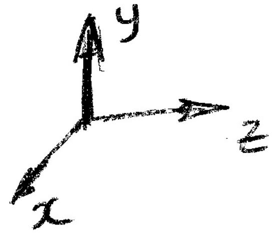

THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves
2025 W1

Midterm 2

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$$E_y = \epsilon_0 E_0 \sin(\omega t - \beta z)$$

Chapter 9 – Maxwell's Equations and Time-Varying Fields
Chapter 10 – Transmission Lines

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1. $\mathbf{E} = E_o \sin(\omega t - \beta z)\,\hat{\mathbf{y}}$ in free space. E_o has units of volts per metre. Given Maxwell's equations in point form, find and give the name of the quantity, and the units of, and the expression for:

   a. **D** [5] ELECTRIC FLUX DENSITY (C/m^2)

   $$\mathbb{D} = \epsilon_0 \mathbb{E} = \epsilon_0 E_0 \sin(\omega t - \beta z)\,\hat{y}$$ ✓

   b. **B** [5] MAGNETIC FLUX DENSITY $(Wb/m^2$ or $T)$

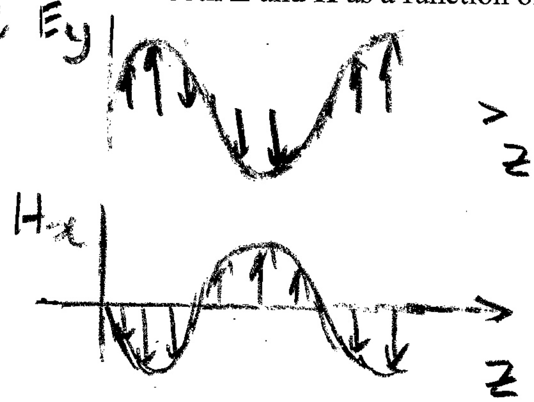
   $$\mathbb{B} = -\frac{\beta E_0}{\omega} \sin(\omega t - \beta z)\,\hat{x}$$

   c. **H** [5] MAGNETIC FIELD STRENGTH (A/m)

   $$\mathbb{H} = \frac{\mathbb{B}}{\mu_0} = -\frac{\beta E_0}{\omega \mu_0} \sin(\omega t - \beta z)\,\hat{x}$$

   d. Sketch both **E** and **H** as a function of z at the instant $t = 0$. [10]

FOR THE WAVE TO PROPAGATE IN THE +Z DIRECTION,
$E_y \times -H_x = S_z$



$$\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t}$$

$$\mathbb{B} = -\int \frac{\partial \mathbb{B}}{\partial t}\, \partial t$$

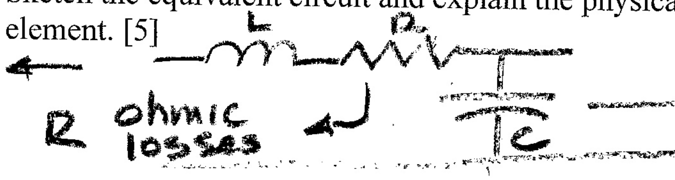
BECAUSE THE FIELDS ARE TIME HARMONIC, $C = 0$.

$Z = 2.59\times10^{-3} + j\omega\, 2.0\times10^{-6}\ \Omega = R + j\omega L$

$Y = j\omega\, 5.56\times10^{-12}\ \Omega = j\omega C$

2. The parameters of a certain transmission line operating at $\omega = 2\pi \times 10^8$ rad/s are $L = 2.0$ uH/m, $C = 5.56$ pf/m, G = 0 μS/m, and R = 2.59×10^{-3} Ω/m. ← VERY LOW RESISTANCE

a. Sketch the equivalent circuit and explain the physical significance of each circuit element. [5]



L accounts for energy in magnetic field that surrounds the lines

R ohmic losses

C accounts for energy in the electric field ~~that surrouds~~ between the lines

b. Find γ, α, β. [5]

$\gamma = \alpha + j\beta = \sqrt{ZY} = 2.159\times10^{-6}\ \text{Np/m} + j2.10\ \text{rad/m}$

α $\qquad$ β

c. Find λ, v_p and Z_0. [5]

$\lambda = \frac{2\pi}{\beta} = 3\,m \qquad v_p = \frac{\omega}{\beta} = 2.99\times10^{8}\ m/s$

d. Find the total attenuation in both Np and dB of a 10-metre length of this transmission line. [5]

$\alpha = 2.159\times10^{-6}\ \text{Np/m} \qquad \alpha = 1.875\times10^{-5}\ \text{dB/m}$

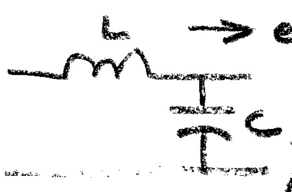
$A = 2.159\times10^{-5}\ \text{Np} \qquad A = 1.875\times10^{-4}\ \text{dB}$

e. Find the operating frequency and the physical length of a quarter wavelength of this line. [5]

$f = 10^8\ Hz = 100MHz \qquad \frac{\lambda}{4} = 0.75\,m = \frac{3}{4}\,m$

$Z_0 = \sqrt{\frac{Z}{Y}} = (600 - j\,6.18\times10^{-4})\ \Omega$

IF THIS WERE LOSSLESS,



L → energy stored in magnetic field that surrounds the line

C energy stored in the electric field between the conductors

$\alpha = 0$ in Np and dB

$f = 10^8\ Hz = 100MHz$

$\gamma = \alpha + j\beta = j\beta$

$\beta = \omega\sqrt{LC} = 2.10\ \text{rad/m}$

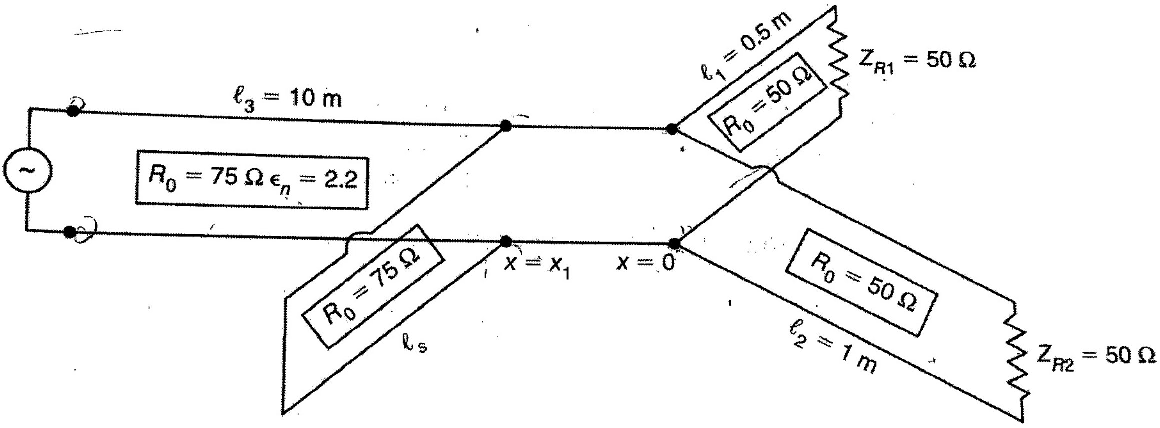
$\lambda = \frac{2\pi}{\beta} = 3.00\,m$

$\frac{\lambda}{4} = 0.75\,m$

$v_p = \frac{\omega}{\beta} = 2.99\times10^{8}\ m/s$

$Z_0 = \sqrt{\frac{L}{C}} = 600\,\Omega$

3. A 0.15 GHz signal is applied to a 10-m-long 75-Ω coaxial line terminated in a composite load consisting of the parallel connection of two 50-Ω lines of lengths 0.5 m and 1 m, each terminated in a 50-Ω resistance. All lines are lossless with $\epsilon_r = 2.2$. See the figure below.



a. Find the phase velocity and wavelength on the main feedline. [5]

$v_p = \frac{c}{\sqrt{2.2}} = 2.02 \times 10^8$ m/s $\quad \lambda = \frac{v_p}{f} = 1.35$ m $\quad$ 2

b. Find the effective load impedance at $x = 0$. [5]

$Z_L = 25\Omega \quad z_L = 0.3 = \frac{Z_L}{Z_0}$

c. Use a Smith Chart to determine the length ℓ_s of a shunt short-circuit stub that will produce minimum VSWR on the feed line. The stub should be as close as possible to the load. [5]

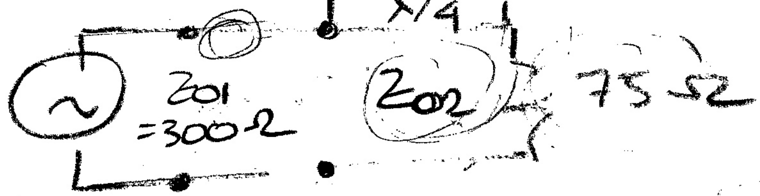
$x_1 = 0.3325 - 0.25\lambda = 0.0835\lambda$
$= 0.1127$ m

d. Use a Smith Chart to determine the connection point x_1 of a shunt short-circuit stub that will produce minimum VSWR on the feed line. The stub should be as short as possible. [5]

$\ell_s = (0.25 + 0.1345\lambda) = 0.3845\lambda$
$= 0.519$ cm

e. What are the two main disadvantages of a single-stub matching network? [5]

- ONLY WORKS AT A SINGLE FREQUENCY
- MAY BE DIFFICULT TO ATTACH A STUB TO THE TRANSMISSION LINE AT AN ARBITRARY DISTANCE, PARTICULARLY IF THE TRANSMISSION LINE IS A COAXIAL CABLE



4. A quarter-wave section of transmission line is used to match a 75-Ω load to a 300–Ω transmission line. Use a Smith Chart to determine:

   a. The characteristic impedance of the quarter-wave section. [5]

$$Z_0 = \sqrt{Z_L Z_{in}} = \sqrt{75 \times 300} = 150\,\Omega$$

EXPLANATION: $\frac{Z_L}{Z_0}$ must equal $\frac{Z_0}{Z_{in}} = \frac{1}{\frac{Z_{in}}{Z_0}}$

   b. The VSWR on the quarter-wave section. [5]

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1.333}{0.666} = 2 = \frac{V_{max}}{V_{min}} = VSWR$$

$|\Gamma| = 0.333$

   c. The voltage reflection coefficient on the quarter-wave section [5]

$$\Gamma(z) = \Gamma_0 e^{j2\beta z} = -0.333\, e^{j\left(2 \cdot \frac{2\pi}{\lambda} \cdot z\right)}$$

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$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\cancel{300}\,75 - 150}{\cancel{300}\,75 + 150} = \frac{\cancel{150}\,-75}{\cancel{450}\,225} = -\frac{1}{3} = -0.333$$

   e. What are the principal advantages and disadvantages of this matching technique? [5]

ADVANTAGES

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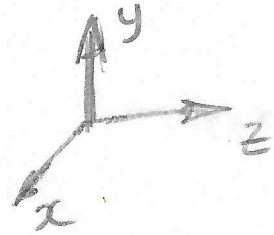

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Midterm 2
SOLUTIONS

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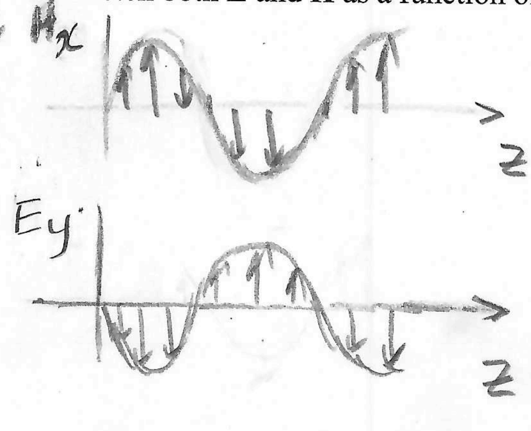

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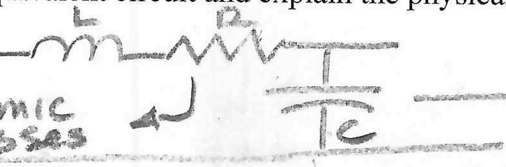

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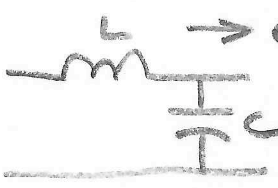

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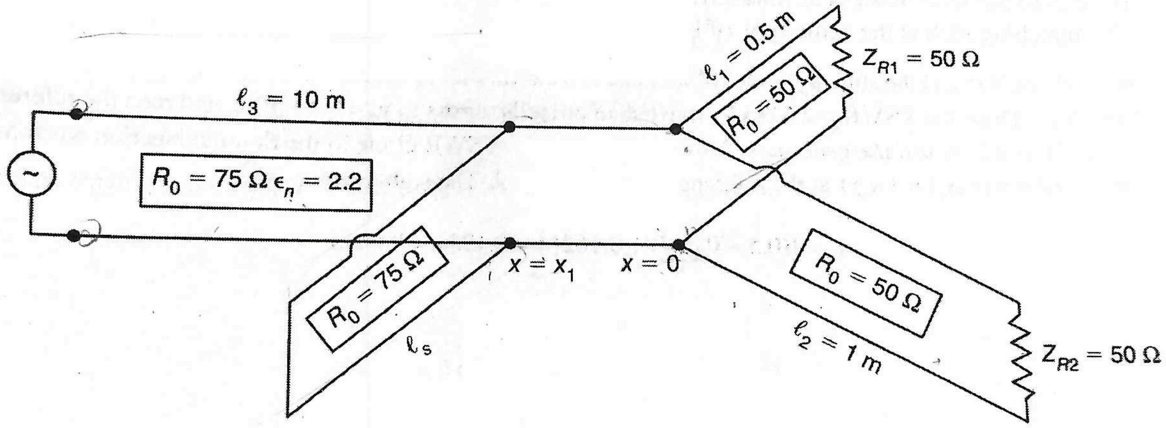

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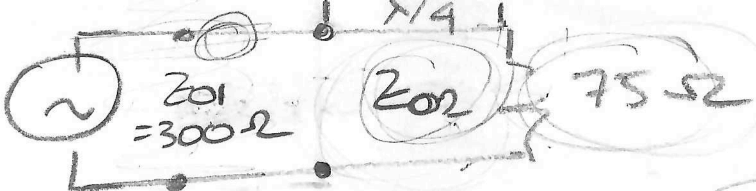

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