THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering

ELEC 311 – Electromagnetic Fields and Waves
2025 W1

Practice Final Exam 1

Read the entire question before answering. Answers should be short and to the point. ***Use sketches to explain your solution as required****. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Numerical answers should include the symbol, quantity, and units, e.g.,* $\boldsymbol{\alpha} = \mathbf{5\ Np/m}$*, and be inserted within the large square brackets. The actual exam will be printed on tabloid-size paper.*

1. Time-Varying Fields and Materials [25]

A non-magnetic material has $\sigma = 5.0$ S/m and $\epsilon_r = 1$. The electric field is directed in the z direction and has intensity $E = 250 \sin 10^{10} t\ V/m$.

a. [5] Find expressions for:

i. the conduction current density: [$J_c = \sigma E = 1250 \sin 10^{10} t \ A/m^2$]

ii. the displacement current density: [$J_D = \frac{\partial D}{\partial t} = 22.1 \cos 10^{10} t \ A/m^2$]
$= \frac{\partial}{\partial t}(\epsilon_0 \epsilon_r 1250 \sin 10^{10} t)$

b. [5] Find the frequency at which the conduction and current densities are equal:

$\omega : \sigma = \omega \epsilon$ [$\omega = \frac{5.0}{8.85 \times 10^{-12}} = 5.65 \times 10^{11}$ rad/s]
$f = 89.9$ GHz

c. [5] Define the loss tangent and give a numerical value at this frequency:

$\tan\delta = \frac{\sigma}{\omega\epsilon} = \frac{J_c}{J_D}$ [$\tan\delta = 1$; RATIO OF CONDUCTION CURRENT TO DISPLACEMENT CURRENT]
LOSS TANGENT

A non-magnetic material has $\sigma = 5 \times 10^{-3}$ S/m and $\epsilon_r = 8$.

d. [5] Would the material be considered a dielectric or conductor at 1 MHz? Justify your answer.

$\tan\delta = \frac{\sigma}{\omega\epsilon} = \frac{5 \times 10^{-3}}{2\pi \times 10^6 \times 8 \times 8.85 \times 10^{-12}}$ [CONDUCTOR; $\tan\delta = 11.24$]

e. [5] At what frequencies may it be considered a 'very good' dielectric with $\tan\delta < 0.01$.

$0.01\omega\epsilon = \sigma = \frac{2\pi f \epsilon}{\cancel{100}\ 50}$ [$f \geq 1.12$ GHz]

$f = \frac{50\sigma}{\pi\epsilon} = \frac{50 \times 5 \times 10^{-3}}{\pi \times 8 \times 8.85 \times 10^{-12}}$

$l = 25\text{cm} = \frac{\lambda_0}{8}$

$\lambda_0 = \frac{c}{f} = 2.0\text{ m} = 200\text{ cm}$

2. Transmission Lines [25]

Consider a lossless air-filled line where $Z_0 = 50\ \Omega$ and the load is $Z_L = 25 + j\,25\ \Omega$. The line is 25 cm long. The signal generator outputs 1 W at 150 MHz.

a. [5] Use a Smith Chart to find the input impedance and voltage reflection coefficient at the load:

CHECK:

$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.2 + j0.4$

$= 0.45\angle 117^\circ$

$z_{in}(0) = z_L = 0.5 + j0.5$

[$Z_{in}(0) = Z_L = 25 + j25\ \Omega$]

[$\Gamma_0 = \Gamma(0) = 0.45\angle 117^\circ$]

b. [5] Use a Smith Chart to find the input impedance and voltage reflection coefficient at the generator:

$\Gamma(-25\text{cm}) = \Gamma_0 e^{j2\beta z}$

$= \Gamma_0 e^{j\frac{4\pi}{8}2}$

$= j\Gamma_0$

$z_{in}(l) = z_{in} = 2 + j$

[$Z_{in}(l) = 100 + j50\ \Omega$]

[$\Gamma(l) = \Gamma_{in} = 0.45\angle 27.5^\circ$]

c. [5] Find the power absorbed by, and reflected from, the load:

LOSSLESS LINE SO

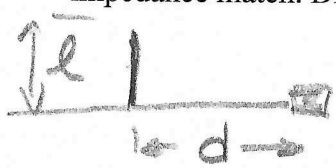
$P_{ABS} = P_{in}(1 - |\Gamma|^2)$

$P_{REF} = P_{in}(|\Gamma|^2)$

[$P_{abs} = 0.798\text{ W}$]

[$P_{ref} = 0.202\text{ W}$]

d. [5] Use a Smith Chart to find the distance and length of a single-stub that will achieve an impedance match. Draw a sketch with dimensions.



[$d = 0.324\lambda = 0.648\text{ m}$]

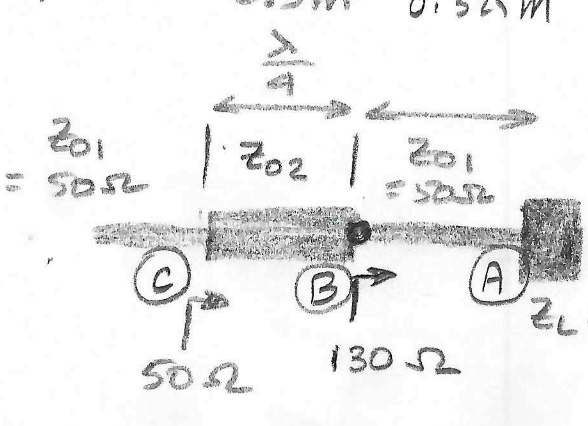
[$l = 0.125\lambda = 0.250\text{ m}$]

THIS IS THE SHORTEST SOLUTION.

e. [5] Find the distance, length, and impedance of a quarter-wave section that will achieve an impedance match. Draw a sketch with dimensions.

$Z_{02} = \sqrt{130 \cdot 50}$

l_2 0.5m $\quad l_1$ 0.325m



$Z_L = 25 + j25\ \Omega$

[$l_1 = 0.325\text{ m}$]

[$l_2 = 0.5\text{ m}$]

[$Z_{02} = 80.6\ \Omega$]

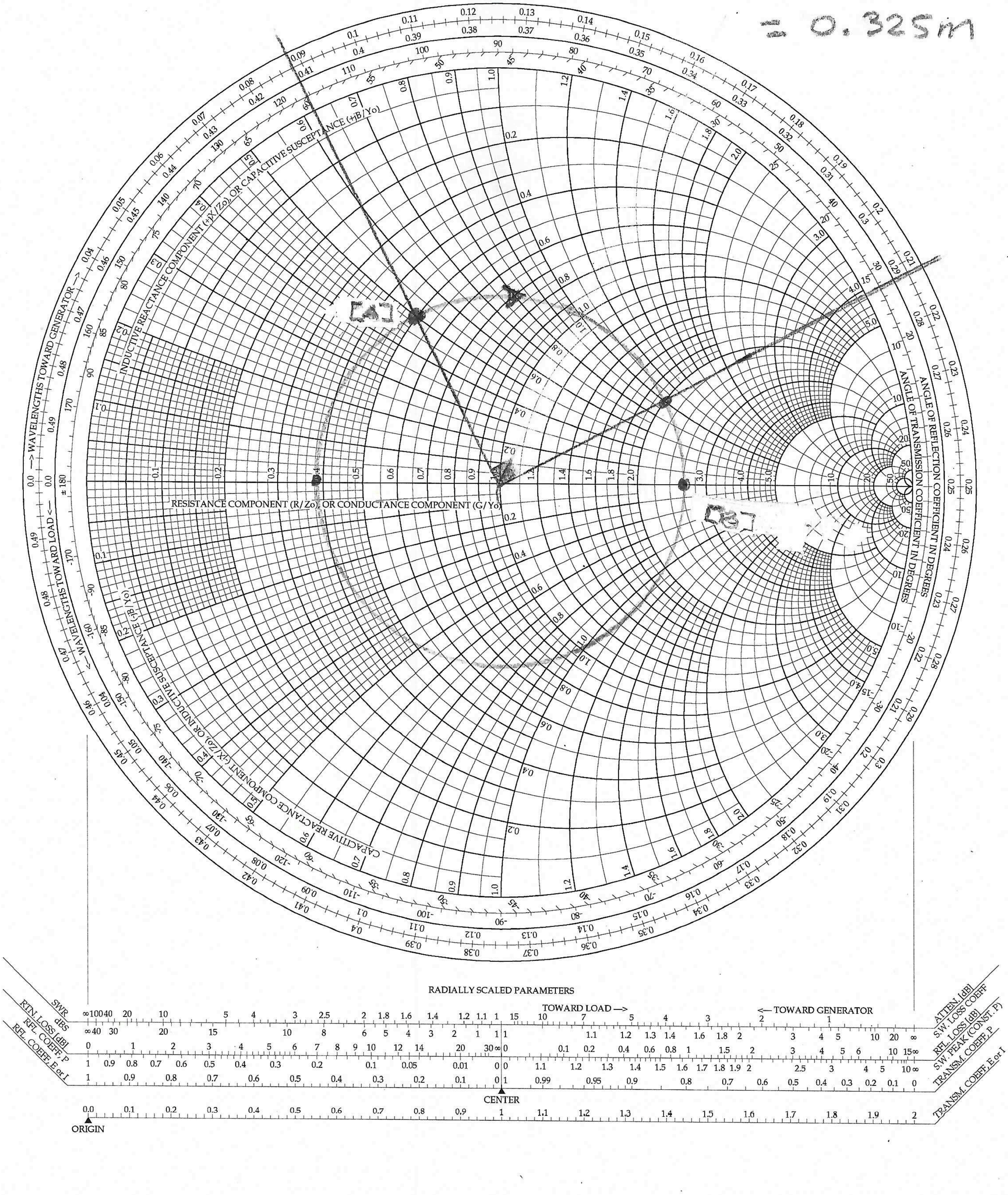
a, b and e

Smith Chart

$\lambda = 2.0\,m$

$117^\circ = 0.1625\lambda$

$= 0.325\,m$



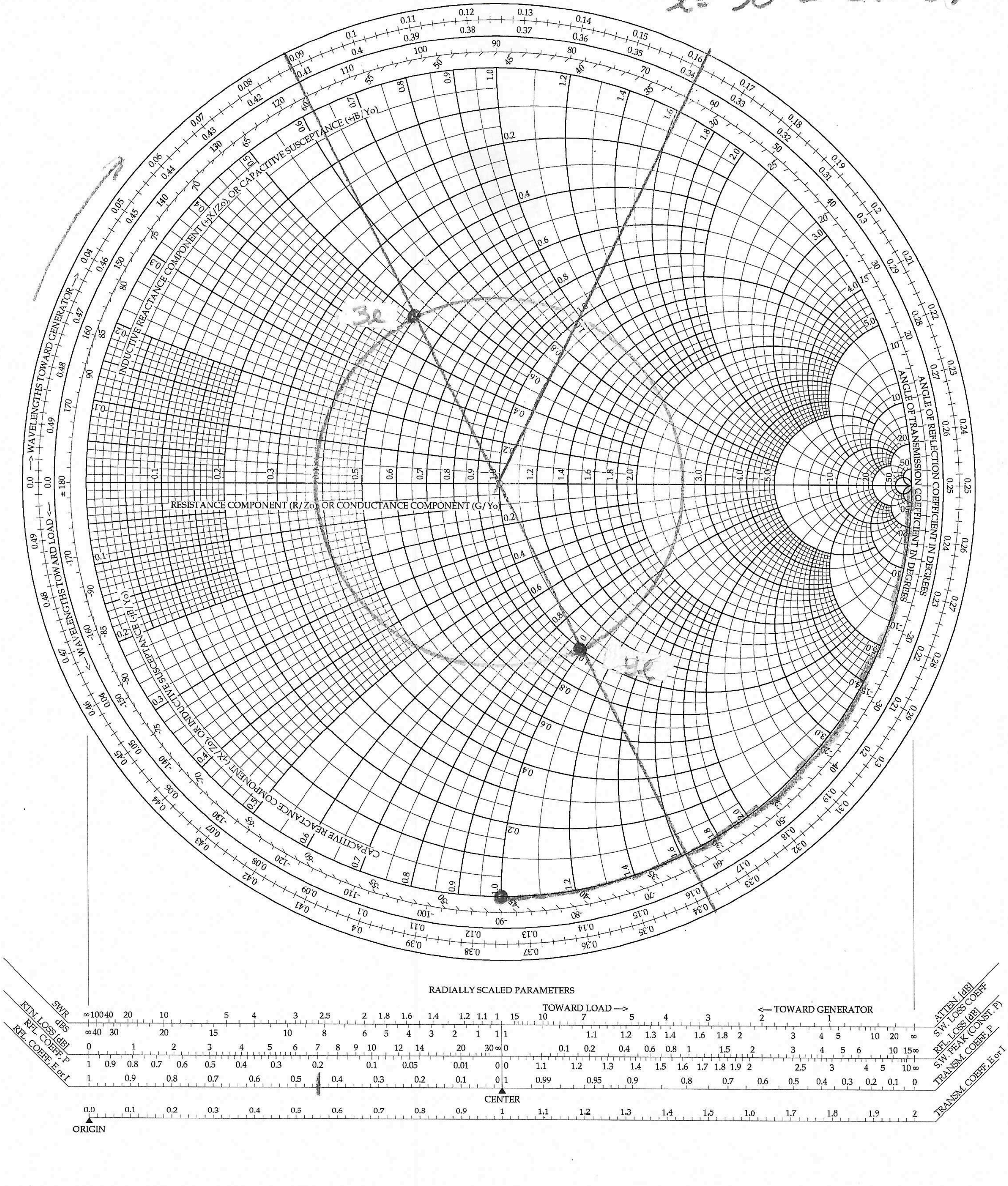
C.

Smith Chart

SINGLE-STUB MATCH

$d = 233° = 0.324\lambda$

$l = 90° = 0.125\lambda$

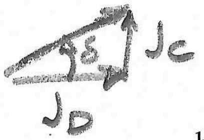


$$\tan\delta = \frac{3.82\times10^{7}}{2\pi\times1.6\times10^{6}\times 8.85\times10^{-12}}$$

3. Electromagnetic Waves [25]

An electromagnetic plane wave with a frequency of 1.6 MHz is propagating in aluminum where the conductivity is 38.2 MS/m and the relative permeability is 1.

a. [5] Define the loss tangent and give a numerical value at this frequency:



$\tan\delta = \frac{\sigma}{\omega\varepsilon} = \frac{J_C}{J_D}$ [$\tan\delta =$]

LOSS TANGENT $\rightarrow \delta$ $\rightarrow d = 5\delta$

b. [5] Define the skin depth and penetration depth of the material and give numerical values at this frequency:

THIS IS A VERY GOOD CONDUCTOR

$\alpha = \sqrt{\pi f \mu \sigma}$

$= 1.55\times10^{4}$ Np/m

[$\delta = \frac{1}{\alpha} = 6.44\times10^{-5}$ m $= 64.4\ \mu m$]

[$d = 5\delta = 32.2\times10^{-5}$ m $= 322\ \mu m$]

$\rightarrow$ ATTENUATION CONSTANT, α

$\rightarrow$ PHASE CONSTANT, B

c. [5] Find the real and imaginary components of the propagation constant:

$\gamma = \alpha + jB$

same numerical value

[$\alpha = 1.55\times10^{4}$ Np/m]

[$B = 1.55\times10^{4}$ rad/m]

d. [5] Find the velocity of propagation in the material:

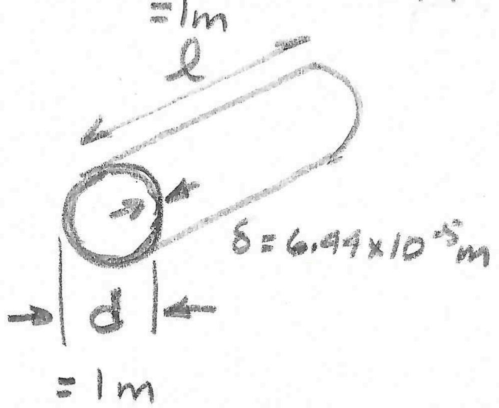
$v = \frac{\omega}{B} = \frac{2\pi\times1.6\times10^{6}}{1.55\times10^{4}}$ [$v = 648.5$ m/s]

e. Find the resistance of an aluminum wire of length 1 metre and diameter 1.0 mm at both DC and 1.6 MHz:

$R = \frac{L}{\sigma A}$

[$R_{dc} = 0.0333\ \Omega$]

[$R_{ac} = 0.1294\ \Omega$]



$R_{dc} = \frac{L}{\sigma\pi\left(\frac{d}{2}\right)^{2}}$ $R_{ac} = \frac{L}{\sigma\pi d\delta}$

4. Oblique Incidence [25]

An electromagnetic wave propagates through a dielectric material toward the interface with free space. A researcher uses a measurement apparatus to show that the critical angle is 20 degrees.

a. [5] Find the relative permittivity of the dielectric material:

$\theta_c = \arcsin\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}$; $\epsilon_{r2} = 1$

$\sin^2\theta_c = \frac{1}{\epsilon_{r2}}$

[$\epsilon_{r2} = 8.55$]

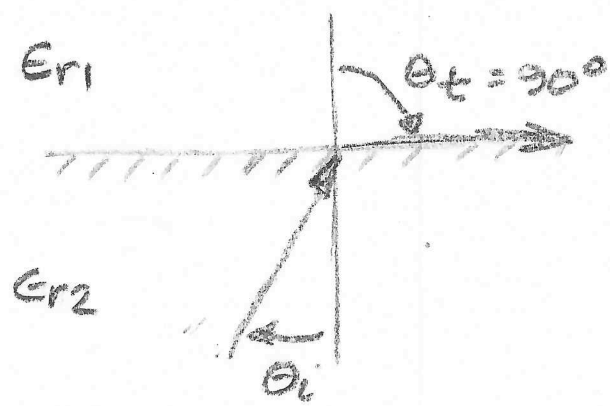
b. [5] Find the angle of reflection.

[$\theta_r = 20°$]

c. [5] Find the angle of transmission (or refraction).

[$\theta_t = 90°$ (BY DEF'N)]

d. [5] Sketch the problem scenario and indicate the relevant material media, waves, and directions:



e. [5] Suppose the same wave propagates in free space towards the dielectric. What is the critical angle in this case?

[IN THIS CASE, $\theta_t < \theta_i$ FOR ALL θ_i, SO θ_t CAN NEVER $= 90°$
AS A RESULT, THE CRITICAL ANGLE CANNOT EXIST FOR THIS CASE.]