Elec 315 Assignment 2 Solutions

1. a) $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \rightarrow -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)XYZ = EXYZ \} \rightarrow$

Separation of variables: $\psi = XYZ$

$$-\frac{\hbar^2}{2m}\left[\frac{X''}{X}+\frac{Y''}{Y}+\frac{Z''}{Z}\right] = E \rightarrow \begin{cases} -\frac{\hbar^2}{2m}\frac{X''}{X} = E_x \\ -\frac{\hbar^2}{2m}\frac{Y''}{Y} = E_y \\ -\frac{\hbar^2}{2m} Z''/Z = E_z \end{cases}, \quad E = E_x + E_y + E_z$$

For each direction : for example x: $-\frac{\hbar^2}{2m}\frac{X''}{X} = E_x \rightarrow$

$X'' + \frac{2mE_x}{\hbar^2}X = 0 \rightarrow X(x) = A e^{jk_x x} + B e^{-jk_x x}$, $k_x = \sqrt{\frac{2mE_x}{\hbar^2}}$

$X(0) = 0 \rightarrow A + B = 0 \rightarrow X(x) = A' \sin k_x x$

$X(L_x) = 0 \rightarrow k_x L_x = n_x \pi \rightarrow k_x = \frac{n_x \pi}{L_x} \rightarrow$

So if we do this for all x, y, z: $E = \frac{\hbar^2}{2m_0}\left(k_x^2 + k_y^2 + k_z^2\right)$, $k_x = \frac{n_x \pi}{L_x}$

$k_y = \frac{n_y \pi}{L_y}$, $k_z = \frac{n_z \pi}{L_z}$

b) $E = \frac{\hbar^2\pi^2}{2m_0 L_x^2}\left(n_x^2 + n_y^2 + \frac{n_z^2}{100^2}\right) = 0.377\text{ eV}\left(n_x^2 + n_y^2 + n_z^2/10^4\right)$

lowest energies $\rightarrow E_{111} = 0.75388\text{ eV} \rightarrow$ ground state $\rightarrow \psi(x,y,z,t) = A \sin\left(\frac{\pi x}{L_x}\right)\sin\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right) e^{-j\frac{0.75388t}{\hbar}}$

$E_{112} = 0.75399\text{ eV}$

$E_{113} = 0.75418\text{ eV}$

$E_{114} = 0.75445\text{ eV}$

$E_{115} = 0.75479\text{ eV}$

$E_{116} = 0.75520\text{ eV}$

c) $\psi_{112} \rightarrow \psi_{111} \rightarrow h\nu = E_{112} - E_{111} = 0.113\text{ meV} \rightarrow \lambda = \frac{hc}{E} = 1.099\text{ cm}$

$\psi_{121} \rightarrow \psi_{111} \rightarrow h\nu = E_{121} - E_{111} = 1.13\text{ eV} \rightarrow \lambda = \frac{hc}{E} = 1.099\ \mu\text{m}$

d) $E < 1.885\text{ eV}$

$n_x = 1, n_y = 1, n_z = ? \rightarrow 2 + \frac{n_z^2}{10^4} < 5.00103 \rightarrow n_z^2 < 3.00103\times10^4$, $n_z < 173.2 \rightarrow n_{z\,max} = 173$

$n_x = 1, n_y = 2, n_z = ? \rightarrow 5 + n_z^2/10^4 < 5.00103 \rightarrow n_z^2 < 0.00103\times10^4$, $n_z < 3.2 \rightarrow n_{z\,max} = 3$

$n_x = 2, n_y = 1, n_z = ?$

$n_x = 2, n_y = 2$, n_z not acceptable

Number of states $= 173 + 2\times3 = \boxed{179}$

2-a) $\psi_1 = A_1 e^{jk_1x} + B_1 e^{-jk_1x}$ $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$

$\psi_2 = A_2 e^{k_2x} + B_2 e^{-k_2x}$ $k_2 = \sqrt{\frac{2m(U_{barrier} - E)}{\hbar^2}}$

$\psi_3 = A_3 e^{jk_3x}$ $k_3 = k_1$ B_3 is zero

$x=0 \begin{cases} \psi_1(0) = \psi_2(0) \rightarrow A_1 + B_1 = A_2 + B_2 \\ \psi_1'(0) = \psi_2'(0) \rightarrow jk_1(A_1 - B_1) = k_2(A_2 - B_2) \end{cases}$

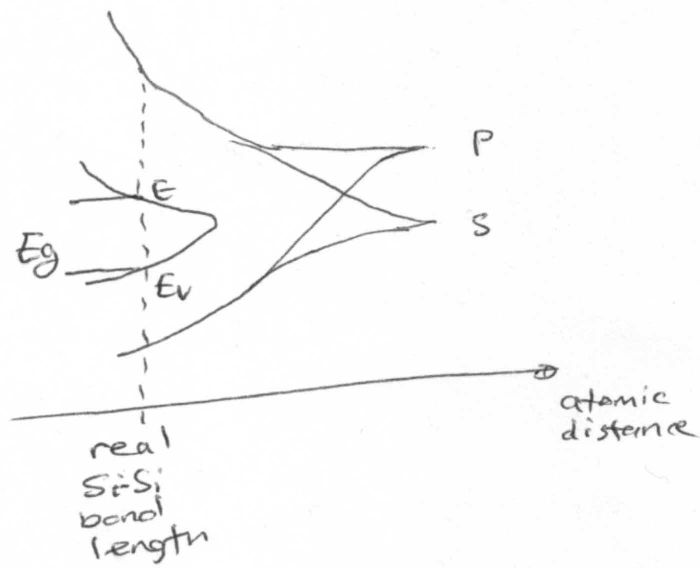
$x=W \begin{cases} \psi_2(W) = \psi_3(W) \rightarrow A_2 e^{k_2W} + B_2 e^{-k_2W} = A_3 e^{jk_1W} \\ \psi_2'(W) = \psi_3'(W) \rightarrow A_2k_2 e^{k_2W} - B_2k_2 e^{-k_2W} = jk_3A_3 e^{jk_3W} \end{cases}$

b) $D = 1.225$, $k_2 = 1.14 \times 10^{10}$ 1/m , $T = 4.33 \times 10^{-20} \rightarrow I = 6.98 \times 10^{-30}$ A

if $W = 0.8$ nm $\rightarrow$ $T = 3.66 \times 10^{-8} \rightarrow I = 5.85 \times 10^{-18}$ A

3. $T\uparrow \rightarrow E_g \downarrow$

$T\downarrow \rightarrow E_g \uparrow$



$\rightarrow$
increase T
increase atomic distane