ELEC 315: Dielectrics

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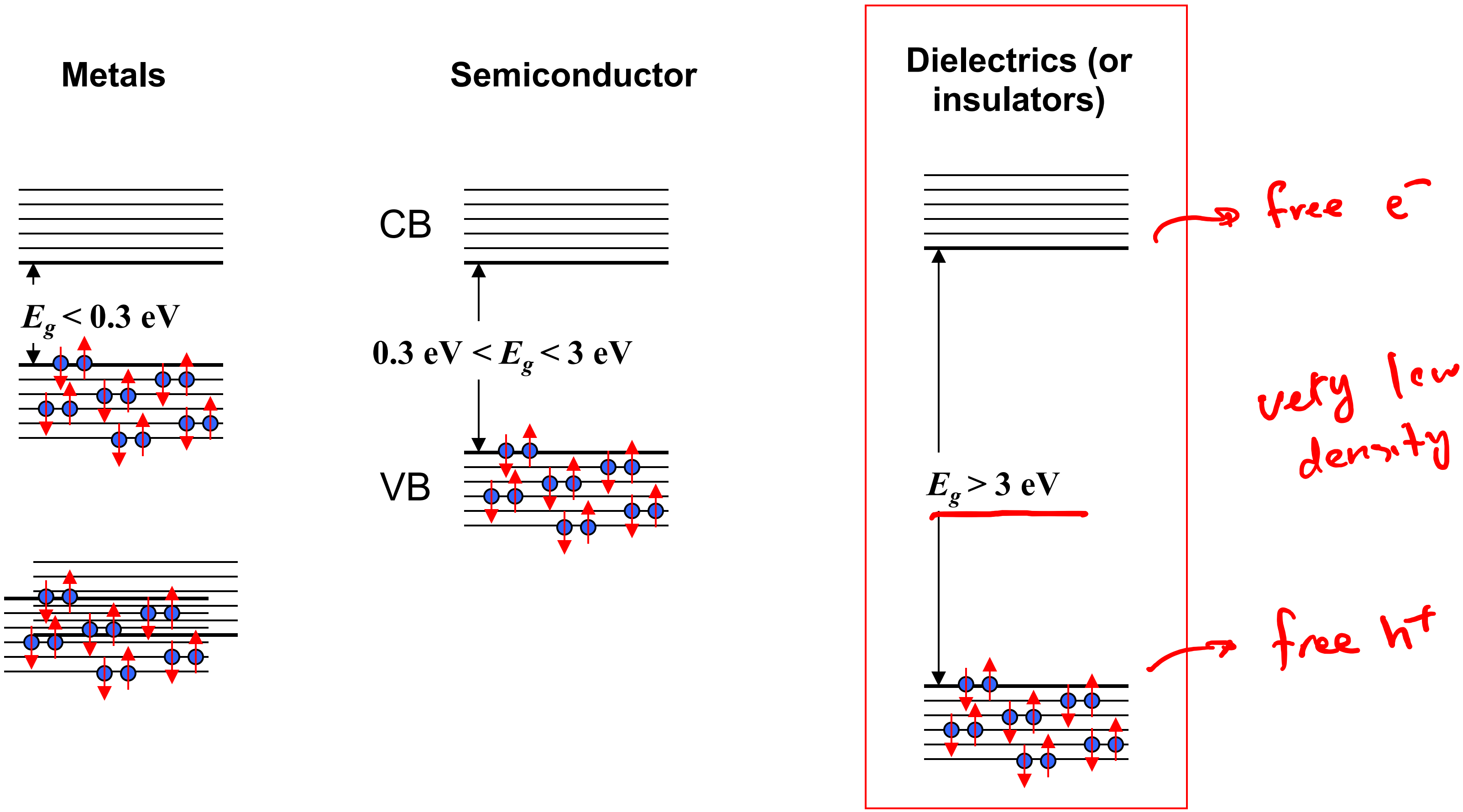
Dielectrics

Dielectrics (or insulators) have large band gaps.

A large band gap means small number of carriers and low conductivity.

Light absorption:

Visible light wavelength is 400 to 780 nm (energy per photon: 3.1 eV to 1.6 eV).
So the energy is not enough to create electron-hole pairs (EHP) in dielectrics.
As a result, a lot of dielectrics look transparent to visible light.



Conductivity

Conductivity is due to the presence of free carriers: $\sigma = q\mu_n n + q\mu_p p$

To find the density of free electrons and holes similar to semiconductors we can write:

$$n(E_F) = \int_{E_C}^{\infty} g(E) f(E)\, dE = N_C \exp\left(\frac{E_F - E_C}{kT}\right) \qquad p(E_F) = \int_{0}^{E_V} g(E)\left[1 - f(E)\right] dE = N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

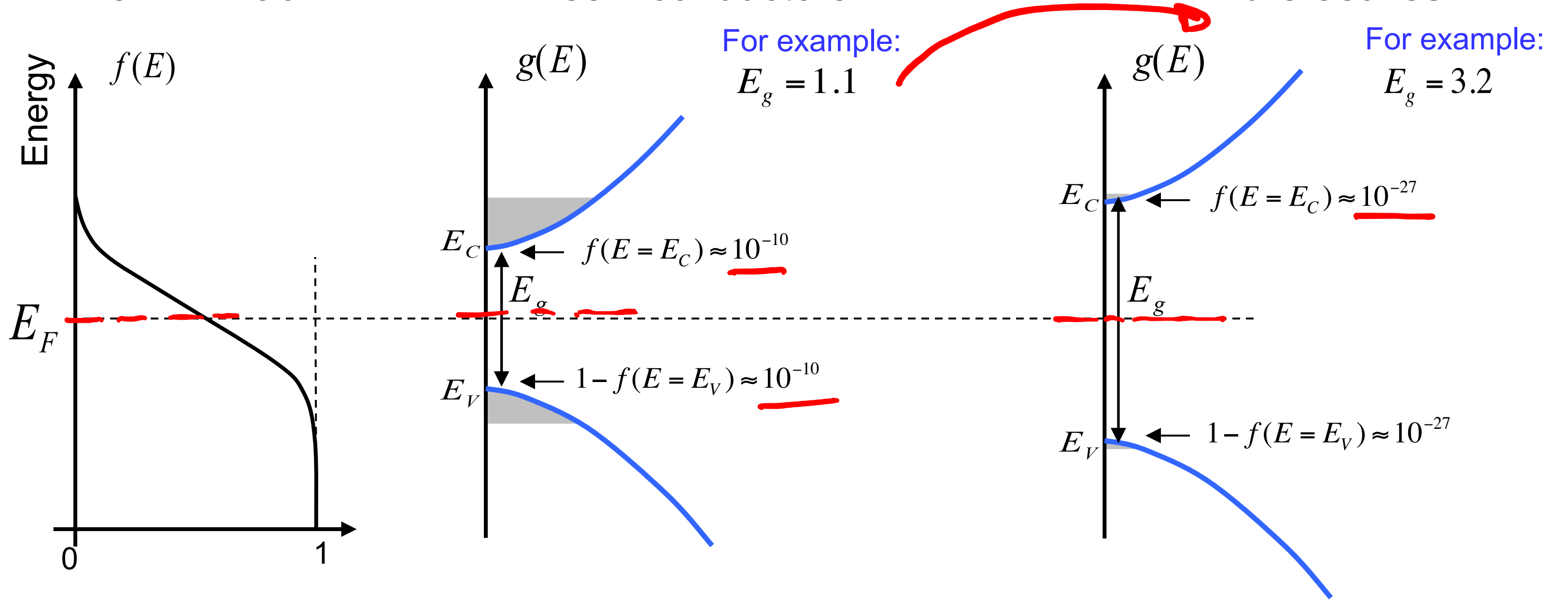
$g(E)$ is the density of states

If we assume Fermi energy is in the middle of the gap for a semiconductor and a dielectric:

$f(E)$ the probability for occupation of states in conduction band

1-$f(E)$ the probability for unoccupied states in valence band

Fermi-Dirac semiconductors 2.1 eV **dielectrics**



Dielectric Constant

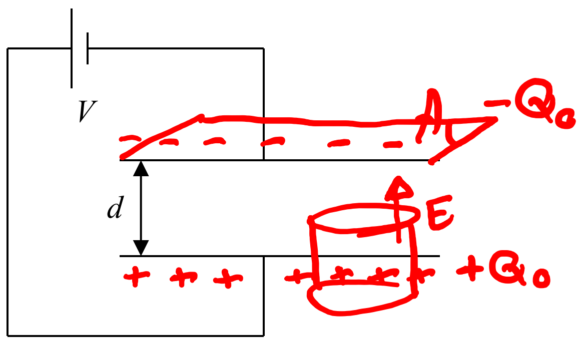
Dielectric constant ϵ relates the electric flux density D to electric field:

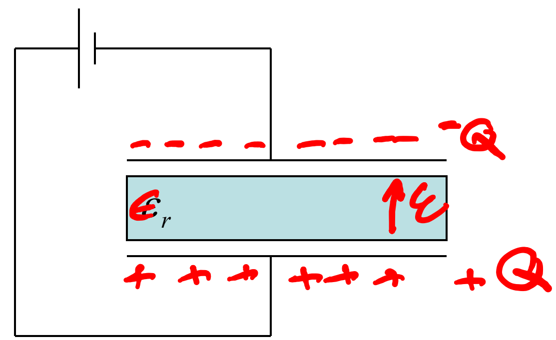
relative dielectric constant

$$D = \epsilon E = \epsilon_0 \epsilon_r E$$

vacuum dielectric

Experiment on parallel plate capacitance:





For charge and capacitance per unit area we have:

charge surface density

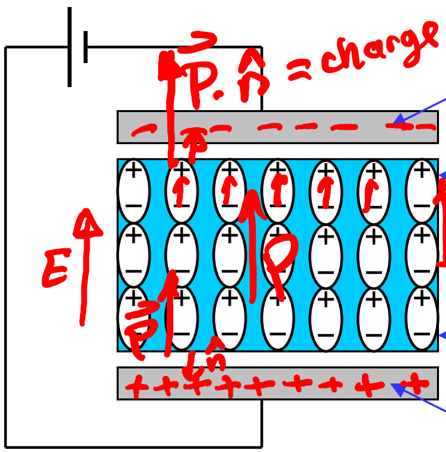
$$\frac{Q_0}{A} = \epsilon_0 E = \epsilon_0 \frac{V}{d} \qquad \frac{Q}{A} = \epsilon_0 \epsilon_r \frac{V}{d}$$

$$C_0 = \frac{Q_0}{V} = \frac{\epsilon_0 A}{d} \qquad C = \frac{Q}{V} = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$\boxed{\epsilon_r = \frac{Q}{Q_0} = \frac{C}{C_0}}$$

Polarization

The increase in capacitance or charge is due to polarization of the dielectric medium:



$\vec{P}\cdot\hat{n}$ = charge density

$$-Q = -Q_0 - Q_P$$

Q_P → surface charge

$$-Q_P$$

$$Q = Q_0 + Q_P$$

$\vec{E} \Rightarrow$ dipole

cause

Electric dipole

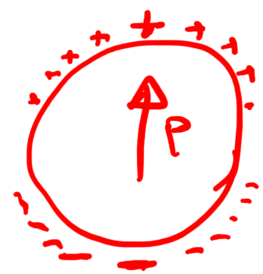
$\vec{p} = q\vec{d}$

Polarization vector

$$\vec{P} = \frac{\sum_i \vec{p}_i}{Volume}$$

$$P = \epsilon_0 \chi \mathcal{E}$$

Dielectric surface charge density P:



$$P = D - \epsilon_0 E$$

$$P = \epsilon_0(\epsilon_r - 1)E = \epsilon_0 \chi E$$

$U = c/n$

Dielectric susceptibility:

$$\chi = \epsilon_r - 1$$

Refractive index:

$$n = \sqrt{\epsilon_r \mu_r} = \sqrt{\epsilon_r}$$

$\mu_r = 1$ for most materials that transmit light.

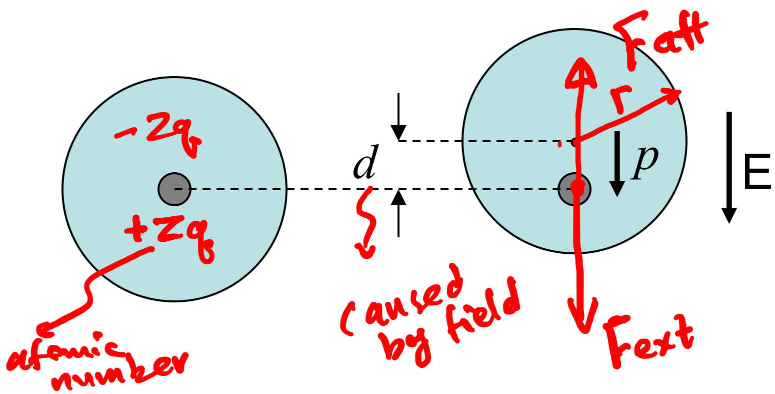
Microscopic Polarization

At microscopic level, different mechanisms are responsible for polarization:

1. Electronic polarization: Atoms are surrounded with electron cloud. Electrons have much smaller mass and respond to electric field easily. Under external electric field, the centre of mass for electrons is displaced by d resulting in a dipole p:

$$p = Zqd$$

$$\boxed{\vec{p} = \alpha \vec{E} \qquad \alpha \text{ is called } \textbf{polarizability}.}$$



Attraction force for electron cloud and nuclei

$$F_{att} = \frac{(Zq)^2 d}{4\pi\varepsilon_0 r^3}$$

External electric field force

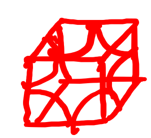
$$F_{ext} = \underline{ZqE}$$

$$d = \frac{4\pi\epsilon_0 r^3}{Zq} E$$

$$p = Zqd = 4\pi\epsilon_0 r^3 E$$

$$\alpha = \frac{p}{E} = 4\pi\epsilon_0 r^3$$

Example: cubic unit cell



$$N = \frac{1 \text{ atom}}{(2r)^3} = \frac{1}{8r^3}$$

$$\chi = \frac{N\alpha}{\epsilon_0} = \frac{1}{8r^3} \times \frac{4\pi\epsilon_0 r^3}{\epsilon_0} = \frac{\pi}{2}$$

$$\epsilon_r = 1 + \pi/2 = 2.57$$

$$n = \sqrt{\epsilon_r} = 1.6$$

To find the macroscopic polarization vector and dielectric constant we should write: (N is the density of atoms per unit volume, cm^{-3})

$$\vec{P} = N\vec{p} = N\alpha\vec{\mathcal{E}}$$

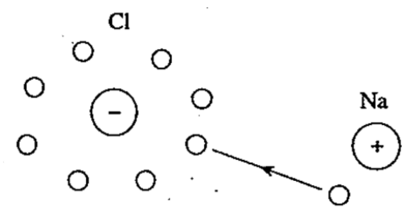
$$\chi = \frac{P}{\epsilon_0 \mathcal{E}} = \frac{N\alpha}{\epsilon_0} = \epsilon_r - 1$$

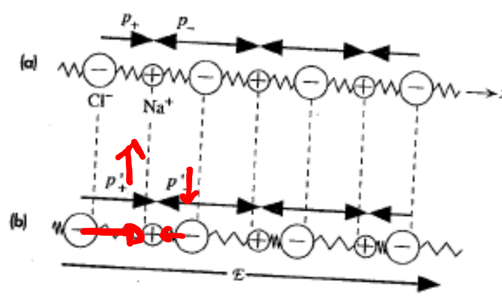
$$\boxed{\epsilon_r = 1 + N\frac{\alpha}{\epsilon_0}}$$

Microscopic Polarization - 2

2. Molecular and Ionic polarization: Bonds between atoms are stretched by electric field in multi-atom systems such as molecules or lattices.

For example, in NaCl, small deformations of the ionic bond will occur when a field is applied, increasing the dipole moment of the lattice.





No field

With field

Non-polar materials: No permanent dipoles. Examples: Si, Ge, C (diamond).

Polar materials: The lattice or molecules have permanent dipoles:
- Lattice: GaAs, GaP, …, The lattice is a mass of permanent dipoles. (Group V: positive, Group III: negative).
- Ionic bonded materials: NaCl, CsCl
- Hydrocarbons: C_6H_6, paraffins
- H_2O: Permanent dipoles and total polarization is determined by orientational polarizability.

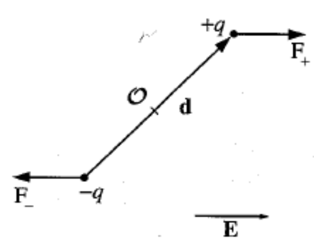
3. Orientational polarization: In gases and liquids, the molecules with permanent or induced dipoles move in-line with electric field. Example: Water.

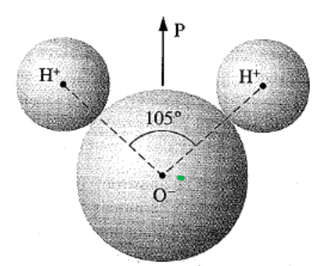
This alignment with field is temperature dependent. Higher T means more random movement and less average dipole $\langle p \rangle$.

water dipole

electric field

$$\langle p \rangle = \frac{p_0^2 E}{3kT} \qquad \langle E \rangle = p_0 E$$





Clausius-Mossotti

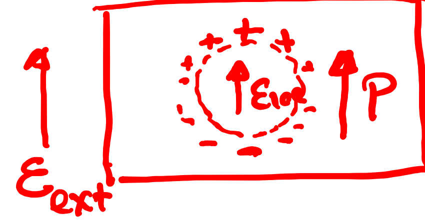
No effect of neighbors

$$\boxed{\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}}$$

N → density

α → polarizability

Local field is different from the external field due to the presence of local charge and dipole distribution.



Clausius-Mossotti equation uses the simplest case for a material with cubic crystal or a liquid to approximate the local field:

neighbor polarization field

$$E_{loc} = E_{ext} + \frac{P}{3\varepsilon_0}$$

$$P = N\alpha E_{loc}$$

$$\frac{P}{N\alpha} = E_{ext} + \frac{P}{3\varepsilon_0} \rightarrow P = \frac{E_{ext}}{\left(1/N\alpha - 1/3\varepsilon_0\right)}$$

$$\varepsilon_r - 1 = \chi = \frac{P}{\varepsilon_0 E_{ext}} = \frac{1}{\varepsilon_0\left(1/N\alpha - 1/3\varepsilon_0\right)} = \left(\frac{\varepsilon_0}{N\alpha} - \frac{1}{3}\right)$$

$$\boxed{\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}}$$

with neighbors

$$\boxed{\varepsilon_r = \frac{1 + \frac{2N\alpha}{3\varepsilon_0}}{1 - \frac{N\alpha}{3\varepsilon_0}}}$$

Example: Given a polarizability of $\alpha = 4.17\times10^{-36}$ Fcm² for Si. What is Si dielectric constant? $N_{Si} = 5\times10^{22}$ cm⁻³. What is the ratio of the local field to the external electric field?

If No neighbors: $\varepsilon_r - 1 = \frac{N\alpha}{\varepsilon_0} = 5\times10^{22} \times \frac{4.17\times10^{-36}}{8.85\times10^{-14}} = 2.36 = \chi \rightarrow \varepsilon_r = 3.36$

with effect ″ : $\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0} = \frac{2.36}{3} = 0.78 \longrightarrow \varepsilon_r = \frac{2\times0.78+1}{1-0.78} = 11.64$

$$\varepsilon_r - 1 = \chi = \frac{P}{\varepsilon_0 E_{ext}} = \frac{N\alpha E_{loc}}{\varepsilon_0 E_{ext}} \rightarrow \frac{E_{loc}}{E_{ext}} = \frac{\varepsilon_0 \chi}{N\alpha} = \frac{\varepsilon_0(\varepsilon_r - 1)}{N\alpha} = \frac{10.64}{2.36} = 4.5$$

Frequency Response

Different polarization mechanisms such as electronic, ionic, directional have different speeds in responding to AC electric field.

Electronic polarization α_e is the fastest as electrons have small effective mass and can even respond to visible light oscillations.
Ionic polarization α_i is much slower than electronic polarization as atoms are more sluggish in movement.
Orientational polarization α_d is even slower. Thermal vibration and local rotation of dipoles due to neighbor molecules oppose alignment of polar molecules in liquids or gases.

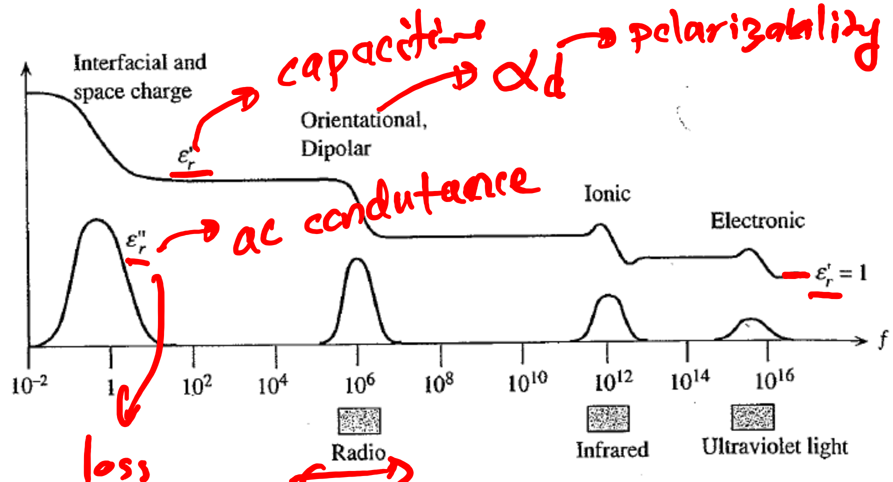


Figure 7.14 The frequency dependence of the real and imaginary parts of the dielectric constant in the presence of interfacial, orientational, ionic, and electronic polarization mechanisms.

$$E = E_0 e^{j\omega t}$$

$\omega = 2\pi f$ → frequency of oscillation

$$\alpha_d(\omega) = \frac{\alpha_d(0)}{1 + j\omega\tau_d}$$

$\alpha_d(0)$ → maximum value of dipolar polarization

$\tau_d = 1/\omega_d$ → ~~Resonant~~ frequency

$$\varepsilon(\omega) = \varepsilon_r\varepsilon_0 = \varepsilon_0 + \frac{N\alpha_d(0)}{1 + j\omega\tau_d}$$

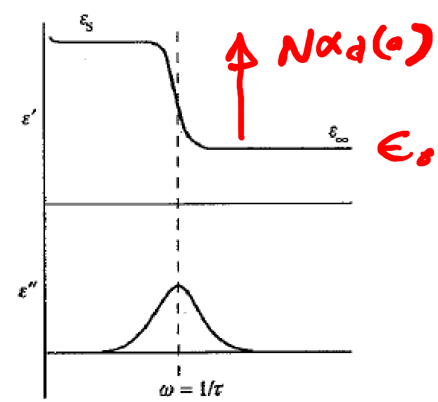


Fig. 10.9
Frequency variation predicted by th
Debye equations.

Total Polarization

To find the total polarization we should add all different components including electronic, ionic, and orientational polarizations.

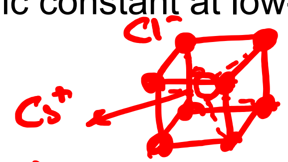
$$\alpha_e(\omega) = \frac{\alpha_e(0)}{1+j\omega\tau_e} \qquad \alpha_d(\omega) = \frac{\alpha_d(0)}{1+j\omega\tau_d}$$

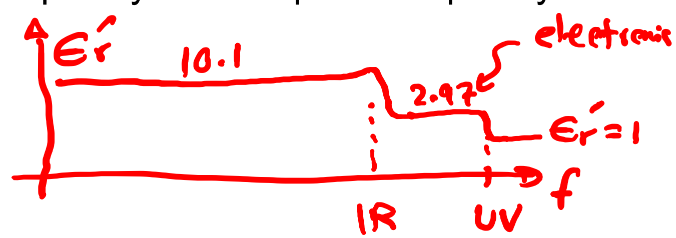
$$\alpha_i(\omega) = \frac{\alpha_i(0)}{1+j\omega\tau_i}$$

$$p_{av} = \alpha_e E + \alpha_i E + \alpha_d E$$

$$\boxed{\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N}{3\epsilon_0}\left(\alpha_e(\omega) + \alpha_i(\omega) + \alpha_d(\omega)\right)}$$ based on Clausius-Mossotti equation.

Example: For CsCl crystal (lattice parameter a of 0.4 nm), the electronic polarizability of Cs^+ and Cl^- ions is 3.35×10^{-36} Fcm^2 and 3.40×10^{-36} Fcm^2, respectively, and the mean ionic polarizability per ion pair is 6×10^{-36} Fcm^2. What is the dielectric constant at low-frequency and at optical frequency? What is the refractive index?





Optical f: only electronic

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3\epsilon_0}\left(N_{Cs^+}\cdot\alpha_{Cs^+} + N_{Cl^-}\cdot\alpha_{Cl^-}\right)$$

$$= \frac{1}{3\epsilon_0 a^3}\left(\alpha_{Cs^+} + \alpha_{Cl^-}\right) = 0.39 \rightarrow \epsilon_r' = 2.97$$

At low f: electronic + ionic

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N}{3\epsilon_0}\left[\alpha_e + \alpha_i\right] = 0.39 + \frac{1}{3\epsilon_0 a^3} \times 6\times10^{-36} = 0.75 \rightarrow \epsilon_r = 10.1$$

ion pair per unit cell

Complex Dielectric Constant

The dielectric constant can be written as a complex number:

$$\varepsilon = \varepsilon_0(\varepsilon_r' - j\varepsilon_r'')$$

$$\varepsilon(\omega) = \varepsilon_0 + \frac{N\alpha_d(0)}{1 + j\omega\tau_d}$$

$$= \epsilon_0\left(1 + \frac{N\alpha(\omega)}{\epsilon_0}\right)$$

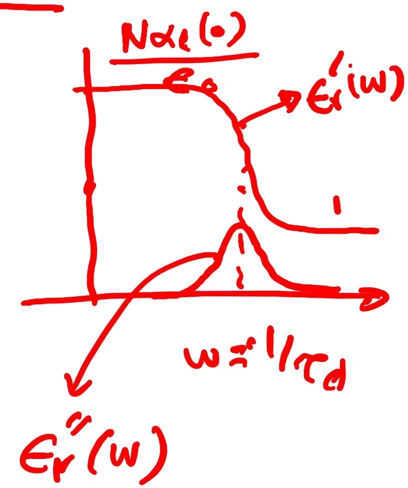
$$\epsilon(\omega) = \epsilon_0 + \frac{N\alpha_d(0)(1 - j\omega\tau_d)}{1 + \omega^2\tau_d^2}$$

capacitive

$$\varepsilon_r'(\omega) = 1 + \frac{N\alpha_d(0)}{\varepsilon_0(1 + \omega^2\tau_d^2)}$$

loss

$$\varepsilon_r''(\omega) = \frac{\omega\tau_d N\alpha_d(0)}{\varepsilon_0(1 + \omega^2\tau_d^2)}$$



$$n = \sqrt{\varepsilon_r'}$$

The real part of the complex dielectric constant is the normal dielectric constant and is related to **refractive index** and **capacitance**.

$$\varepsilon_r'' = \frac{\sigma}{\varepsilon_0\omega}$$

The imaginary part is proportional to conductivity. In dielectrics the conductivity is not desirable and represents **leakage** or **loss**.

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$

Loss tangent

Equivalent Admittance

The equivalent admittance $Y(\omega)$ of a slab of dielectric between two metallic plates is:

$$I(\omega) = Y(\omega)V(\omega)$$

complex dielectric constant

$$Z^{-1}(\omega) = Y(\omega) = \frac{A}{d}\, j\omega\varepsilon(\omega)$$

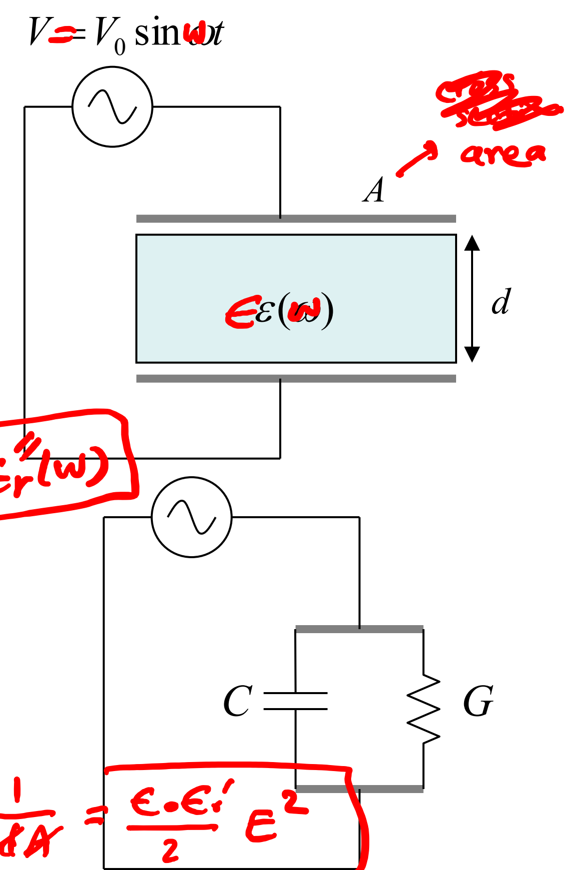
$$Y(\omega) = \frac{A}{d}\, j\omega\left[\epsilon_0\epsilon_r'(\omega) - j\epsilon_0\epsilon_r''(\omega)\right]$$

$$Y(\omega) = \frac{A}{d}\, j\omega\epsilon_0\epsilon_r'(\omega) + \frac{A}{d}\,\omega\epsilon_0\epsilon_r''(\omega)$$

$$Y(\omega) = j\omega C + G$$

$$\boxed{C = \epsilon_0\epsilon_r'(\omega)\frac{A}{d} \qquad G = \omega\epsilon_0\epsilon_r''(\omega)\frac{A}{d} = \sigma\frac{A}{d}}$$

$$\boxed{\sigma = \omega\epsilon_0\epsilon_r''(\omega)}$$



Energy stored per unit volume and power loss per unit volume are:

$$E_{vol} = \frac{\text{Stored energy}}{\text{Volume}} = \frac{1}{2}CV^2 \times \frac{1}{dA} = \frac{1}{2}\epsilon_0\epsilon_r'\frac{A}{d}(E\cdot d)^2 \times \frac{1}{dA} = \boxed{\frac{\epsilon_0\epsilon_r'}{2}E^2}$$

$$W_{vol} = \frac{\text{Power loss}}{\text{Volume}} = GV^2 \times \frac{1}{dA} = \omega\epsilon_0\epsilon_r''\frac{A}{d}(E\cdot d)^2 \times \frac{1}{dA} = \omega\epsilon_0\epsilon_r''E^2$$

$$\boxed{W_{vol} = E^2\omega\epsilon_0\epsilon_r'(\omega)\tan\delta}$$

Dielectric Breakdown

Dielectric breakdown is very important for capacitor design as it sets the upper limit for charge storage. Sudden increase in current when the voltage exceeds a critical value V_{BR} (This breakdown is very fast 10^{-8} s in solids). This critical voltage is related to breakdown field E_{BR} (or dielectric strength).

1. Intrinsic breakdown: Happens when the few electrons in the conduction band are accelerated by electric field and gain enough energy to ionize the lattice atoms. This is similar to avalanche breakdown in pn diodes.

2. Thermal breakdown: Heating in dielectric can increase number of electrons in the conduction band, and as a result, decrease the breakdown voltage. This heating can be due to dielectric loss (not only by free electrons but higher ϵ_r'' at *resonance frequencies, dipoles*). Polyethylene: $E_{BR} \approx 5$ MV/cm drops to ≈ 0.05 MV/cm at 1 MHz.

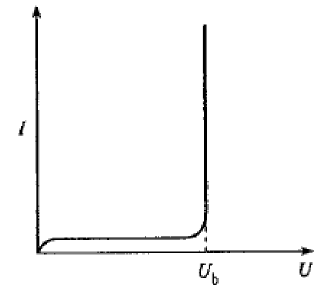


Fig. 10.11
Current voltage characteristics for an insulator. The current increases very rapidly at the breakdown voltage, U_b.

Solymar

3. Discharge breakdown: Gas within porous materials such as mica or porous ceramics is ionized and accelerates breakdown. What is plasma?

Based on breakdown field we can find maximum energy that can be stored safely in a capacitor (safety parameter η, $E_{max} = E_{BR}/\eta$):

$$E_{vol} = \frac{\epsilon_0 \epsilon_r' E_{BR}^2}{2\eta^2}$$

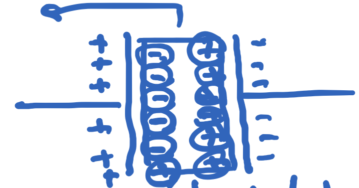
$$W_{vol} = \frac{E_{BR}^2}{\eta^2} \omega \epsilon_0 \epsilon_r' \tan\delta$$

Typical dielectrics

Properties such as breakdown field and dielectric constant determine the suitability of the use of dielectrics for capacitances.

supercapacitor

double layer capacitance



electrolyte

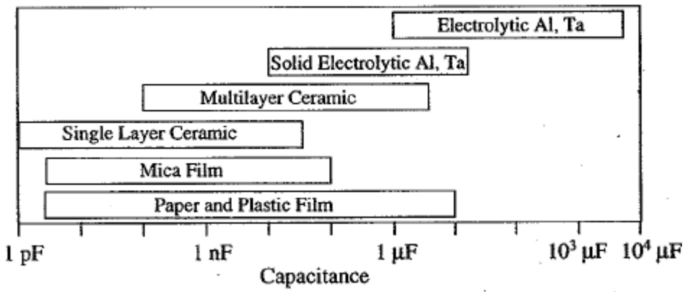


Figure 7.27 Examples of dielectrics that can be used for various capacitance values.

Table 7.6 Comparison of dielectric properties at 60 Hz for capacitor applications: typical values. (Assume $\eta = 2$, PS = polystyrene, PET = polyethyleneterephthalate. X7R is the name of a particular ceramic solid solution.)

	Polymer Film PS	Polymer Film PET	Mica	Ceramic Al_2O_3	Ceramic TiO_2	High-*K* Ceramic ($BaTiO_3$ based)
Name	Polystyrene	Polyester	Mica	Anodized alumina film	Polycrystalline titania	X7R
ε_r'	2.5	3.2	6.9	8.5	90	1,800
$\tan\delta$	3×10^{-4}	5×10^{-3}	5×10^{-4}	1×10^{-3}	4×10^{-4}	5×10^{-2}
$\mathcal{E}_{br}$ (kV cm^{-1})	200–250	150–200	1000	1000	50–100	100
d (typical minimum)	1–2 μm	1–2 μm	2.5 μm	0.1 μm	10 μm	10 μm
C_{vol} (μF cm^{-3})	22.1	28	10	7500	7.5	159.4
E_{vol} (mJ cm^{-3})	1.1	0.80	76	94	2.5	200
W_{vol} (W cm^{-3})	0.00025	0.0030	0.029	0.071	0.00075	7.5
Polarization	Electronic bond	Electronic bond and dipolar	Ionic	Ionic	Ionic	Large ionic displacement

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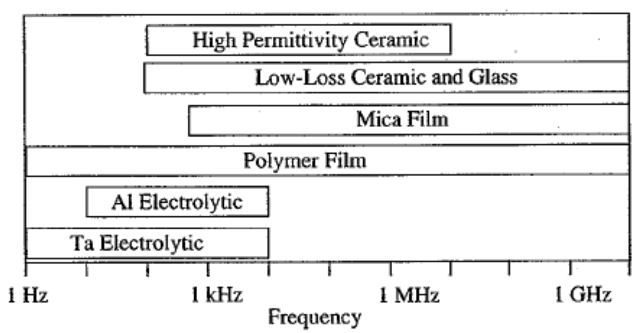

Figure 7.28 Examples of dielectrics that can be used in various frequency ranges.

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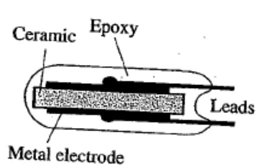
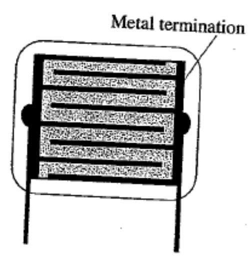
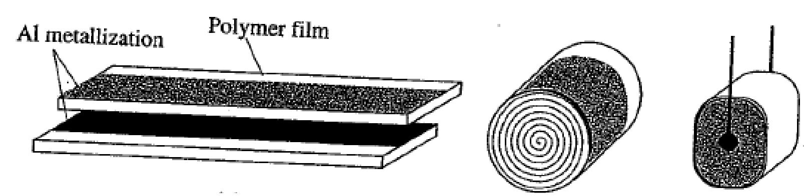


(a) Single-layer ceramic capacitor (e.g. disk capacitors)



(b) Multilayer ceramic capacitor (stacked ceramic layers)



Piezoelectricity

Piezoelectric effect: Mechanical strain will produce polarization (and as a result electric field or voltage), or an applied electric field will cause mechanical strain.

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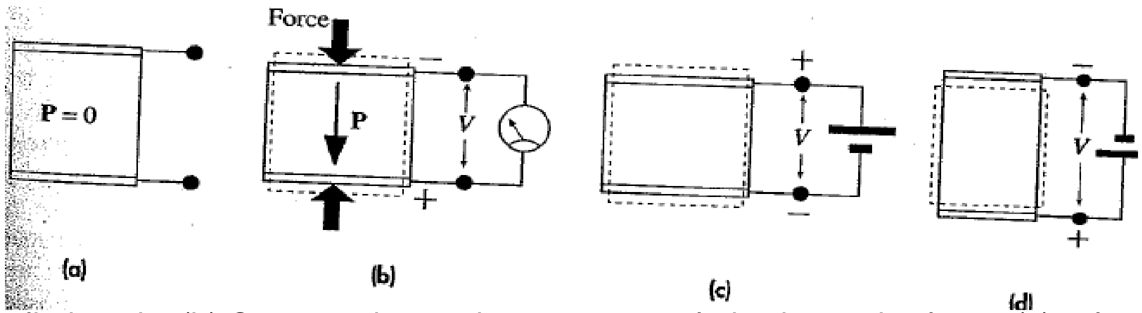

(a) no applied strain, (b) Compressive strain generates polarization and voltage, (c) voltage generates compressive strain (contraction), (d) reverse voltage generates tensile strain (extension).

Some crystals show piezoelectric effects such as quartz (crystalline SiO_2) and $BaTiO_3$ (ceramic). Pizeoelectric crystals have no centre of symmetry (unit cell *noncentrosymmetric*). Applied mechanical stress (or force) T_j in some j direction can cause polarization P_i in other crystal directions i. So in summarized form:

$$P_i = d_{ij} T_j$$

d_{ij} **Piezoelectric coefficients**

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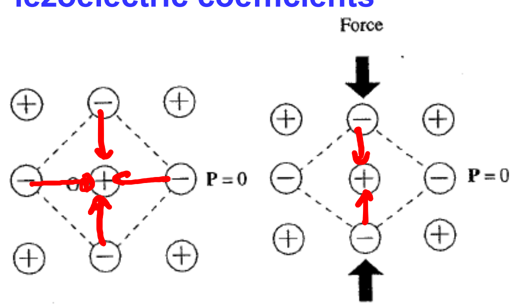
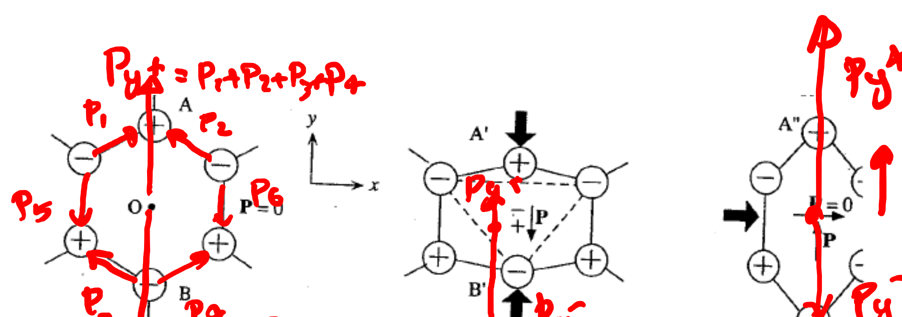


NaCl: cubic lattice, with centre of symmetry, **no piezoelectric effect.**



Hexagonal unit cell, no centre of symmetry, **piezoelectric effect.**

Piezoelectricity - 2

Converse piezoelectric effect: Applied electric field $\mathcal{E}_i$ in some i direction induce mechanical strain S_j in other crystal directions j. So in summarized form:

$$S_j = d_{ji} E_i$$

d_{ij} **Same piezoelectric coefficients**

Pizeoelectric crystals are mechanical transducers.
Applications: microphones, electro-mechanical filters, …

Quartz Oscillators:
The quartz is used for frequency control in oscillators and filters. The length of the crystal is accurately set so the oscillation frequency matches the resonance frequency of the crystal.

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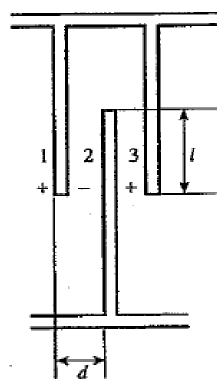
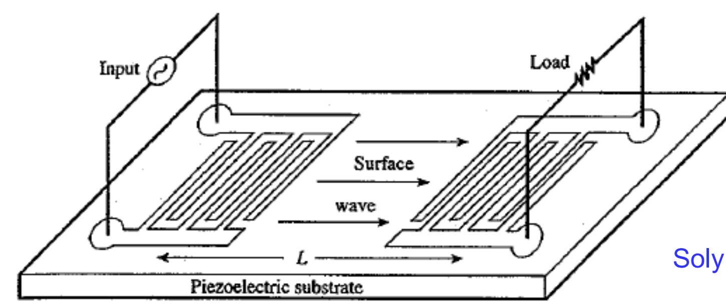
Surface Acoustic Wave (SAW) filters: Interdigital lines on piezoelectric substrate convert electric oscillations to acoustic surface wave that travels and is converted back to electric signal by another interdigital lines.

$$\frac{\lambda}{2} = d \qquad f_s = \frac{v_{sound}}{\lambda}$$





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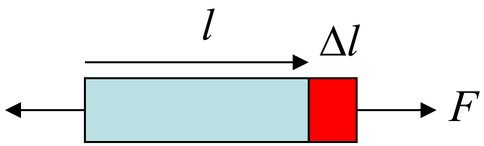
Tensors, Strain, Stress

In one dimension mechanical strain S_i and stress T_i can be defined as:

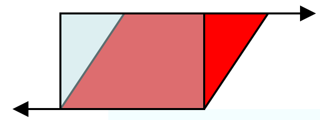


$$S_i = \Delta l / l \qquad \text{Change in length/Length}$$

$$T_i = F / A \qquad \text{Force/Unit area}$$

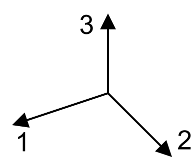
Crystal	d (m V^{-1})	k
Quartz (crystal SiO_2)	2.3×10^{-12}	0.1
Rochelle salt ($NaKC_4H_4O_6 \cdot 4H_2O$)	350×10^{-12}	0.78
Barium titanate ($BaTiO_3$)	190×10^{-12}	0.49
PZT, lead zirconate titanate ($PbTi_{1-x}Zr_xO_3$)	480×10^{-12}	0.72
Polyvinylidene fluoride (PVDF)	18×10^{-12}	—

Shear strain and stress are also shown here:



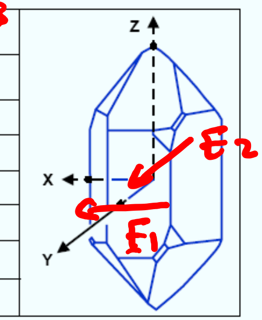
In 3 D:

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} = \mathbf{YT} + \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \\ d_{41} & d_{42} & d_{43} \\ d_{51} & d_{52} & d_{53} \\ d_{61} & d_{62} & d_{63} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}$$



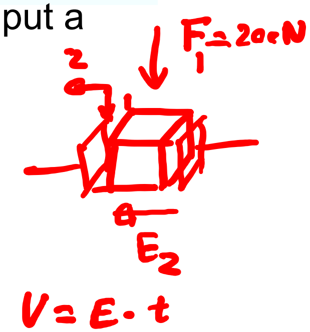
The Piezoelectric Effect in Quartz

			FIELD along:	
		E_1	E_2	E_3
STRAIN		X	Y	Z
EXTENSIONAL along:	X (S_1)	√ d_{11}		
	Y (S_2)	√ d_{21}		
	Z (S_3)			
SHEAR about:	X (S_4)	√ d_{41}		
	Y (S_5)		√ d_{52}	
	Z (S_6)		√ d_{62}	



In quartz, the five strain components shown may be generated by an electric field. The modes shown on the next page may be excited by suitably placed and shaped electrodes. The shear strain about the Z-axis produced by the Y-component of the field is used in the rotated Y-cut family, including the AT, BT, and ST-cuts.

Example: Given that for quartz $d = 2.3 \times 10^{-10}$ cm/V, what is the voltage generated if we put a load of 200 N over 1 cm³ quartz crystal?



$$P_2 = d_{21} T_1 = 2.3 \times 10^{-10} \times \frac{200\ N}{1\ cm^2} = 4.6 \times 10^{-8} \ \frac{C}{cm^2}$$

$$P_2 = \epsilon_0(\epsilon_r - 1) E_2 \rightarrow E_2 = \frac{P_2}{\epsilon_0(\epsilon_r - 1)}$$

$$V_2 = E_2 \cdot t = \frac{P_2}{\epsilon_0(\epsilon_r - 1)} t = \frac{4.6 \times 10^{-8}}{8.85 \times 10^{-14} \times (3.78 - 1)} = 187\ kV$$

Ferroelectricity

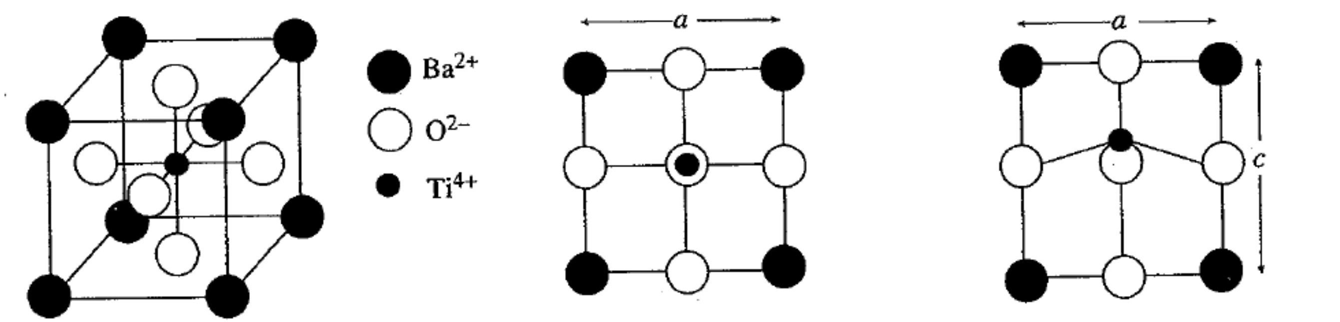
Dielectrics

Piezo

pyroelectric

ferroelectric

Crystals that are permanently polarized are called *ferroelectrics*. No electric field is required to polarize the crystal. For $BaTiO_3$ above the Curie temperature (130°C) the crystal has no total polarization. But below this temperature, a permanent polarization is formed.



(a) $BaTiO_3$ cubic crystal structure above 130 °C

(b) $BaTiO_3$ cubic structure above 130 °C

(c) $BaTiO_3$ tetragonal structure below 130 °C

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Using electric field we can set the direction of permanent polarization. This is like *ferromagnetism*. There is hysteresis in P-E graph. In ferromagnetic materials this is H-B graphs.

Permanent polarization

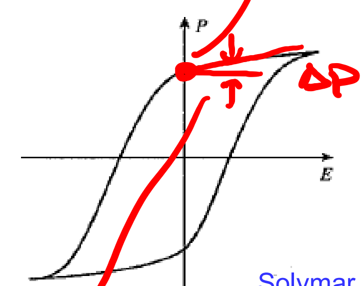

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Fig. 10.18
Ferroelectric hysteresis loop.

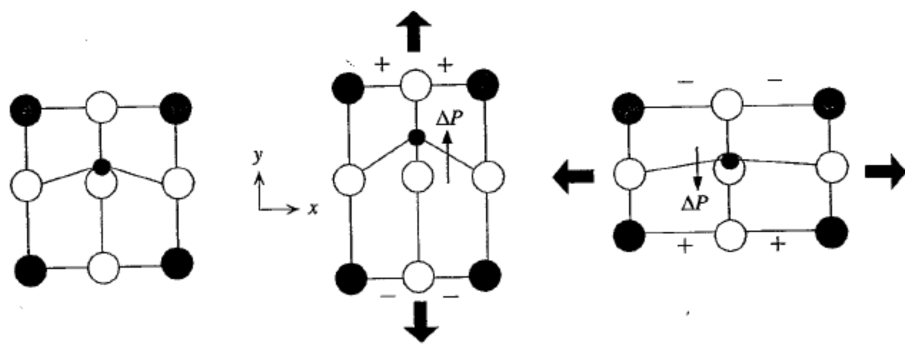
Since there is an initial polarization P_0, the dielectric equation will become:

$$\Delta P = \epsilon_0 \chi \Delta E$$

$$\epsilon_r - 1 = \chi = \frac{\Delta P}{\epsilon_0 \Delta E}$$

Pyroelectricity

Change in temperature causes change in P.



(a) $BaTiO_3$ tetragonal structure below 130 °C

(b) $BaTiO_3$ crystal under stress along *y*

(c) $BaTiO_3$ crystal under stress along *x*

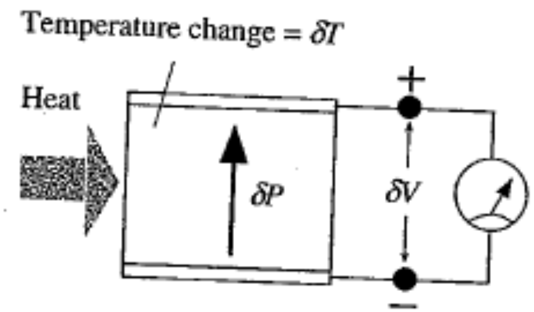


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Figure 7.43 The heat absorbed by the crystal increases the temperature by δT, which induces a change δP in the polarization.
This is the pyroelectric effect. The change δP gives rise to a change δV in the voltage that can be measured.

Pyroelectric coefficient: $p = \dfrac{dP}{dT}$

Table 7.8 Some pyroelectric (and also ferroelectric) crystals and typical properties

Material	ε_r'	tan δ	Pyroelectric Coefficient ($\times 10^{-6}$ C m^{-2} K^{-1})	Curie Temperature (°C)
$BaTiO_3$	4100 ⊥ polar axis; 160 // polar axis	7×10^{-3}	20	130
PZT modified for pyroelectric	290	2.7×10^{-3}	380	230
PVDF, polymer	12	0.01	27	80

Used in temperature and light sensors.

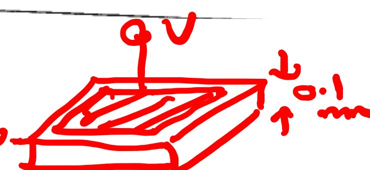
Example: For 1 mK change in temperature, what is the voltage generated by PZT with thickness 0.1 mm?

$\Delta T = 1\ \text{mK} \rightarrow \Delta P = p \cdot \Delta T = 380 \times 10^{-6} \frac{C}{m^2 K} \cdot 10^{-3} K$

$= 380 \times 10^{-9}\ C/m^2$

$\Delta V = t \cdot \Delta E = t \dfrac{\Delta P}{\varepsilon_0(\varepsilon_r - 1)} = 0.1 \times 10^{-3} \times \dfrac{380 \times 10^{-9}}{8.85 \times 10^{-12}(290-1)} = 15\ \text{mV}$

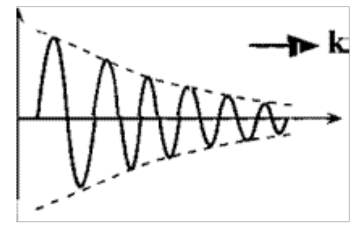
P↓ E↓



Complex Refractive Index

Monochromatic light wave can be represented by a traveling electric field wave:

$$E = E_0 \exp j\left(wt - kz\right)$$



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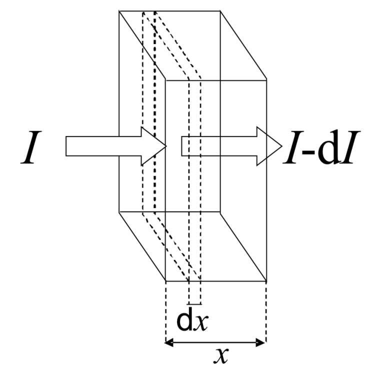
When there is loss mechanism and absorption involved then we can expect a complex propagation wave vector:

$$k = k' - jk''$$

absorption

This means that the wave equation will be in form of:

$$E = E_0 \exp\left(-k''z\right)\exp j\left(wt - k'z\right)$$



Now for intensity:

$$I \propto \left|E\right|^2 \propto \exp\left(-2k''z\right) \qquad v = \omega / k'$$

$$\frac{dI}{dz} = -2k''I$$

absorption coefficient

The complex refractive index can be defined as

$$N = n - jK = \frac{k}{k_0} = \frac{k'}{k_0} - j\frac{k''}{k_0}$$

Extinsion Coefficient K

The extinsion coefficient of a material can be defined as

$$K = \frac{k''}{k_0}$$

In the absence of loss we know that the refractive index can be defined as

$$n = \sqrt{\varepsilon_r}$$

complex dielectric constant

When we have loss and non-zero extinsion coefficient

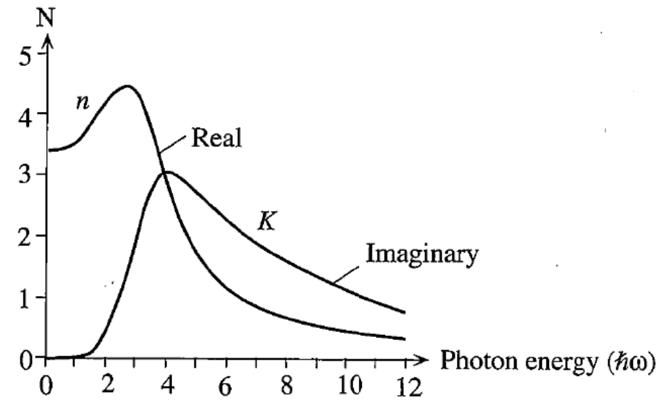
$$N = n - jK = \sqrt{\varepsilon_r} = \sqrt{\varepsilon_r' - j\varepsilon_r''}$$

Which means

$$n^2 - K^2 = \varepsilon_r' \qquad 2nK = \varepsilon_r''$$

Reflection coefficient for a normal (90°) incident light is given by:

$$R = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}$$



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Example

Spectroscopic elipsometry measurements on a silicon wafer at a wavelength of 827 nm show the real and imaginary parts of the complex relative permittivity are 13.5 and 0.04. Find the complex refractive index, the reflectance and the absorption coefficient at this wavelength and the wave velocity.

$\epsilon = \epsilon_0 (\epsilon_r' - j\epsilon_r'')$

$\epsilon_r' = 13.5$

$\epsilon_r'' = 0.04$

$N = n - jK \longrightarrow \begin{cases} n^2 - K^2 = \epsilon_r' \\ 2nK = \epsilon_r'' \end{cases} \longrightarrow \begin{cases} n^2 - K^2 = 13.5 \\ 2nK = 0.04 \end{cases} \to n^2 - \left(\frac{0.04}{2n}\right)^2 = 13.5$

$n =$

$K =$

$N = n - jK$

$R = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2} =$

$A = 1 - R$