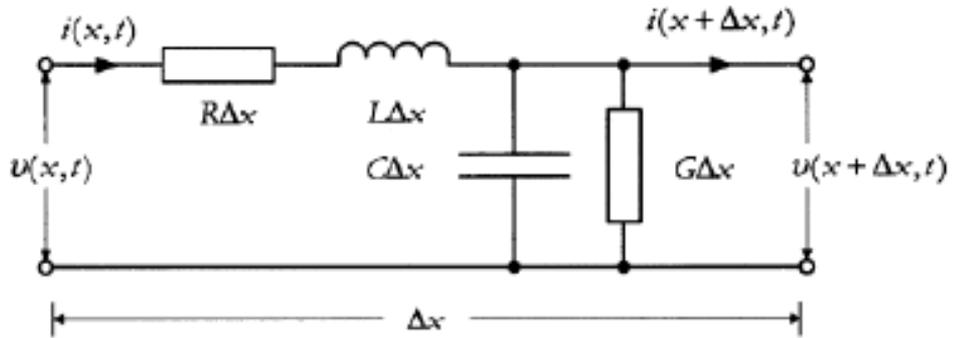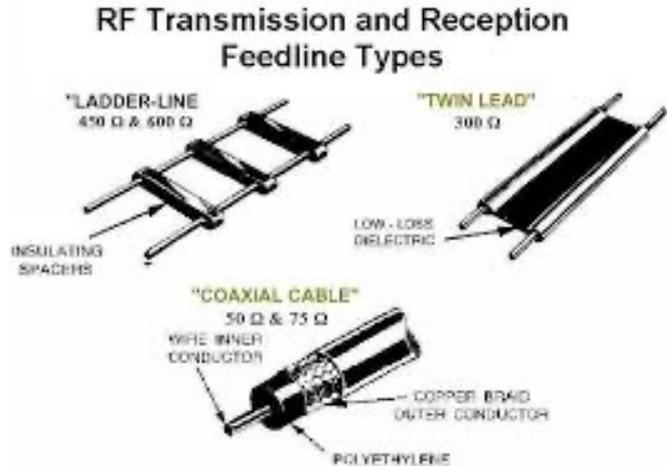
Given a physical description of a transmission line, how to estimate the equivalent circuit parameters?

Fields Around a Parallel-Wire Transmission Line

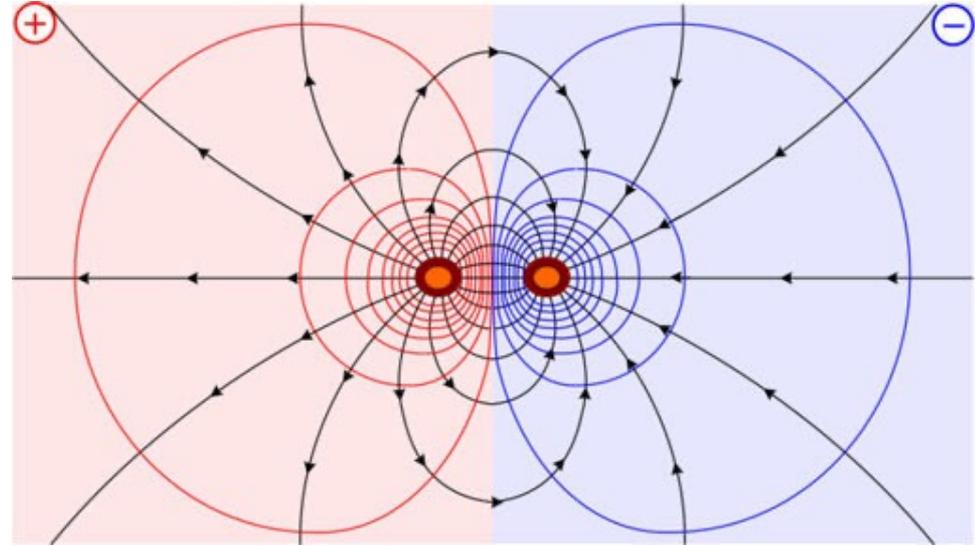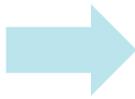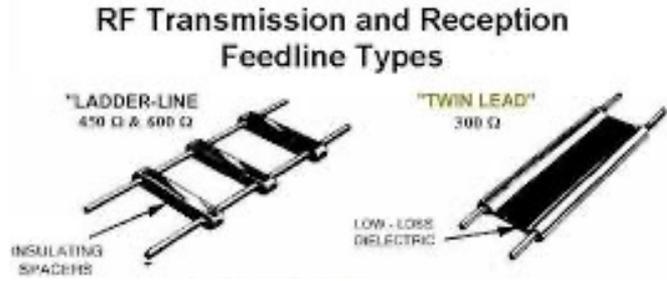Given a physical description of a transmission line, how can resolving the structure of the electric and magnetic fields allow us to estimate the equivalent circuit parameters?

ELEC 311
2025 W1

Equivalent Circuit Model of a Transmission Line

Phasor Notation – time dependence ($e^{j\omega t}$) suppressed)

$$V_i(z) = V_{i0}e^{-\gamma z} = V_{i0}\,e^{-(\alpha+j\beta)z}$$

Maxwell's equations allows us to relate fields and waves to voltages and currents.
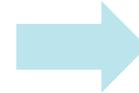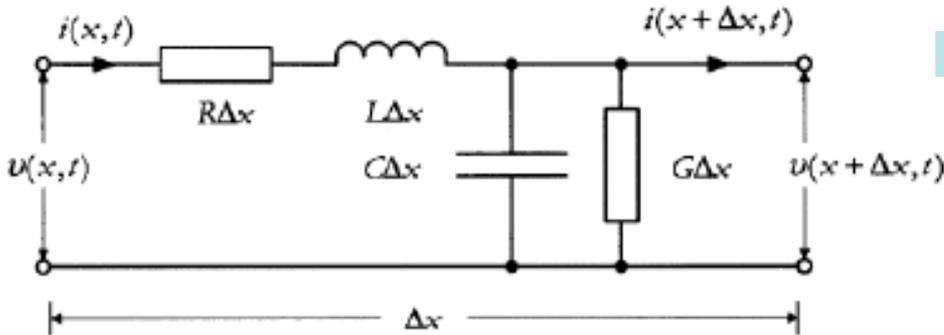
$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{Z}{Y}}$$

$$v = \frac{\omega}{\beta} \qquad \lambda = \frac{2\pi}{\beta}$$

Given the equivalent circuit parameters, we can determine the propagation constants and transmission parameters that describe waves traveling along a transmission line.

ELEC 311
2025 W1

COURSE TEXT (will be followed closely)

- W. H. Hayt, Jr. and J. A. Buck, *Engineering Electromagnetics*, 9th ed., McGraw-Hill, Chapters 9-13, 2019.
- available as both an e-book and a printed book

COURSE OUTLINE

- Time-Varying Fields and Maxwell's Equations (Chap. 9)
- Transmission Lines (Chap. 10)
- The Uniform Plane Wave (Chap. 11)
- Plane Wave Reflection and Dispersion (Chap. 12)
- Guided Waves (Chap. 13)
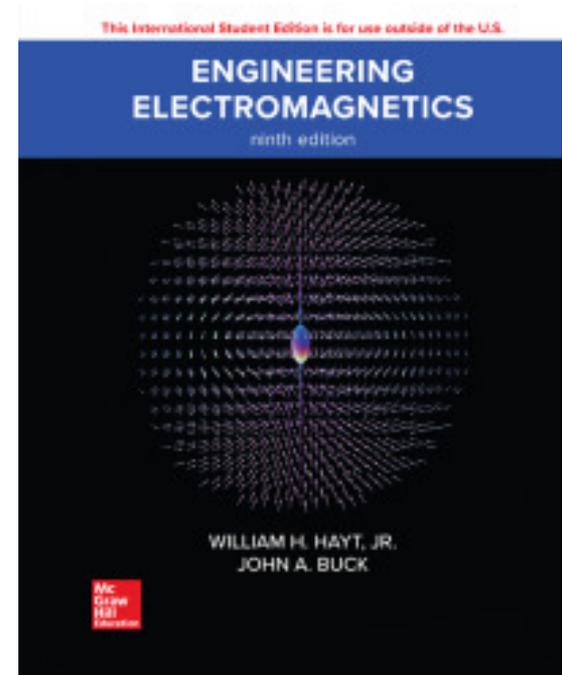
Chapter 9

Time-Varying Fields and Maxwell's Equations

Introduction

- In previous courses, you were introduced to the manner in which time-varying electric and magnetic fields are coupled to each other, as described by Maxwell's equations.

- This module serves as a brief review of this material before we start discussing propagating waves on transmission lines and in unbounded media.

- Chapter Outline:

    9.1 – Faraday's Law

    9.2 – Displacement Current

    9.3 – Maxwell's Equations in Point Form

    9.4 – Maxwell's Equations in Integral Form

    9.5 – The Retarded Potentials   (not a focus in ELEC 311, more relevant to ELEC 411)

Performance Objectives

1. Give Faraday's law in customary, point and integral form and determine the EMF associated with a time-varying magnetic field, a time-constant flux, & a moving closed path.

2. Give expressions for the time-variation version of Ampère's law in both point and integral form and calculates the magnitude of the displacement current in practical scenarios.

3. Give Maxwell's equations in point and integral form and the constitutive relations or auxiliary equations, explain the significance of the Helmholtz theorem to these results, and the relationship between the SI and previous systems of electromagnetic units.

4. Demonstrate the relationship between Maxwell's equations in point and integral form, and the boundary conditions imposed on field strength and flux density across material boundaries.

Pedagogy

Phase 1 - Read (several times in various sequences):

- the *Chapter Brief* – both for Performance and Enabling Objectives and for context and motivation
- the *Chapter Lecture Notes* – for additional insights
- the *Book Chapter* - use SQ3R to enhance your efficiency
- the *Chapter Supplement* – for additional or clarifying material
- the *Chapter Review Questions* – to self-assess

Phase 2 - Solve:

- the Example Problems (and Solutions) – to become acquainted with the subtleties of actual problems
- The Take Home Assignment – to practice problem solving without reference

Your goal for phase 1 is to be able to correctly interpret and apply the key formulas in obvious ways.

KNOWLEDGE

⇕

INTUITION or INSIGHT

Your goal for phase 2 is to be able to correctly interpret and apply the key formulas in non obvious ways.

Survey, Question, Read, Recite, Review

SQ3R

Survey Question Read Recite Review

A 'best practice' for assimilating the contents in detailed technical documents!

- **Survey** the document highlights, including section headings, key definitions, key equations, key figures, and example problems.

- Turn section headings and key issues into **Questions**.

- **Read** each section with an aim to answering these questions and mastering specific types of problems.

- After reading a section, **Recite** (or Recall) the key points and techniques without referring to the text or your notes.

- **Review** the entire document, then, as applicable, attempt to:
    (1) answer the review questions and
    (2) solve the example problems without reference.

ELEC 311
2025 W1

Interactive Sessions

We will use Interactive Sessions during the term to stress Best Practices when solving problems.

1. Draw and label the problem geometry.

2. Identify both the known (or given) and unknown (or desired) quantities.

3. Identify known relationships and applicable formulas.

4. Develop a strategy for solving for the unknown quantities.

   o Use bullet points to explain briefly!

5. Solve for the unknown quantities.

6. Clearly identify the solution including the units.

The Vital Role of Freehand Writing and Sketching
(or the Importance of Informal Technical Communications)

- As an engineer, you won't always have the luxury of preparing technical notes or documents using a word processor or drawing program.

- At meetings, in the shop, or in the field, you will often be required to record data or prepare memos, notes, plans, and other documents freehand.

- First impressions count, and your technical writing often precedes you.

- Lay out your freehand documents so they are easy to read and understand.

- Learn to:

  o organize your thoughts,

  o block or speed print quickly and neatly, and,

  o draw a straight line, box, or circle without a ruler or compass.

Block or Speed Printing
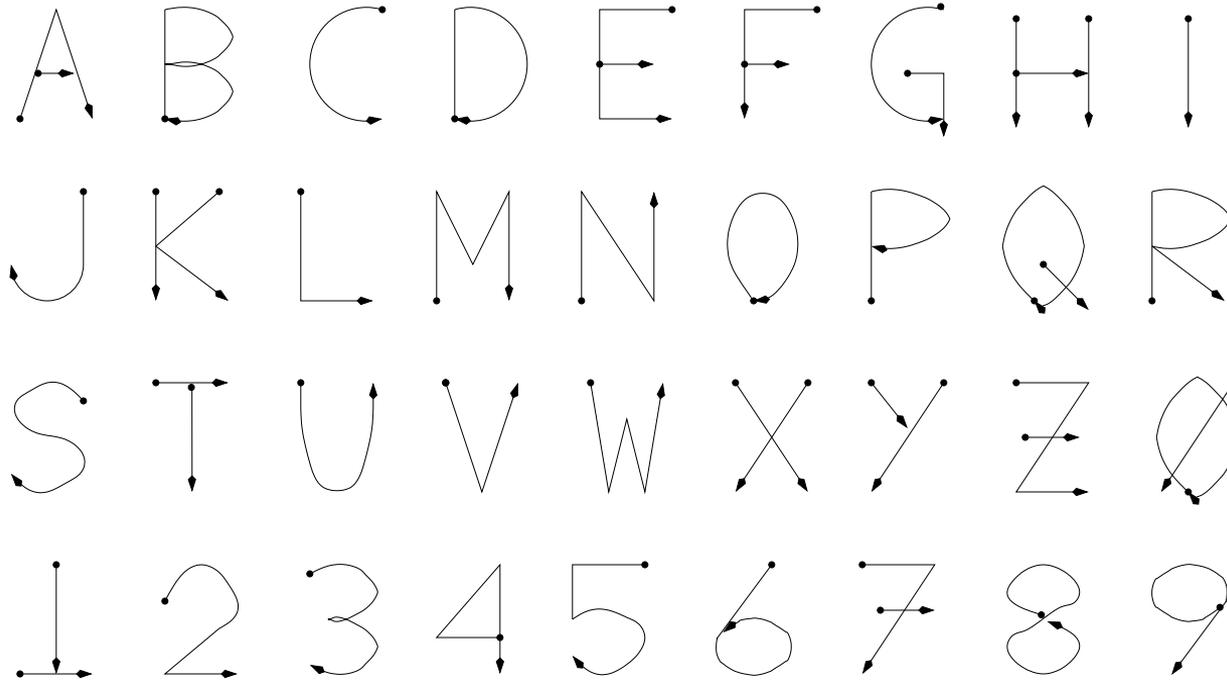
PO* 1 - Faraday's Law

Give Faraday's law in customary, integral, and point form, and determine the EMF associated with a time-varying magnetic field, a time-constant flux, & a moving closed path. (§9.1)

$$\Phi = \int \mathbf{B} \cdot d\mathbf{S}$$

The relevant directions of circulation and the normal are defined by the right-hand rule

* Performance Objective
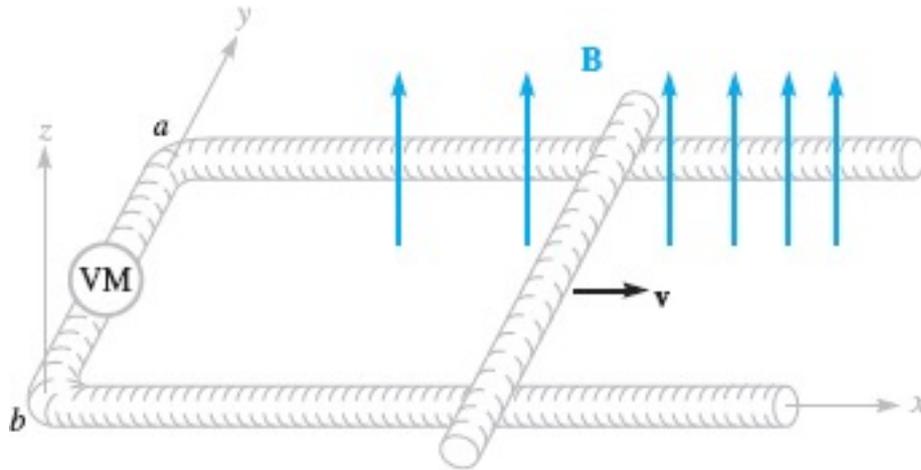
Recall

- the customary form: $emf = -N \, d\Phi/dt$
- the integral form: $emf = \oint \mathbf{E} \cdot d\mathbf{L} = -d/dt \int \mathbf{B} \cdot d\mathbf{S}$
- the point form: $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$

- that force on a charge is given by $\mathbf{F} = Q\mathbf{v} \times \mathbf{B}$
- that the motional electric field intensity is given by $\mathbf{E}_m = \mathbf{v} \times \mathbf{B}$
- that including both the transformer and the motional emf yields

$$emf = \oint \mathbf{E} \cdot d\mathbf{L} = -\int \partial \mathbf{B}/\partial t \cdot d\mathbf{S} + \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{L}$$

We can't emphasize enough that a complete description of the above formulas requires:

- a definition and the units of each parameter, and
- a sketch that describes the relevant geometry or scenario.

PO 2 - Time-variation Version of Ampère's Law

Give expressions for the time-variation version of Ampère's law in both point and integral form and calculate the magnitude of the displacement current in practical scenarios. (§9.2)

- Recall the point form, $\nabla \times \mathbf{H} = \mathbf{J}_c + \partial \mathbf{D} / \partial t$

- Recall the integral form, $\oint \mathbf{H} \cdot d\mathbf{L} = \int \int (\mathbf{J}_c + \partial \mathbf{D}/\partial t) \cdot d\mathbf{S}$

- Recognize that the magnitude of the displacement current is given by

$$|J_D| = \left|\frac{\partial \mathbf{D}}{\partial t}\right| = \epsilon_r \epsilon_0 \left|\frac{\partial \mathbf{E}}{\partial t}\right|$$

Note that a complete description of the above formulas requires a definition and the units of each parameter, and a sketch that describes the relevant geometry or scenario.

PO 3 - Maxwell's Equations

Give Maxwell's equations in point and integral form and the constitutive relations or auxiliary equations, explain the significance of the Helmholtz theorem to these results, and describe the relationship between the SI and previous systems of electromagnetic units. (§9.3 and §9.4)

- The Helmholtz theorem (or the fundamental theorem of vector calculus) tells us that any vector field can be resolved into irrotational and rotational components

- The irrotational component is completely specified by the divergence of the field and the rotational component is completely specified by the curl of the field.

- The general form of Maxwell's equations in point form is therefore predictable and inevitable with the right-hand side simply being experimental observations.

ELEC 311
2025 W1

Vector Fields

- A vector field is an assignment of a vector to each point in a space.
- Vector fields can be used to model the speed and direction of a moving fluid or the strength and direction of electric or magnetic *flux* or *force*.
- Hydraulic analogies are surprisingly useful when one attempts to visualize concepts such as:
  - a *source* or a *sink*, or,
  - the *divergence* or *curl* of a field.
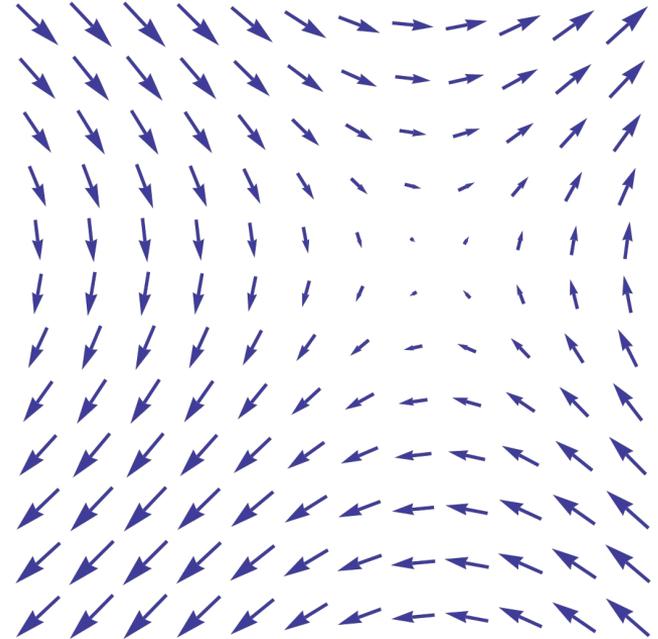
Electric and Magnetic Fields

- The E fields and B fields due to electric charges (black/white) and magnetic poles (red/blue).

  ○ E fields due to stationary electric charges and B fields due to stationary magnetic charges (note in nature N and S monopoles do not exist).

  ○ In motion (velocity v), an electric charge induces a B field while a magnetic charge (not found in nature) would induce an E field.

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$
$$\nabla \times \mathbf{H} = \mathbf{J}_c + \partial \mathbf{D}/\partial t$$
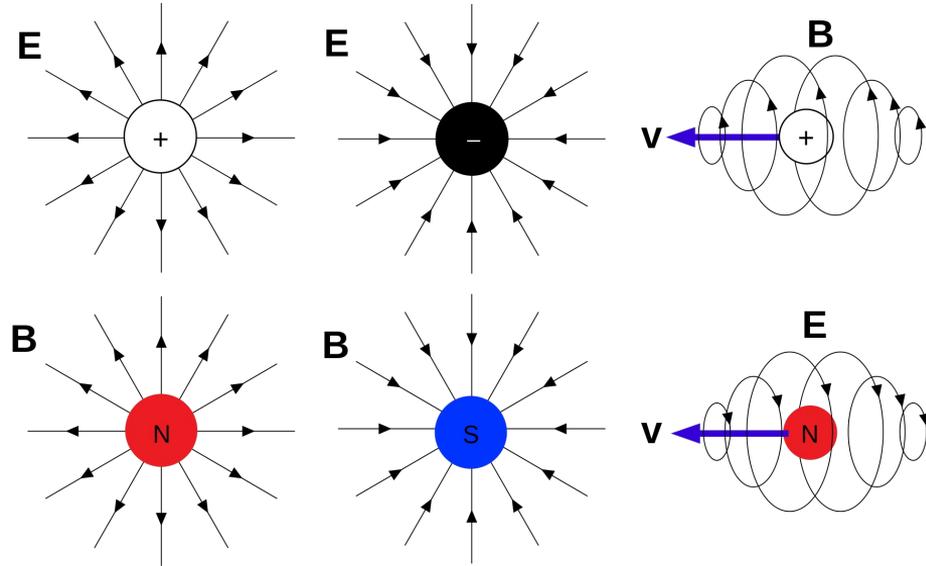
$$\mathbf{D} = \epsilon_r \epsilon_0 \mathbf{E}$$
$$\mathbf{B} = \mu_r \mu_0 \mathbf{H}$$
$$\mathbf{J} = \sigma \mathbf{E}$$

- The point forms are perhaps easier to visualize but assume continuous derivatives and cannot be used to analyze transitions across boundaries.

- The constitutive relations relate field quantities concerning flux to other field quantities concerning electric or magnetic field strength.

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$
$$\nabla \times \mathbf{H} = \mathbf{J}_c + \partial \mathbf{D}/\partial t$$

Spatial derivatives   Temporal derivatives

$$\mathbf{D} = \epsilon_r \epsilon_0 \mathbf{E}$$
$$\mathbf{B} = \mu_r \mu_0 \mathbf{H}$$
$$\mathbf{J} = \sigma \mathbf{E}$$

- Equations have a story to tell beyond algebraic manipulation

- Note the structure inherent to these equations

  - Spatial derivatives on the LHS

  - Temporal derivatives on the RHS

  - Coupling between the Curl equations

- The curl of a vector is always perpendicular to the original vector.

  - Thus, the third and fourth equations suggest that coupled electric and magnetic fields will be perpendicular to each other.

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$
$$\nabla \times \mathbf{H} = \mathbf{J}_c + \partial \mathbf{D}/\partial t$$

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho \, dv$$
$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$
$$\oint_C \mathbf{E} \cdot d\ell = \int_S -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$
$$\oint_C \mathbf{H} \cdot d\ell = \int_S \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$$

- Stokes' Theorem and the Divergence Theorem allow us to transform between the point and integral forms of Maxwell's equations.

- The integral forms are more general than the point forms because they do not assume continuous derivatives.

- As a result, the integral forms can be used to analyze transitions across boundaries.

- SI (formerly known as rationalized MKS) units are based on the fundamental units of metres, kilograms and seconds while the various CGS systems of units are based on the fundamental units of centimetres, grams and seconds.

- Conversion between CGS and SI units is not straightforward due to fundamental differences in their formulation.

- In most cases, referring to CGS implies CGS Gaussian.

  - SI units are predominantly used in engineering applications and physics education

  - Gaussian CGS units are commonly used in theoretical physics, descriptions of microscopic systems, relativistic electrodynamics, and astrophysics.

$\gamma = c \left[\dfrac{cm}{s}\right]$

$\mu_0 = 4\pi$

$\varepsilon_0 = \dfrac{1}{4\pi}$

Gaussian CGS system

$\gamma = c \left[\dfrac{cm}{s}\right]$

$\mu_0 = 1$

$\varepsilon_0 = 1$

Heaviside-Lorentz system

$\gamma^2 = c^2 * \mu_0 * \varepsilon_0$

$\gamma = 1$

$\mu_0 = \dfrac{4\pi}{c^2} \left[\dfrac{s^2}{cm^2}\right]$

$\varepsilon_0 = \dfrac{1}{4\pi}$

Electrostatic CGS system

$\gamma = 1$

$\mu_0 = 4\pi$

$\varepsilon_0 = \dfrac{1}{4\pi} * \dfrac{1}{c^2} \left[\dfrac{s^2}{cm^2}\right]$

Electromagnetic CGS system

$\gamma = 1$

$\mu_0 = \dfrac{4\pi}{10^7} \left[\dfrac{Vs}{Am}\right]$

$\varepsilon_0 = \dfrac{10^7}{4\pi} * \dfrac{1}{c^2} \left[\dfrac{As}{Vm}\right]$

International MKSA system
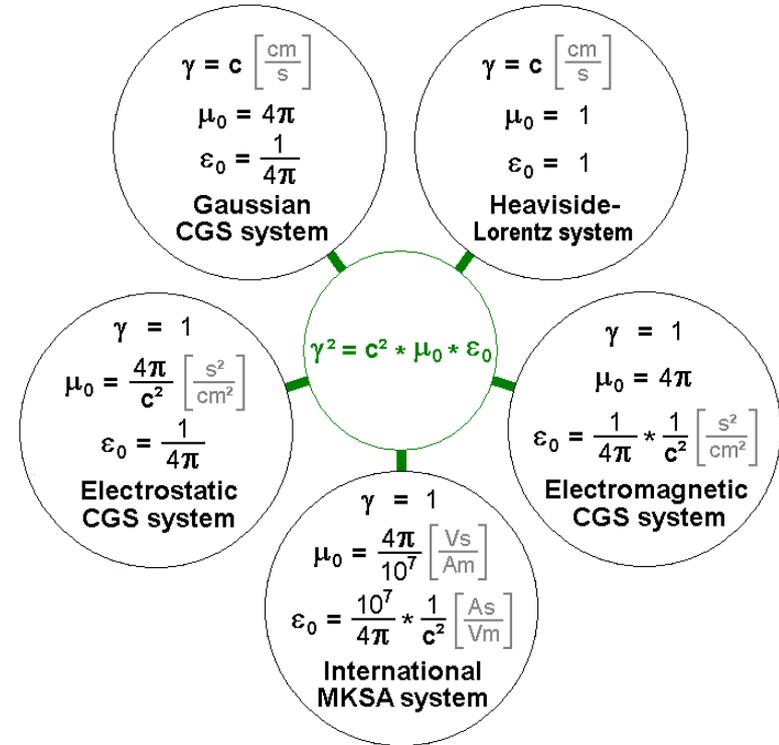
$1\ N = 1\ kgm/s^2 = 10^5\ gcm/s^2 = 10^5\ dyn$

$1\ J = 1\ Ws = 1Nm = 10^5\ dyn * 10^2\ cm = 10^7\ erg$

$c \approx 2{,}998 * 10^8\ m/s = 2{,}998 * 10^{10}\ cm/s$
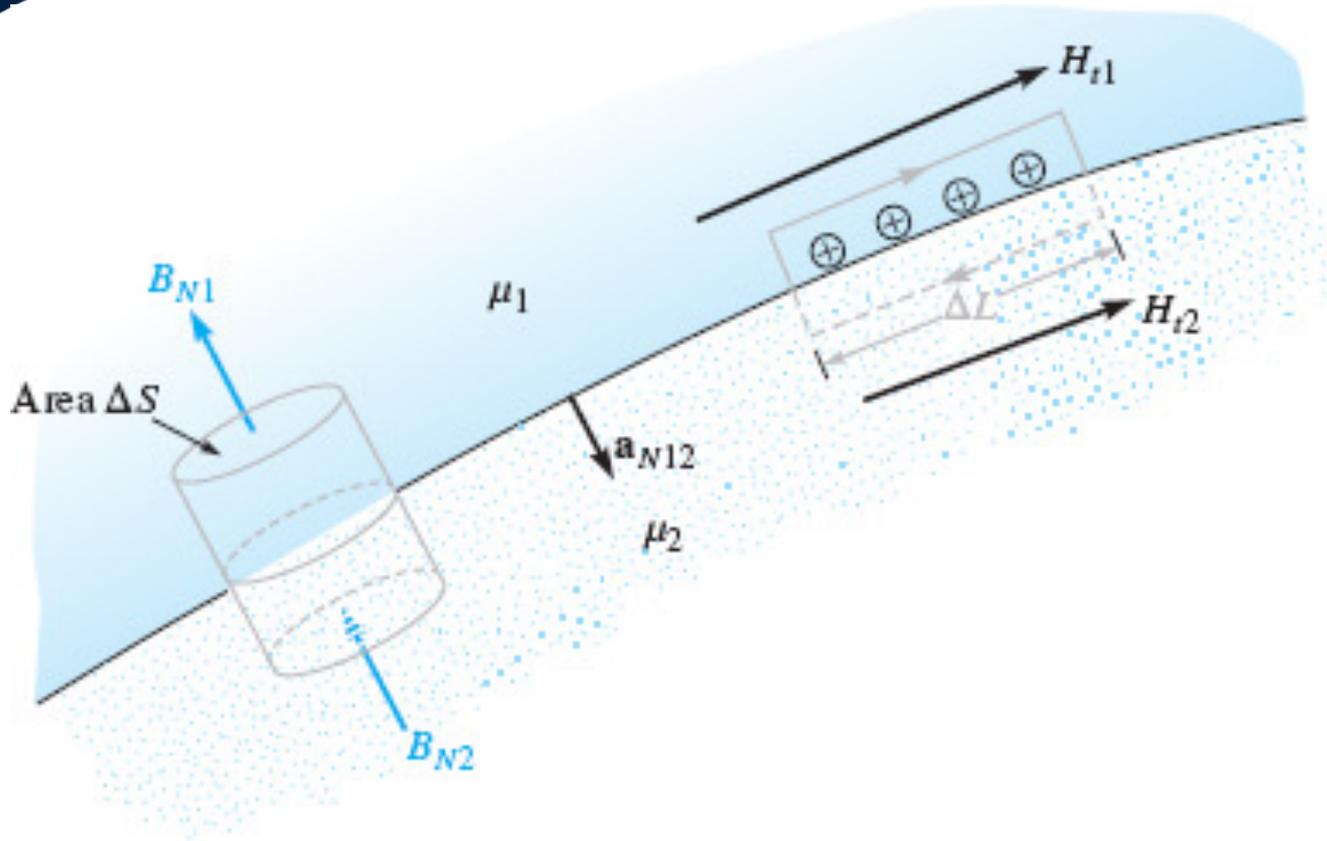
PO 4 – Boundary Conditions

Demonstrate the relationship between Maxwell's equations in point and integral form, and the boundary conditions imposed on field strength and flux density across material boundaries. (§9.3 and §9.4)

- Recognize that:
  - the *divergence theorem* links the divergence equations in **D** and **B**, *i.e.,* the flux densities, to the corresponding closed surface integrals.
  - these closed surface integrals can be used to deduce the continuity of the *normal component of the flux density* across a material interface.
  - *Stokes' theorem* links the curl equations in **E** and **H**, *i.e.,* the field strengths, to the corresponding closed line integrals
  - these closed line integrals can be used to deduce the continuity of the *tangential component of the field strength* across a material interface.

We will see the details in later example problems.
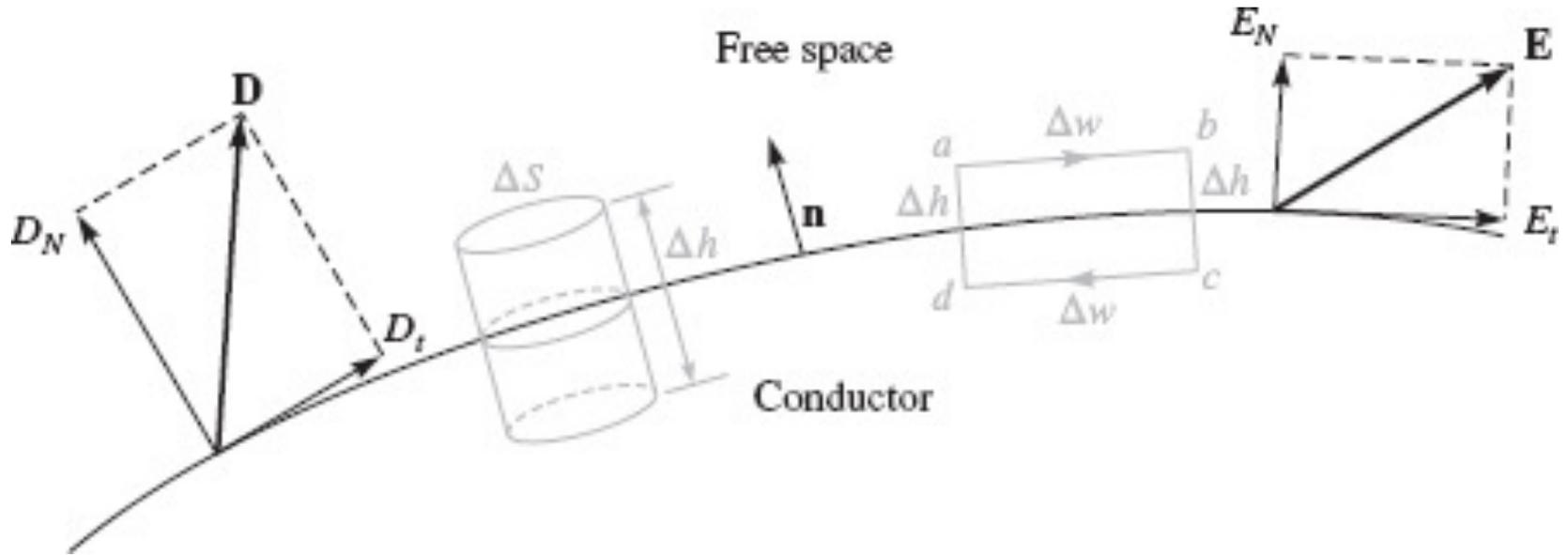
An appropriate closed path and gaussian surface are used to determine boundary conditions at a boundary between a conductor and free space; $E_t = 0$ and $D_N = \rho_S$.

A gaussian surface and a closed path are constructed at the boundary determine the boundary conditions $B_{N1} = B_{N2}$ and $H_{t1} - H_{t2} = K$, the component of the surface current density directed into the page.

Example Problems – A two-phase approach

Phase One – Understand the problem (intuition)

- *Draw* sketch(es)
- *Identify* key formulas or relationships
- *Devise* and *communicate* a strategy for obtaining a desired quantity in light of the given quantities.

Phase Two – Solve the Problem (execution)

- *Execute* the strategy
  - *derive,*
  - *substitute,*
  - *calculate*
- *Communicate* the answer.

Pedagogy

Phase 1 - Read (several times in various sequences):

- the *Chapter Brief* – both for Performance and Enabling Objectives and for context and motivation
- the *Chapter Lecture Notes* – for additional insights
- the *Book Chapter* - use SQ3R to enhance your efficiency
- the *Chapter Supplement* – for additional or clarifying material
- the *Chapter Review Questions* – to self-assess

Phase 2 - Solve:

- the Example Problems (and Solutions) – to become acquainted with the subtleties of actual problems
- The Take Home Assignment – to practice problem solving without reference

Your goal for phase 1 is to be able to correctly interpret and apply the key formulas in obvious ways.

KNOWLEDGE

⇕

INTUITION

Your goal for phase 2 is to be able to correctly interpret and apply the key formulas in non obvious ways.
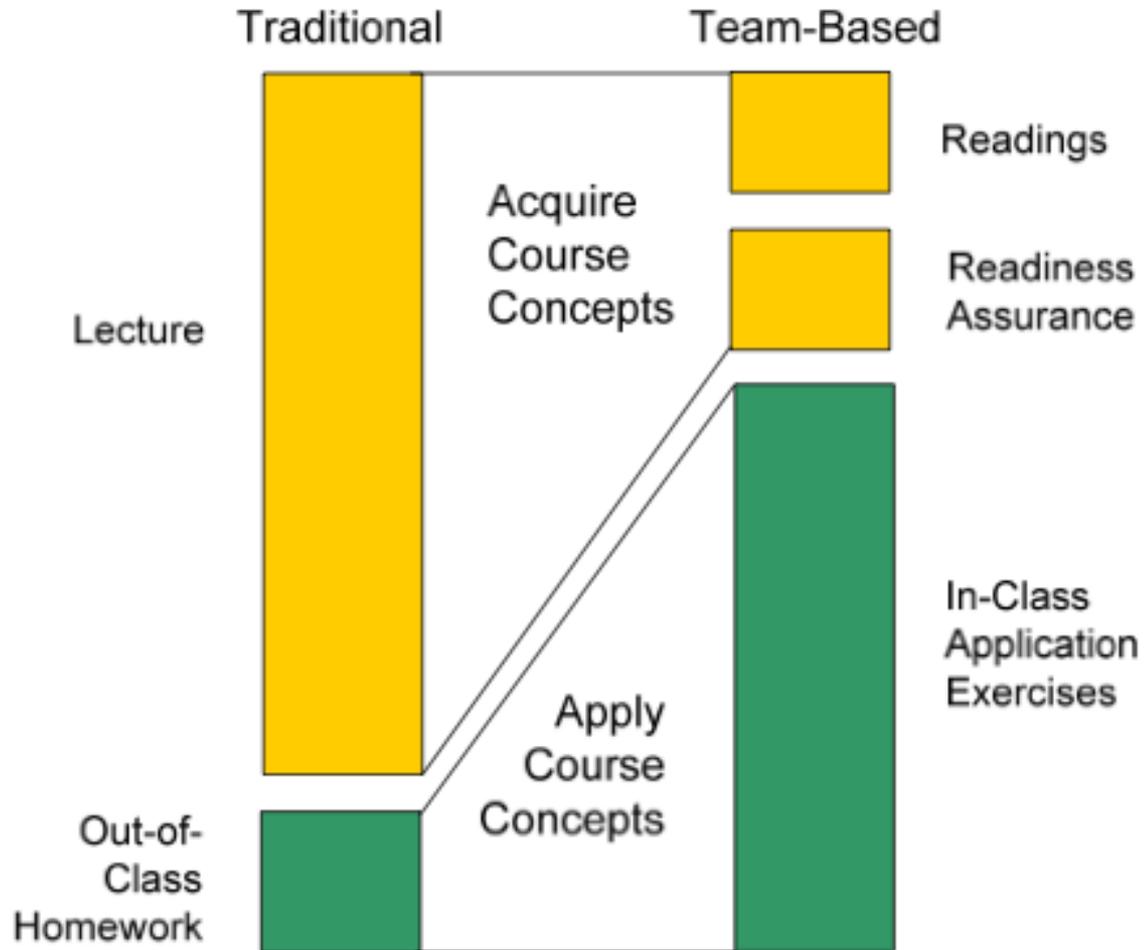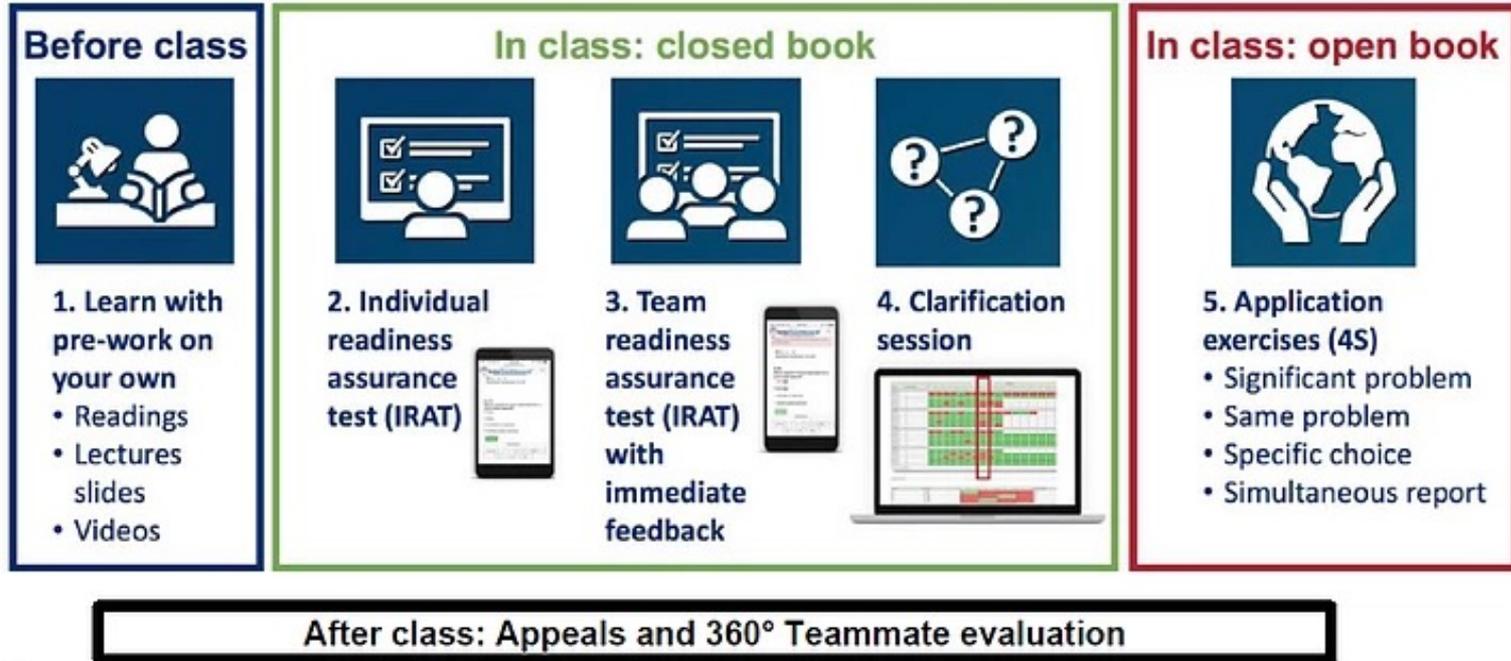
Survey, Question, Read, Recite, Review

A 'best practice' for assimilating the contents in detailed technical documents!

- **Survey** the document highlights, including section headings, key definitions, key equations, key figures, and example problems.

- Turn section headings and key issues into **Questions**.

- **Read** each section with an aim to answering these questions and mastering specific types of problems.

- After reading a section, **Recite** (or Recall) the key points and techniques without referring to the text or your notes.

- **Review** the entire document, then, as applicable, attempt to:
  (1) answer the review questions and
  (2) solve the example problems without reference.

The Team-Based Learning Process

Before class

1. Learn with pre-work on your own
   - Readings
   - Lectures slides
   - Videos

In class: closed book

2. Individual readiness assurance test (IRAT)

3. Team readiness assurance test (IRAT) with immediate feedback

4. Clarification session

In class: open book

5. Application exercises (4S)
   - Significant problem
   - Same problem
   - Specific choice
   - Simultaneous report

After class: Appeals and 360° Teammate evaluation

Next:

Chapter 10
Transmission Lines

W. H. Hayt, Jr. and J. A. Buck, *Engineering Electromagnetics*, 9th ed.,
McGraw-Hill, Chapters 10, 2019.