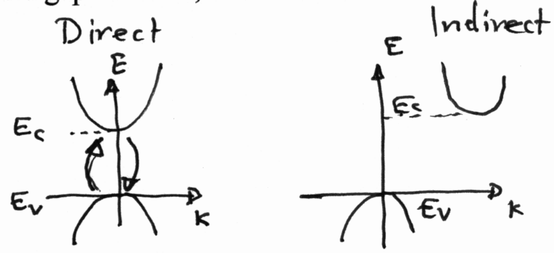
1. (/13) For GaAs,

(a) (/8) what is the type of the unit cell? How many Ga and As atoms are there in each unit cell? Explain the difference between a direct and indirect band gap semiconductor. Does GaAs have a direct or indirect band gap? Considering band gap of GaAs, what electronic applications is GaAs used for?



Unit cell is diamond

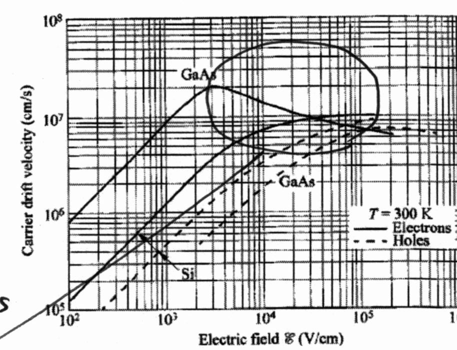
4 Ga, 4 As atoms

Direct bandgap is when in E-k diagram the bottom of conduction band (CB) is aligned with the top of valence band (VB). Thus, direct transition of electron from VB ⇄ CB is highly probable. GaAs is direct bandgap.

Indirect bandgap is when minimum of CB and maximum of VB are not aligned. Indirect transition is more probable.

Since GaAs has direct bandgap, it is very good for optoelectronic applications.

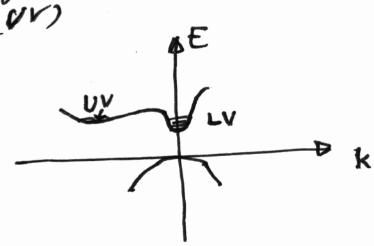
(b) (/5) Write the relationship between carrier drift velocity and electric field and explain why the electron velocity saturates at high electric field (as shown in figure). Briefly explain negative differential conductivity and why it occurs in GaAs.



$v_d = \mu_d \mathcal{E}$

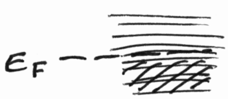
At high electric field, the v_d saturates due to ~~increased~~ the fact that time between collisions is not enough for electrons to gain speed.

For GaAs, the ~~conductivity~~ mobility drops with increasing electric field. This is because at high field electrons populate also in the upper valley (UV) that has much lower curvature and as a result heavy electrons.

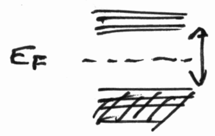


2. (/10) **(a) (/5)** Using a schematic, briefly explain the difference between density of states, bandgaps, and Fermi energy levels for a metal, a semiconductor, and a dielectric.

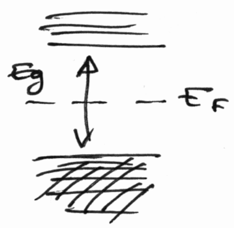
Metal



no bandgap
E_F in middle of band.

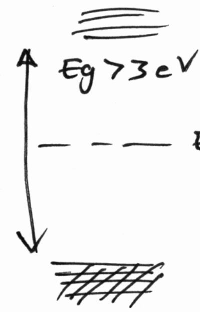


Semiconductor



$0.3eV < E_g < 3eV$

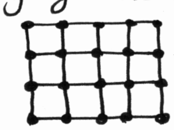
Dielectric



Fermi energy is in the middle of the band gap.

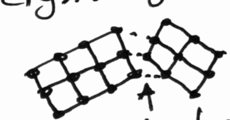
(b) (/5) Using a schematic explain the difference between single crystalline, polycrystalline and amorphous materials. In which material you expect to have a higher electron mobility? Explain why.

Crystalline
highly ordered



long range order

polycrystalline
crystal grains

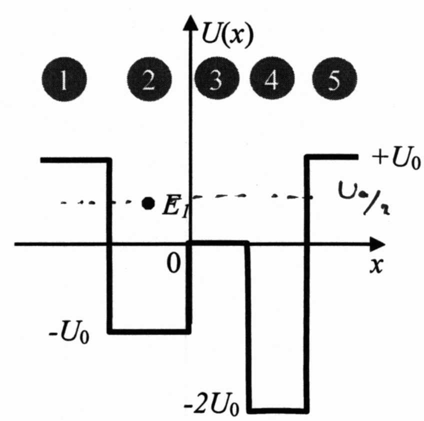


Amorphous

No long range order
Only ~~and~~ some order with neighbors

Due to long range order, crystalline materials have low scattering and electron mobility.

3. (/10) In the given potential energy $U(x)$, we have an electron with the total energy of $E_1 = +U_0/2$.



(a) (/2) In which region does the electron has the smallest wavelength? Explain why.

$\lambda \sim \frac{h}{p}$

highest kinetic energy in region 4

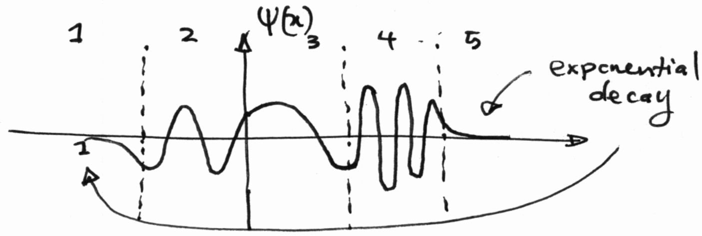
$\Rightarrow$ max $p \Rightarrow$ min λ

(b) (/5) Which regions have sinusoidal wave function solutions? Find the wavenumber for wave functions in these regions.

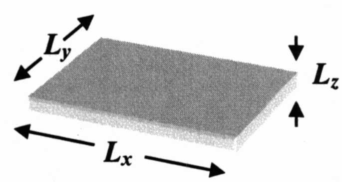
Sinusodial solution for when $KE > 0$ or $E > U(x)$

Regions : 2, 3, 4

(c) (/3) Draw an approximate solution for the wave function of this electron in all regions.



4. (**/10**) A sheet of graphene can be modeled as a three dimensional (3-D) potential well ($U(x, y, z) = 0$ inside the box, $U(x, y, z) = \infty$ outside) with sides of $L_x = L_y = 100$ nm and $L_z = 1$ nm.



(a) (/7) Write the Schrodinger equation for an electron inside the well and using separation of variables, write the equation for energy of electron, in terms of quantum numbers as

$$E_{n_x,n_y,n_z} = \frac{h^2}{8\pi^2 m_0}\left(k_x^{\,2} + k_y^{\,2} + k_z^{\,2}\right), \text{ where } k_x = \frac{n_x\pi}{L_x},\ k_y = \frac{n_y\pi}{L_y}, \text{ and } k_z = \frac{n_z\pi}{L_z}.$$

Calculate the lowest six lowest energy levels in this material. Are there any degenerate states?

$U(x,y,z) = 0$ inside

$$-\frac{\hbar^2}{2m_0}\nabla^2\psi(x,y,z) = E\psi(x,y,z) \qquad \psi(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$-\frac{\hbar^2}{2m_0}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) XYZ = E\,XYZ \rightarrow$$

$$-\frac{\hbar^2}{2m_0}\left(X''YZ + XY''Z + XYZ''\right) = E\,XYZ \rightarrow$$

$$E = -\frac{\hbar^2}{2m_0}\left(\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z}\right) \longrightarrow \begin{cases} X''/X = -k_x^2 \\ Y''/Y = -k_y^2 \\ Z''/Z = -k_z^2 \\ E = \frac{\hbar^2}{2m_0}\left(k_x^2 + k_y^2 + k_z^2\right) \end{cases}$$

$$X''(x) + k_x^2 X(x) = 0 \rightarrow X(x) = Ae^{jk_x x} + Be^{-jk_x x}$$

$$X(0) = 0 \rightarrow X(x) = A'\sin k_x x, \quad X(L_x) = 0 \rightarrow \sin(k_x L_x) = 0 \rightarrow k_x = \frac{n_x\pi}{L_x}$$

Similarly, $k_y = \frac{n_y\pi}{L_y}$, $k_z = \frac{n_z\pi}{L_z}$

$$E = \frac{h^2}{8m_0 L_z^2}\left(n_x^2 + n_y^2 + 10^4 n_z^2\right) \rightarrow E_{111}$$

n_x	n_y	n_z
1	1	1
1	2	1
2	1	1
2	2	1
1	3	1
3	1	1

$E_{111} = 0.37768$ eV

$E_{121} = E_{211} = 0.37779$ eV

$E_{221} = 0.37791$ eV

$E_{131} = E_{311} = 0.37798$ eV

degenerate

(b) (/3) What is the wavelengths of the photon emitted when the electron makes the transition $\psi_{1,1,2} \rightarrow \psi_{1,1,1}$. Is this photon visible?

$$\Delta E = \frac{h^2\, 10^4}{8 m_o L_x^2}(4-1) = 1.13 \text{ eV} = h\nu = h\frac{c}{\lambda}$$

$\lambda = 1097$ nm IR not visible

5. (/7) A semiconductor has an intrinsic carrier density of $n_i = 10^{10}$ cm^{-3} and conduction and valence band densities of $N_C = 4N_V = 10^{19}$ cm^{-3} at room temperature T=300 K.

(a) (/3) What is the band gap E_g of this semiconductor at room temperature?

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} \longrightarrow -E_g/kT = 2\ln\frac{n_i}{\sqrt{N_c N_v}} \longrightarrow$$

$$E_g = 2kT \ln\frac{\sqrt{N_c N_v}}{n_i} = 1.1 \text{ eV}$$

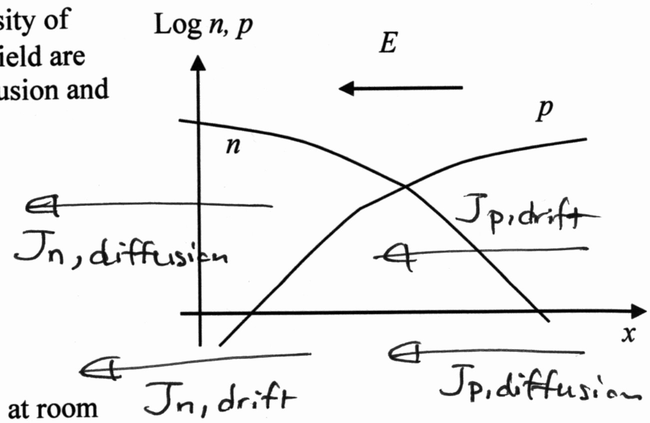
(b) (/4) Find the intrinsic Fermi energy with respect to the middle of the gap at room temperature.

$$\left.\begin{array}{l} n_i = N_c\, e^{\frac{E_{Fi}-E_c}{kT}} \longrightarrow \ln\frac{n_i}{N_c} = \frac{E_{Fi}-E_c}{kT} \\ n_i = p_i = N_v\, e^{\frac{E_V - E_{Fi}}{kT}} \longrightarrow \ln\frac{n_i}{N_v} = \frac{E_V - E_{Fi}}{kT} \end{array}\right\} \longrightarrow$$

$$\frac{2E_{Fi}}{kT} - \frac{E_c+E_v}{kT} = \ln\frac{N_v}{N_c} \longrightarrow E_{Fi} - E_i = \frac{kT}{2}\ln\frac{N_v}{N_c}$$

$$E_{Fi} - E_i = \boxed{-0.017 \text{ eV}}$$

6. (/4) In a piece of semiconductor, the density of electrons and holes and the direction of electric field are shown in the figure. Plot the direction of the diffusion and drift currents for electrons and holes.



7. (/12) For a semiconductor $n_i = 10^{11}$ cm^{-3} at room temperature and $\mu_n = 2\mu_p = 2000$ cm^2/Vs.

(a) (/4) Find the density of majority and minority carriers and the conductivity of a wafer of this semiconductor if it is doped with a donor density of $N_D = 2\times10^{15}$ cm^{-3}.

majority e^- : $n_0 = N_D = 2\times10^{15}$ cm^{-3}

minority h^+ : $p_0 = \frac{n_i^2}{N_D} = 5\times10^6$ cm^{-3}

$\sigma \simeq q\mu_n n_0 = 0.64$ S/cm

(b) (/4) Find the density of acceptor dopants that is required to be added to the sample in part (a) (with the donor atoms) to increase its resistivity by 10 times, while keeping the final semiconductor n-type.

$\sigma' = \frac{\sigma}{10} \rightarrow n_0' = \frac{n_0}{10} = 2\times10^{14} = N_D - N_A \rightarrow$

$N_A = 2\times10^{15} - 2\times10^{14} = 1.8\times10^{15}$ cm^{-3}

(c) (/4) If under light illumination a steady density 10^{14} cm^{-3} of excess electrons and holes is generated what is the new resistivity of the two samples in part (a) and (b)? Which sample is better for building a photodetector?

a) $\begin{cases} n = n_0 + \hat{n} = 2\times10^{15} + 10^{14} = 2.1\times10^{15} \text{ cm}^{-3} \\ p = p_0 + \hat{p} = 5\times10^6 + 10^{14} = 10^{14} \text{ cm}^{-3} \end{cases} \rightarrow \sigma = 0.688$ S/cm

b) $\left.\begin{cases} n = n_0 + \hat{n} = 2\times10^{14} + 10^{14} = 3\times10^{14} \text{ cm}^{-3} \\ p = p_0 + \hat{p} = 5\times10^7 + 10^{14} = 10^{14} \text{ cm}^{-3} \end{cases}\right\} \rightarrow \sigma = 0.112$ S/cm

$\sigma_0 = 0.08$ S/cm

↑ much higher change

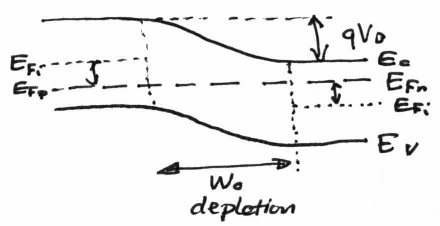
better for photodetectors

8. (/18) A silicon wafer (Si: $n_i = 10^{10}\text{cm}^{-3}$ and $\varepsilon_r = 12$) is used to make a PN junction over a square area $a \times a$ with the side of $a = 1$ mm. Aluminum is used as a p-type dopant with density of 10^{17} cm^{-3}, and Arsenic as an n-type dopant with density of 10^{15} cm^{-3}.

(a) (/2) Find the Fermi energies E_{Fn} and E_{Fp} in the n-type and p-type regions far from the junction with respect to the intrinsic Fermi energy E_{Fi}.

$$E_{Fn} - E_{Fi} = kT \ln \frac{N_D}{n_i} = 297 \text{ meV}$$
$$E_{Fi} - E_{Fp} = kT \ln N_A/n_i = 416 \text{ meV}$$

(b) (/3) Draw the band diagram for the PN junction, and find the built-in voltage V_0.



$$V_0 = \frac{E_{Fn} - E_{Fi} + E_{Fi} - E_{Fp}}{q} = 713 \text{ mV}$$

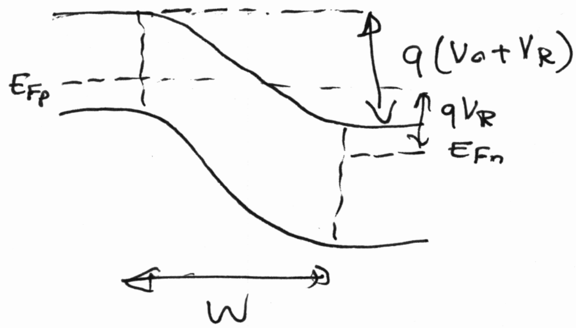
(c) (/3) Find the depletion width W_0 and the maximum electric field $\mathscr{E}_0$ in the depletion region.

$$W_0 = \sqrt{\frac{2\varepsilon}{q} \frac{N_A + N_D}{N_A N_D} V_0} = 978 \text{ nm} \qquad \mathcal{E}_0 = -\frac{2V_0}{W_0} = -1.46 \text{ MV/m}$$

(d) (/3) Given $\mu_n = 2\mu_p = 1000$ cm^2/Vs and $\tau_n = \tau_p = 1$ μs, find the diode reverse saturation current I_0 and the diode current at a forward voltage $V = 0.5$ V.

$$D_n = \frac{kT}{q}\mu_n = 25.8 \text{ cm}^2/\text{s} \qquad L_n = \sqrt{D_n \tau_n} = 50.7 \,\mu\text{m} \qquad p_{n0} = \frac{n_i^2}{N_D} = 10^5 \text{ cm}^{-3}$$
$$D_p = \frac{kT}{q}\mu_p = 12.9 \text{ cm}^2/\text{s} \qquad L_p = \sqrt{D_p \tau_p} = 35.9 \,\mu\text{m} \qquad n_{p0} = \frac{n_i^2}{N_A} = 10^3 \text{ cm}^{-3}$$
$$I_0 = qA\left(\frac{D_p}{L_p} p_{n0} + \frac{D_n}{L_n} n_{p0}\right) = 5.83 \times 10^{-13} \text{ A}$$
$$I = I_0 e^{V_a/V_{th}} = 1.52 \times 10^{-4} \text{ A}$$

(e) (/2) Draw the band diagram when a negative bias is applied to the junction.



(f) (/5) Name the difference mechanisms of breakdown in a PN junction and explain their difference. Which one occurs in lightly doped junction and which one in highly doped junctions?

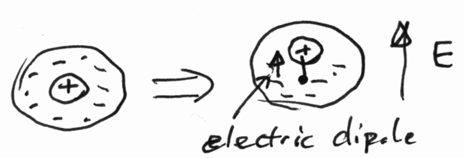
Avalanche: Over a wide depletion region electrons are accelerated and collide with other electrons that are ionized, and added to electron current, like an avalanche, happens in lightly doped junctions.
Zener: Over a narrow depletion, when the voltage is high, so the triangular energy barrier becomes narrower for electrons and holes to funnel to the other side. It can be reversible unlik avalanche and happens in highly doped junctions

9. (/13) A dielectric has $N = 2\times10^{22}$ atoms per cm^3 and an electronic polarizability of $\alpha_e = 10^{-36}$ F/cm^2.

(a) (/4) Explain the electronic polarization mechanism in a couple of sentences using a schematic. Find the dielectric constant ε_r' of this dielectric at optical frequencies using Clausius-Mossotti equation. What are the refractive index and the speed of light in this dielectric.

Electronic cloud around an atom is polarized by external electric field. The center of electron cloud moves away from the nucleus.

$$\frac{\varepsilon_r'-1}{\varepsilon_r'+2} = \frac{N}{3\varepsilon_0}(\alpha_e)$$



$= 0.0753$

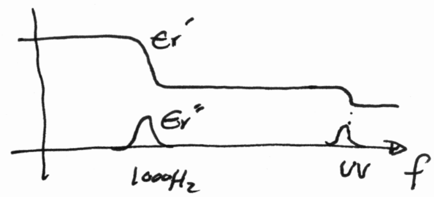
$\varepsilon_r' = 1.24$

$n = \sqrt{\varepsilon_r'} = 1.11$

$v = \frac{c}{n} = 2.69\times10^{10}$ cm/s

(b) (/5) At very lower frequencies (<1000 Hz) we observe that the dielectric constant increases to 20. Draw an approximate plot showing ε_r' and ε_r'' as a function of frequency. What is the best guess for the mechanism responsible for the change in dielectric constant? Find the polarizbility associated to each atom for this polarization mechanism based on Clausius-Mossotti equation.

Interfacial and space charge mechanism
very slow



$$\alpha = \frac{3\epsilon_0}{N}\left(\frac{\epsilon_r' - 1}{\epsilon_r' + 2}\right) - \alpha_e$$
$$= 1.05 \times 10^{-35}\ F/cm^2$$

(c) (/4) If for this dielectric, we have a breakdown field of 10 MV/cm, find the maximum electrical energy per unit volume that can be stored using this dielectric at a frequency of 60 Hz assuming a safety factor of 1.5. What is the energy loss per unit volume if the loss tangent is 0.1.

$$E_{store} = \frac{\epsilon_0 \epsilon_r'}{2} E^2 = 39.3\ J/cm^3$$
$$W_{loss} = \epsilon_0 \epsilon_r' \tan\delta\, \omega E^2 = 2965\ J/cm^3$$

10. (/8) (a) (/5) Explain the difference between a paramagnetic, diamagnetic and ferromagnetic material. Provide an example for each type of material

Paramagnetic: Have small but positive susceptibility $\chi_m \sim 10^{-5} - 10^{-4}$
Attracted by magnetic field, examples oxygen, Li, Mg

Diamagnetic: Small but negative $\chi_m \sim 10^{-6}$
Repels external magnetic field. Examples Si, Ge, Au

Ferromagnetic: Large positive χ_m, permanent magnetic domains, hysterisis in B-H curve.

(b) (/3) On the hysteresis curve on the right show coercive field, and retentive and saturation magnetic fields.