ELEC 315:
Metals and Semiconductors

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Outline

- Silicon atom
- Bonding
- Energy bands
- Metal, semiconductor, insulator

Quantum numbers and periodic table

$$\Psi_{nlm_l}(r,\theta,\phi) = R_n(r)\Theta_l(\theta)\Phi_{m_l}(\phi) \qquad n \geq 1,\ 0 \leq l \leq n-1,\ -l \leq m_l \leq l$$

n : principal quantum number, defines shells

l : orbital quantum number

m_l: magnetic quantum number

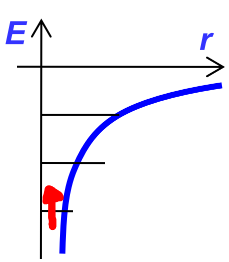
Spin: Angular momentum of each electron

s : spin number (1/2 or -1/2)

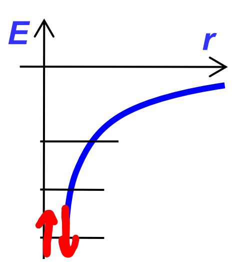
Pauli Exclusion Principle

For each value of n, l, m_l, only two electrons with opposite spins occupy each energy level.

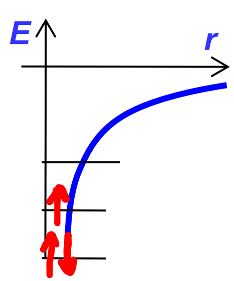
$$E = \frac{-13.6Z^2}{n^2}$$



H



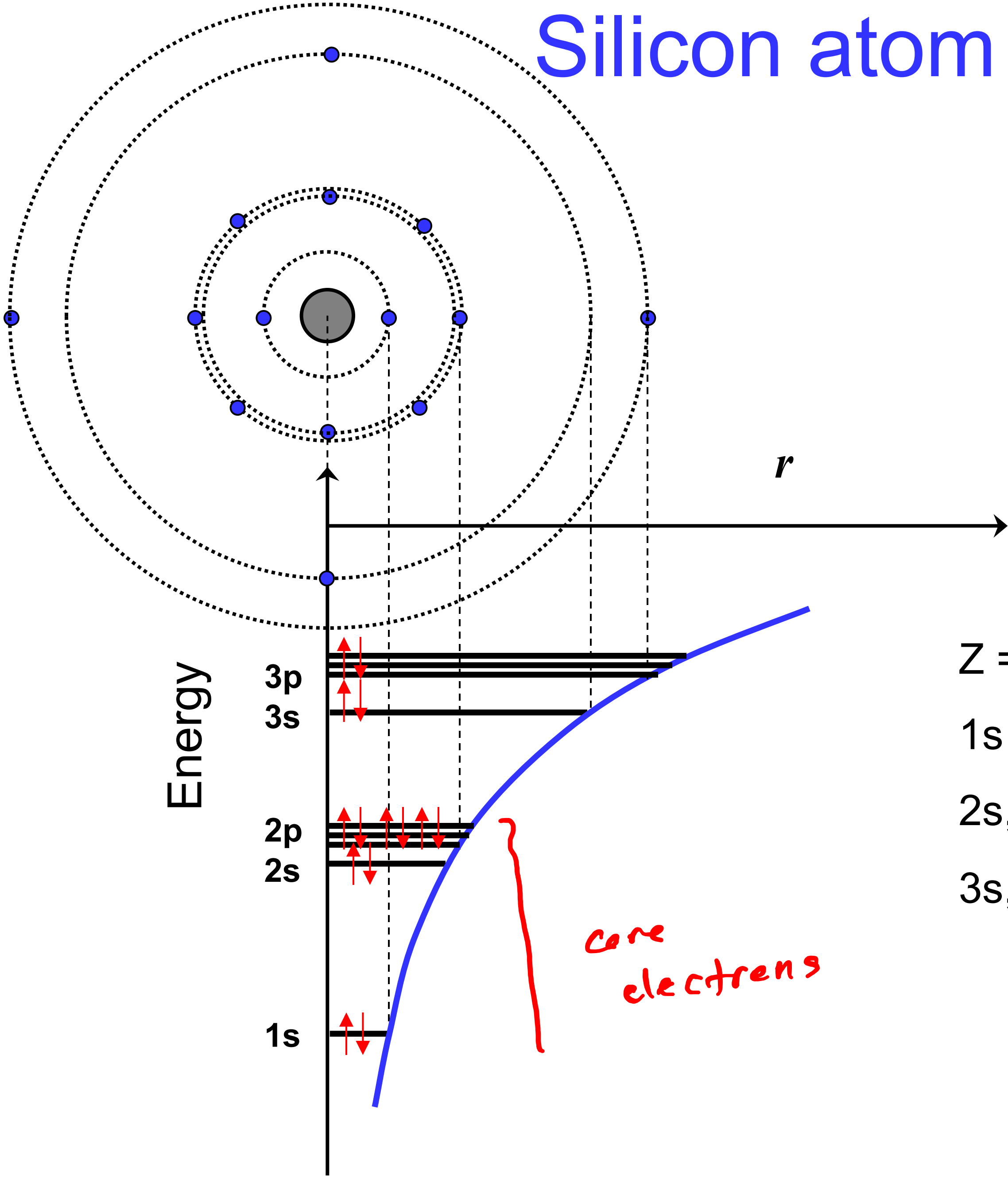
He



Li

Atomic number (Z)	Element	n = 1, l = 0: 1s	2, 0: 2s	2, 1: 2p	3, 0: 3s	3, 1: 3p	3, 2: 3d	4, 0: 4s	4, 1: 4p	Shorthand notation
		Number of electrons								
1	H	1								$1s^1$
2	He	2								$1s^2$
3	Li	helium core, 2 electrons	1							$1s^2\ 2s^1$
4	Be		2							$1s^2\ 2s^2$
5	B		2	1						$1s^2\ 2s^2\ 2p^1$
6	C		2	2						$1s^2\ 2s^2\ 2p^2$
7	N		2	3						$1s^2\ 2s^2\ 2p^3$
8	O		2	4						$1s^2\ 2s^2\ 2p^4$
9	F		2	5						$1s^2\ 2s^2\ 2p^5$
10	Ne		2	6						$1s^2\ 2s^2\ 2p^6$
11	Na	neon core, 10 electrons			1					[Ne] $3s^1$
12	Mg				2					$3s^2$
13	Al				2	1				$3s^2\ 3p^1$
14	Si				2	2				$3s^2\ 3p^2$
15	P				2	3				$3s^2\ 3p^3$
16	S				2	4				$3s^2\ 3p^4$
17	Cl				2	5				$3s^2\ 3p^5$
18	Ar				2	6				$3s^2\ 3p^6$
19	K	argon core, 18 electrons						1		[Ar] $4s^1$
20	Ca							2		$4s^2$
21	Sc						1	2		$3d^1\ 4s^2$
22	Ti						2	2		$3d^2\ 4s^2$
23	V						3	2		$3d^3\ 4s^2$
24	Cr						5	1		$3d^5\ 4s^1$
25	Mn						5	2		$3d^5\ 4s^2$
26	Fe						6	2		$3d^6\ 4s^2$
27	Co						7	2		$3d^7\ 4s^2$
28	Ni						8	2		$3d^8\ 4s^2$
29	Cu						10	1		$3d^{10}\ 4s^1$
30	Zn						10	2		$3d^{10}\ 4s^2$
31	Ga						10	2	1	$3d^{10}\ 4s^2\ 4p^1$
32	Ge						10	2	2	$3d^{10}\ 4s^2\ 4p^2$
33	As						10	2	3	$3d^{10}\ 4s^2\ 4p^3$
34	Se						10	2	4	$3d^{10}\ 4s^2\ 4p^4$
35	Br						10	2	5	$3d^{10}\ 4s^2\ 4p^5$
36	Kr						10	2	6	$3d^{10}\ 4s^2\ 4p^6$

Silicon atom



Z = 14

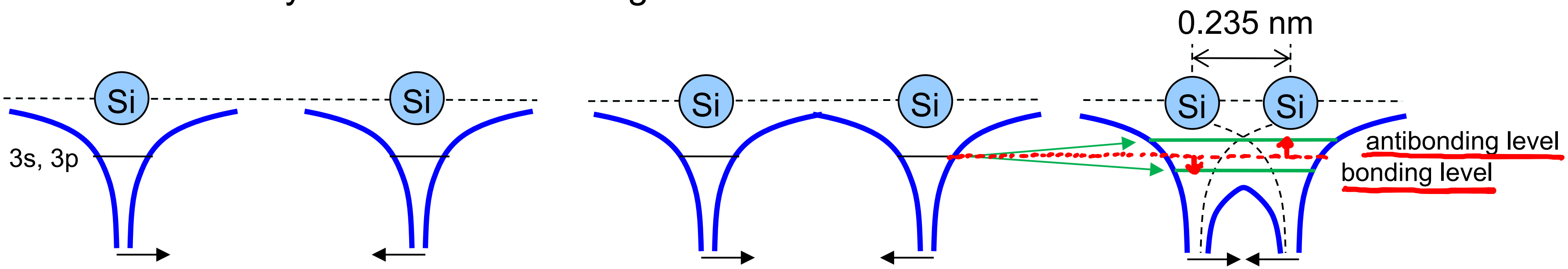
1s : 2e

2s, 2p: 8e

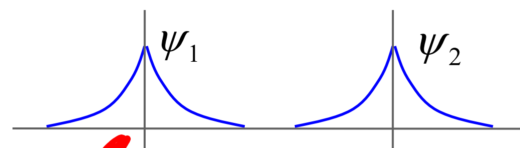
3s, 3p: 4e

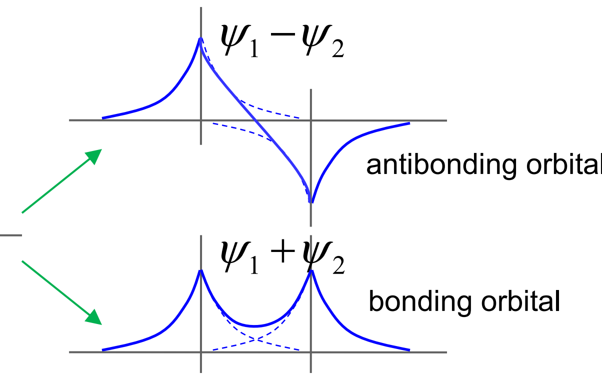
Bonding and Crystalline Silicon

- To form a crystal we need to bring atoms close to each other.

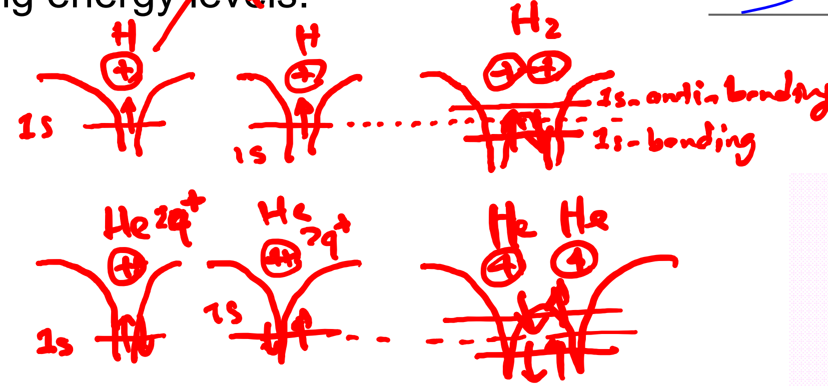


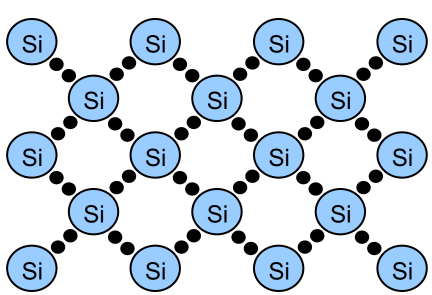
- The potential well of the two atoms mix, resulting in the mixing of the orbitals that form separate antibonding and bonding orbitals.
- The antibonding orbital energy level is at higher energy and bonding orbital is at lower energy.
- There is a separation between bonding and antibonding energy levels.

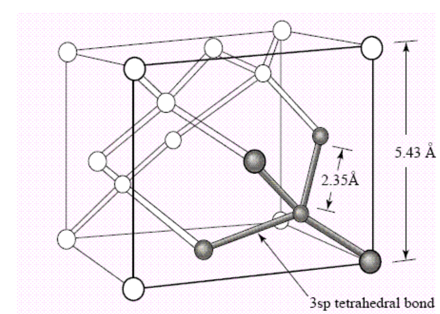




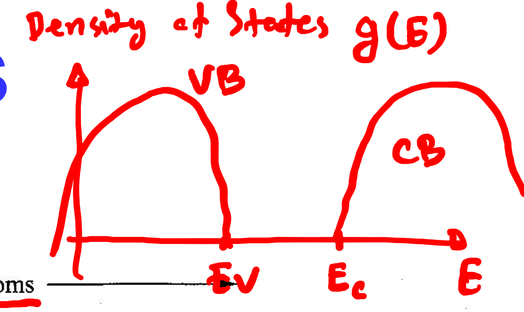
wave function



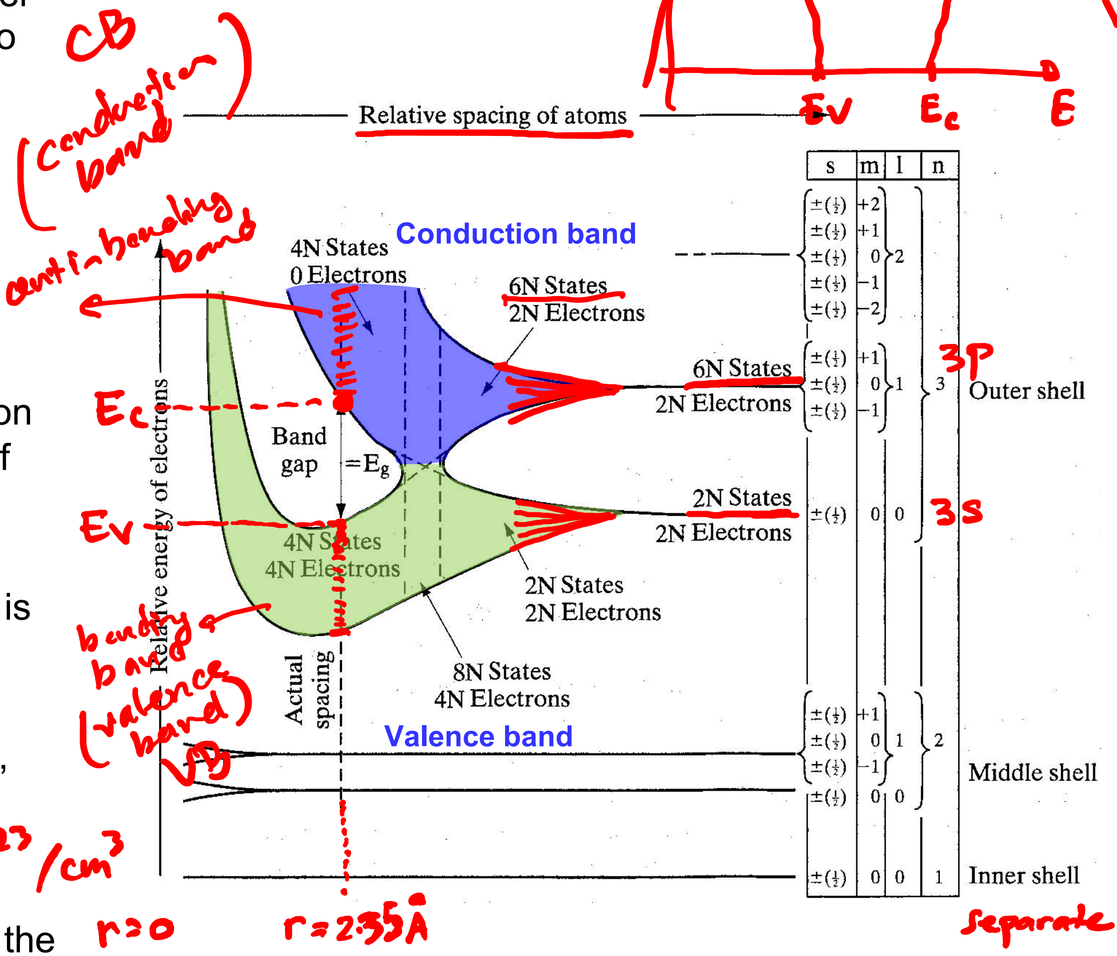




Silicon energy bands



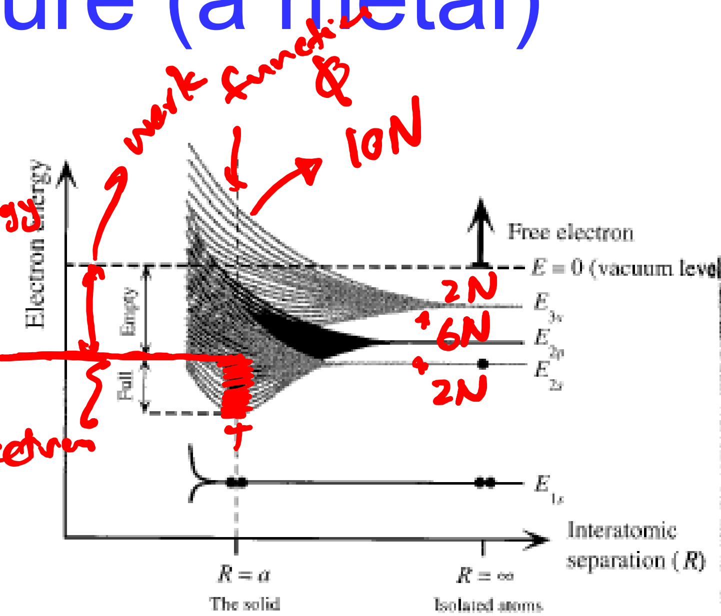
- When more silicon atoms are brought closer to each other, 3s-3p orbitals are combined to form a bonding (valence) band and an antibonding (conduction) band, at actual spacing in the figure. Bands (shaded in the figure) are a collection of a large number of energy levels for electrons.
- These bands are separated in energy. Between these bands there are no energy levels available for electrons. This separation in the bands is referred to as the bandgap of silicon E_g.
- The number of energy levels in the conduction and valence bands is equal and is given by $4N$, where N is the number of Si atoms.
- Given the density of Si: $N_{Si} = 5\times10^{22}$ /cm³, the number of energy levels (states) in valence and conduction bands are: 2×10^{23} /cm³
- The last four electrons of each atom fill up the bonding (or valence) band. This band is full with 2×10^{23} /cm³ electrons.
- The antibonding (or conduction) band is empty.

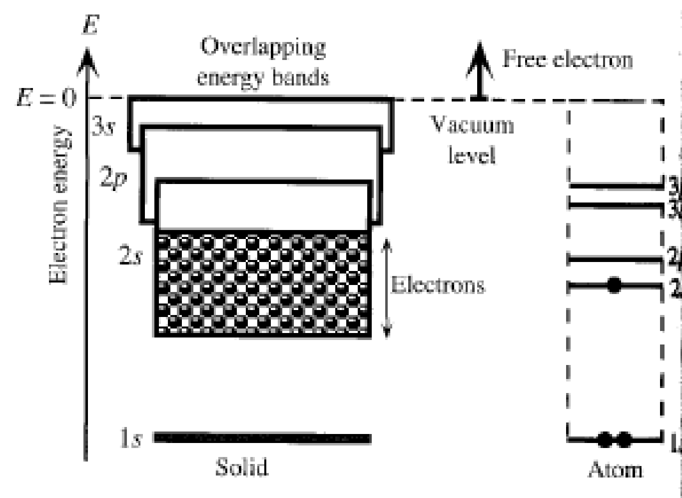


Streetman & Banerjee, Solid State Electronic Devices

Lithium band structure (a metal)

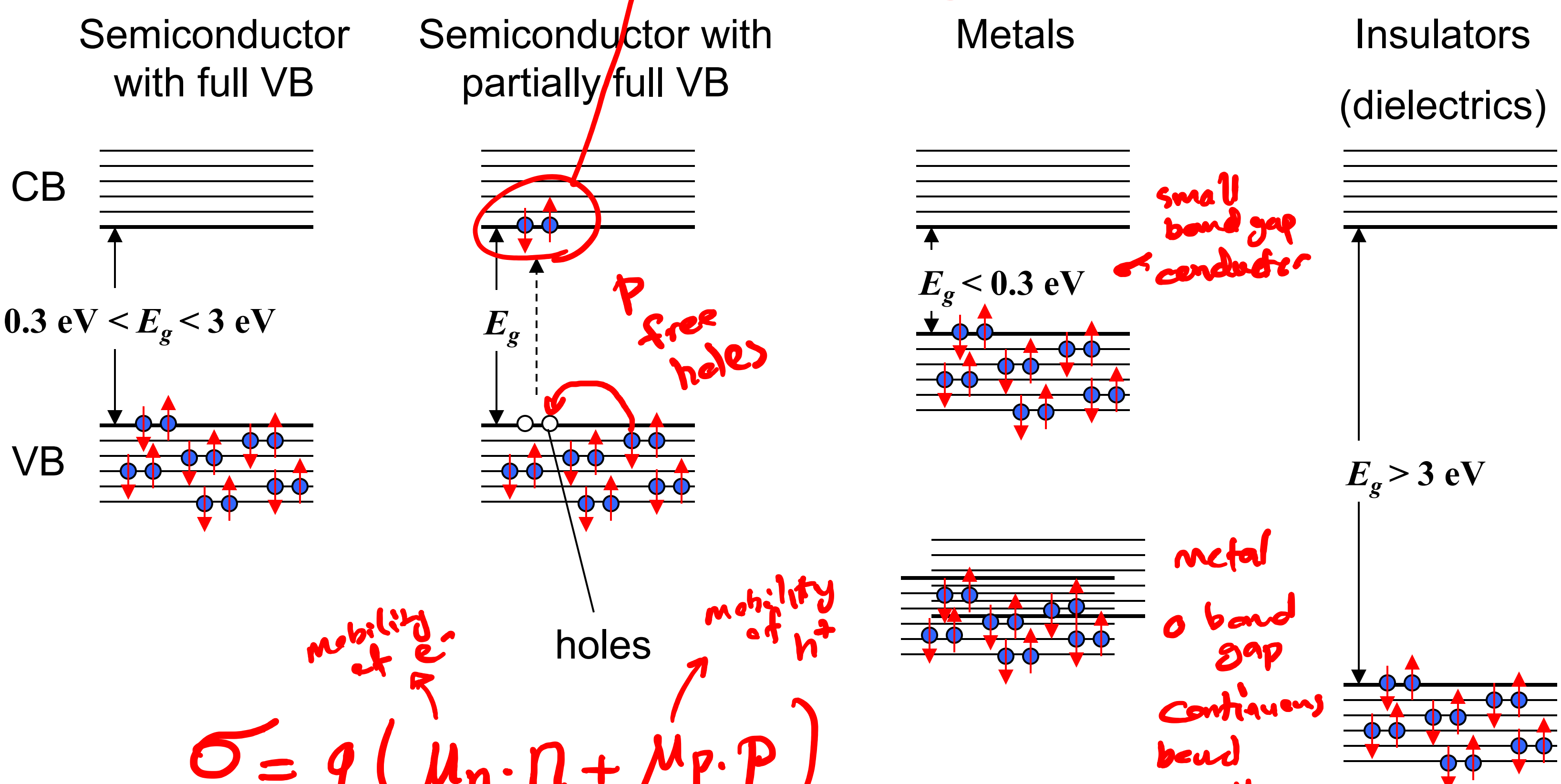
- For a metal like lithium atoms have one electron in the last shell: $1s^2\ 2s^1\ 3s^0$
- When lithium atoms are brought closer to each other, 2s, 2p and 3s orbitals combine and form some energy bands.
- However, these three bands overlap and there is no separation between these bands in lithium, in contrast to silicon.
- The electron in the last shell of each lithium atom fill energy levels from the bottom to the middle of the continuous band.
- The resulting band structure is half full and half empty.





Bandgap and Material Type

T = 300 K (27°C) room temperature



mobility of e^- mobility of h^+

$$\sigma = q(\mu_n \cdot n + \mu_p \cdot p)$$

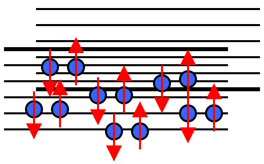
free e^- free h^+

B.V. Zeghbroeck, online book, Chapter 2.3, 2.4, 2.5

P&T, Chapter 2,3,4

Metals

partially-filled band



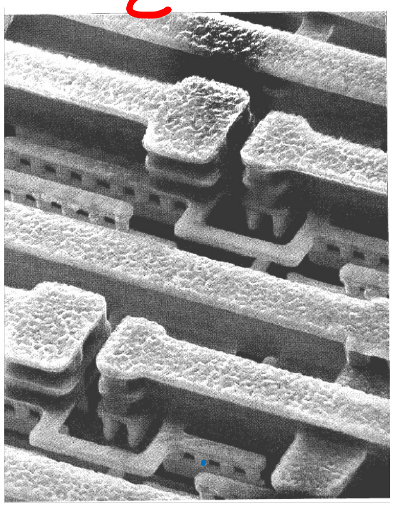
Why metals have such a high conductivity?

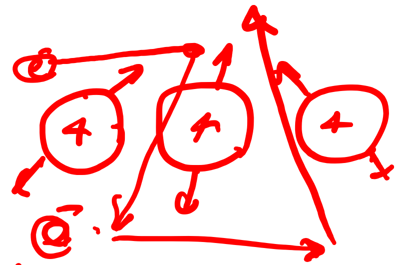
n free electrons are very high

What is the temperature dependence of conductivity in metals?

$\sigma = q \mu_n n$

$\mu_n = \frac{\langle V_d \rangle}{\mathcal{E}}$





lots of collisions → scattering

$T\uparrow \Rightarrow$ more vibration of atoms
more scattering
$\Rightarrow \mu_n \downarrow \Rightarrow \sigma \downarrow$

What is electromigration?

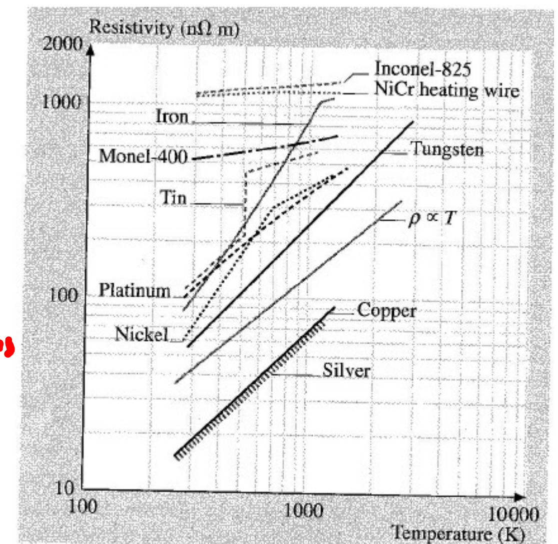


Fig. 2.5 The resistivities of various metals as a function of temperature above 0 °C. Tin melts at 505 K, whereas nickel and iron go through a magnetic-to-nonmagnetic (Curie) transformation at about 627 K and 1043 K, respectively. The theoretical behavior ($\rho \propto T$) is shown for reference.

Streetman & Banerjee, Solid State Electronic Devices

Kasap & Capper, Handbook on Electronic and Photonic Materials

Mobility and Scattering

Generally the electric current density ($J_x(t)$) can be written in terms of carrier density and their drift velocity:

$$J_x(t) = qnv_{dx}(t)$$

electrons → speed of electrons → cm/s

(C) /cm^3

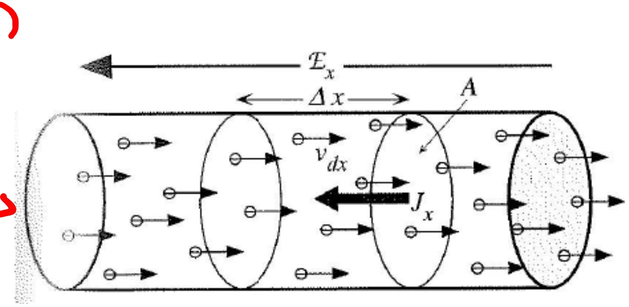
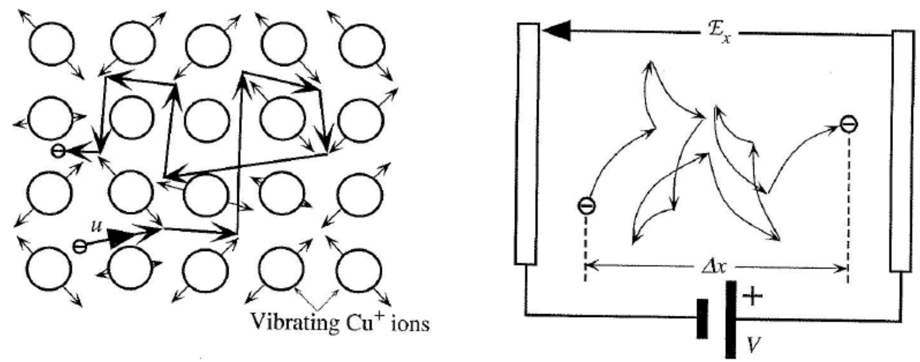


Figure 2.1 Drift of electrons in a conductor in the presence of an applied electric field.

Electrons drift with an average velocity v_{dx} in the x direction.

In conductors the electric field accelerates the electrons and as a result increases their average speed:

$$\langle v_{dx}\rangle = \mu_d E_x \qquad \mu_d = \frac{\langle v_{dx}\rangle}{E_x}$$



(a) A conduction electron in the electron gas moves about randomly in a metal (with a mean speed u) being frequently and randomly scattered by thermal vibrations of the atoms. In the absence of an applied field there is no net drift in any direction.

(b) In the presence of an applied field, $\mathcal{E}_x$, there is a net drift along the x direction. This net drift along the force of the field is superimposed on the random motion of the electron. After many scattering events the electron has been displaced by a net distance, Δx, from its initial position toward the positive terminal.

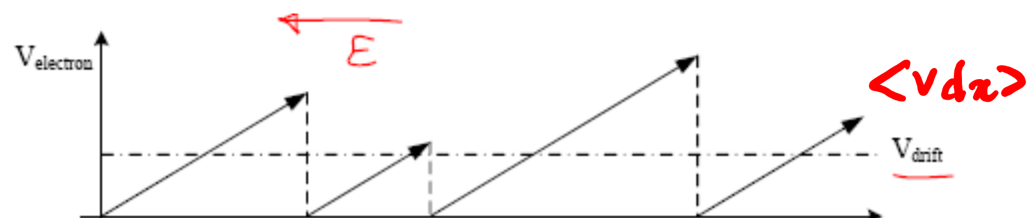
Figure 2.2 Motion of a conduction electron in a metal.

The mobility is proportional to mean time between collisions or mean scattering time τ : $\langle v_{dx}\rangle \propto \tau$

$$\mu_d = \frac{q\tau}{m}$$

τ → scattering time

m → mass of electron



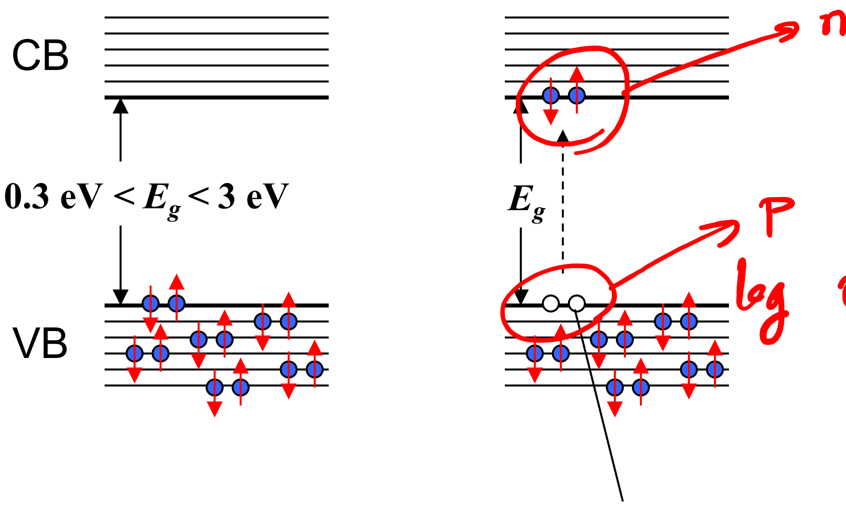
$$\sigma = q\mu_d n$$

metal → semiconductor

$$\sigma = q(\mu_n n + \mu_p p), \quad \mu_n = \frac{q\tau_n}{m_n}, \quad \mu_p = \frac{q\tau_p}{m_p}$$

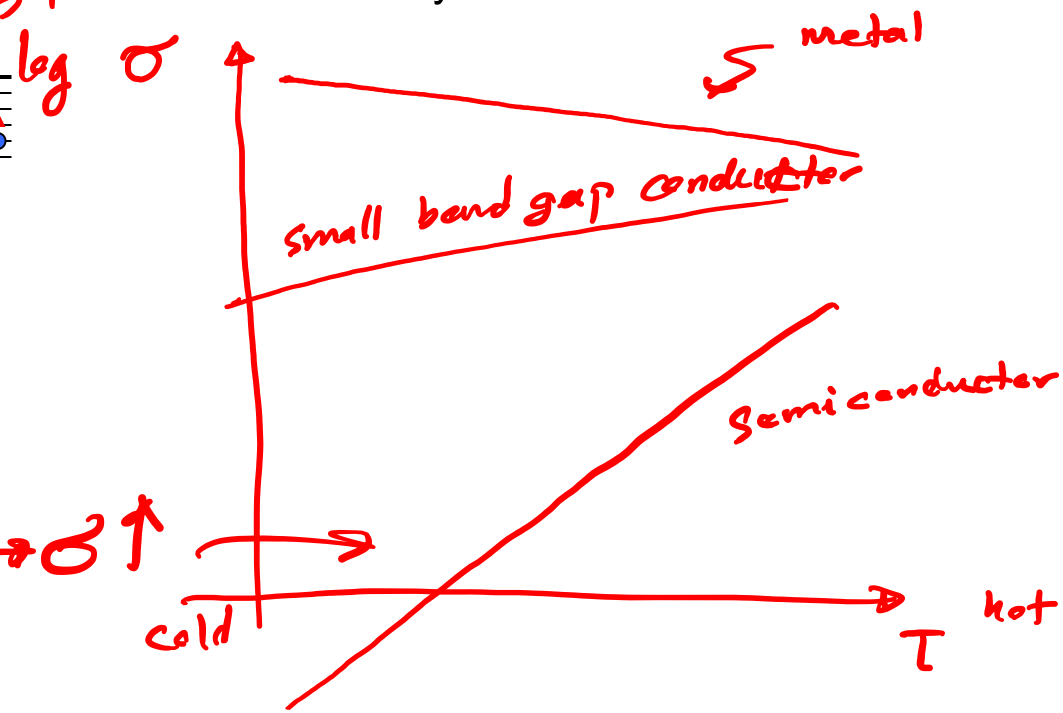
Semiconductors

Semiconductor with full VB	Semiconductor with partially full VB



What is the range of conductivity for semiconductors?

What is the temperature dependence of conductivity in semiconductors?

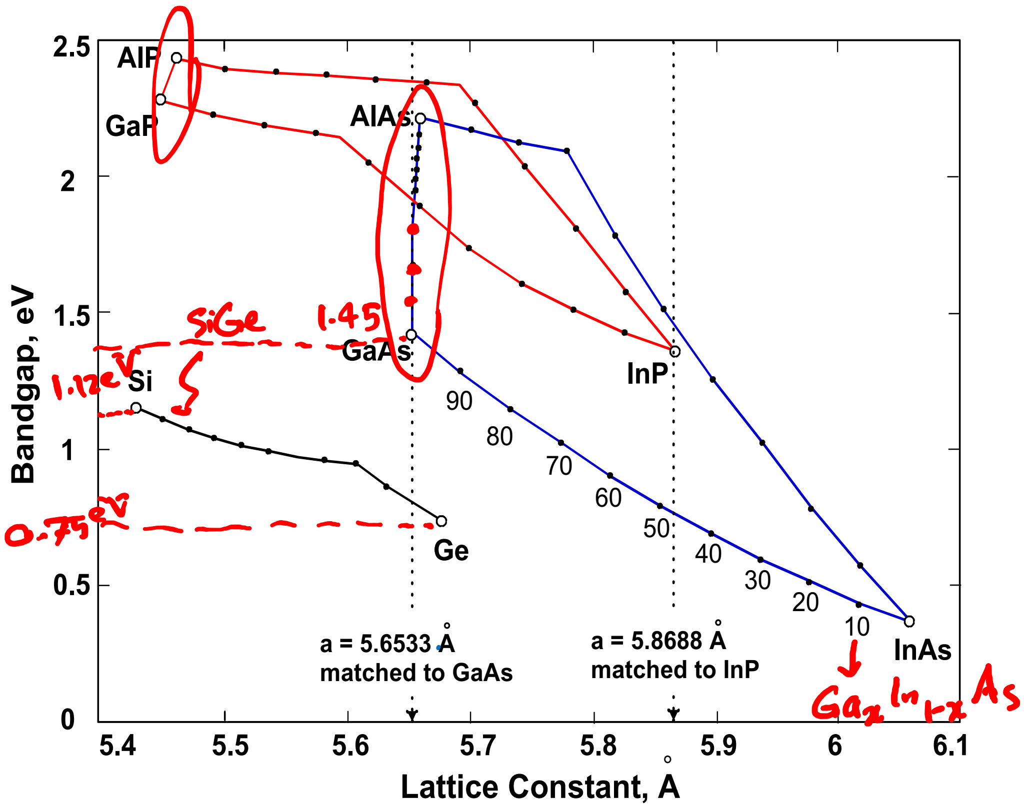


$$\sigma = q(\mu_n n + \mu_p p)$$

$T\uparrow \rightarrow n, p \uparrow$ exponentially $\rightarrow \sigma \uparrow$

Bandgaps of Semiconductors

- The conductivity of semiconductors is mainly determined by their bandgap
- The bandgap is a function of the composition and structure of a semiconductor.
- In compound semiconductors (such as SiGe, GaAs, and GaP), the bandgap changes with the relative ratio of the two materials.



Examples:

- GaP - green LEDs
- GaAs - LNA MESFETs
- AlGaAs/GaAs - HBTs, lasers
- InGaAsP - HEMTs
- SiGe - HBTs, biCMOS
- Si - everything else!

From J.J. Liou

Intrinsic Semiconductors

$n_i = p_i$

- The density (number per unit volume) of carriers (electrons in conduction band (CB) and holes in valence band (VB)) in semiconductors determine their conductivity.

$$\sigma = q\mu_n n + q\mu_p p \longrightarrow \sigma_i = q\mu_n n_i + q\mu_p p_i = q(\mu_n + \mu_p) n_i$$

- In intrinsic (pure) semiconductors (very low impurity and defects), the density of conduction electrons and valence holes at 0K is theoretically zero.

- As temperature increases, thermal energy is enough to move some electrons from VB to CB. As a result, the number of electrons and holes increase exponentially according to Boltzmann Equation. Note that number of holes and electrons are equal.

$$n_i = p_i = f(T) = \overbrace{B}^{\text{constant}} \exp\left(\frac{-E_g}{2kT}\right)$$

$E_g \rightarrow$ band gap
$T \rightarrow$ Temperature
$k \rightarrow$ Boltzmann's constant

T=300 K

$y = Be^{-E_g/2kT} \rightarrow$

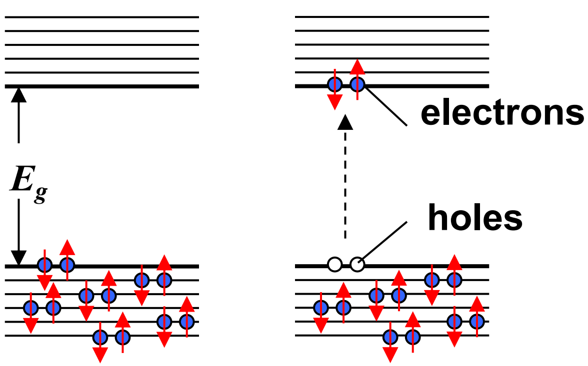
$$\log_{10} y = \log_{10} B - \frac{E_g}{2kT}\log_{10} e = \log_{10} B - \frac{E_g \times \log_{10} e}{2k \times 1000} \times \boxed{\frac{1000}{T}}$$

$x = \frac{1000}{T}$

$$\log_{10} y = a + bx \longrightarrow b = \text{slope} = -\frac{E_g \times \log_{10} e}{2k \times 1000}$$

n

p



T = 0 K

Lower T

Higher T

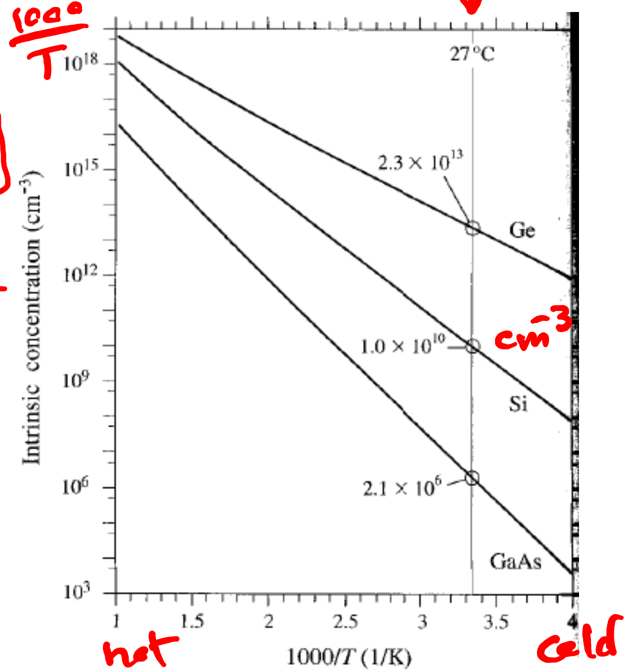


Figure 5.16 The temperature dependence of the intrinsic concentration.

Example

- Find the resistivity of silicon at 100 K and room temperature, knowing its bandgap is 1 eV, density of intrinsic electrons is 10^{10} cm^{-3}, and mobility of electron and holes are 1000 and 500 cm^2/Vs, respectively.

$$\sigma_{300} = q(\mu_n + \mu_p) n_i = 1.6\times10^{-19}(1000+500)\,10^{10} = 2.4\times10^{-6}\ \text{S/cm}\ \ \Omega^{-1}\text{cm}^{-1}$$

$$\rho_{300} = 1/\sigma_{300} = 4.16\times10^{5}\ \Omega.\text{cm}$$

thermal energy $kT = k\times300 = 25.9\ \text{meV}$

thermal voltage $kT/q = V_{th} = 25.9\ \text{mV}$

$$\begin{cases} n_i(100k) = B\, e^{-E_g/2k\times100} \\ n_i(300k) = B\, e^{-E_g/2k\times300} \end{cases}$$

$$n_i(100k) = n_i(300k)\times e^{-\frac{E_g}{2k}\left(\frac{1}{100}-\frac{1}{300}\right)}$$

$$= 10^{10}\times e^{-\frac{E_g}{2k\times300}\left(\frac{300}{100}-1\right)}$$

$$= 10^{10}\times e^{-\frac{1\,\text{eV}}{\cancel{2}\times25.9\,\text{meV}}(\cancel{2})}$$

$$n_i(100) = 1.47\times10^{-7}\ \text{cm}^{-3}$$

$$\sigma_{100} = 3.53\times10^{-23}\ \Omega^{-1}\text{cm}^{-1}$$

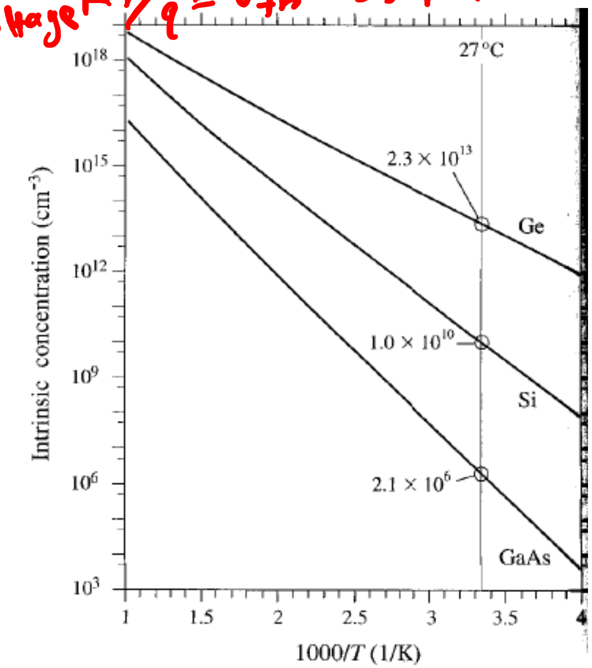


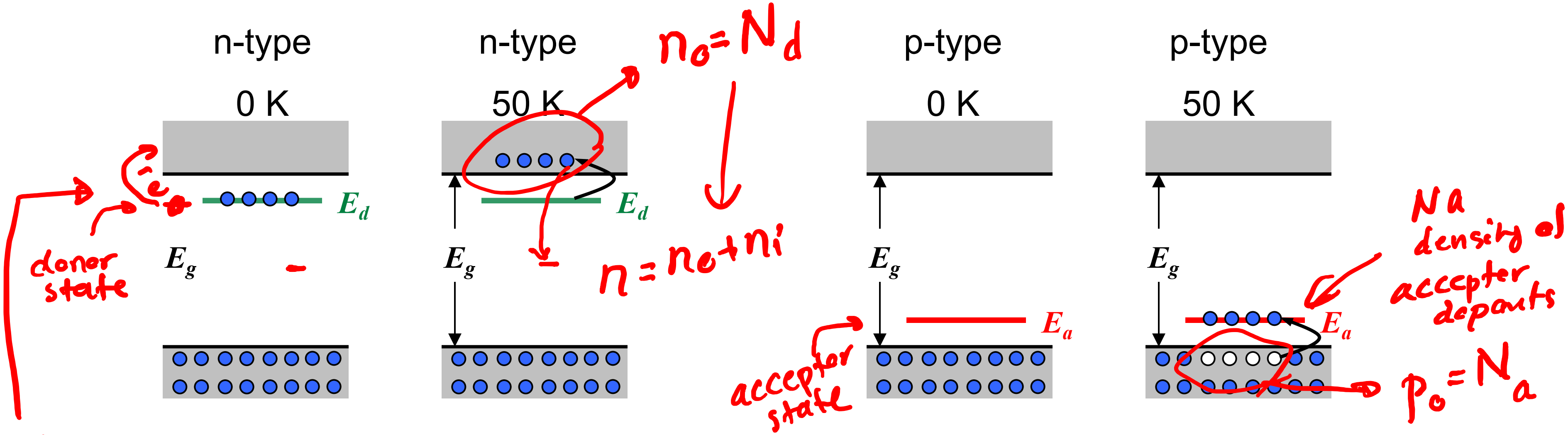
Figure 5.16 The temperature dependence of the intrinsic concentration.

Extrinsic Semiconductor

lithography mask

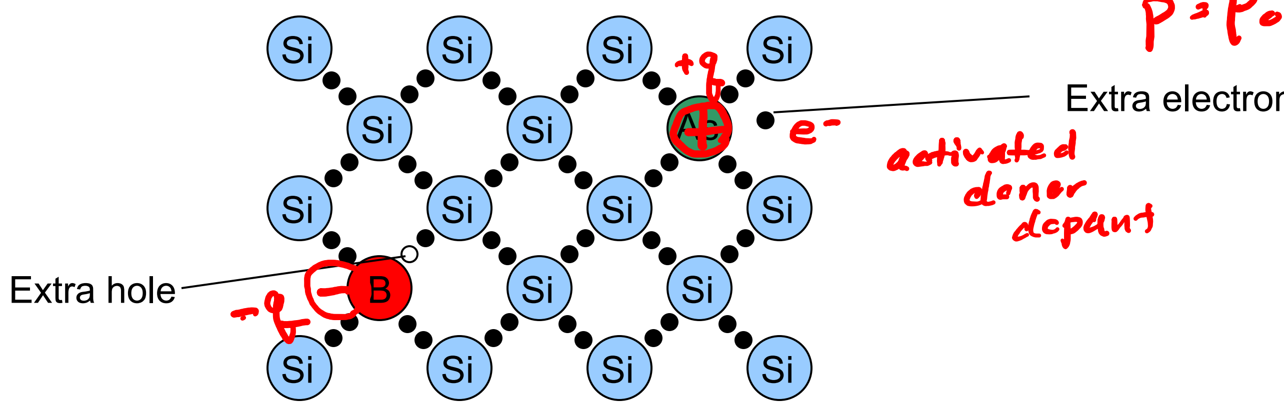
ion implantation

- Adding impurity atoms (B, P, Al, As, …) in silicon crystal modifies the number of electrons and holes in the network.
- N-type dopants have an extra electron. P-type dopants need an electron. They give an electron (n-type) or hole (p-type) to the semiconductor network at a temperature like 50 K.



N_d (density of donor atoms)

$p = p_0 + p_i$



Extrinsic Semiconductor

$\sigma = q\mu_n n + q\mu_p p$

Different type of dopants for silicon are:

n-type: donors for Si and Ge: P, As, and Sb

p-type: acceptors for Si and Ge: B and Al

• It is important to note that when a dopant is ionized (gives electron or hole to the network), there will be a fixed ionic charge (positive for n-type and negative for p-type) at the place of the dopant.

• So if a semiconductor is doped with N_A p-type and N_D n-type dopants, the charge neutrality equation will be in the form of:

intrinsic — donor dopant e^-

total $n = n_i + n_o$ $\qquad p = p_i + p_o$ $\qquad p_o = N_A$

$\approx n_i + N_D$ constant $\qquad \approx n_i + N_A$

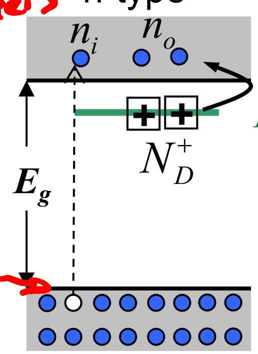
Charge neutrality $n + N_A^- = p + N_D^+ \rightarrow n - p = N_D^+ - N_A^-$

total electrons — total holes

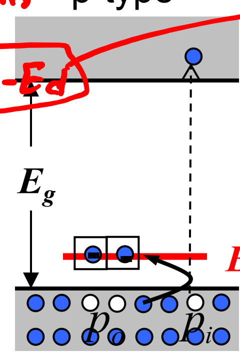
majority carriers

$n_o = N_D$

minority carriers



$\Delta E = E_c - E_d$



minority

majority

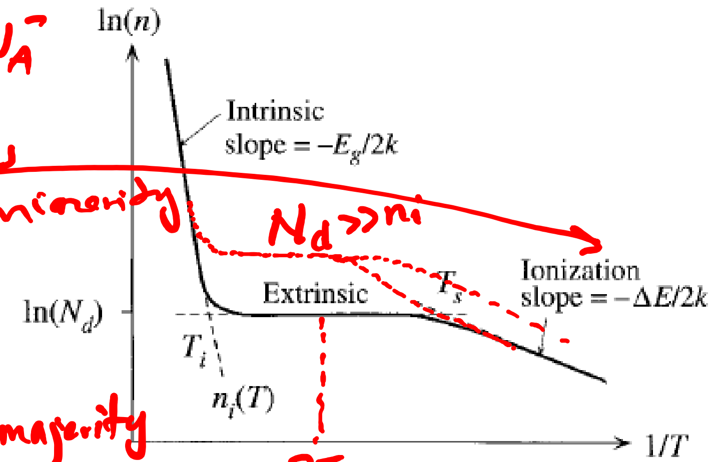


Figure 5.15 The temperature dependence of the electron concentration in an n-type semiconductor.

Fermi-Dirac Distribution

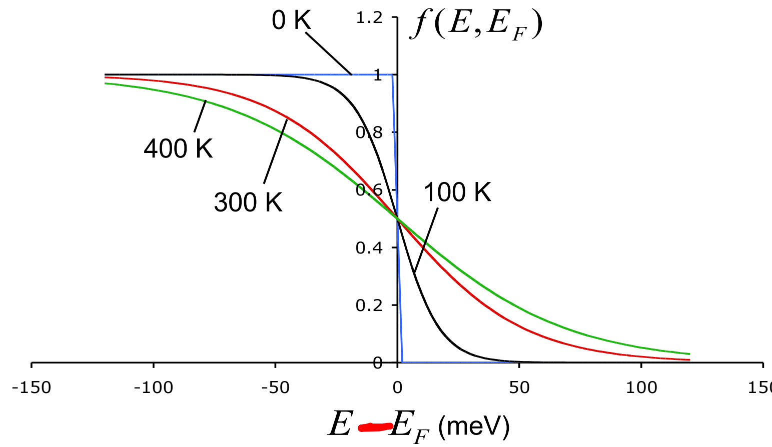
Fermi-Dirac Distribution function $f(E,E_F)$ is used to calculate the density of electrons and holes. This function yields the probability for a state at energy E to be occupied by an electron. This function for a given Fermi energy E_F at temperature T is given by:

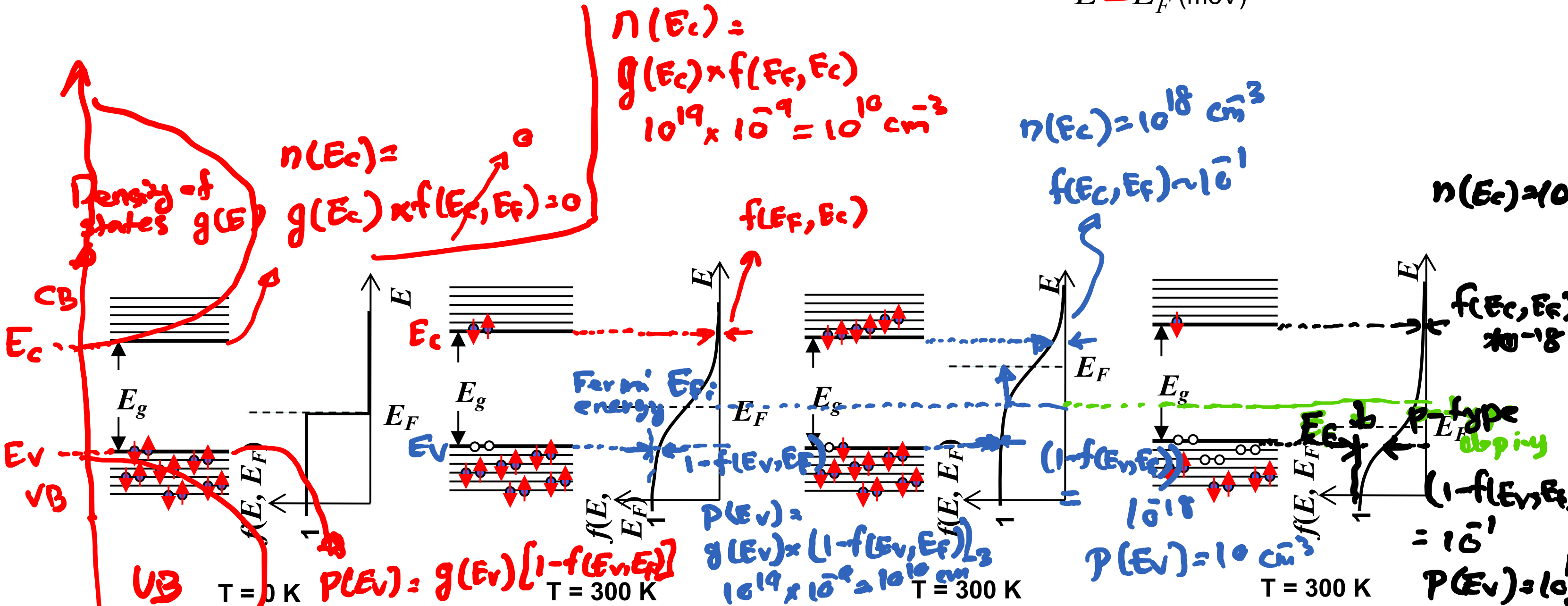
$$f(E,E_F) = \frac{1}{1+e^{(E-E_F)/kT}}$$

Boltzmann's constant

$k = 8.62 \times 10^{-5}$ eV/K

$k = 1.38 \times 10^{-23}$ J/K





At higher temperature   n-type doping   p-type doping

Carrier Concentration

top of CB

Based on Fermi Dirac Distribution, the density of electrons in the conduction band as a function of Fermi energy can be calculated as:

Isoelectronic model

$n(E) = g(E) f(E, E_F) \longrightarrow n_{\text{in CB}} = \int_{E_C}^{\infty} g(E) f(E, E_F)\, dE$

$25.9\ \text{meV} = kT$ thermal energy

assume $E_C - E_F \gg kT$

$$n \approx N_C f(E_C, E_F) = N_C \frac{1}{1+e^{(E_C-E_F)/kT}} \approx N_C e^{-(E_C-E_F)/kT}$$

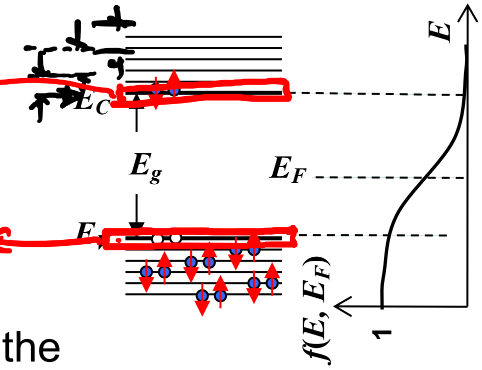
Density of states of the conduction band N_C is given by:

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$\boxed{n = N_C\, e^{\frac{E_F - E_C}{kT}}}$$

N_C

N_V



where effective electron mass $m_n^* = 1.38\, m_0$ and $m_0 = 9.1\times 10^{-31}$ kg is the mass of free electron. For silicon $N_C = 3.22\times 10^{19}$ /cm³ at room temperature.

Density of holes is given by

$p(E) = g(E)\,[1 - f(E, E_F)] \rightarrow p_{\text{in VB}} = \int_{-\infty}^{E_V} g(E)\,[1 - f(E, E_F)]\, dE$

$$p \approx N_V[1 - f(E_V, E_F)] \approx N_V\left[1 - \frac{1}{1+e^{(E_V-E_F)/kT}}\right] \approx N_V e^{-(E_F-E_V)/kT}$$

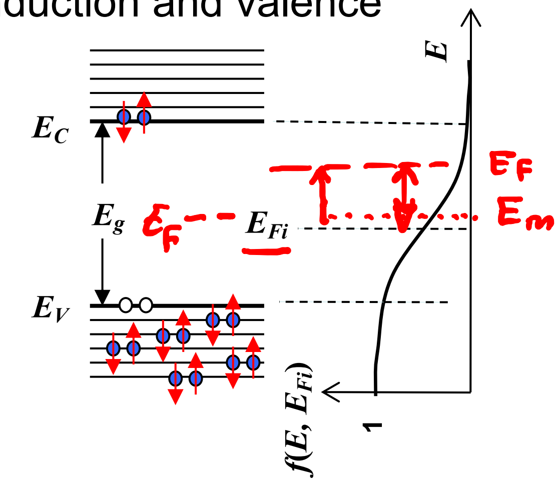
$E_F - E_V \gg kT$

$$\boxed{p = N_V\, e^{-\frac{(E_F - E_V)}{kT}}}$$

where $N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$ is density of states in the valence band and effective hole mass $m_p^* = 0.946\, m_0$. For silicon $N_V = 1.83\times 10^{19}$ /cm³ at room temperature.

Example

Find the Fermi energy E_{Fi} for an intrinsic semiconductor with conduction and valence band densities of N_C and N_V, respectively.



$$n = N_C e^{(E_F - E_C)/kT} \xrightarrow[\text{intrinsic}]{\text{for}} n_i = N_C e^{(E_{Fi} - E_C)/kT}$$

$$p = N_V e^{(E_V - E_F)/kT} \xrightarrow[\text{''}]{\text{''}} p_i = N_V e^{(E_V - E_{Fi})/kT}$$

$$n_i = p_i \rightarrow N_C e^{(E_{Fi} - E_C)/kT} = N_V e^{(E_V - E_{Fi})/kT}$$

$$\ln N_C + \frac{E_{Fi} - E_C}{kT} = \ln N_V + \frac{-E_{Fi} + E_V}{kT} \rightarrow$$

middle of bandgap

$$E_{Fi} = \frac{E_C + E_V}{2} + \frac{kT}{2} \ln \frac{N_V}{N_C}$$

Density of electrons and holes for an arbitrary Fermi energy can be written with respect to E_{Fi} :

$$\boxed{n = N_C e^{(E_F - E_C)/kT}} \text{ (all cases)}$$

$$n_i = N_C e^{(E_{Fi} - E_C)/kT} \text{ (intrinsic)}$$

$$\rightarrow \frac{n}{n_i} = e^{(E_F - E_{Fi})/kT} \rightarrow \boxed{n = n_i e^{\frac{E_F - E_{Fi}}{kT}}}$$

$$\boxed{p = N_V e^{(E_V - E_F)/kT}} \text{ (all cases)}$$

$$p_i = N_V e^{(E_V - E_{Fi})/kT} \text{ (intrinsic)}$$

$$\rightarrow \frac{p}{p_i} = e^{(E_{Fi} - E_F)/kT} \rightarrow \boxed{p = n_i e^{\frac{E_{Fi} - E_F}{kT}}}$$

Find the product $np =$ $n_i e^{\cancel{\frac{E_F - E_{Fi}}{kT}}} \times n_i e^{\cancel{\frac{E_{Fi} - E_F}{kT}}} = n_i^2$

Fermi Energy for Doped Semiconductors

The Fermi energy for a n-type semiconductor doped by dopant atoms with a density of N_D can be written as:

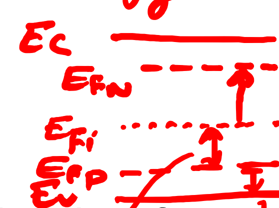
$n = N_D$

$n = N_D = N_C e^{(E_F - E_C)/kT} \longrightarrow \ln N_D = \ln N_C + \frac{E_F - E_C}{kT} \longrightarrow$

kT @ 300 K = 25.9 meV thermal energy

$$\boxed{E_F = E_C - kT \ln \frac{N_C}{N_D}}$$

$n = N_D = n_i e^{\frac{E_F - E_{Fi}}{kT}} \longrightarrow \boxed{E_F = E_{Fi} + kT \ln \frac{N_D}{n_i}}$



The Fermi energy for a p-type semiconductor doped by dopant atoms with a density of N_A can be written as:

$p = N_A = N_V e^{(E_V - E_F)/kT} \longrightarrow \ln N_A = \ln N_V + \frac{E_V - E_F}{kT} \longrightarrow$

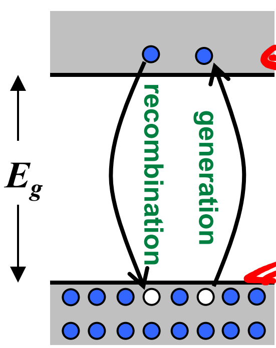
$$\boxed{E_F = E_V + kT \ln \frac{N_V}{N_A}}$$

$N_A = p = n_i e^{\frac{E_{Fi} - E_F}{kT}} \longrightarrow \boxed{E_F = E_{Fi} - kT \ln \frac{N_A}{n_i}}$

Recombination and Generation

Thermal generation is the process of electrons gaining energy and moving to the conduction band. Electrons in the conduction band can recombine with holes in the valence band through recombination process. The equilibrium between these two processes leads to a stable electron and hole density.

		intrinsic	extrinsic
For intrinsic material:	$p_i = n_i$		
Generation rate: density of electrons generated /s/cm^3	$g_{th}(T) = g_i(T)$ thermal generation	$g_{th}(T)$	$g_{th}(T)$
Recombination rate: density of electrons recombined /s/cm^3	$r(T) = \alpha_r np$ coefficient of recombination	$\alpha_r n_i p_i = \alpha_r n_i^2$	$\alpha_r n_o p_o$
Equilibrium: condition	$g_{th}(T) = r(T)$	$g_{th}(T) = \alpha_r n_i^2$	$g_{th}(T) = \alpha_r n_o p_o$



$$\frac{dn}{dt} = g - r$$

electron hole pair

$$\frac{dp}{dt} = g - r$$

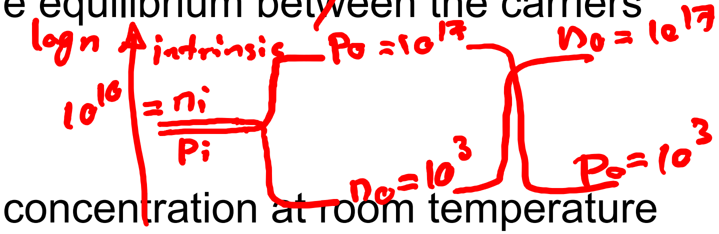
$$\alpha_r n_i^2 = \alpha_r n_o p_o \rightarrow$$

$$\boxed{n_o p_o = n_i^2}$$

Majority and Minority Carriers

In an extrinsic semiconductor, the dopants determine the number of majority carriers. The number of minority carriers is determined by the equilibrium between the carriers through recombination.

p-type doping



$$n_o p_o = n_i^2$$

Example: What is the minority and majority carrier concentration at room temperature in silicon doped with: $n_i = 10^{10}\ cm^{-3}$

	Boron	Phosphorus
1)	$N_a = 10^{17}$ cm^{-3}	–
2)	–	$N_d = 10^{17}\ cm^{-3}$
3)	10^{17}	10^{19}

$p_o = N_a = 10^{17}\ cm^{-3}$ majority

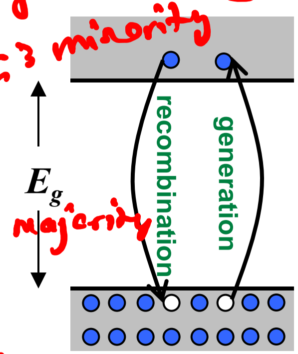
$p_o n_o = n_i^2 \rightarrow n_o = \frac{n_i^2}{p_o} = \frac{(10^{10})^2}{p_o} = \frac{10^{20}}{10^{17}} = 10^3\ cm^{-3}$

$\sigma = q\mu_n n_o + q\mu_p p_o = \boxed{q\mu_p p_o}$ minority

$n_o = N_d = 10^{17}\ cm^{-3}$ majority

$p_o = \frac{n_i^2}{n_o} = \frac{10^{20}}{10^{17}} = 10^3\ cm^{-3}$ minority

$\sigma = q\mu_n n_o$



$$\boxed{\begin{array}{l} n_o + N_A^- = p_o + N_D^+ \\ p_o n_o = n_i^2 \end{array}}$$

$\rightarrow n_o = N_D^+ - N_A^- = 10^{19} - 10^{17} = 9.9\times10^{18}\ cm^{-3}$

$\rightarrow p_o = \frac{n_i^2}{n_o} = \frac{10^{20}}{9.9\times10^{18}} = 11\ cm^{-3}$

$\sigma = q\mu_n n_o$

Excess Carriers, Direct Recombination

If we have a semiconductor under equilibrium: $n_0 p_0 = n_i^2$

External energy sources such as light can also give energy to electrons to move from valence to conduction band. This generates an electron and a hole. We call these carriers excess carriers and the condition for equilibrium is not held anymore: $np \neq n_i^2$

Holes: $p = p_0 + \hat{p}$

$r(T) = \alpha_r np$

Electrons: $n = n_0 + \hat{n}$

n-type

Recombination rate: $r(T) = \alpha_r (p_0 + \hat{p})(n_0 + \hat{n})$

$= \alpha_r p_0 n_0 + \alpha_r p_0 \hat{n} + \alpha_r n_0 \hat{p} + \alpha_r \hat{p}\hat{n}$ → high carrier injection term

$\boxed{g_{th} - \alpha_r n_0 p_0 = 0}$

$\approx \alpha_r p_0 n_0 + \alpha_r p_0 \hat{n} + \alpha_r n_0 \hat{p}$

p-type

n-type: $n_0 = N_D$

p-type $p_0 = N_A$

light energy → $\hat{n}(t) = \hat{p}(t)$

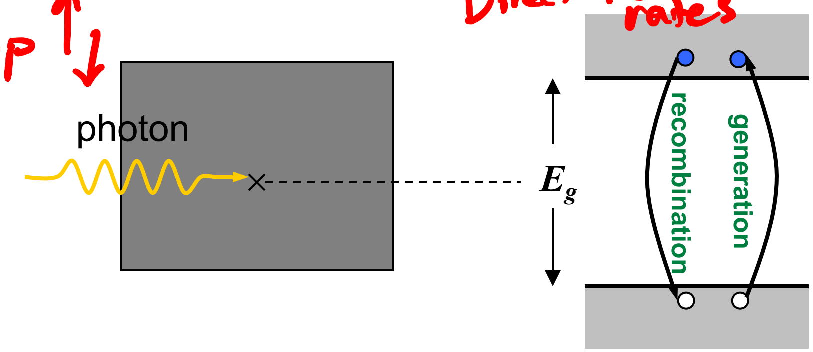
$r(T) = \alpha_r N_D \hat{p}$

$r(T) = \alpha_r N_A \hat{n}$

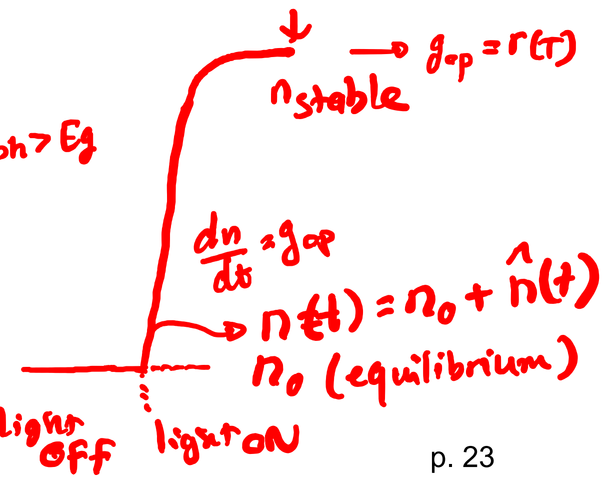
Direct recombination rates

$g = g_{th} + g_{op}$ ↑↓

$\frac{dn}{dt} = g - r$



$E_{ph} > E_g$



Indirect Recombination

Direct recombination is the process of electrons in the conduction band recombining directly with holes in the valence band.

Another mechanism of recombination is indirect recombination. The electrons from conduction band move a defect (trap) level in the middle of the gap and then recombine with a hole in the valence band.

density of trap states

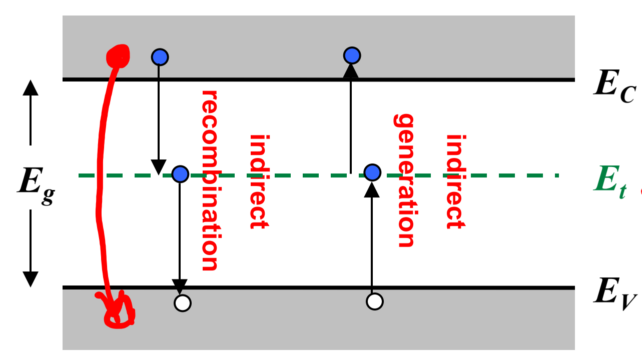
Recombination rate:

$$r_n(T) = \alpha_n N_t \hat{n}$$

$$r_p(T) = \alpha_p N_t \hat{p}$$

radiative recombination

non-radiative recombination



trap states
deep states

Quasi Fermi Level

When a semiconductor is under light illumination, the photons give energy to electrons to move them from valence band to conduction band. This leaves a pair of electron and hole. As a result of this the conductivity of semiconductor increases upon exposure of light.

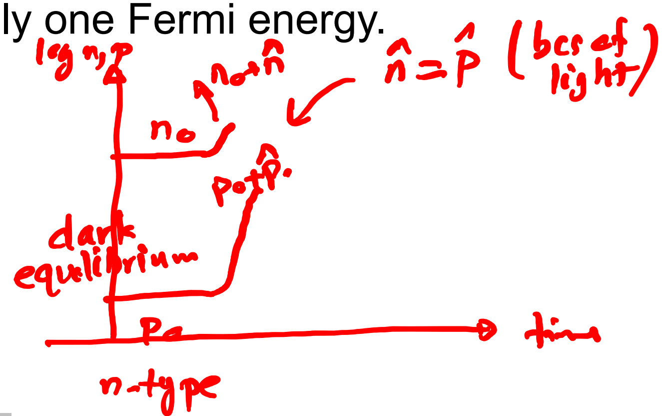
For this non-equilibrium conditions, we can define quasi Fermi energies for both electrons and holes. Remember in equilibrium there is only one Fermi energy.

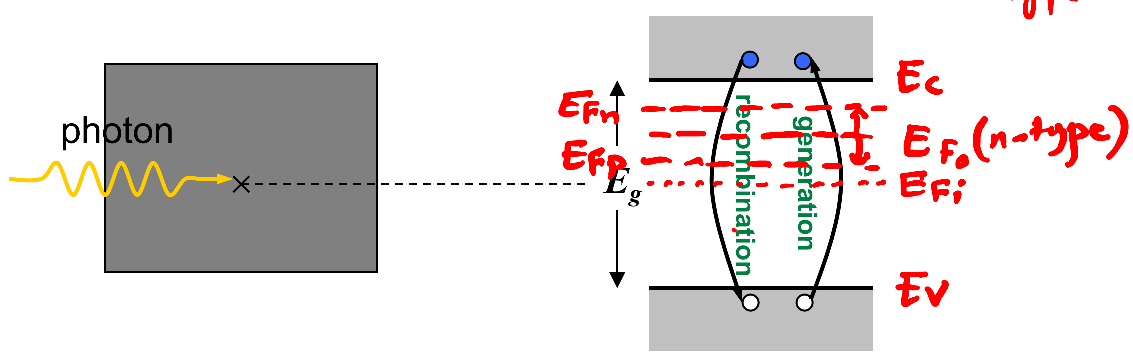
$$E_{Fn} = E_{Fi} + kT \ln \frac{n}{n_i} = E_{Fi} + kT \ln \frac{n_0 + \hat{n}}{n_i}$$

$n = n_0 + \hat{n}$

$p = p_0 + \hat{p}$

$$E_{Fp} = E_{Fi} - kT \ln \frac{p}{n_i} = E_{Fi} - kT \ln \frac{p_0 + \hat{p}}{n_i}$$





Minority Carrier Lifetime

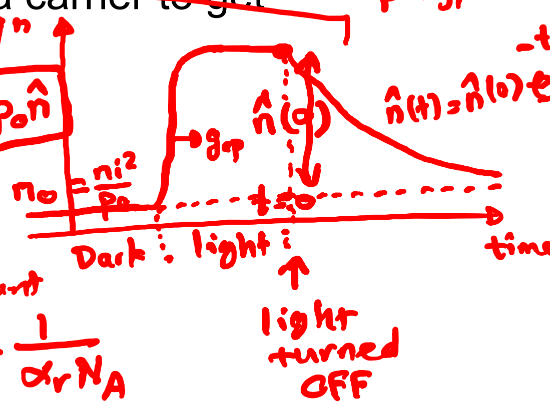
Excess carriers (electrons or holes) generated in a semiconductor under non-equilibrium conditions, the carriers remain until they get recombined. The transient response of excess carrier density is characterized by a parameter called *carrier lifetime*. Carrier lifetime τ is the average time that takes for a carrier to get recombined.

$p_o = N_A$ p-type

$n = n_o + \hat{n}(t)$ $\quad g - r < 0$

For total electron density: $\frac{dn(t)}{dt} = g(T) - r(T) = g_{th} - \alpha_r n_o p_o - \boxed{\alpha_r p_o \hat{n}}$

$\frac{d\hat{n}}{dt} = -\alpha_r p_o \hat{n}$



Excess electron density:(p-type)

$$\frac{d\hat{n}(t)}{dt} = -\alpha_r p_0 \hat{n}(t)$$

$$\hat{n}(t) = \hat{n}(0)e^{-t/\tau_n} \qquad \tau_n = \frac{1}{\alpha_r p_o} = \frac{1}{\alpha_r N_A}$$

time constant

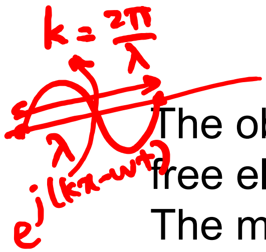
Excess hole density:(n-type)

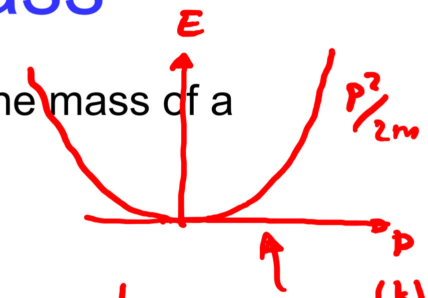
$$\frac{d\hat{p}(t)}{dt} = -\alpha_r n_0 \hat{p}(t)$$

$$\hat{p}(t) = \hat{p}(0)e^{-t/\tau_p} \qquad \tau_p = \frac{1}{\alpha_r n_o} = \frac{1}{\alpha_r N_D}$$

	Direct recombination	Indirect recombination
electrons	$\tau_{nd} = \frac{1}{\alpha_r N_A}$	$\tau_n = \frac{1}{\alpha_r N_A + \alpha_n N_t}$ (direct; indirect)
holes	$\tau_{pd} = \frac{1}{\alpha_r N_D}$	$\tau_p = \frac{1}{\alpha_r N_D + \alpha_p N_t}$ (direct; indirect)

Density of States Effective Mass





$\frac{d^2E}{dp^2} = \frac{1}{m}$

The observed mass of electrons in a semiconductor is different from the mass of a free electron m_0.

The momentum is given by $p = \underbrace{\hbar k}_{\text{quantum}} = \underbrace{m^* v_e}_{\text{classical}}$ → mass → speed

$$\frac{1}{2} m^* v_e^2 = E = \frac{p^2}{2m^*} \qquad \frac{dE}{dk} = \frac{2p}{2m^*}\frac{dp}{dk} = \frac{p}{m^*}\hbar = v_e \hbar \qquad v_e = \frac{1}{\hbar}\frac{dE}{dk} \;\rightarrow\; \text{speed}$$

Using classical formula to define the mass of a particle:

acceleration

$$F_{ext} = \boxed{m^*\frac{dv_e}{dt} = \frac{dp}{dt} = \hbar\frac{dk}{dt}} \qquad m^*\frac{dv_e}{dk}\cdot\cancel{\frac{dk}{dt}} = \hbar\cancel{\frac{dk}{dt}} \qquad m^*\cdot\left(\frac{1}{\hbar}\frac{d^2E}{dk^2}\cdot\right)\frac{dk}{dt} = \hbar\frac{dk}{dt}$$

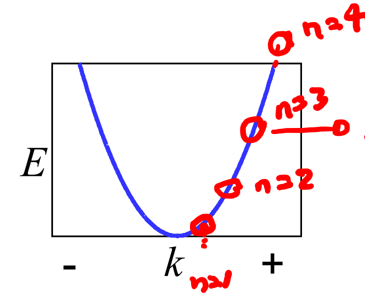
$$\boxed{m^* = \frac{\hbar^2}{d^2E/dk^2}}$$

For example, in an infinite potential well:

quantum numbers

$$k = \frac{n\pi}{L} \qquad E_n = \frac{n^2\hbar^2\pi^2}{2mL^2} \qquad E_n = \frac{k_n^2\hbar^2}{2m}$$

$$m^* = \frac{\hbar^2}{d^2E/dk^2} = \frac{\hbar^2}{\hbar^2/m} = m$$



$\frac{\hbar^2 k^2}{2m}$

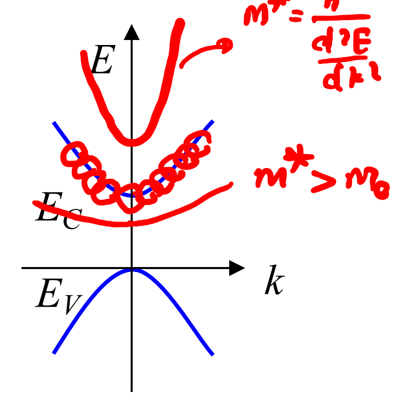
light weight

$m^* < m_0$

$m^* = \frac{\hbar^2}{d^2E/dk^2}$

$m^* > m_0$

In a semiconductor, we can define effective masses for electrons m_n^* and holes m_p^*:



Indirect and Direct Bandgap

Si and Ge has indirect bandgaps, the minimum of conduction band and maximum of valence band do not line up in E-k diagram. GaAs has a direct bandgap.

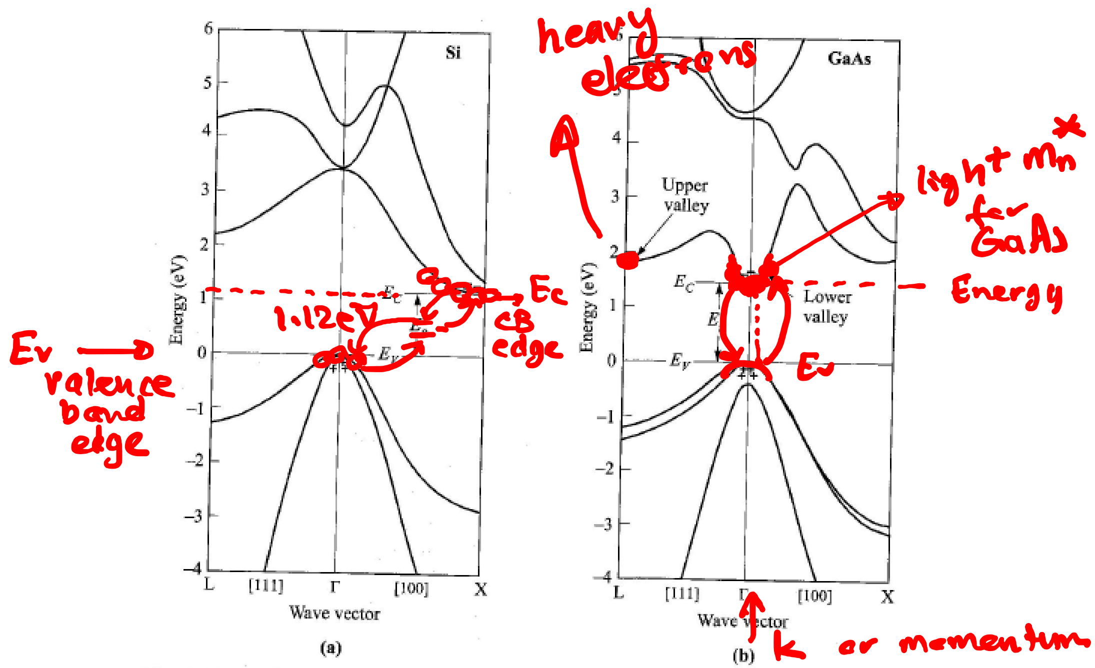


Fig. 4 Energy-band structures of (a) Si and (b) GaAs, where E_g is the energy bandgap. Plus signs (+) indicate holes in the valence bands and minus signs (–) indicate electrons in the conduction bands. (After Ref. 20.)

Different conduction band minima correspond to different effective masses.

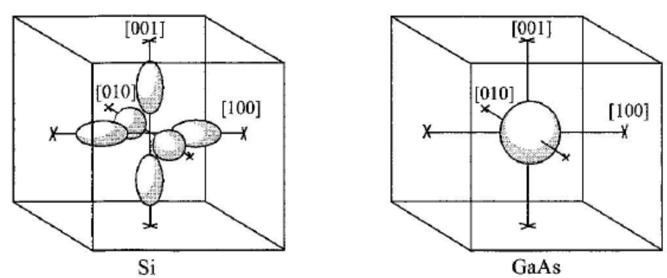


Fig. 5 Shapes of constant-energy surfaces for electrons in Si and GaAs. For Si there are six ellipsoids along the ⟨100⟩-axes with the centers of the ellipsoids located at about three-fourths of the distance from the Brillouin zone center. For GaAs the constant-energy surface is a sphere at zone center. (After Ref. 21.)

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