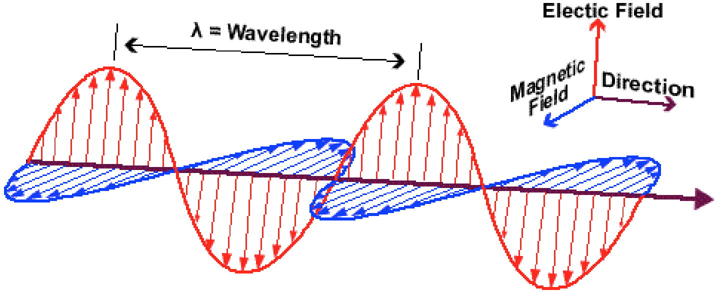


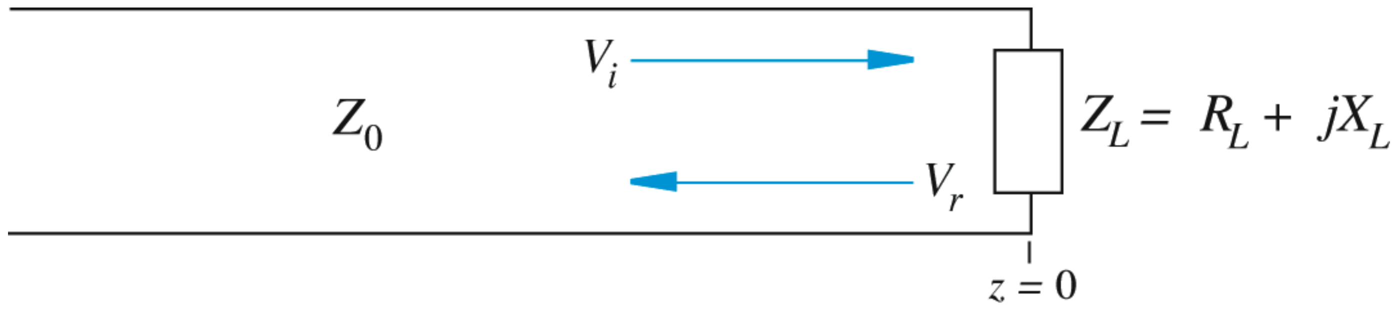
The Smith Chart

Waves and propagation; Maxwell's equations; applications including transmission lines; impedance matching and Smith charts; reflection and refraction; waveguides and antennas.. [4-0-0]

Prof. David G. Michelson

- During this lecture, the instructor will bring up many points and details not given on these slides. Accordingly, it is expected that the student will annotate these notes during the lecture.
- The lecture only introduces the subject matter. Students must review these notes after class and complete the reading assignments and problems if they are to master the material.

Discontinuities on Transmission Lines



1. What are the boundary conditions at $z = 0$?
2. Can we show that $\Gamma = \frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$?
3. Is wave impedance given by $Z_w = \frac{V_i + V_r}{I_i + I_r}$?

On the nature of characteristic impedance, Z_0

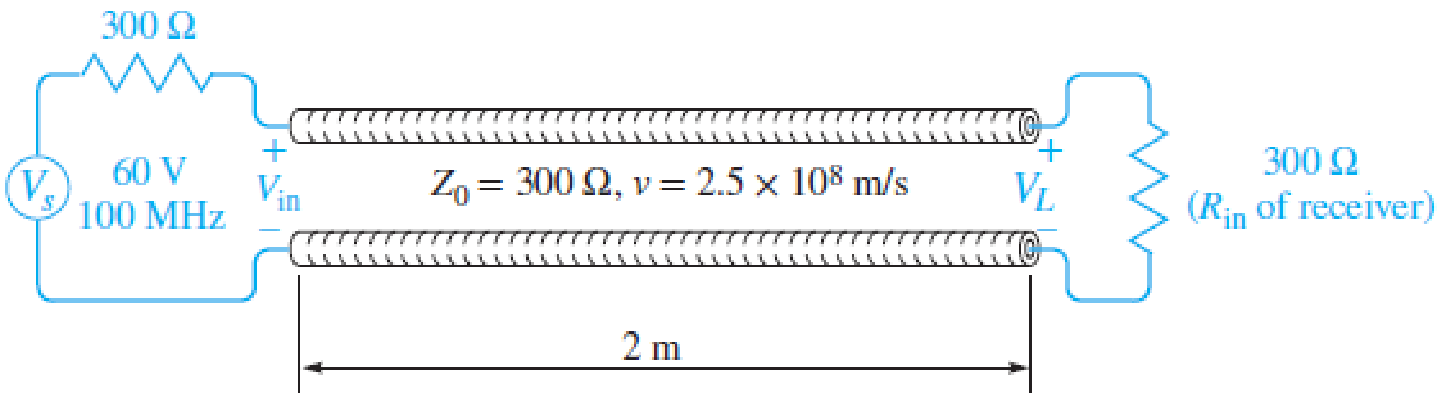


Figure 10.8 A transmission line that is matched at both ends produces no reflections and thus delivers maximum power to the load.

Characteristic impedance, Z_0,

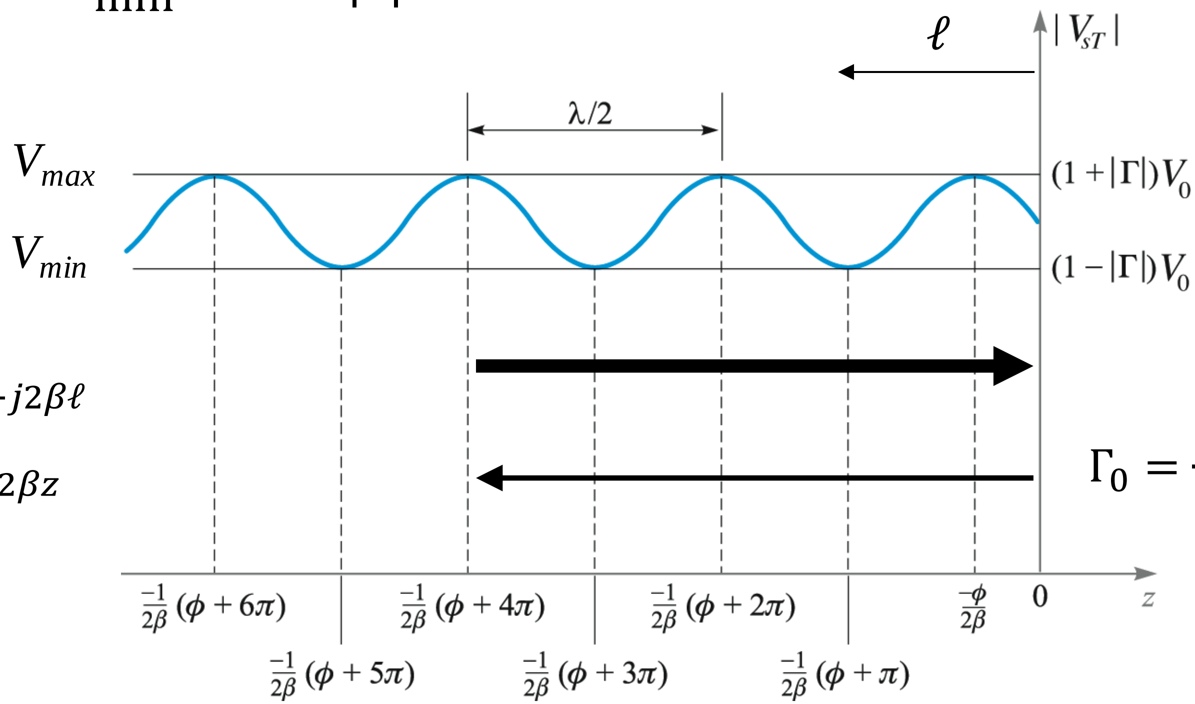
- is not a resistance nor does it represent dissipation of energy
- depends ONLY on the cross-sectional geometry of the transmission line

Voltage Standing Waves

$$S = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$V_i(z) = V_{i0}e^{-j\beta z} \qquad V_r(z) = V_{r0}e^{j\beta z}$$

$$\beta = \frac{2\pi}{\lambda}$$



$$\Gamma(\ell) = \Gamma_0 e^{-j2\beta\ell}$$

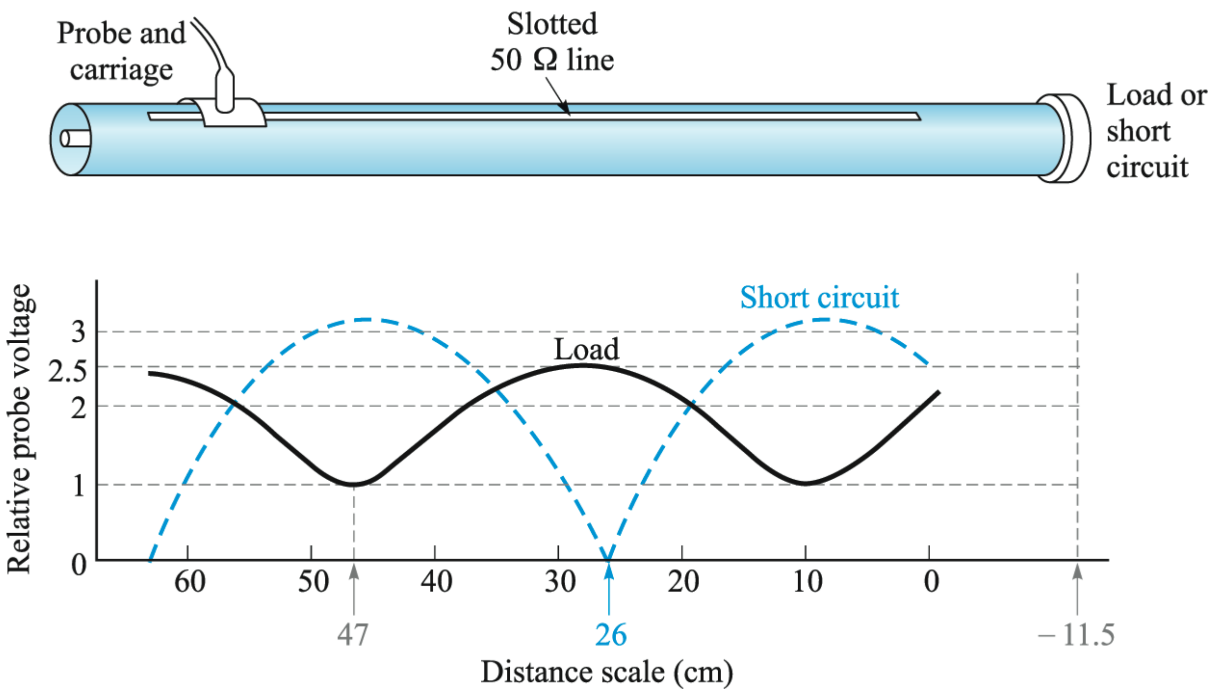
$$\Gamma(z) = \Gamma_0 e^{j2\beta z}$$

$$\Gamma_0 = \frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in}(z) = Z_0\,\frac{Z_L - jZ_0\tan(\beta z)}{Z_0 - jZ_L\tan(\beta z)}$$

$$Z_{in}(\ell) = Z_0\,\frac{Z_L + jZ_0\tan(\beta\ell)}{Z_0 + jZ_L\tan(\beta\ell)}$$

Using a Slotted Line to Observe a Standing Wave



Performance Objectives (in brief)

1. Given $\wp_0$, plot $\wp(\ )$ on a Smith chart.
2. Given Z_0 and Z, plot Z/Z_0 (normalized Z) on a Smith chart.
3. Given Z_0 and Z, find $\wp$ using a Smith chart.
4. Given Z_0, Z and d, find $\wp(d)$ and $\wp(-d)$ using a Smith chart.
5. Given Y_0, plot Y/Y_0 (normalized Y) on a Smith chart.
6. Given Z, plot Y on a Smith chart.
7. Analyze $\lambda/2$ and $\lambda/4$ transformers using a Smith chart.
8. Given a short- or open-circuit stub of length ℓ, find Y_{in} & Z_{in}.
9. Given Z_L, find d and ℓ required to achieve a single-stub match using a shunt stub.
10. Given Z_L, find d, ℓ_1 and ℓ_2 required to achieve a double-stub match using shunt stubs.

The Smith Chart

- The Smith chart was developed in the 1930's by Phillip Smith, a Bell Labs engineer.
- The Smith chart is simply a mapping from the Z (or impedance) plane onto the $\wp$ (or reflection coefficient) plane.
- Any given value of (normalized) load impedance on the chart is at the location that gives the corresponding complex reflection coefficient, i.e.,

$$\Gamma_0 = \frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- This conformal mapping is immediately recognized as a *bilinear transformation*.

The Smith Chart

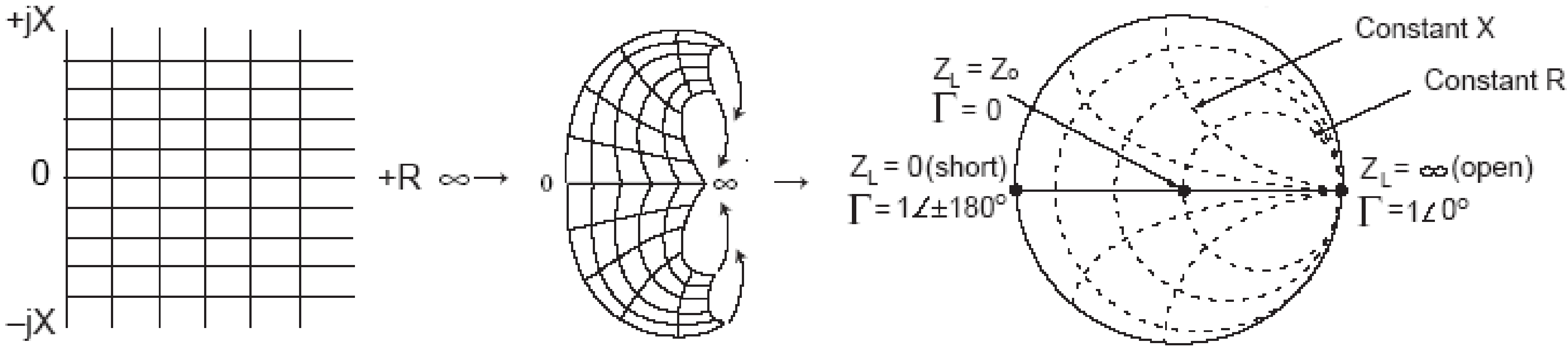
- The reverse mapping is given by

$$z_L = \frac{Z_L}{Z_0} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

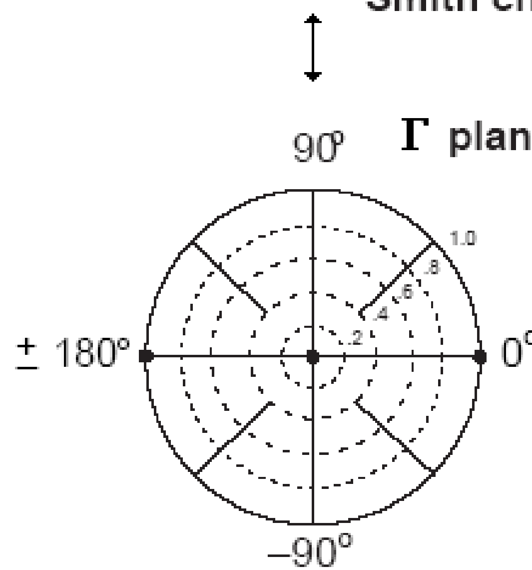
Prove this!

- Because the mapping is a bilinear transformation, lines of constant resistance and reactance in the z-plane are transformed into circles of different radii in the $\wp$ -plane.
- Because the mapping is conformal, resistance circles and reactance circles are everywhere orthogonal to each other.
- Because $\wp(z)$ describes a circle with constant radius, the Smith chart permits one to easily visualize the manner in which Z is transformed as one moves either forward or backward along a transmission line.

Mapping the *Z*-plane onto the Γ–plane



Rectilinear impedance plane **Bilinear transformation** **Smith chart**



$$\frac{Z_L}{Z_0} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

$$\Gamma_0 = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

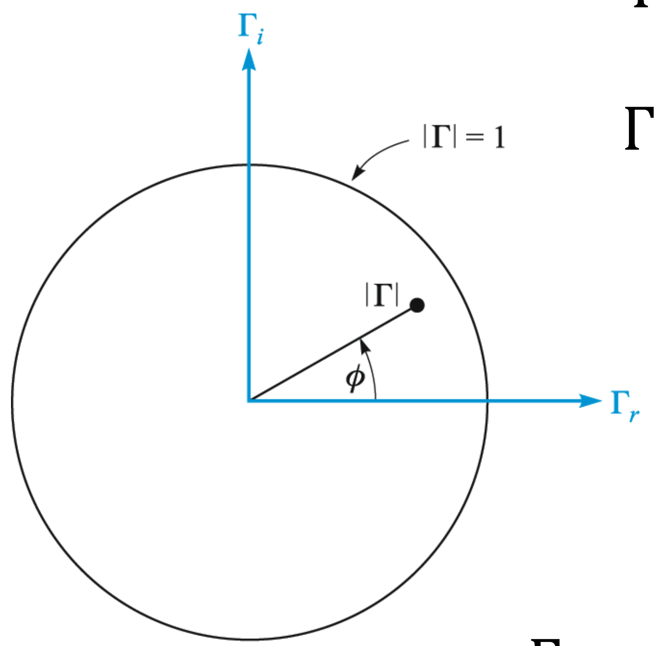
Properties of the Reflection Coefficient

$$\Gamma(z) = \Gamma_0 e^{j2\beta z}$$

$$\Gamma(\ell) = \Gamma_0 e^{-j2\beta\ell}$$

- The reflection coefficient can be expressed as a complex exponential and defined by its amplitude and phase.

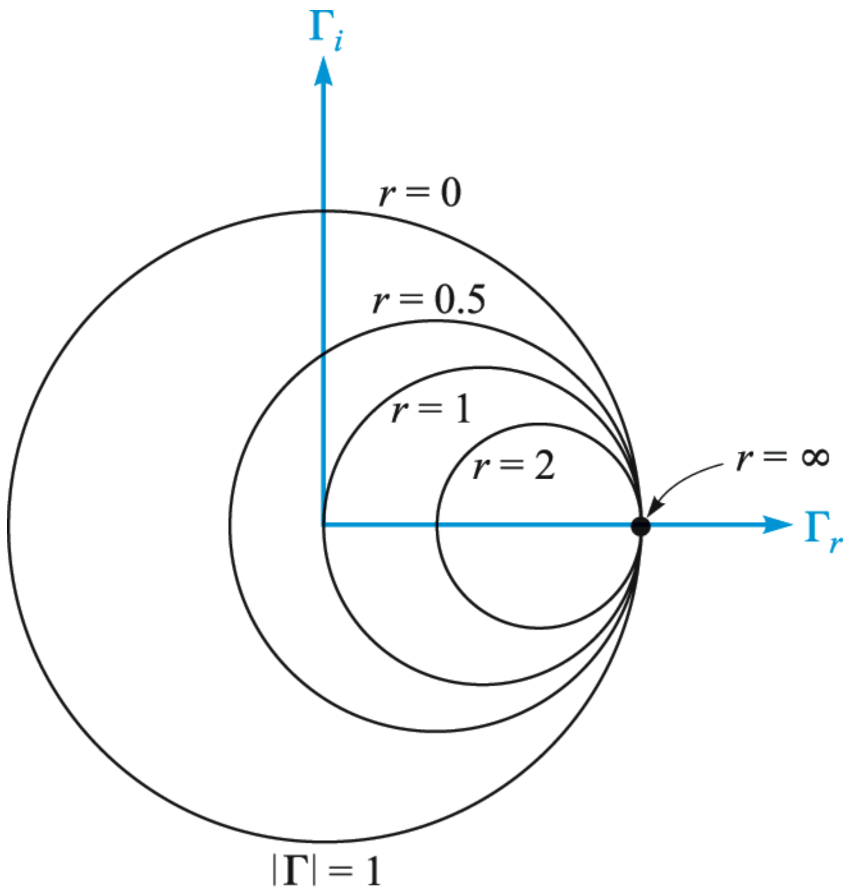
- Why must | Γ | be constant with distance if the line is lossless?



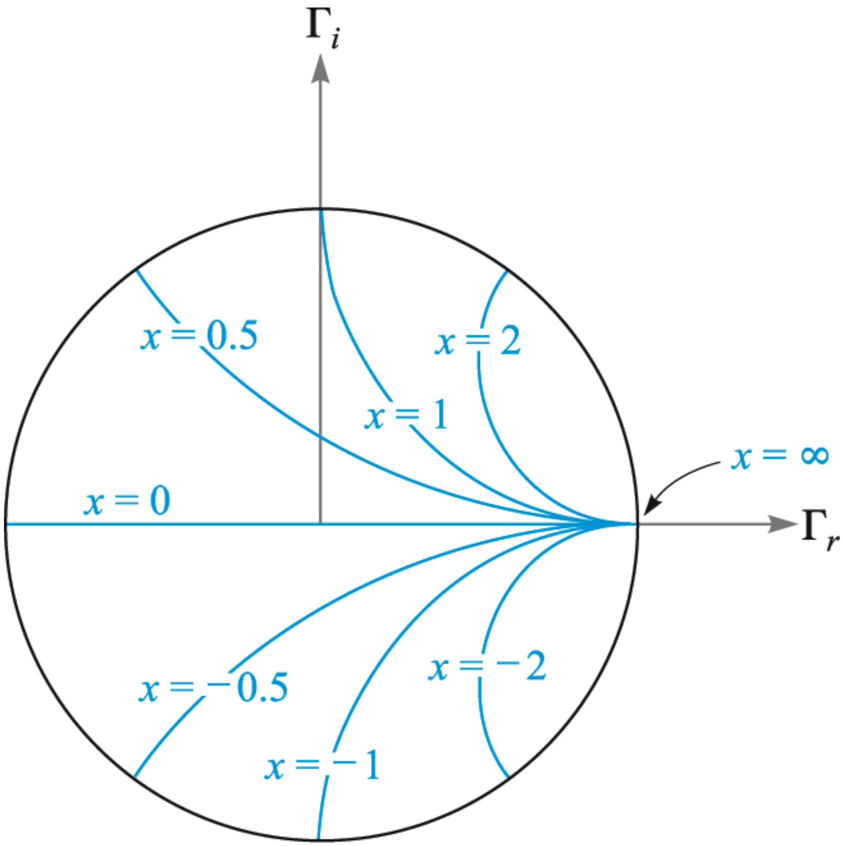
$$\Gamma_0 = \frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

1. Given $\wp_0$, plot $\wp(\;\;)$ on a Smith chart

Lines of Constant Resistance

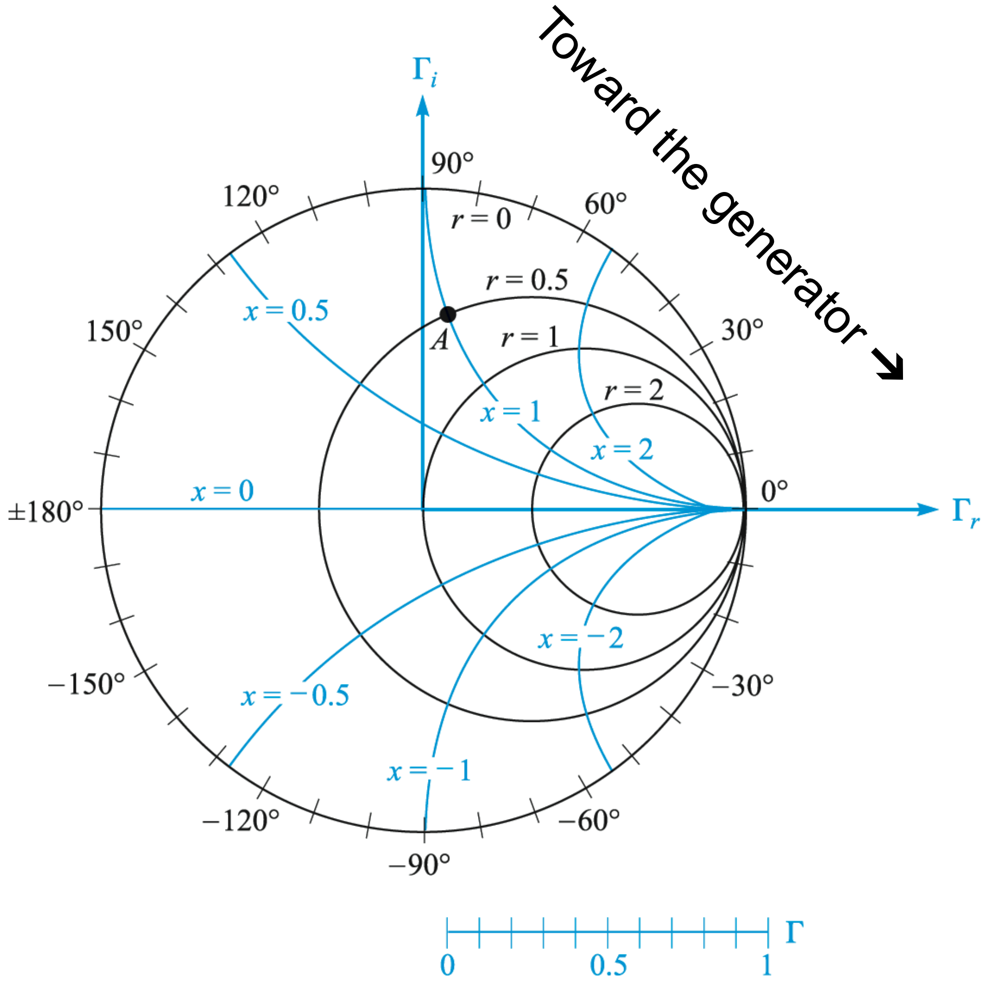


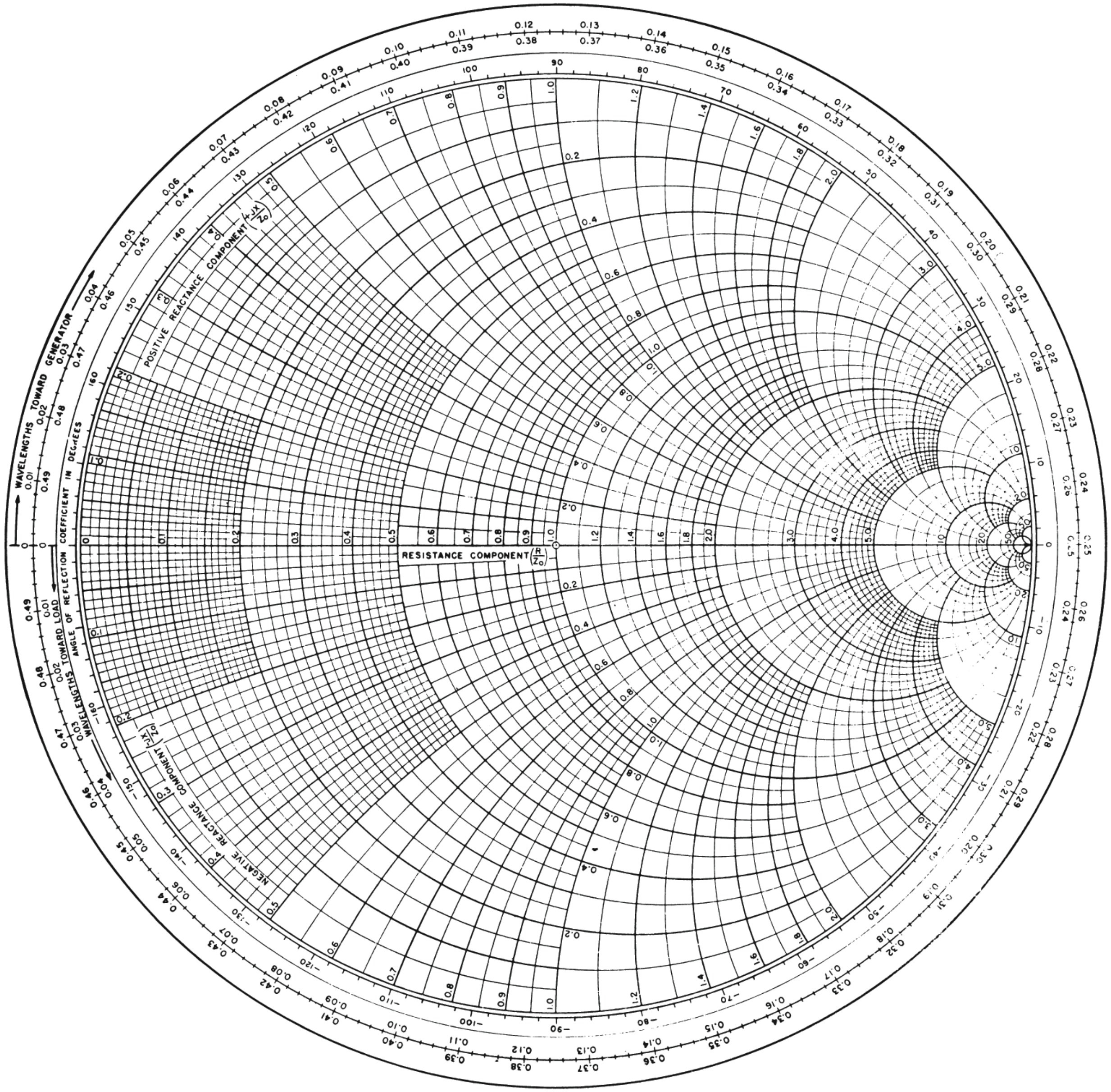
Lines of Constant Reactance



Plotting Impedance A on a Smith Chart

2. Given Z_0 and Z, plot Z/Z_0 (normalized Z) on a Smith chart.

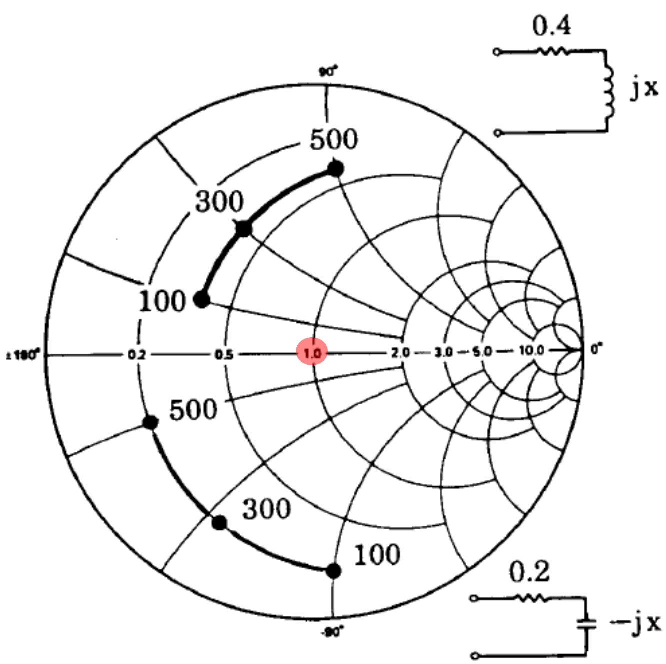


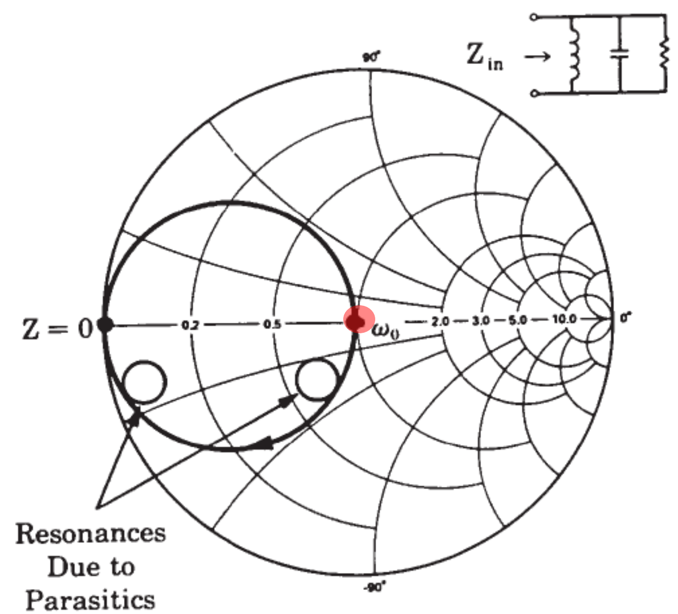


The Smith Chart as commonly depicted.

Fun Facts

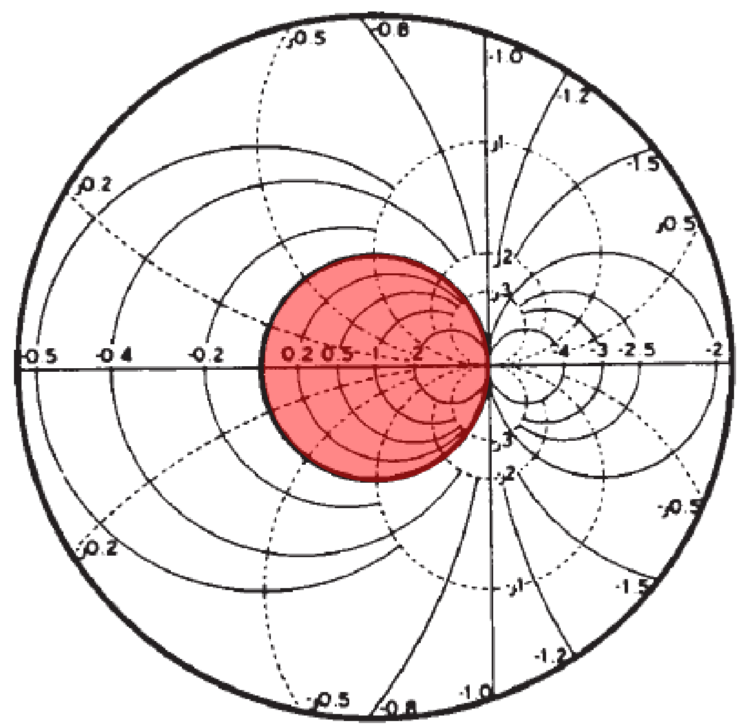
- As a consequence of Foster's reactance theorem, Z tends to traverse the reflection plane in a clockwise direction as frequency increases.
- Lossy resonant circuits describe loops that encircle prime ($Z = 1$), marked here with a red dot, (*overcoupled*), touch prime (*critically coupled*), or fall short of prime (*undercoupled*)





Fun Facts

- The conventional Smith chart (shown in red) lies within the circle defined by $|\wp| = 1$ or $R = 0$.
- The region beyond $|\wp| = 1$ corresponds to cases where resistance is negative.

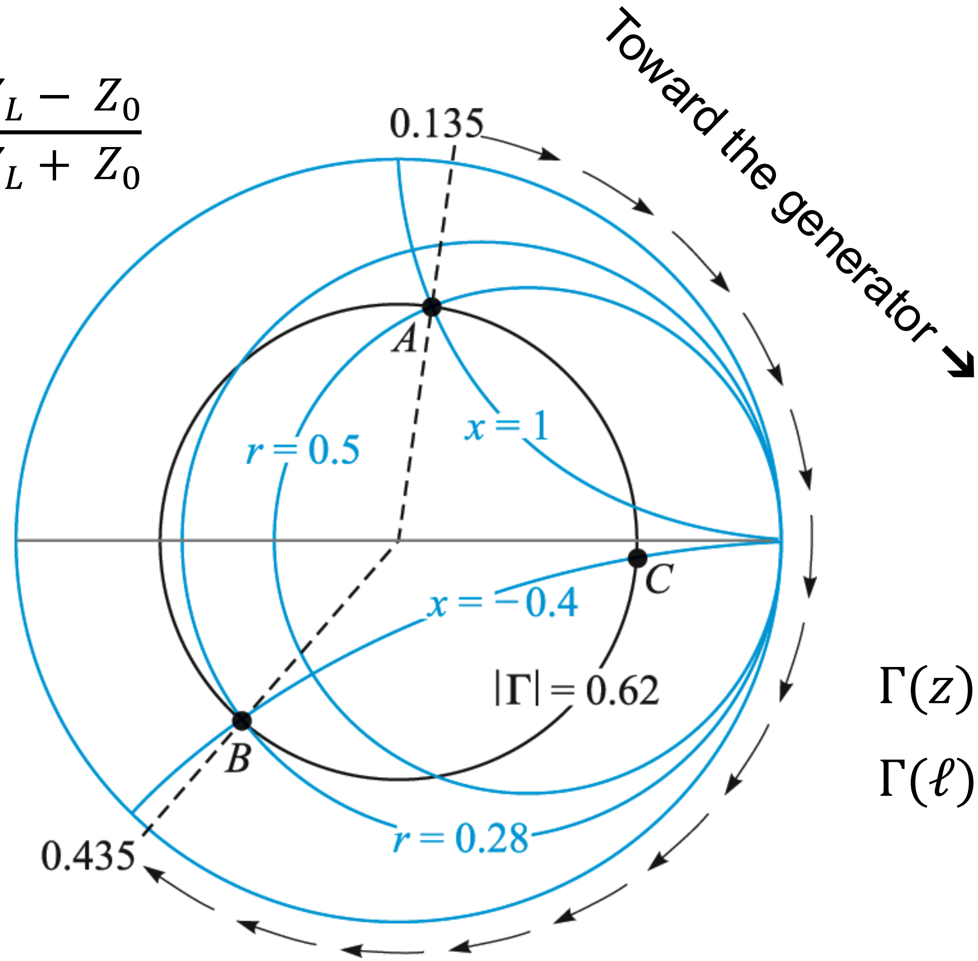


Using the Smith Chart to show Wave Impedance as a function of z

$$\Gamma_0 = \frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

3. Given Z_0 and Z, find $\wp$ using a Smith chart.

4. Given Z_0, Z and d, find $\wp(d)$ and $\wp(-d)$ using a Smith chart.



$$\Gamma(z) = \Gamma_0 e^{j2\beta z}$$

$$\Gamma(\ell) = \Gamma_0 e^{-j2\beta \ell}$$

Impedance and Admittance

5. Given Y_0, plot Y/Y_0 (normalized Y) on a Smith chart.
6. Given Z, plot Y on a Smith chart.

$$\Gamma = \frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1/Y_L - 1/Y_0}{1/Y_L + 1/Y_0} = \frac{1 - Y_L/Y_0}{1 + Y_L/Y_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L}$$

Conclusion: If one assigns Y to the grid rather than Z, then the Γ grid is rotated by 180 degrees.

Question: Does this suggest a simple way to calculate Y given Z or vice versa?

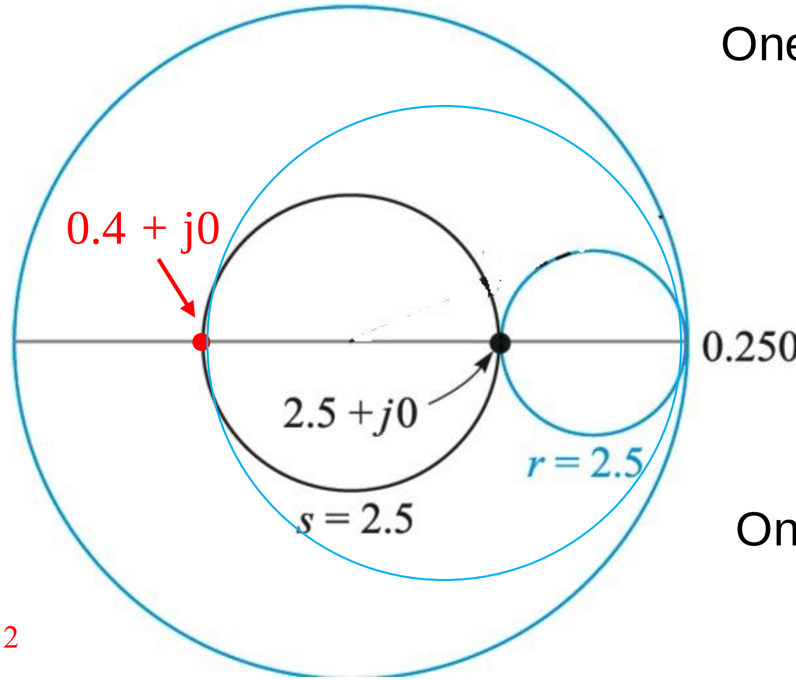
We will make use of this when we design single-stub matching networks!

Performance Objectives

1. Given Γ_0, plot $\Gamma(\ell)$ on a Smith chart. ✔
2. Given Z_0 and Z, plot Z/Z_0 (normalized Z) on a Smith chart ✔
3. Given Z_0 and Z, find □ using a Smith chart ✔
4. Given Z_0, Z and d, find □(d) and □($-d$) using a Smith chart ✔
5. Given Y_0, plot Y/Y_0 (normalized Y) on a Smith chart ✔
6. Given Z, plot Y on a Smith chart ✔
7. Analyze $\lambda/2$ and $\lambda/4$ transformers using a Smith chart.
8. Given a short- or open-circuit stub of length ℓ, find Y_{in} & Z_{in}.
9. Given Z_L, find d and ℓ required to achieve a single-stub match using a shunt stub.
10. Given Z_L, find d, ℓ_1 and ℓ_2 required to achieve a double-stub match using shunt stubs.

λ/2 and λ/4 transformers or sections

7. Analyze λ/2 and λ/4 transformers (or sections) using a Smith chart.



One half revolution corresponds to λ/4 of travel.

$Z_{in} = Z_0^2 / Z_L$

$Z_{in} Z_L = Z_0^2$

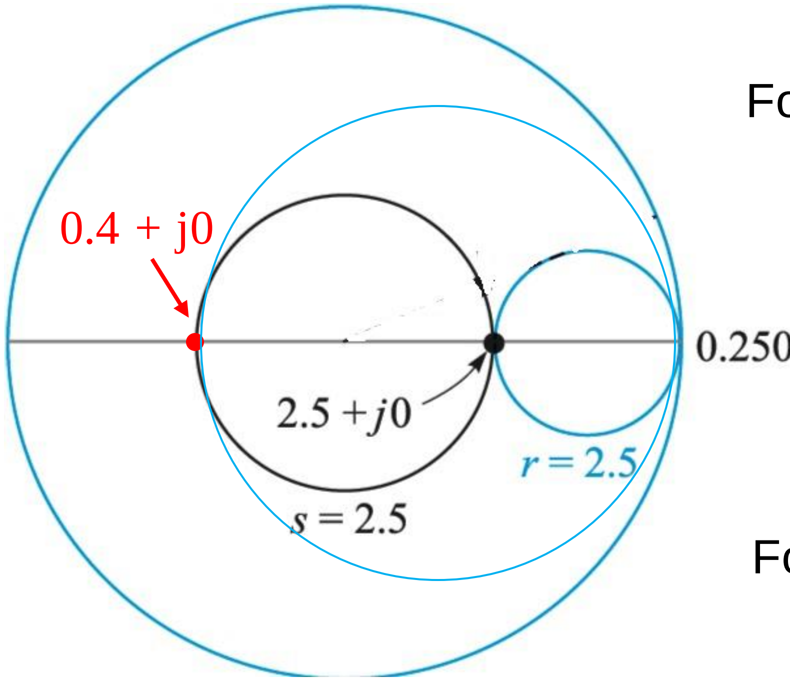
$0.4 \times 2.5 = 1^2$

One full revolution corresponds to λ/2 of travel.

$Z_{in} = Z_L$

λ/2 and λ/4 transformers

7. Analyze λ/2 and λ/4 transformers using a Smith chart.



Problem:
For a λ/4 transformer and given Z_{in} and Z_L, find Z_0

Ans. $Z_0^2 = Z_{in} Z_L$

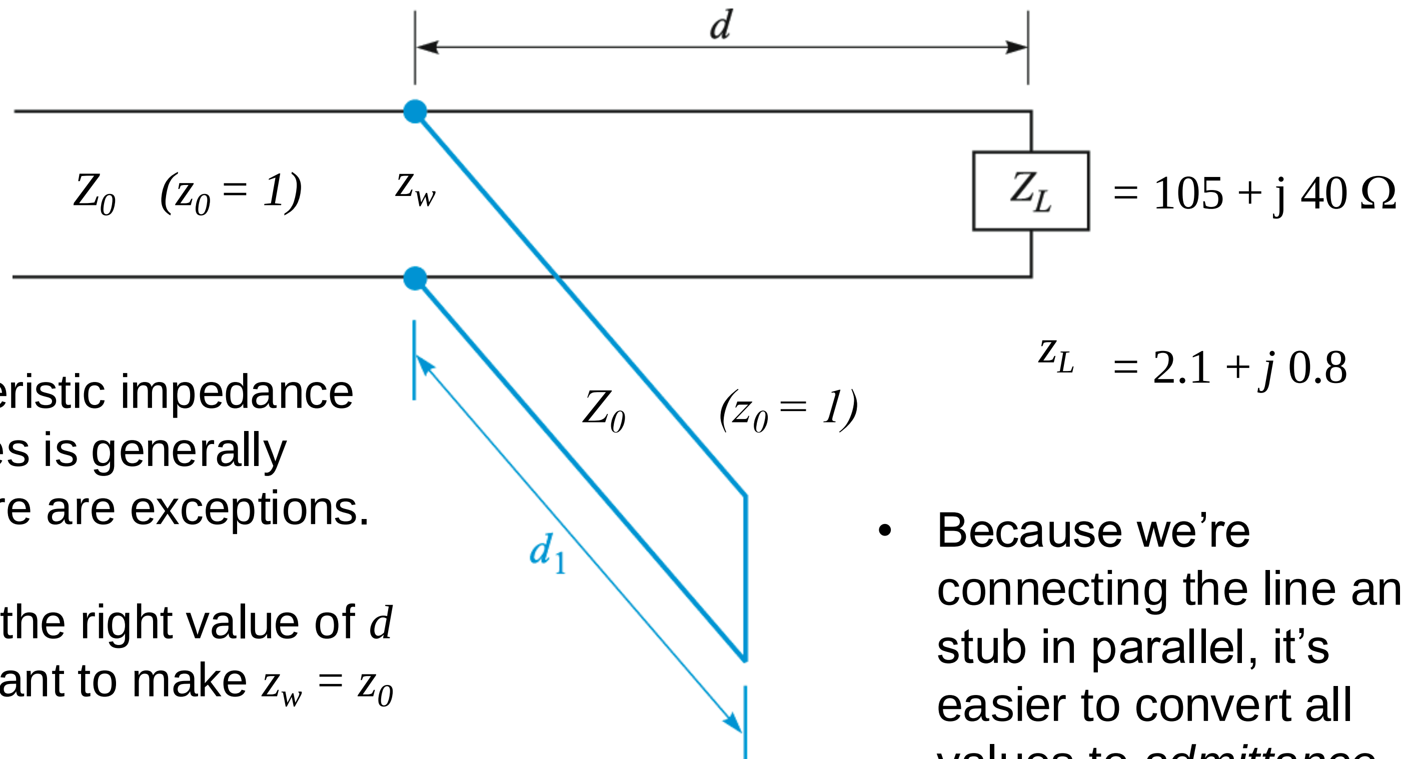
Problem:
For a λ/4 transformer and given Z_L and Z_O, find Z_{in}

Ans. $Z_{in} = Z_0^2 / Z_L$

Single-Shunt-Stub Match

8. Given a short- or open-circuit stub of length d_1, find Y_{in} & Z_{in}.

9. Given Z_L, find d and d_1 required to achieve a single-stub match using a shunt stub.



- The characteristic impedance (Z_0) of all lines is generally 50 Ω but there are exceptions.
- By selecting the right value of d and d_1, we want to make $z_w = z_0 = 1$

- Because we're connecting the line and stub in parallel, it's easier to convert all values to *admittance*.

Single-Stub Match

8. Given a short- or open-circuit stub of length d_1, find Y_{in} & Z_{in}.

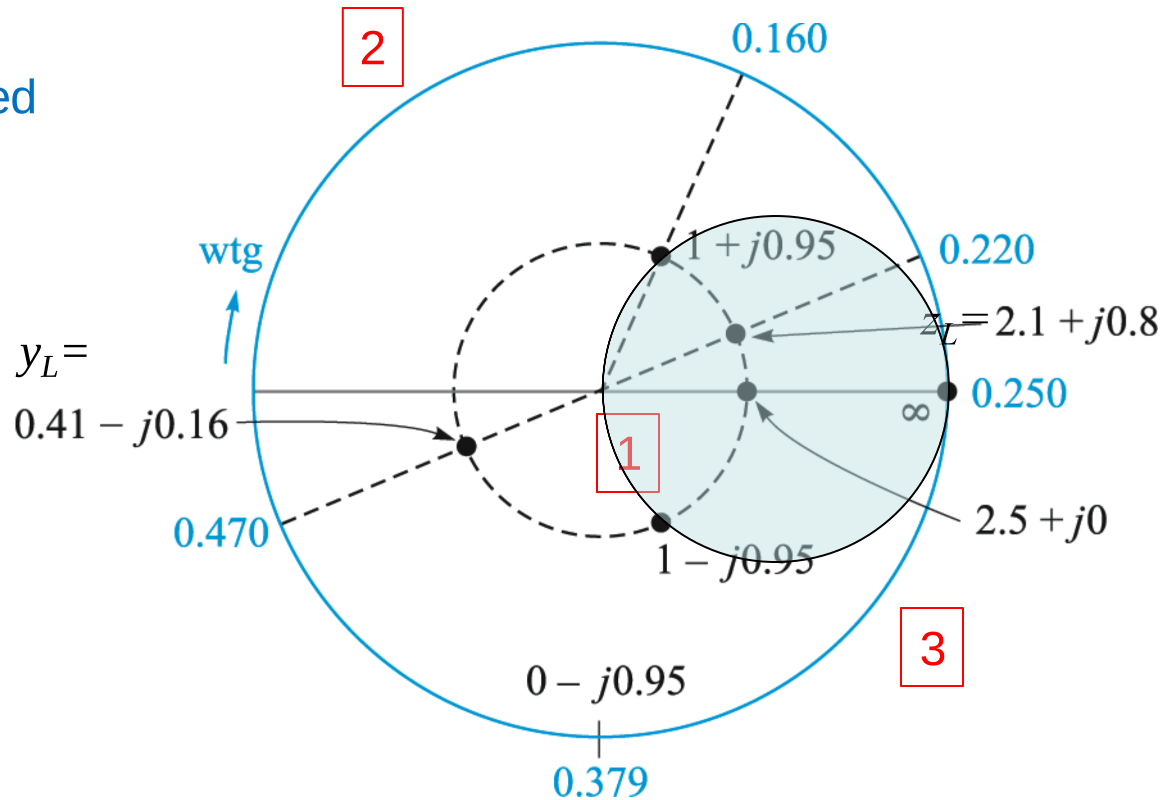
9. Given Z_L, find d and d_1 required to achieve a single-stub match using a shunt stub.

This example is best followed using a real Smith chart!

1. Convert from z to y

Once we convert z_L to y_L, we'll interpret the Smith Chart as an admittance chart rather than an impedance chart.

That is, instead of $z = r + jx$, we're plotting $y = g + jb$



Single-Stub Match

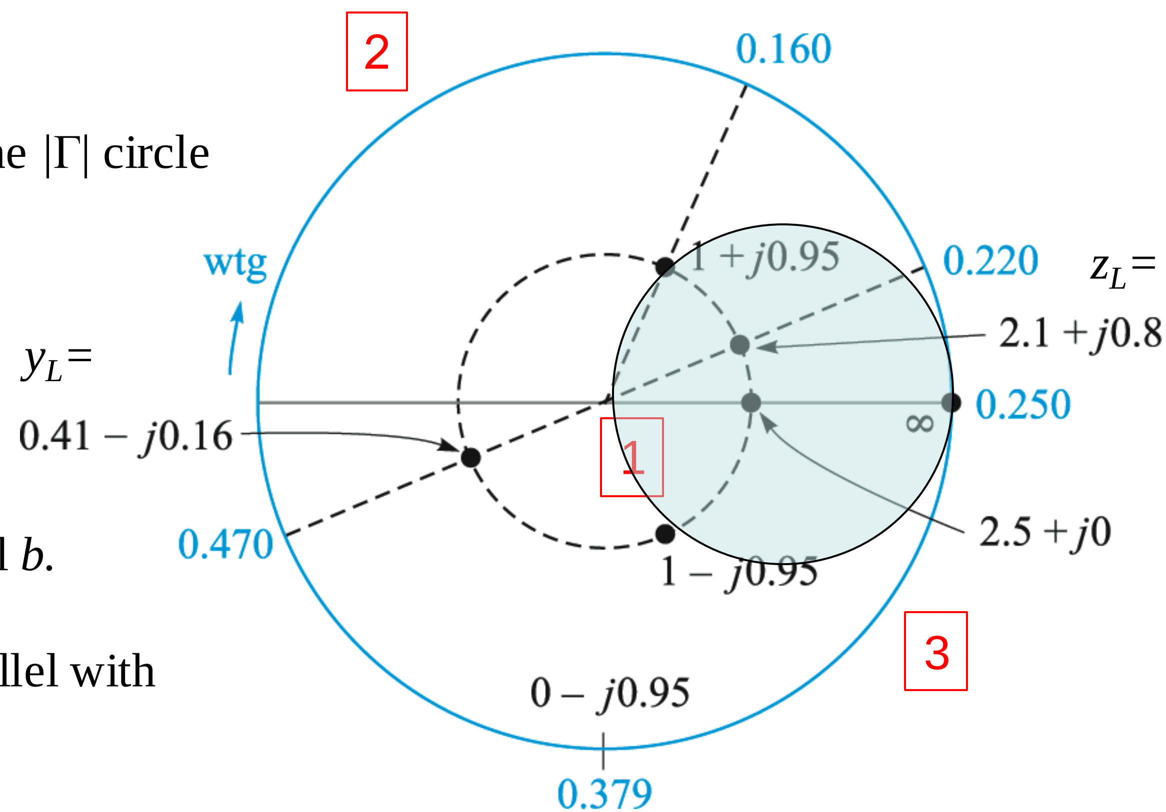
This example is best followed using a real Smith chart!

1. Convert from z to y

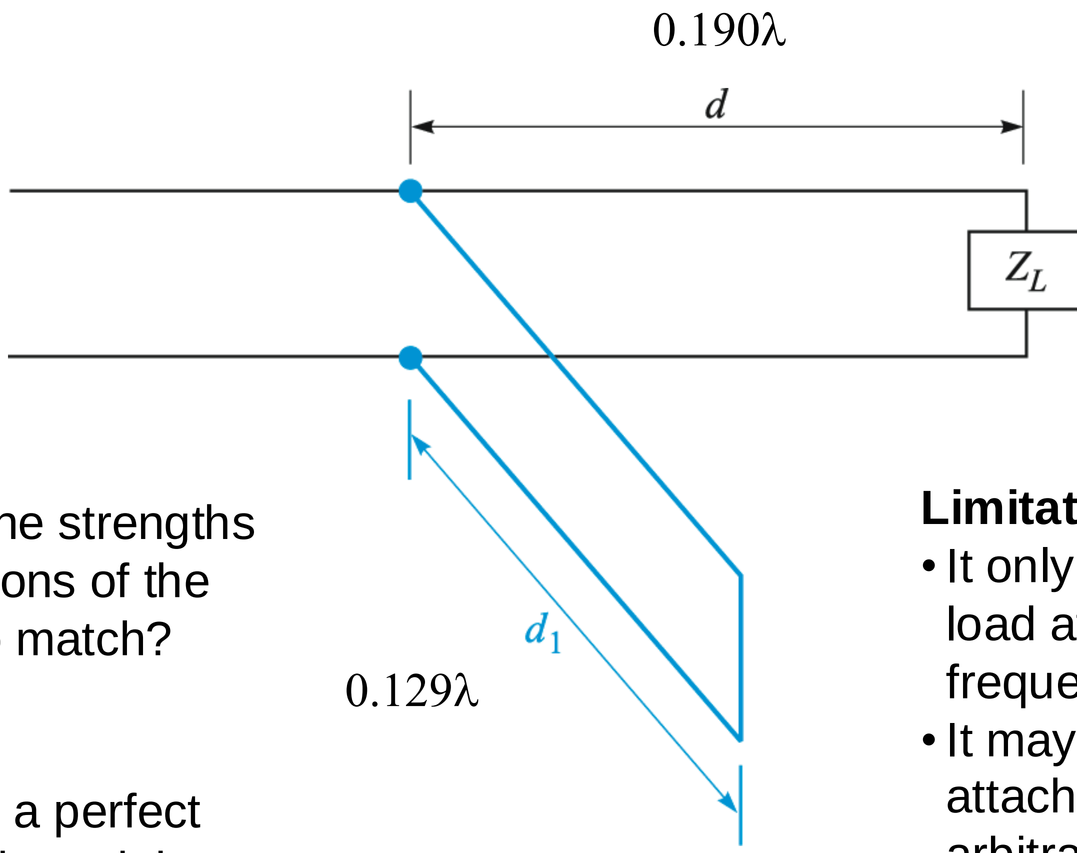
2. To find d, go back along the $|\Gamma|$ circle toward the generator (*wtg)* until $g = 1 +/- jb$. *There are two solutions!*

3. To find d_1, find the length of the stub that will give $y = 0 = -/+ jb$, *i.e.*, cancel b.

4. If the stub is added in parallel with the line at d, y_{in} will = 1.



Single-Stub Match



What are the strengths and limitations of the single-stub match?

Strengths

- It delivers a perfect match with a minimum of extra hardware.

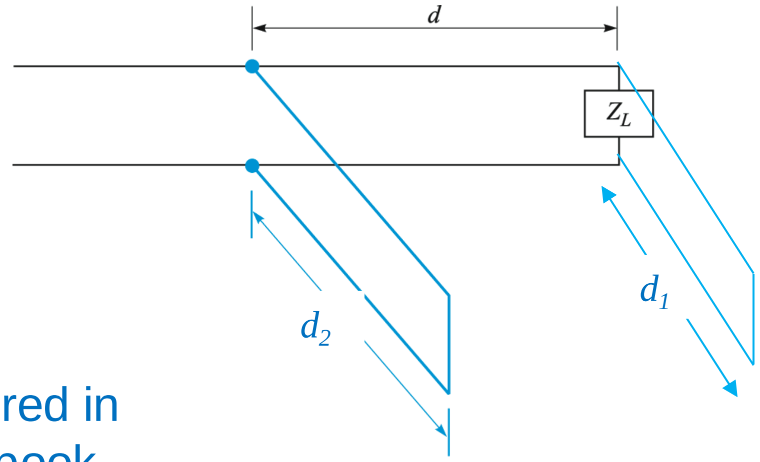
Limitations

- It only matches the load at a single frequency.
- It may be difficult to attach the stub at an arbitrary point in the line.

Double-Stub Match

- For practical reasons, it is convenient to match an arbitrary load using a pair of adjustable stubs attached at predefined locations rather than a single adjustable stub attached at a location TBD..

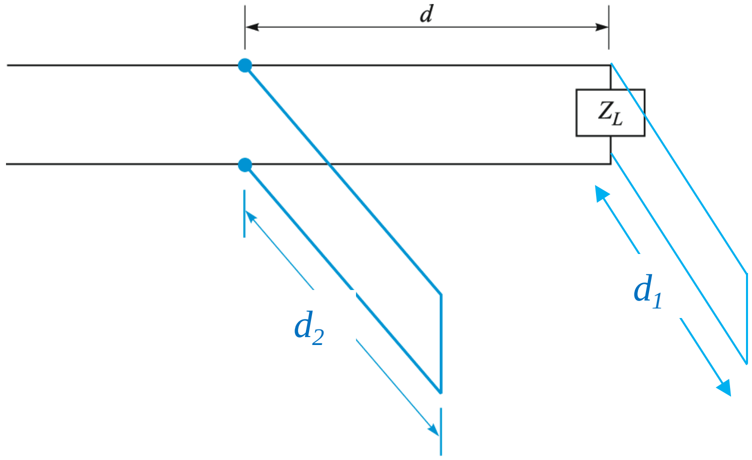
10. Given Z_L, find d, ℓ_1 and ℓ_2 required to achieve a double-stub match using shunt stubs.



This is not covered in the course textbook.

Double-Stub Match

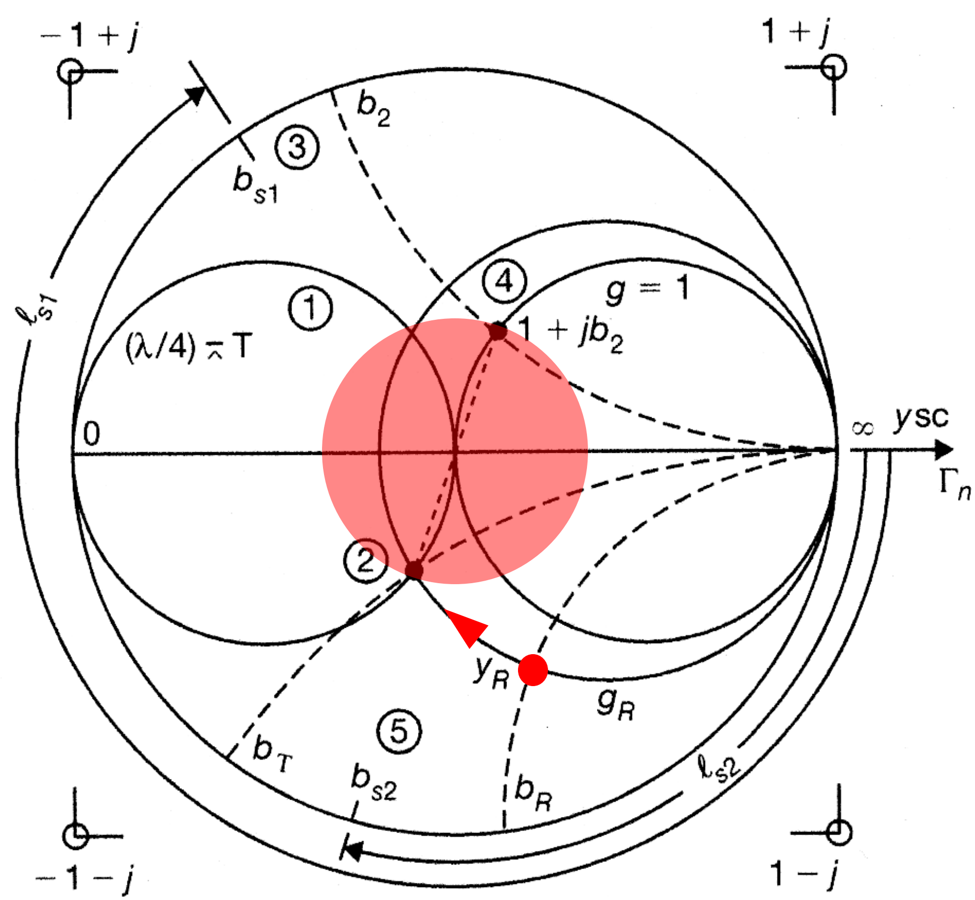
- The stub at location 1 is adjusted until the wave impedance at the second stub's location is given by $1 + jB$.
- This is accomplished by adding or subtracting susceptance from the impedance at location 1 so that the G circle gets bigger or smaller.
- The stub at location 2 is adjusted so that its susceptance cancels out the jB.
- The separation between stubs, d, is typically $\lambda/4$ or $3\lambda/8$.
- The first stub can be located anywhere along the line; one is simply matching to the wave impedance at that point rather than the load impedance itself.
- For convenience, the first stub can be located some distance away from the load.



Double-Stub Match with λ/4 Separation

The offered load is $y_R = g_R - jb_R$

1. Plot the λ/4 tuner circle.
2. Find the value of b_T that will cause $g_R + b_T$ to intersect the tuner circle.
3. Stub 1 at $z = 0$ changes b_R to b_T.
4. Travel λ/4 toward the generator and find $1 + jb_2$.
5. Cancel the suspectance b_2 by adjusting stub 2 to yield $y = 1 + j0$



This example is best followed using a real Smith chart!

Double-Stub Match

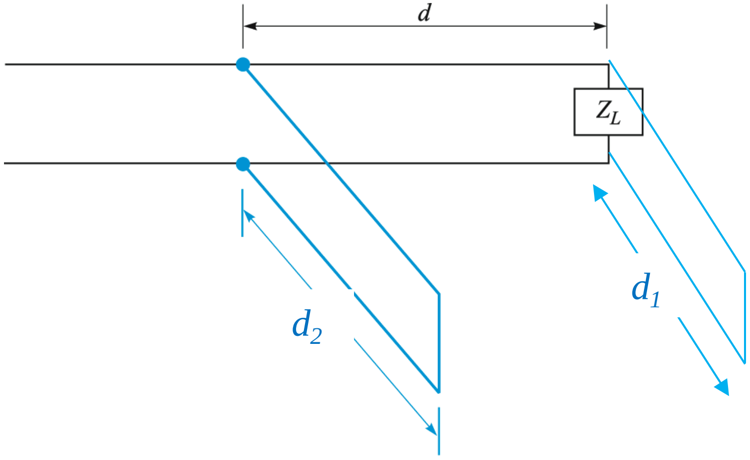
What are the strengths and limitations of the double-stub match?

Strengths

- It delivers a perfect match with a minimum of extra hardware.
- The connection points for the two stubs are predefined.

Limitations

- It only matches the load at a single frequency.
- A solution may not exist!
- *e.g.*, for a $\lambda/4$ tuner with $g_R > 1$, the conductance circle does not intersect the tuner circle.



A triple-stub tuner combines the advantage of predefined connection points with the guarantee of a solution, and is more commonly used.