THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves
2025 W1

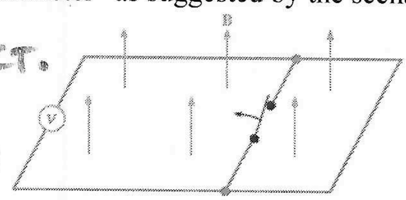
Midterm 1
Chapter 9 – Maxwell's Equations and Time-Varying Fields
Chapter 10 – Transmission Lines
SOLUTIONS

The purpose of this midterm exam is to assess your mastery of the fundamental techniques used to analyze time-varying fields and transmission lines.

Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Make both explicit.

1. Concept Questions [25 in total]

   a. Suppose the magnetic flux density in the figure below is constant. Justify or refute the claim that "An apparent increase in flux linkages does not lead to an induced voltage when one part of a circuit is simply substituted for another by opening the switch. No indication will be observed on the voltmeter" as suggested by the scenario below. [5]



THE CLAIM IS CORRECT. MERELY OPENING & CLOSING A SWITCH DOES NOT ADD ENERGY TO THE SYSTEM, SO CANNOT INDUCE A VOLTAGE.

   b. What is the unit of electric charge? If magnetic monopoles exist, what would be the unit of magnetic charge? [5]

UNIT OF ELECTRIC CHARGE = COULOMB

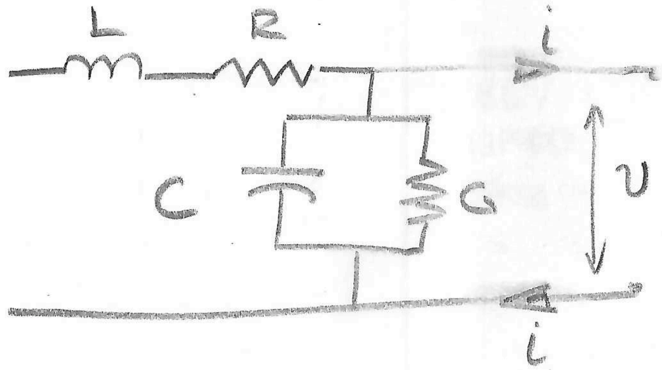
UNIT OF MAGNETIC CHARGE = WEBER

IN S.I.

L ACCOUNTS FOR ENERGY STORAGE IN THE MAGNETIC FIELD THAT SURROUNDS EACH CONDUCTOR

C ACCOUNTS FOR ENERGY STORAGE IN THE ELECTRIC FIELD THAT EXISTS BETWEEN EACH CONDUCTOR.

c. Sketch the equivalent circuit of a uniform transmission line. Explain the physical significance of each circuit element. [5]



R ACCOUNTS FOR OHMIC LOSSES IN EACH CONDUCTOR

G ACCOUNTS FOR DIELECTRIC LOSSES BETWEEN THE CONDUCTOR.

d. Recite the telegraphist's equations, explain in general terms how they are derived, and explain why they are useful. [5]

- THE TELEGRAPHIST'S EQUATIONS RELATE THE CHANGE IN VOLTAGE WITH DISTANCE TO THE CHANGE IN CURRENT WITH TIME AND VICE VERSA.
- THEY ARE DERIVED BY APPLYING KIRCHHOFF'S CURRENT AND VOLTAGE LAWS TO THE EQUIVALENT CIRCUIT OF A TRANSMISSION LINE.

e. Why is a reflected wave generated when the load impedance doesn't match the characteristic impedance of the transmission line? [5]

$$\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t}\right) \qquad \frac{\partial I}{\partial z} = -\left(GV + C\frac{\partial V}{\partial t}\right)$$

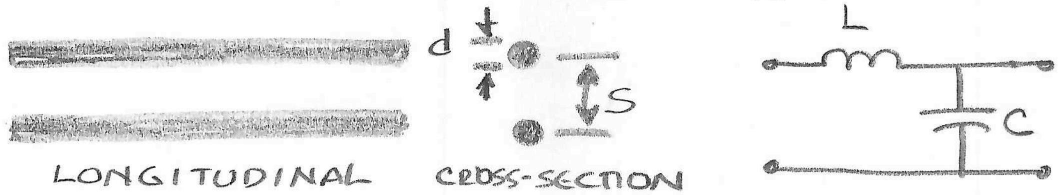
- APPLYING $\frac{\partial}{\partial z}$ TO THE FIRST AND $\frac{\partial}{\partial t}$ TO THE SECOND ALLOWS US TO EQUATE THEM VIA THE COMMON TERM THAT CONTAINS $\partial I/\partial t \partial z$
- THIS LEADS TO THE GENERAL WAVE EQUATION FOR THE VOLTAGE & CURRENT ON TRANSMISSION LINES.

A REFLECTED WAVE OF APPROPRIATE AMPLITUDE AND PHASE WILL CAUSE THE WAVE IMPEDANCE @ $z=0$ TO BE EQUAL TO THE LOAD IMPEDANCE AT $z=0$.

4. The equivalent circuit parameters of a certain two-conductor "twin-lead" parallel-wire transmission line with an air dielectric and wire diameter d of 1 mm and that operates at $\omega = 6\times10^8$ rad/s are:

$$C = 60 \text{ pf/m},\ G = 0\ \mu\text{S/m, and } R = 0\ \Omega\text{/m}.$$

a. Sketch the transmission line both longitudinally (length-wise) and in cross section based upon the available information. Sketch the equivalent circuit and explain the physical significance of each circuit element. [5]



b. For this case, find f, v_p, and λ. [5]

$\omega = 6\times10^8$ rad/s

$f = \frac{\omega}{2\pi} = 95.5$ MHZ

$v_p = \frac{c}{\sqrt{\epsilon_r}} = c = 3\times10^8$ m/s

ϵ_r = 1 for an air dielectric

$\lambda = c/f = 3.14$ m $= \pi$ m

c. For this case, find the value of L in H/m and Z_0. [5]

$v_p = \frac{1}{\sqrt{LC}} = c$

$LC = \frac{1}{c^2}$

$L = \frac{1}{Cc^2} = 185.2$ nH/m

$Z_0 = \sqrt{\frac{L}{C}} = 55.6\ \Omega$

d. For this case, find γ, α, and β. [5]

$\gamma = \alpha + j\beta$ $\quad$ $\alpha = 0$ $\quad$ $\beta = \omega\sqrt{LC} = 2\pi/\lambda = 2$ rad/m

e. What is the disadvantage of using two-conductor "twin-lead" transmission line in practical situations?

DISADVANTAGE: FIELDS ARE NOT CONTAINED WITHIN A PROTECTIVE SHIELD SO: 1) T/L CAN BE DISTURBED BY OBJECTS IN THE VICINITY, 2) T/L ISN'T SUITABLE FOR HIGH POWER APPLICATIONS.

3. A 50-Ω high-frequency air-filled lossless line is used at a frequency where $\lambda = 15$ cm with an open-circuit load at $z = 0$. Use the Smith Chart to find (5 marks each for a total of 25):

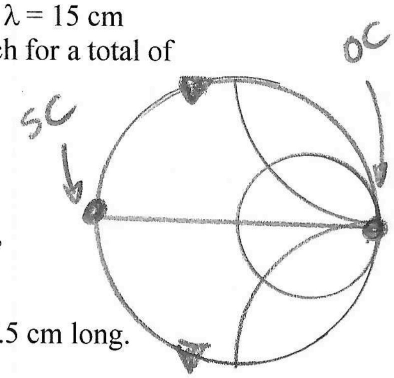
   a. Γ_R,

   b. VSWR,

   c. Distance to the first voltage maximum and minimum from the load,

   d. The wave impedance at V_{max} and V_{min},

   e. The input impedance (and admittance) for a section of line that is 7.5 cm long.



a. $\Gamma_R = \Gamma_0 = +1$ (VOLTAGE REFLECTION COEFFICIENT AT THE LOAD.)

b. VSWR = ∞

c. $z = 0, -\frac{\lambda}{2}, -\frac{n\lambda}{2}$ ARE ALL VOLTAGE MAXIMA. $= 2V_0$

$z = -\frac{\lambda}{4}, -\frac{\lambda}{4} - \frac{n\lambda}{2}$ ARE ALL VOLTAGE MINIMA $= 0$

d. Z_w @ V_{max} = OPEN CIRCUIT

Z_w @ V_{min} = 0 = SHORT-CIRCUIT

e. $7.5\text{ cm} = \frac{\lambda}{2}$ OR ONE COMPLETE ROTATION OF THE SMITH CHART.

$\therefore Z_{in}(-\frac{\lambda}{2}) = \infty\ \Omega$

$Y_{in}(-\frac{\lambda}{2}) = 0\ \Omega$