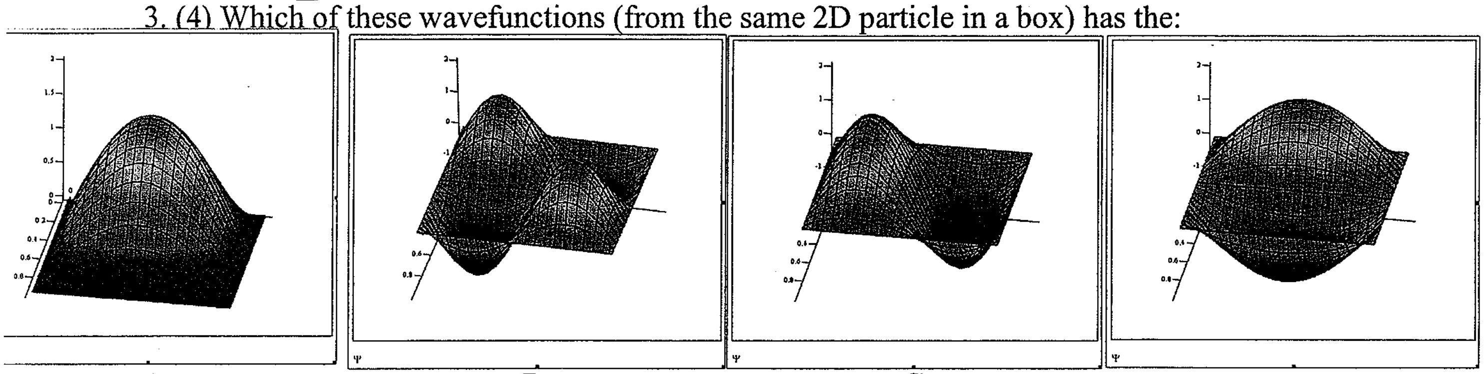
ANSWERS

EECE 352
Midterm October 2006
Closed Book. Only pens, pencils allowed. No calculators. Time: 50 minutes.
Equation list and periodic table provided on last 2 pages.
Total marks: **95**

1. (2) What is the conductivity of a semiconductor at 0 Kelvin?
   $\sigma = 0$ since no e^-'s are thermally promoted to CB
2. (2) A photon strikes a metal of work function W. Which of the following is true of the kinetic energy of the ejected electron, KE (circle the best answer):

   (i) $KE = h\nu - W$ $KE_{max} = h\nu - W$ THIS IS TRUE, BUT GENERALLY NOT ALL $h\nu$ IS CONVERTED TO KE.
   (ii) $KE > h\nu - W$
   (iii) $KE = h\nu + W$ $KE \le h\nu - W$
   (iv) $KE < h\nu - W$

3. (4) Which of these wavefunctions (from the same 2D particle in a box) has the:

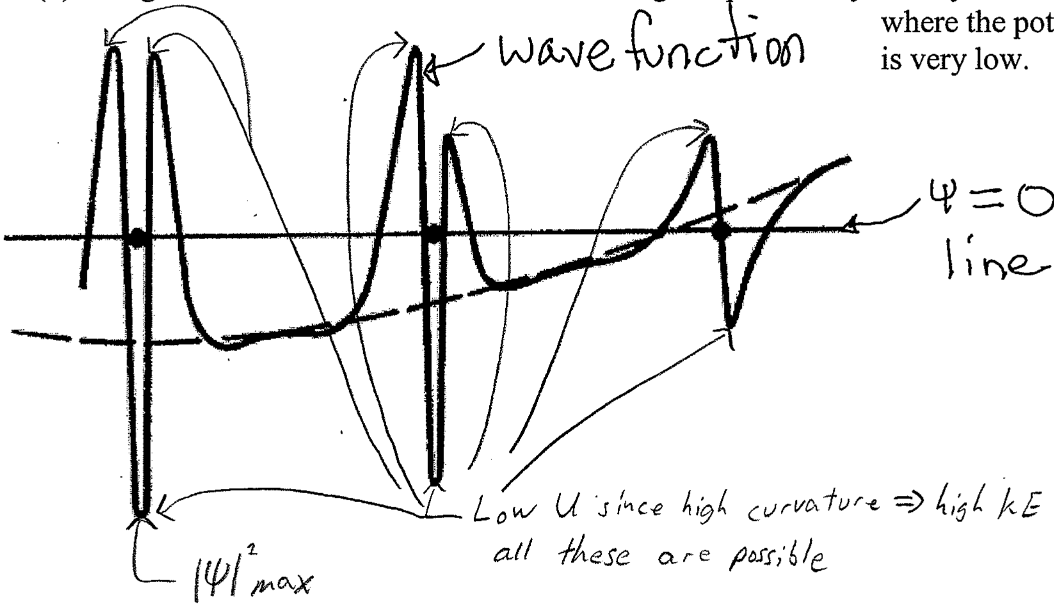


A B C D

(a) Highest kinetic energy? (circle) A B C D None most $\frac{1}{2}$ waves in the box

(b) Largest average momentum vector? A B C D None $P_{avg} = 0$ over the box

4. (4) Using an arrow show the location with the highest probability density. Point to a location where the potential energy is very low.



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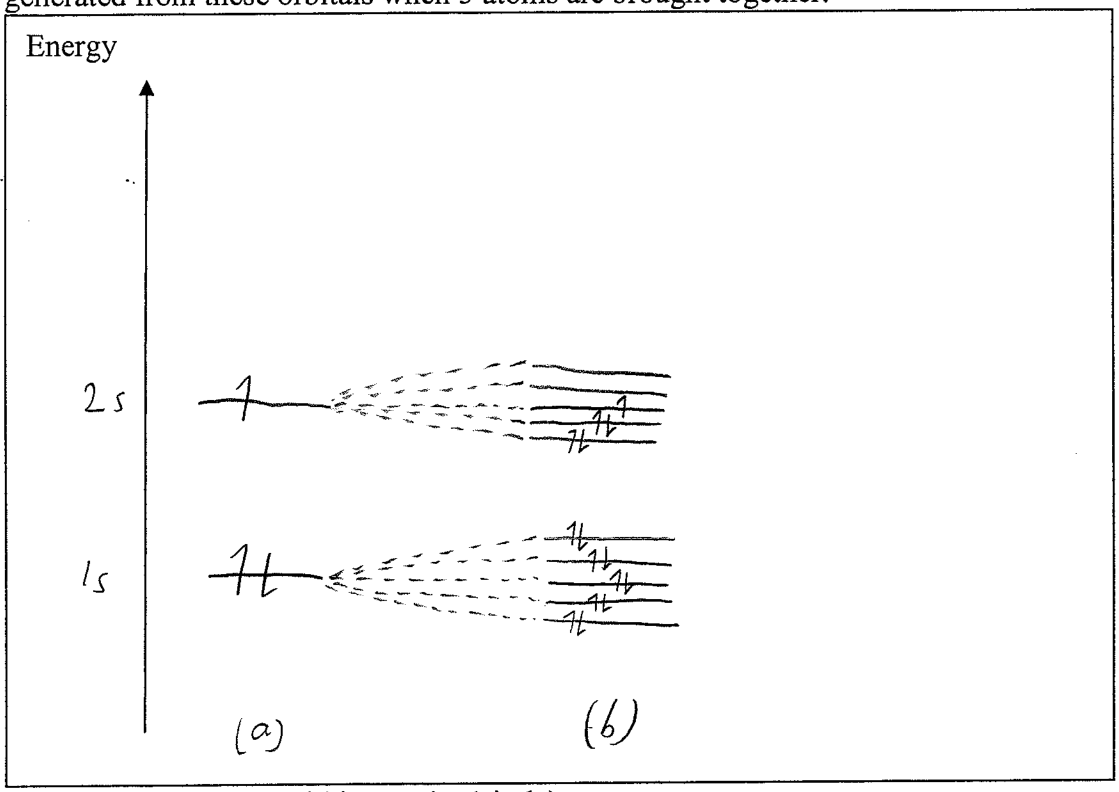
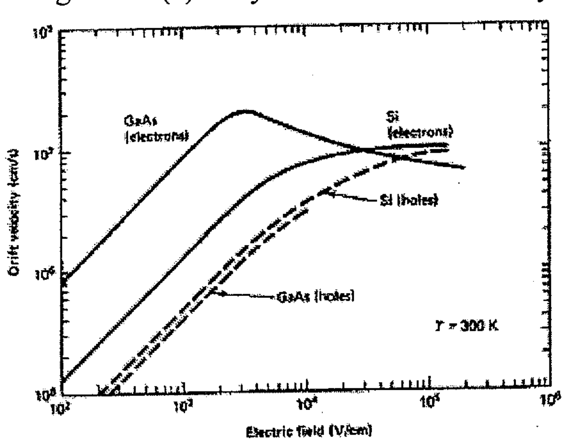
5. (a) (3) Show the filling of the 1s and 2s orbitals in Lithium (Li) by sketching horizontal lines representing their approximate relative energies and filling (for the ground state) with arrows representing electrons with spin. (b) (4) Show how these orbitals split by depicting the states generated from these orbitals when 5 atoms are brought together.



(c) (2) Do you expect Lithium to be (circle) an:

(a) Insulator
(b) Semiconductor
(c) Metal   trend shows a ½ filled band from 2s orbital
(d) Can't tell because bands may overlap.

6. (4) The figure below shows that the drift velocity reaches saturation, and may even drop with increasing field. (a) Why does the drift velocity reach a limit (circle the best answer)?



(i) Because carriers reach a terminal velocity, so increasing field cannot increase speed.
(ii) The field distorts the crystal, pulling nuclei to one side and electrons to the other, thereby slowing electrons down.
(iii) The field begins to significantly affect the average speed and kinetic energy of electrons, leading to a heating of the lattice and increased scattering.
(iv) At high speeds the electrons tunnel past impurities and lattice scattering points.

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7. (5) True or False (Wrong subtracted from Right). Mobility typically
   i. is inversely proportional to effective mass. T F
   ii. is reduced by increasing doping density. T ~~F~~
   iii. increases with increasing temperature. T F
   iv. depends on field when the applied field is small. T F
   v. drops at very high fields. T F

T is correct at very low temperature (which is not typically the operating condition)

8. (a) (3) A very wide and thick crystalline block of material sits at $x \geq 0$. Assume that for $x < 0$ the potential energy U is constant. Which of the following equations best approximates the wavefunction of an electron in the valence band when it is outside the crystal ($x<0$)? $(U>E)$
   (i) $A \cdot e^{kx} + B \cdot e^{-kx}$
   (ii) $A \cdot e^{jkx} + B \cdot e^{-jkx}$ tunneling $\Rightarrow$ decaying exponential
   (iii) $A \cdot e^{kx}$
   (iv) $A \cdot \sin kx$
   (v) The electron cannot be found outside the material because its total energy is less than the potential energy.

9. A semiconductor is doped with 10^{12} cm^{-3} Phosphorous (P) atoms and 10^{17} cm^{-3} Boron (B) atoms.

(a) (4) What two equations must be satisfied in order to solve for the density of electrons in the conduction band and the density of holes in the valence band?

$$n_o p_o = n_i^2 \quad , \quad N_A^- + n_o = N_D^+ + p_o$$

(b) (4) What are the densities of electrons and holes in the conduction and ~~electron~~ valence bands, respectively? (Provide approximate numbers.) ($n_i = 10^{10}$ cm^{-3}).

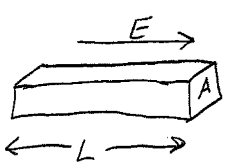
$$p_o \simeq N_A^- - N_D^+ = 10^{17}\,cm^{-3} - 10^{12}\,cm^{-3} = 10^{17}\,cm^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{10^{20}\,cm^{-6}}{10^{17}\,cm^{-3}} = 10^{3}\,cm^{-3}$$

(c) (4) Write an expression for the position of the Fermi level relative to the valence band edge.

$$p_o = N_v \exp\left(-\frac{(E_F - E_v)}{kT}\right) \Rightarrow E_F = E_v - kT\ln\left(\frac{p_o}{N_v}\right)$$

(d) (4) A field E is applied to the semiconductor along length, L and through a cross-sectional area, A. Derive an expression for the total current. Express this current in terms of n_o, p_o, fundamental charge $+e$, the length L, the area A, the mobilities μ_e, and μ_h of the electrons and holes, and the voltage, V.



$$V = IR \Rightarrow I = \frac{V}{R}$$

$$I = V\sigma\frac{A}{L}$$

$$\boxed{I = Ve(\mu_e n_o + \mu_h p_o)\frac{A}{L}}$$

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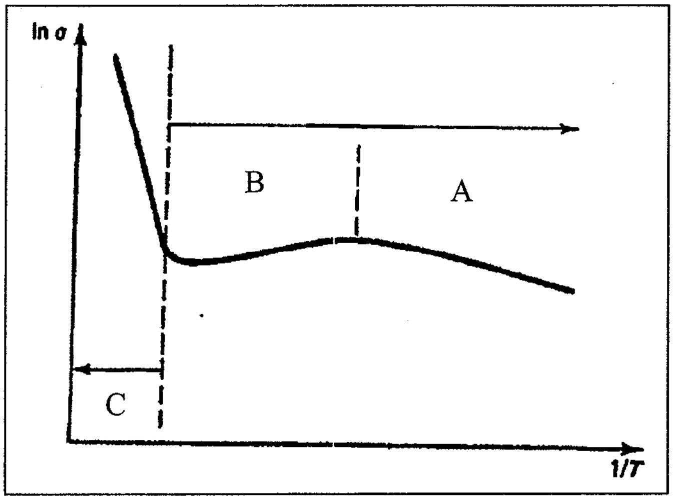
10. (9) Explain why conductivity in a doped semiconductor increases from low temperatures (A), then decreases (B) and finally increases again (C).



Region A:
Very low temperatures 1) Not all of the dopant atoms are ionized. As T is increased, dopant atoms are ionized, releasing carriers into the semiconductor. This increases σ until a temperature at which all dopants are ionized.
2) Another effect in region A is the scattering of e^-/h^+ off the ionized impurities, which decreases when T is increased.
See Pulfrey & Tarr, pp. 46-52.

Region B:
As the temperature is increased further, the increased thermal energy results in more vibration in the lattice atoms. This means more e^-/h^+ scattering from the lattice and decreases the 'mean free path' and the mobility of e^- and h^+. Since the number of carriers is constant in region B, a decrease in mobility decreases the conductivity.

Region C:
At very high T, electrons of the valance band gain enough energy to cross the band gap into the conduction band, increasing the conductivity sharply. (nearly exponentially)

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11. (2) (a) If $f(E)$ is the probability of finding an electron in a state having energy E, what is the probability of there not being an electron in that state?

$1 - f(E)$

(b) (4) Under certain conditions the probability of finding an electron in a state of energy E in the conduction band is $\exp\left(-\frac{[E-E_F]}{kT}\right)$. Derive a similar expression for holes in the valence band.

$$f_h = 1 - \frac{1}{1+e^{\frac{E-E_F}{KT}}} = \frac{e^{\frac{E-E_F}{KT}}}{1+e^{\frac{E-E_F}{KT}}} = \frac{1}{e^{-\frac{E-E_F}{KT}}+1}$$

In the valence band: $E \ll E_F \Rightarrow e^{-\frac{E-E_F}{KT}} \gg 1$

$$\Rightarrow \boxed{f_h = e^{\frac{E-E_F}{KT}}}$$

(c) (2) Under what conditions does the approximation in part b apply?

$|E-E_F| \gg KT$

12. (2) In a metal at extremely low temperatures (very close to absolute zero) what is the number of electrons having energies higher than the Fermi level?

$n =$ Zero.

See the definition of Fermi level.

More mathematically:

$$f(E) = \frac{1}{1+e^{\frac{E-E_F}{KT}}}$$

$$\left.\begin{array}{l} E-E_F>0 \\ E-E_F \gg KT \end{array}\right\} \longrightarrow e^{\frac{E-E_F}{KT}} \gg 1$$

$$1+e^{\frac{E-E_F}{KT}} \gg 1$$

$$\Rightarrow f(E) \simeq 0$$

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13. An $x=L$ wide, $y=t$ long, by $z=t$ thick slab of material is modeled as an infinite square well, with a potential inside of $U=0$.

(a) (6) Write out differential equations for the time independent wavefunctions in each direction, $\psi_x(x)$, $\psi_y(y)$ and $\psi_z(z)$. In doing so, express the total energy, E, as the sum of the energies E_x, E_y, and E_z. You do not need to show your steps (these equations are the result of separation of variables).

$$E\psi_x\psi_y\psi_z = -\frac{\hbar^2}{2m}\left[\left(\frac{\partial^2\psi_x}{\partial x^2}\right)\psi_y\psi_z + \psi_x\psi_z\frac{\partial^2\psi_y}{\partial y^2} + \psi_x\psi_y\frac{\partial^2\psi_z}{\partial z^2}\right]$$

$$E_x+E_y+E_z = \frac{-\hbar^2}{2m}\left[\underbrace{\frac{1}{\psi_x}\frac{\partial^2\psi_x}{\partial x^2}}_{f(x)} + \underbrace{\frac{1}{\psi_y}\frac{\partial^2\psi_y}{\partial y^2}}_{f(y)} + \underbrace{\frac{1}{\psi_z}\frac{\partial^2\psi_z}{\partial z^2}}_{f(z)}\right] \Rightarrow \text{must be indep.}$$

3 Equations

$$\boxed{E_x\psi_x = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_x}{\partial x^2},\quad E_y\psi_y = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_y}{\partial y^2},\quad E_z\psi_z = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_z}{\partial z^2}}$$

(b) (6) Use the boundary conditions to solve for $\psi_x(x)$ in terms of quantum number n_x (show your work). Write out the solutions for $\psi_y(y)$, & $\psi_z(z)$ in terms of the boundary conditions and n_y or n_z (no normalization required). Note that the dimension in the x direction is L, and it is t in the y and z directions.

$$(*)\ E_x\psi_x = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_x}{\partial x^2}, \quad \psi_x(0)=\psi_x(L)=0$$

$\psi_x = A\sin kx + B\cos kx$. Since $\psi_x(0)=0 \Rightarrow B=0$

$\psi_x = A\sin kx$. $\psi_x(L)=0 \Rightarrow k=\frac{\pi n}{L}$ $n=1,2\ldots$

$$\boxed{\psi_x = A\sin\frac{\pi n x}{L}} \Rightarrow \text{plug into } (*) \text{ above } E_x = \frac{+\hbar^2}{2m}\frac{\pi^2 n^2}{L^2}$$

$$\boxed{\psi_y = A\sin\frac{\pi n_y y}{t} \qquad \psi_z = A\sin\frac{\pi n_z z}{t}}$$

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(c) (2) Write out the normalization condition for ψ(x,y,z) showing the limits of integration (no need to solve).

$$\int_0^L \int_0^t \int_0^t |\psi_x(x)\psi_y(y)\psi_z(z)|^2 \, dx\,dy\,dz$$

$\frac{E_z 2m}{\hbar^2}$

(d) (4) Show that $E = \frac{\hbar^2}{2m}\left[k_x^2 + k_y^2 + k_z^2\right]$, and express this in terms of $\cancel{n_x}$, n_y and n_z.

$$\cancel{E\psi} = \cancel{E_x\psi_x + E_y\psi_y + E_z\psi_z} = -\frac{\hbar^2}{2m}\left[\frac{1}{\psi_x}\frac{\partial^2\psi_x}{\partial x^2} + \underbrace{\frac{1}{\psi_y}\frac{\partial^2\psi_y}{\partial y^2}} + \underbrace{\frac{1}{\psi_z}\frac{\partial^2\psi_z}{\partial z^2}}\right]$$

$\frac{E_y 2m}{\hbar^2}$ $\frac{E_x 2m}{\hbar^2}$

plug in ~~plug in ψ_x~~

$\psi_x = A\sin k_x x$

$\psi_y = A'\sin k_y y$

$\psi_z = A''\sin k_z z$

$$E = +\frac{\hbar^2}{2m}k_x^2 + \frac{\hbar^2}{2m}k_y^2 + \frac{\hbar^2}{2m}k_z^2$$

$$E = \frac{\hbar^2\pi^2}{2m}\left[\frac{n_x^2}{L^2} + \frac{n_y^2}{t^2} + \frac{n_z^2}{t^2}\right]$$

(e) (6) Write out the lowest 6 energy states assuming that L=1000·t. Which term(s) in the expression in (d) can be neglected near the bottom of the band?

$L \gg t \Rightarrow$ therefore $\frac{n^2}{L^2} \ll \frac{n^2}{t^2}$.

Lowest energy states are

1. $\frac{h^2}{8mt^2}\left(\frac{1}{10^6}+2\right)$ (1,1,1)
2. $\frac{h^2}{8mt^2}\left(\frac{4}{10^6}+2\right)$ (2,1,1)
3. $\frac{h^2}{8mt^2}\left(\frac{9}{10^6}+2\right)$ (3,1,1)
4. $\frac{h^2}{8mt^2}\left(\frac{16}{10^6}+2\right)$ (4,1,1)
5. (5,1,1) $\frac{h^2}{8mt^2}\left(\frac{25}{10^6}+2\right)$
6. (6,1,1) $\frac{h^2}{8mt^2}\left(\frac{36}{10^6}+2\right)$

$\boxed{\text{o Terms } \frac{n_y^2}{t^2} \text{ \& } \frac{n_z^2}{t^2} \text{ lead to a constant offset}}$

(e) (3) Which material geometry is best approximated using a 1D Schroedinger Equation:

a. A tiny sphere.
b. A nearly atomically thin wire.
c. A wide, long but very thin layer.
d. A large cube.

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