THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves
2025 W1

Midterm 3
SOLUTIONS
Chapter 11 – Uniform Plane Waves | Chapter 12 – Plane Wave Reflection and Dispersion

The purpose of this midterm exam is to assess your mastery of the fundamental techniques used to analyze plane waves. Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Make both explicit.

Numerical answers should include the symbol, quantity, and units, e.g., $\boldsymbol{\alpha} = \mathbf{5}\ \boldsymbol{Np/m}$.

1. Uniform Plane Waves [50] $f = 30\ MHz$ $\lambda_0 = 10\ m$ $c = 3\times10^8\ m/s$

An electromagnetic plane wave is normally incident from air onto seawater. The amplitude and frequency of the electric field are 1 V/m and 30 MHz, respectively. The relative permittivity and permeability of seawater are 80 and 1, respectively. The conductivity of seawater is 2.5 S/m.

a. [5] What is the intrinsic impedance of:

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$

i. the air? [$\eta_0 = 120\pi = 377\ \Omega$] 2

$\eta_1 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

ii. the seawater? [$\eta_1 = 9.73\angle 43.5^\circ = 7.06 + j6.69\ \Omega$] 2

What is the physical significance of the intrinsic impedance?

[η IS THE RATIO OF ELECTRIC TO MAGNETIC FIELD STRENGTH.] 1

b. [5] What is the loss tangent of:

$\tan\delta = \frac{\sigma}{\omega\epsilon}$

i. the air? [$\tan\delta = 0$] 2

ii. the seawater? [$\tan\delta = 18.73$] 2

What is the physical significance of the loss tangent?

[$\tan\delta$ = THE RATIO OF CONDUCTION TO DISPLACEMENT CURRENT.] 1

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$ IF $\sigma = 0$, $Re(\gamma) = \alpha = 0$

$Im(\gamma) = \omega\sqrt{\mu\epsilon} = \beta$

c. [5] What is the attenuation constant of:

i. the air? [α = 0 Np/m] 2

$\gamma = 16.75 + j\, 17.67$

ii. the seawater? [α = 16.75 Np/m] 2

What is the physical significance of the attenuation conbstant?

[α IS THE RATE AT WHICH FIELD STRENGTH DECAYS] EXPONENTIALLY WITH DISTANCE, $E(z) = E_0 e^{-\alpha z}$. 1

d. [5] What is the phase constant of:

$\beta = \omega\sqrt{\mu\epsilon}$ i. the air? [0.6283 rad/m] 2

$\beta = Im(\gamma)$ ii. the seawater? [17.67 rad/m] 2

What is the physical significance of the phase constant?

[β IS THE RATE AT WHICH THE PHASE ADVANCES] 1 WITH DISTANCE. (ALSO REFERRED TO AS THE SPATIAL FREQUENCY.)

e. [5] What is the velocity of electromagnetic wave propagation in:

$c = \omega/\beta = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ i. the air? [$c = 3\times10^8$ m/s] 2.5

$v_p = \frac{\omega}{\beta}$ ii. the seawater? [$v_p = 1.067\times10^7$ m/s] 2.5

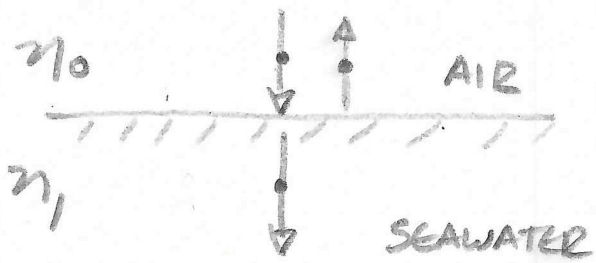
$= \frac{2\pi\times30\times10^6}{17.67} = 1.067\times10^7$ m/s

f. [4] What is the voltage reflection coefficient at the boundary between the air and water?

$$\Gamma = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0} \quad 2$$

$[\Gamma = 0.9633 \angle 178.0 \; \Omega]$ 2

$= -0.9626 + j\,0.0342 \; \Omega$



$\eta_1 = 7.06 + j\,6.69 \; \Omega$

$\eta_0 = 377 \; \Omega$

g. [6] What is the time-averaged power density of the waves that are incident upon, and reflected from, the seawater?

$[\langle S_{inc} \rangle = 1.326 \times 10^{-3} \; W/m^2]$

$[\langle S_{ref} \rangle = 1.231 \times 10^{-3} \; W/m^2]$

$$\langle S_{inc} \rangle = \frac{1}{2} \frac{E_0^2}{\eta_0}$$

$$\langle S_{ref} \rangle = \frac{1}{2} \frac{E_0^2}{\eta_0} |\Gamma|^2$$

$E_0 = 1 \; V/m$

$\eta = 377 \; \Omega$

$|\Gamma| = 0.9633$

$$\langle S_{inc} \rangle = \frac{1}{2} \frac{1^2}{377} = 0.001326 \; W/m^2$$

$$= 1.326 \times 10^{-3} \; W/m^2$$

$$\langle S_{ref} \rangle = \langle S_{inc} \rangle \, 0.9633^2$$

$$= 1.231 \times 10^{-3} \; W/m^2$$

h. [5] For a frequency of 30 MHz, at what depth will the amplitude of the electric field be 1.0 mV/m.

[$z = 0.234$ m]

THE ELECTRIC FIELD JUST INSIDE THE SEAWATER IS $E_o \tau$ WHERE $\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2(7.06 + j6.69)}{7.06 + j6.69 + 377}$

$= 0.0374 + j0.0342$ V/m

$= 0.0506\angle 42.47°$

$E(z) = |E_o \tau| e^{-\alpha z}$

$0.001 = 0.0506\, e^{-16.75 z}$

$\ln \frac{0.001}{0.0506} = -16.75 z$

$z = \ln \frac{0.001}{0.0506} / -16.75 = 0.234$ m.

i. [5] Will the loss tangent of the seawater increase with frequency, decrease with frequency, or stay the same? Explain why.

[THE LOSS TANGENT WILL DECREASE WITH FREQUENCY AS DISPLACEMENT CURRENT INCREASES.]

$\tan \delta = \frac{\sigma}{\omega \epsilon}$

j. [5] Will the velocity of propagation in the seawater increase with frequency, decrease with frequency, or stay the same? Explain why.

[THE VELOCITY OF PROPAGATION WILL INCREASE WITH FREQUENCY. (FOR A GOOD CONDUCTOR, v_p VARIES AS $\sqrt{f}$.)]

$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$

FOR A GOOD CONDUCTOR

$\beta = \sqrt{\pi f \mu \sigma}$ SO $v_p = \frac{2\pi f}{\sqrt{\pi f \mu \sigma}} = \frac{2\sqrt{\pi f}}{\mu \sigma}$

IRRELEVANT

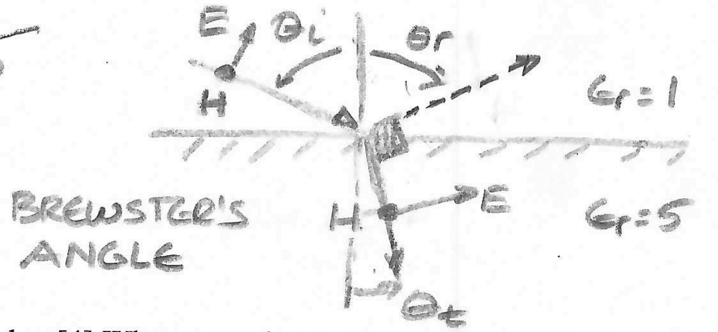
2. Oblique incidence on a plane [25]

REDUNDANT

An electromagnetic plane wave is obliquely incident from air onto a perfect dielectric. The amplitude and frequency of the electric field are 1 V/m and 30 GHz, respectively. The relative permittivity and permeability of the perfect dielectric are 5 and 1, respectively. The conductivity of the perfect dielectric is 0 S/m. A researcher notices that there is no reflection when the wave is polarized in a certain way and for a certain angle of incidence.

a. [5] Sketch the problem geometry and indicate the polarization of the plane wave and all relevant angles and directions. NO REFLECTION

5



$\theta_i = 65.9°$
$\theta_r = 65.9°$
$\theta_t = 24.1°$

b. [4] What name do we give to this polarization and angle that results in no reflection?

[TRANSVERSE MAGNETIC OR TM] 2

[BREWSTER'S ANGLE] 2

c. [6] For this case, what is:

i. the angle of incidence?

$\theta_B = \arctan\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$

[$\theta_i = 65.9°$]

ii. the angle of reflection?

$\theta_r = \theta_i$

[$\theta_r = 65.9°$]

iii. the angle of refraction?

WE CAN USE SNELL'S LAW

$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$

[$\theta_t = 24.1°$]

OR THE OBSERVATION THAT THE DIRECTIONS OF REFLECTION AND REFRACTION (TRANSMISSION) FORM A RIGHT ANGLE. (SEE THE SKETCH.)

5

d. [6] Suppose the experiment is reversed and the electromagnetic plane wave is obliquely incident from the perfect dielectric into the air and, once again, no reflection is observed. For this case, what is:

i. the angle of incidence?

[$\theta_i = 24.1°$]

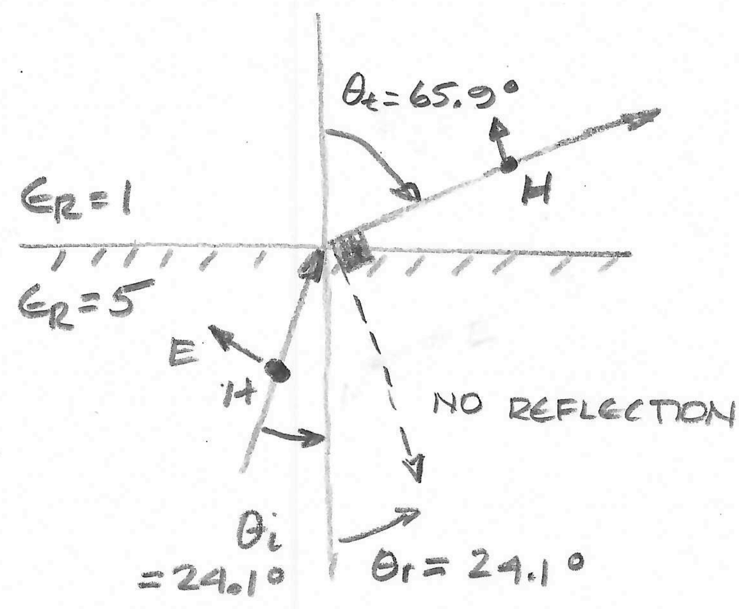
ii. the angle of reflection?

[$\theta_r = 24.1°$]

iii. the angle of refraction?

[$\theta_t = 65.9°$]

e. [4] Sketch the new problem geometry and indicate the polarization of the plane wave and all relevant angles.



CHECK: $\frac{\sin \theta_i}{\sin \theta_r} = \sqrt{\frac{1}{5}} = 0.447$ ✓

3. Propagation through a slab [25] $f = 1\,GHz$ $c = 3\times10^8\,m/s$ $\lambda_0 = 0.3\,m = 30\,cm$

An electromagnetic plane wave is normally incident onto a slab of thickness 7.5 cm. The amplitude and frequency of the electric field are 1 V/m and 1 GHz, respectively. The relative permittivity and permeability of the slab are 4 and 1, respectively. The conductivity of the slab is 0 S/m.

PERFECT DIELECTRIC

$\beta = \omega\sqrt{\mu\epsilon}$ $\lambda = 2\pi/\beta$ $\lambda_0 = \frac{c}{f}$ $\lambda_1 = \frac{c}{f\sqrt{\epsilon_r}}$

a. [4] What is the wavelength of the signal in:

i. the air? [$\lambda_0 = 0.3\,m = 30\,cm$] 2

ii. the slab? [$\lambda_1 = 0.15\,m = 15\,cm$] 2

b. [4] What is the intrinsic impedance of:

i. the air? [$\eta_0 = 120\pi = 377\,\Omega$] 2

ii. the slab? [$\eta_1 = 60\pi = 188.5\,\Omega$] 2

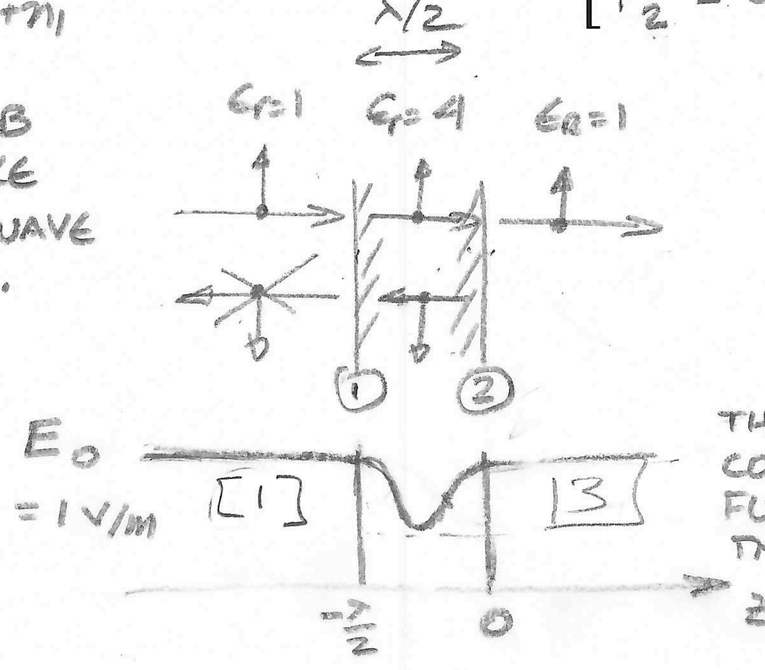
c. [3] What is the reflection coefficient at the first boundary?

$\Gamma = \frac{\eta_{in} - \eta_0}{\eta_{in} + \eta_0} = 0$ [$\Gamma_1 = 0.0$]

d. [3] What is the reflection coefficient at the second boundary?

$\Gamma = \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} =$ [$\Gamma_2 = 0.333$]

THE SLAB ACTS LIKE A HALF-WAVE SECTION.



$V_{max} = |1 + |\Gamma||$
$= 1.3$

$V_{min} = |1 - |\Gamma||$
$= 0.7$

THE STANDING WAVE CORRESPONDS TO ONE FULL ROTATION AROUND THE SMITH CHART

e. [5] Sketch the problem geometry. Include voltage standing wave within the slab and within the free space portions on either side.

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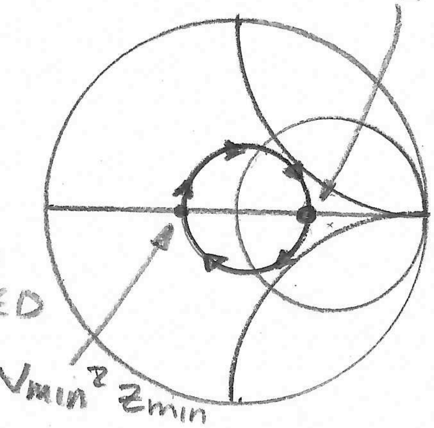

HERE, WE PLOT THE SCENARIO ON A SMITH CHART. THE REFERENCE IMPEDANCE IS THE INTRINSIC IMPEDANCE OF THE SLAB = 188.5 Ω. THE NORMALIZED IMPEDANCE SEEN AT THE SECOND (AND FIRST BOUNDARY, AFTER ONE COMPLETE ROTATION) IS 2 WHICH CORRESPONDS TO 2×188.5 = 377 Ω.

f. [6] What is the VSWR in:

i. the first region?

[S = 1]

ii. the slab?

$$S = \frac{V_{max}}{V_{min}} = \frac{1.3}{0.7} =$$

[S = 1.857]

iii. the third region?

[S = 1]