

Practice Final Exam 3

Read the entire question before answering. Answers should be short and to the point. Use sketches to explain your solution as required. Clarity, conciseness, and presentation all count. Solution = Intuition (strategy) + Execution (calculation). Numerical answers should include the symbol, quantity, and units, e.g., $\alpha = 5 \text{ Np/m}$, and be inserted within the large square brackets. The actual exam will be printed on tabloid-size paper.



1. Maxwell's Equations [25]

Given that $\mathbf{E} = E_0 \cos(\omega t + \beta x) \hat{z}$ in free space.

a. [10] Find \mathbf{H} .

$$\hat{z} \times \hat{y} = -\hat{x} \quad \checkmark$$

$$\mathbf{E} \times \mathbf{H} = \mathbf{S} \quad \checkmark$$

$$[\mathbf{H} = \frac{E_0}{\eta} \cos(\omega t + \beta x) \hat{y}]$$

b. [5] Find \mathbf{D} and \mathbf{B} .

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E}$$

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H}$$

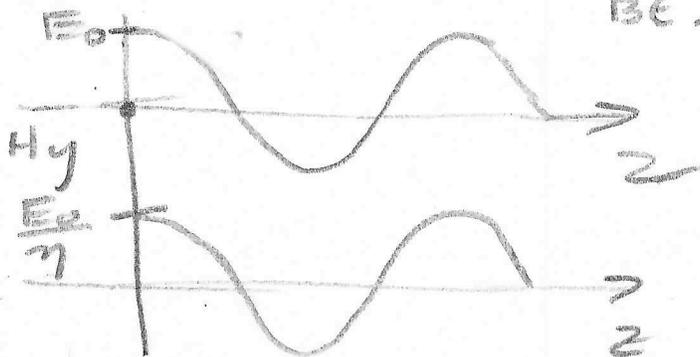
$$[\mathbf{D} = \epsilon_0 E_0 \cos(\omega t + \beta x) \hat{z}]$$

$$[\mathbf{B} = \frac{\mu_0 E_0}{\eta} \cos(\omega t + \beta x) \hat{y}]$$

c. [5] In what direction is the wave propagating?

$$[\text{IN THE } -\hat{x} \text{ DIRECTION}]$$

d. [5] Sketch E and H at $t = 0$. ← IMPLIED THAT THE PLOT SHOULD BE A FUNCTION OF x



wavelength in free space

$$\text{AT } 5 \text{ kHz, } \lambda_0 = 60,000 \text{ m} = 74.5 \text{ miles}$$

$$l = 0.5 \text{ miles} = 805 \text{ m} \ll \lambda_0$$

2. Transmission Lines [25]

Measurements are made at 5 kHz on a 0.5-mile-long transmission line. The results show that the characteristic impedance of the line is $94 \angle -23.2^\circ \Omega$, the total attenuation is 0.06 Np, and the phase shift between the input and output is 8° .

$$8^\circ = 0.1396 \text{ rad} = \theta$$

DO THIS LAST a. [10] Find the R , L , C , and G values per mile for the line:

$$\begin{aligned} R &= \operatorname{Re}(Z) = 20.8 & [R &= 20.8 \Omega/\text{mile}] \\ L &= \operatorname{Im}(Z)/\omega = 19.8/(2\pi \times 5000) & [L &= 6.30 \times 10^{-4} \Omega/\text{mile}] \\ G &= \operatorname{Re}(Y) = 0 & [C &= 1.03 \times 10^{-7} \Omega/\text{mile}] \\ G &= \operatorname{Im}(Y)/\omega = \frac{3.24 \times 10^{-3}}{2\pi \times 5000} & [G &= 0 \text{ S/mile}] \end{aligned}$$

DO THIS FIRST b. [5] Find α and β :

$$\begin{aligned} \alpha &= A/l = 0.06/0.5 & [\alpha &= 0.120 \text{ Np/mile}] \\ \beta &= \theta/l = 0.1396/0.5 & [\beta &= 0.279 \text{ rad/mile}] \end{aligned}$$

DO THIS NEXT

c. [5] Find Z and Y :

SOLVE FOR Z AND Y

$$\sqrt{\frac{Z}{Y}} = 94 \angle -23.2^\circ = Z_0$$

$$[Z = 20.8 + j19.8 \Omega/\text{mile}]$$

$$[Y = j3.24 \times 10^{-3} \Omega/\text{mile}]$$

$$\sqrt{ZY} = 0.120 + j0.279 = \gamma$$

DO THIS NEXT d. [5] Find the wavelength and phase velocity on the line:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.279 \text{ rad/mile}}$$

$$[\lambda = 22.5 \text{ miles}]$$

$$v = \omega/\beta = 2\pi \times 5000 / 0.279$$

$$[v = 112,602 \text{ miles/sec}]$$

Find the power lost when 3 Watts is applied to the input and the line is terminated by a matched load

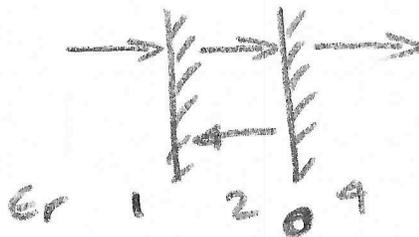
$$P_{\text{out}} = P_{\text{in}} e^{-2\alpha z}$$

$$[P_{\text{lost}} = 0.339 \text{ W}]$$

$$P_{\text{lost}} = P_{\text{out}} - P_{\text{in}}$$

$$= P_{\text{in}} (1 - e^{-2\alpha z})$$

$$= 3(1 - e^{-2 \times 0.120 \times 0.5})$$



3. Electromagnetic Waves [25]

An electromagnetic plane wave with a frequency of 10 GHz is normally incident on a dielectric layer with unknown relative permittivity that is laying atop a second dielectric with relative permittivity = 4.

- a. [5] Find the wavelength in free space and in the second dielectric.

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

FREE SPACE [$\lambda_0 = 3.0 \text{ cm}$]
 SECOND DIELECTRIC [$\lambda_2 = 1.5 \text{ cm}$]

- b. [5] What are the boundary conditions that must be applied at the front and rear surfaces of the dielectric layer? What boundary conditions can be ignored in this case?

[CONTINUITY OF THE TANGENTIAL COMPONENTS OF FIELD STRENGTH ACROSS THE INTERFACE CAN BE IGNORED.]
 [CONTINUITY OF THE NORMAL COMPONENTS OF FLUX DENSITY ACROSS THE INTERFACE CAN BE IGNORED.]

- c. [5] What should the thickness and permittivity of the dielectric layer be to eliminate reflections from the front face?

QUARTER-WAVE SECTION [$t = \frac{\lambda_1}{4} = \frac{2.12}{4} = 0.53 \text{ cm}$]
 $\epsilon_{r1} = \sqrt{\epsilon_{r0} \epsilon_{r2}} = \sqrt{4} = 2$ [$\epsilon_r = 2$]

- d. [5] What will be the VSWR in free space, the dielectric layer, and the second dielectric?

FREE SPACE [$S = 1$ (NO REFLECTED WAVE)]
 DIELECTRIC LAYER [$S = 1.414$]
 SECOND DIELECTRIC [$S = 1$ (NO REFLECTED WAVE)]

- e. [5] What will be the velocity of propagation in free space, the dielectric layer, and the second dielectric?

$v = \frac{c}{\sqrt{\epsilon_r}}$ FREE SPACE [$v_0 = 3 \times 10^8 \text{ m/s}$]
 DIELECTRIC LAYER [$v_1 = 2.12 \times 10^8 \text{ m/s}$]
 SECOND DIELECTRIC [$v_2 = 1.5 \times 10^8 \text{ m/s}$]

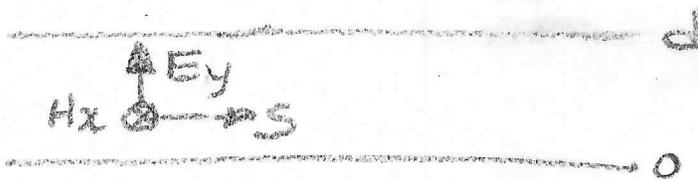
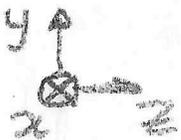
$$\Gamma_0 = \frac{n_2 - n_1}{n_2 + n_1} = \frac{\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{4}} + \frac{1}{\sqrt{2}}} = -0.1716 \quad S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

4. Waveguides [25]

FOR THE TE₀ MODE, E_x WOULD HAVE TO BE ZERO IN ORDER TO SATISFY BOUNDARY CONDITIONS AT y=0, d.

Consider an air-filled parallel-plate waveguide whose width, b , is much greater than the separation between plates, d .

- a. [5] Sketch a parallel plate waveguide and show the direction of the electric and magnetic field vectors of a wave propagating in the TEM mode, also known as the TM₀ mode. Explain why the TE₀ mode cannot exist.



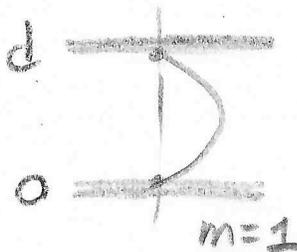
FOR THE TM₀ MODE E_y IS NORMAL TO THE PEC PLATE WHICH DOES NOT VIOLATE BOUNDARY CONDITIONS

- b. [5] Sketch the relationship between k , k_c and k_z in a parallel plate waveguide. Indicate θ_m .



- c. [5] What separation, d , is required for the $m = 1$ mode to have a group velocity $v_g = 0$ at a frequency of 1 GHz? What condition is said to occur when $v_g = 0$?

$f = 1 \text{ GHz}$
 $\lambda_0 = 30 \text{ cm}$



$[d = \frac{\lambda_0}{2} = 15 \text{ cm}]$
 $[\text{THE WAVEGUIDE IS CUT OFF.}]$

- d. If the separation between plates is $d = 30 \text{ cm}$,



- i) [5] Calculate the cutoff frequencies of the first few TE & TM modes. How many TE and TM modes will propagate at 2 GHz? List them.

$m=1$	$f_c = 0.5 \text{ GHz}$	$m=4$	$f_c = 2 \text{ GHz}$
2	$= 1.0 \text{ GHz}$	$= 5$	$= 2.5 \text{ GHz}$
3	$= 1.5 \text{ GHz}$		

- ii) [5] What is the highest frequency at which *only* the TEM mode will propagate if $d = 30 \text{ cm}$?

$[f_{c,1} = 0.5 \text{ GHz}]$

4 TM modes
+ 3 TE modes will propagate at 2 GHz
THE $m=4$ mode will BE CUT-OFF.

AT $f_{c,1}$ AND BELOW, ONLY THE TEM MODE WILL PROPAGATE.