# ELEC 341: Systems and Control

## Lecture 5

## Modeling of DC motor, linearization, and time delay

# Course roadmap

## *Modeling*

- Laplace transform
- Transfer function

Models for systems
- Electrical
- Electromechanical
- Mechanical

Linearization, delay

## *Analysis*

Stability
- Routh-Hurwitz
- Nyquist

Time response
- Transient
- Steady state

Frequency response
- Bode plot

## *Design*

Design specs

Root locus

Frequency domain

PID & Lead-lag

Design examples

*Matlab simulations*

# Transfer function (review)

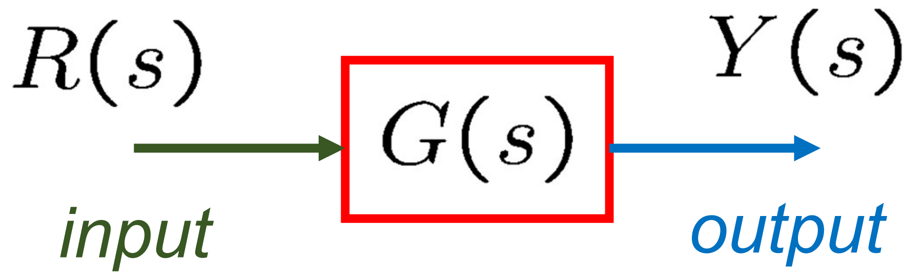
- A transfer function is defined by

$$G(s) = \frac{Y(s)}{R(s)}$$

*Laplace transform of system output*

*Laplace transform of system input*

$$Y(s) = G(s)R(s)$$



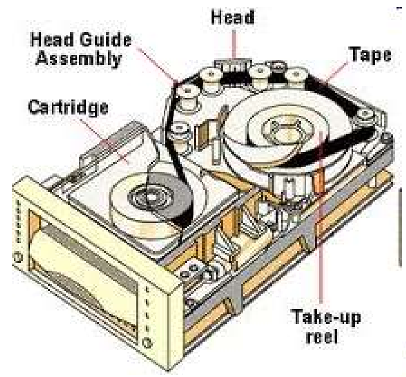

- Transfer function is a generalization of "**gain**" concept.

# DC motor

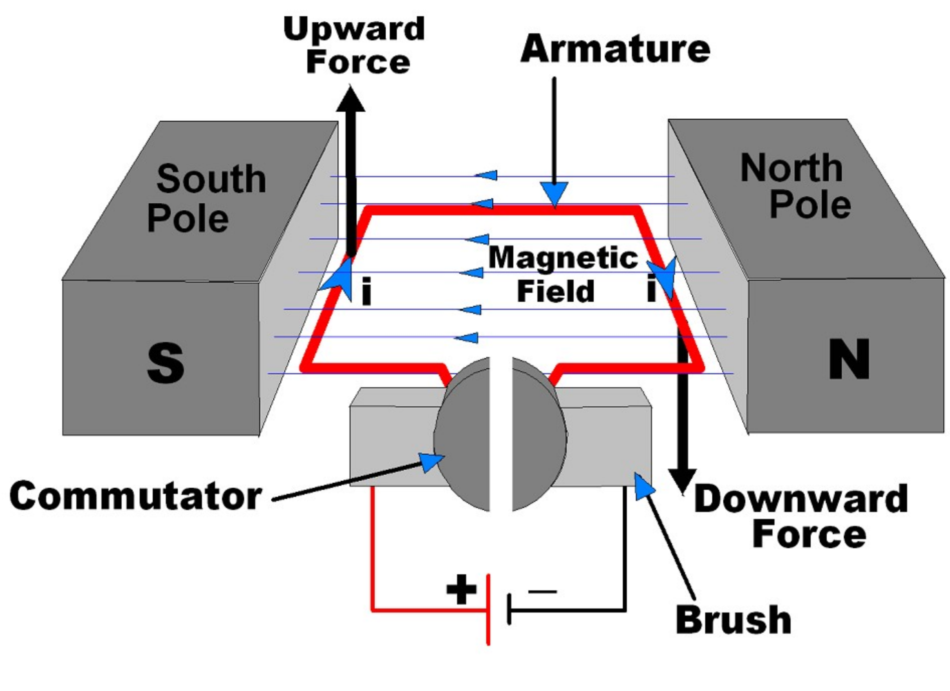
- An **actuator** converts electrical energy into rotational mechanical energy.

# Why DC motor?

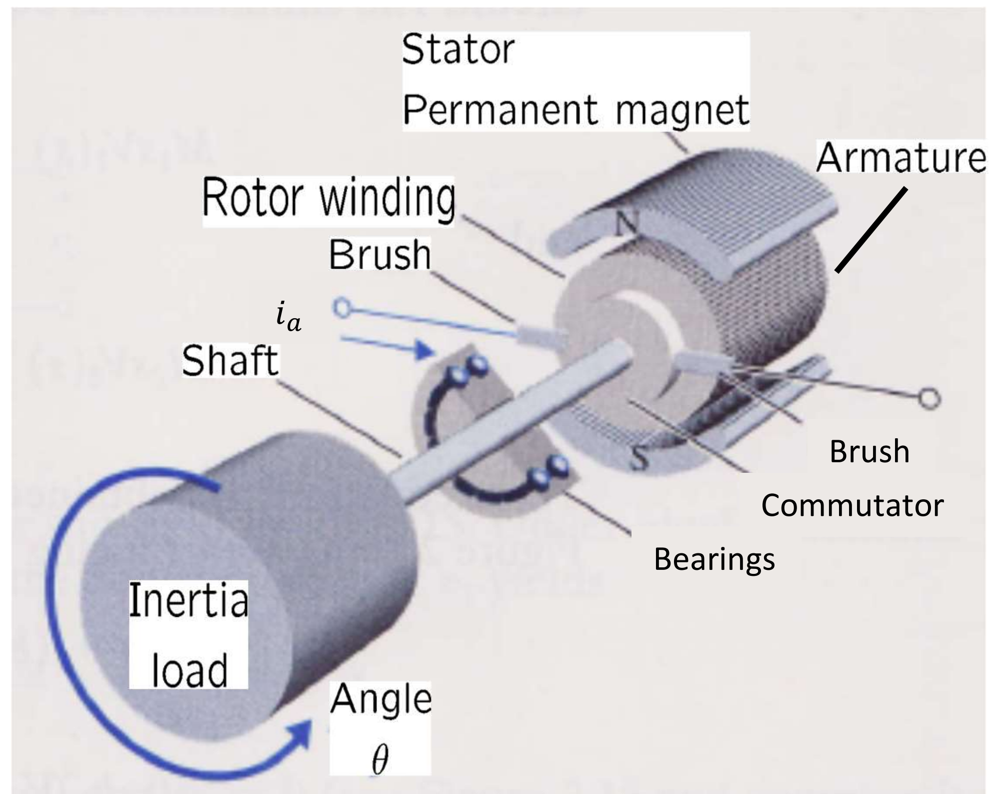
- It has a lot of advantages:
  - High torque
  - Speed controllability
  - Portability, etc.
- Widely used in control applications:
  - Robots
  - Surgical tools
  - Tape drives
  - Printers
  - Machine tool industries
  - Radar tracking systems, etc.



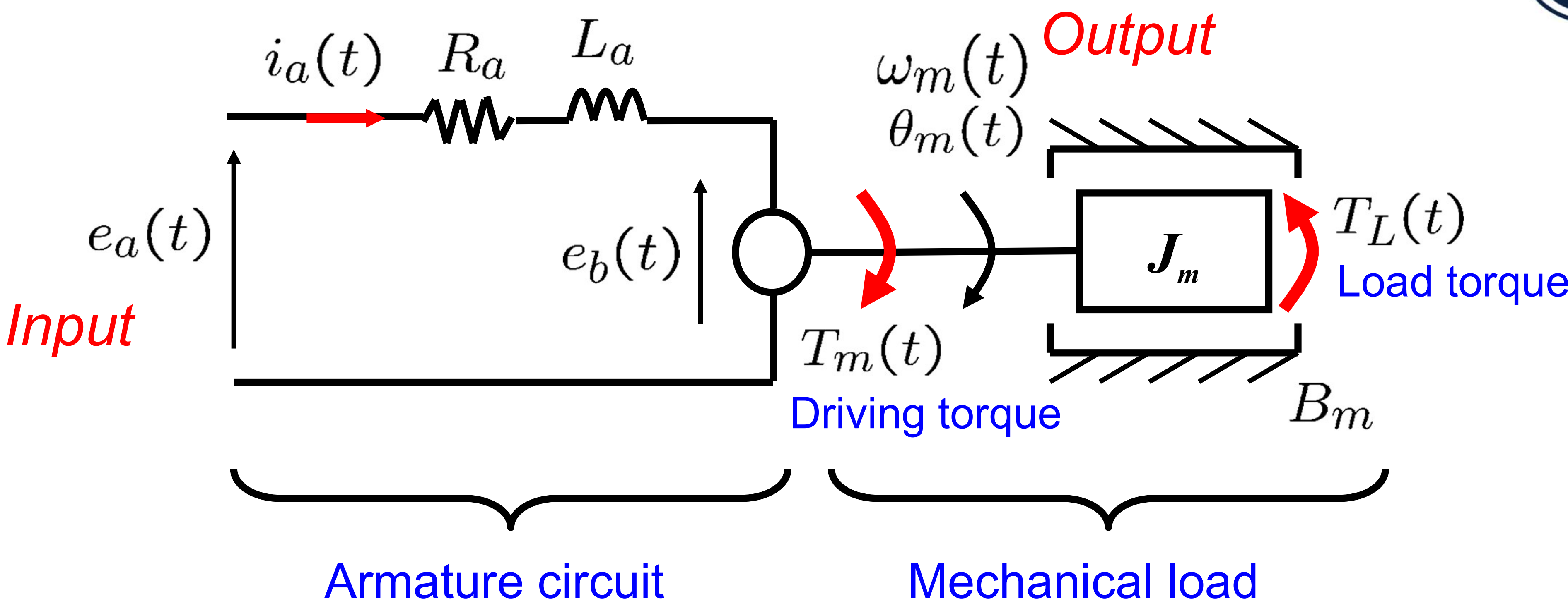

# How does DC motor work?




DC Motor Conceptual Diagram



# Model of DC motor




"$a$": armature

- $e_a$ : applied voltage
- $i_a$ : armature current

"$b$": back EMF

"$m$": mechanical

- $\theta_m$ : angular position
- $\omega_m$ : angular velocity
- $J_m$ : total inertia
- $B_m$ : viscous friction

$$T_i(t) = \tau_i(t)$$

# Modeling of DC motor: *t*-domain

- Armature circuit $$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$$

- Mechanical load $$J_m \dot{\omega}_m(t) = \underbrace{T_m(t)}_{\text{Driving torque}} - B_m \omega_m(t) - \underbrace{T_L(t)}_{\text{Load torque}}$$

- Connection between mechanical/electrical parts
  - Motor torque $$T_m(t) = K_i i_a(t)$$
  - Back EMF $$e_b(t) = K_b \omega_m(t)$$

- Angular position $$\omega_m(t) = \dot{\theta}_m(t)$$

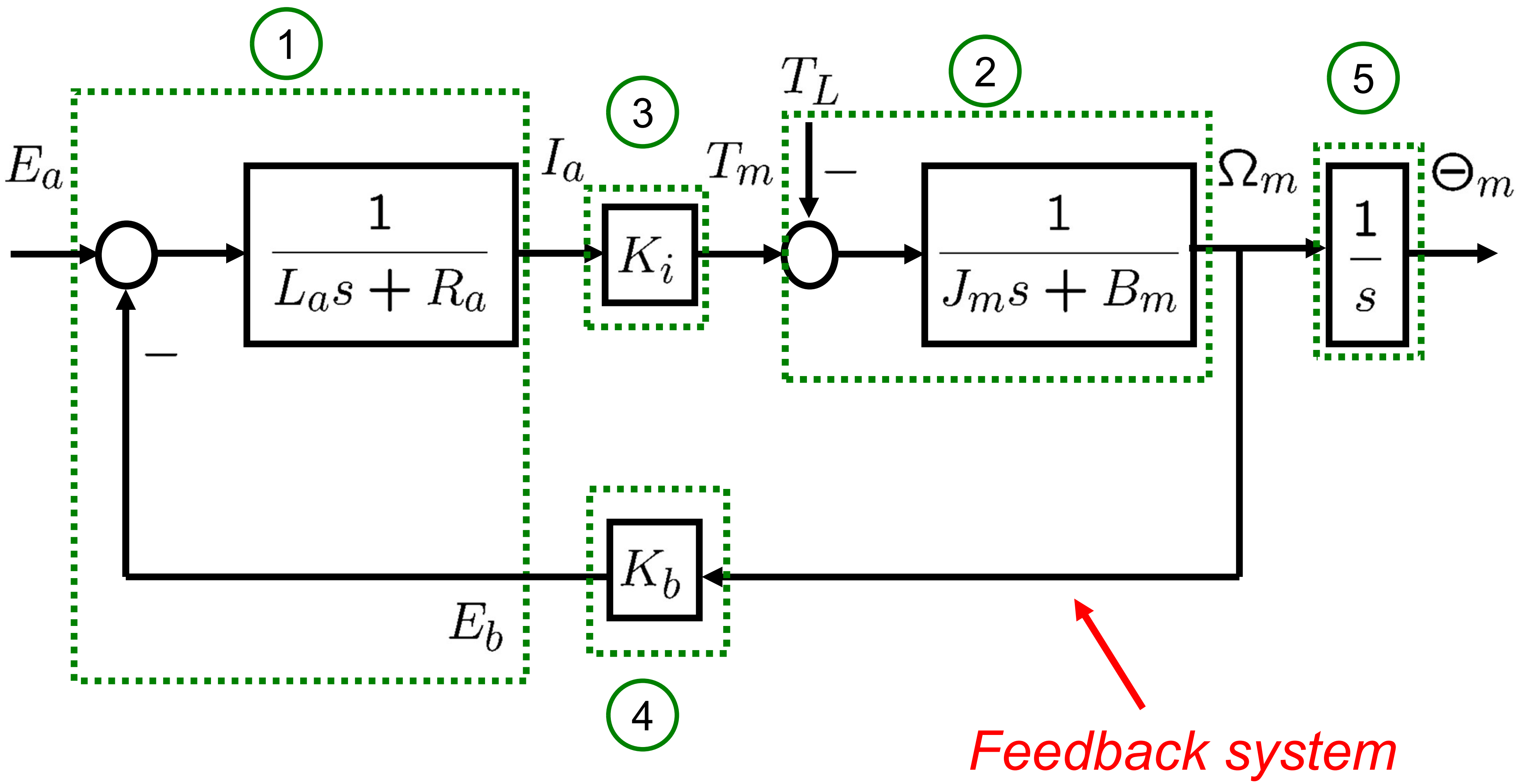
# Modeling of DC motor: *s*-domain

- Armature circuit $$I_a(s) = \frac{1}{L_a s + R_a}(E_a(s) - E_b(s)) \quad (1)$$
- Mechanical load $$\Omega_m(s) = \frac{1}{J_m s + B_m}(T_m(s) - T_L(s)) \quad (2)$$

Note that $\mathcal{L}\{\omega(t)\} = \Omega(s)$ and $\mathcal{L}\{\theta(t)\} = \Theta(s)$.

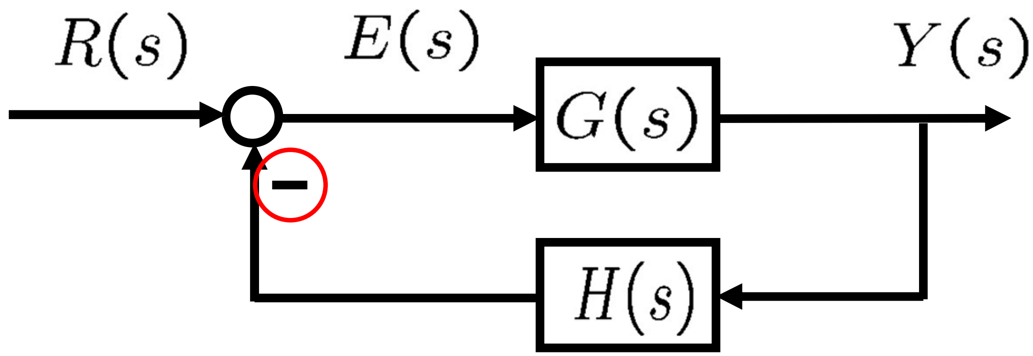
- Connection between mechanical/electrical parts
  - Motor torque $$T_m(s) = K_i I_a(s) \quad (3)$$
  - Back EMF $$E_b(s) = K_b \Omega_m(s) \quad (4)$$
- Angular position $$\Theta_m(s) = \frac{1}{s}\Omega_m(s) \quad (5)$$

# DC motor: Block diagram



# Transfer function (TF) with feedback (Black's formula)

- Negative feedback system




$$\left\{ \begin{aligned} Y(s) &= G(s)E(s) \\ E(s) &= R(s) - H(s)Y(s) \end{aligned} \right\} \quad \xrightarrow{\text{Eliminate } E(s)} \quad \boxed{Y(s) = \frac{G(s)}{1 + G(s)H(s)} R(s)}$$

**OLTF** = Open Loop Transfer Function = Loop Gain
**CLTF** = Closed Loop Transfer Function
**FPTF** = Forward Path Transfer Function = Forward Gain

$$\begin{pmatrix} G(s) & : \text{forward path TF} \\ G(s)H(s) & : \text{open-loop TF} \end{pmatrix}$$

- **Black's Formula: Closed-loop transfer function is given by:** $\dfrac{\textbf{Forward Gain}}{\textbf{1+Loop Gain}} = \dfrac{Y(s)}{R(s)}$ = **CLTF**

# DC motor: Transfer functions

If $T_L = 0$, then $$\frac{\Omega_m(s)}{E_a(s)} = \frac{\dfrac{K_i}{(L_a s + R_a)(J_m s + B_m)}}{1 + \dfrac{K_b K_i}{(L_a s + R_a)(J_m s + B_m)}} = \underbrace{\boxed{\frac{K_i}{(L_a s + R_a)(J_m s + B_m) + K_b K_i}}}_{G_1(s)}$$

If $E_a = 0$, then $$\frac{\Omega_m(s)}{T_L(s)} = -\frac{\dfrac{1}{J_m s + B_m}}{1 + \dfrac{K_b K_i}{(L_a s + R_a)(J_m s + B_m)}} = \underbrace{\boxed{-\frac{L_a s + R_a}{(L_a s + R_a)(J_m s + B_m) + K_b K_i}}}_{G_2(s)}$$

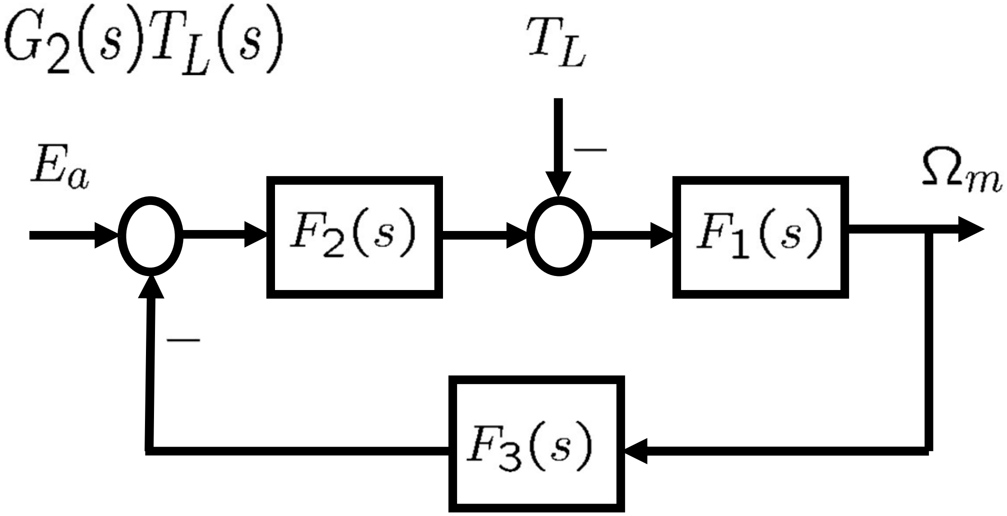
In general, when $T_L \neq 0$ and $E_a \neq 0$, we can prove the following equation (see next slide):

$$\Omega_m(s) = G_1(s)E_a(s) + G_2(s)T_L(s)$$

$$\boxed{\Theta_m(s) = \frac{1}{s}\Omega_m(s) = \frac{1}{s}\left(G_1(s)E_a(s) + G_2(s)T_L(s)\right)}$$

# DC motor: Derivation of TFs

- Why $\Omega_m(s) = G_1(s)E_a(s) + G_2(s)T_L(s)$




$$\Omega_m(s) = F_1(s)\left[-T_L(s) + F_2(s)\left\{E_a(s) - F_3(s)\Omega_m(s)\right\}\right]$$

$$\Rightarrow \left\{1 + F_1(s)F_2(s)F_3(s)\right\}\Omega_m(s) = F_1(s)\left\{-T_L(s) + F_2(s)E_a(s)\right\}$$

$$\Rightarrow \Omega_m(s) = \frac{F_1(s)F_2(s)}{1 + F_1(s)F_2(s)F_3(s)}E_a(s) - \frac{F_1(s)}{1 + F_1(s)F_2(s)F_3(s)}T_L(s)$$

$$\Rightarrow \boxed{\Omega_m(s) = G_1(s)E_a(s) + G_2(s)T_L(s)}$$

# DC motor: TFs (cont'd)

- Note: For DC motors, $L_a s << R_a$ (for moderate values of $s$). Then, if $T_L(s) = 0$, an approximated TF is obtained by setting $L_a = 0$.

2nd order system 1st order system

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_i}{(L_a s + R_a)(J_m s + B_m) + K_b K_i} \approx \frac{K_i}{R_a(J_m s + B_m) + K_b K_i}$$

$$= \frac{K}{Ts+1} \qquad \left(K = \frac{K_i}{R_a B_m + K_b K_i}, \; T = \frac{R_a J_m}{R_a B_m + K_b K_i}\right)$$

2nd order system ⇒ 1st order system

$$\Theta_m(s) = \frac{1}{s}\Omega_m(s) \Rightarrow \boxed{\frac{\Theta_m(s)}{E_a(s)} = \frac{K}{s(Ts+1)}}$$

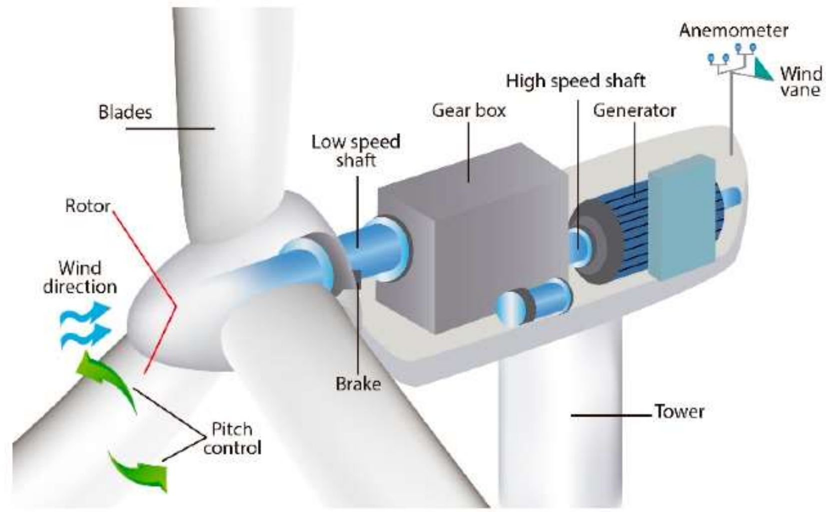
# Gearbox Transfer Function

- A **gearbox** is a mechanical device which is used to increase the output torque or to change the speed (RPM). Below are some of its applications:

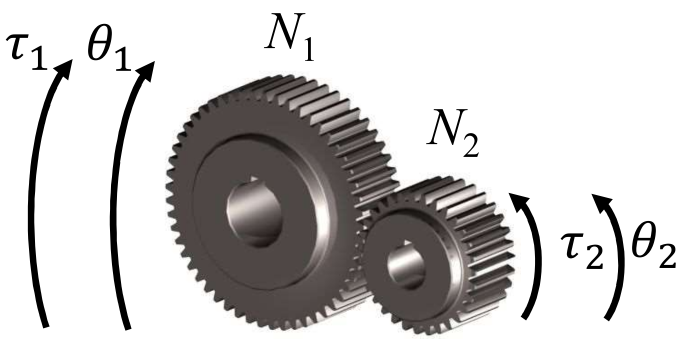
**Car engine and jet engine**

**DC motor**

**Wind turbine**




**Industrial machines, such as paper machine, elevator, conveyor, etc.**

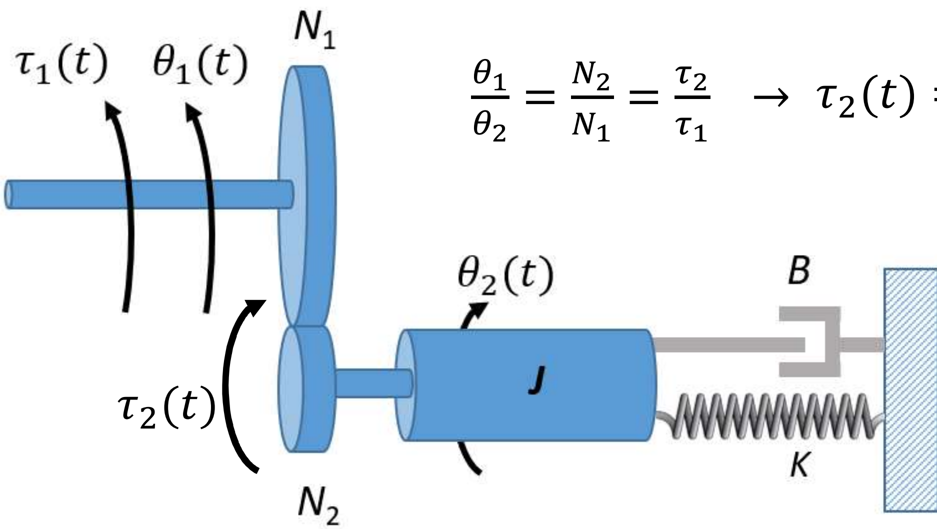


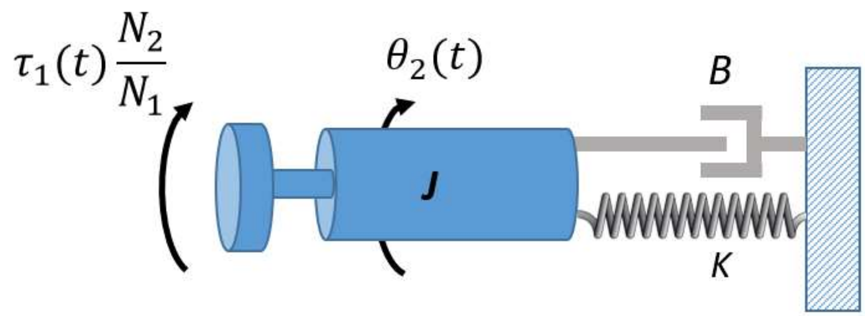

$$\frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} = \frac{\tau_2}{\tau_1}$$

$N_1$, $N_2$: number of teeth on gear 1 and gear 2

# Gearbox Transfer Function

$$\frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} = \frac{\tau_2}{\tau_1} \rightarrow \tau_2(t) = \tau_1(t)\frac{N_2}{N_1}$$




$$J\ddot{\theta}_2(t) = \tau_1(t)\frac{N_2}{N_1} - B\,\dot{\theta}_2(t) - K\theta_2(t) \xrightarrow{\mathcal{L}} Js^2\theta_2(s) + Bs\theta_2(s) + K\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

$$\rightarrow (Js^2 + Bs + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1} \quad (1)$$

$$\frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} = \frac{\tau_2}{\tau_1} \xrightarrow{\mathcal{L}} \theta_2(s) = \theta_1(s)\frac{N_1}{N_2}$$

substitute in (1)

$$\rightarrow (Js^2 + Bs + K)\theta_1(s)\left(\frac{N_1}{N_2}\right)^2 = T_1(s)$$

$$\boxed{\frac{\theta_1(s)}{T_1(s)} = \frac{\left(\frac{N_2}{N_1}\right)^2}{(Js^2 + Bs + K)}}$$

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- Linearization, delay

## *Analysis*

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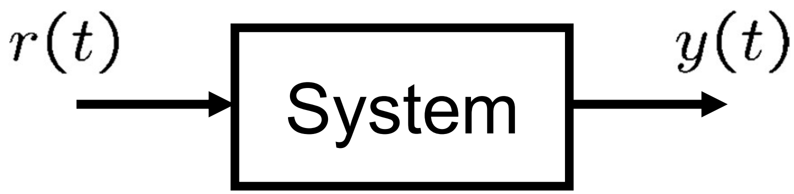
# Linear system

- A linear system satisfies the **Principle of Superposition**:




$$\left.\begin{array}{l} r_1(t) \rightarrow y_1(t) \\ r_2(t) \rightarrow y_2(t) \end{array}\right\} \Rightarrow \alpha_1 r_1(t) + \alpha_2 r_2(t) \rightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

$$\forall \alpha_1, \alpha_2 \in \mathbb{R} \quad (\text{'}\forall\text{' means 'for all'})$$

A nonlinear system does not satisfy the principle of superposition.

# Why linearization?

- Real systems are inherently nonlinear because system parameters like stiffness $K$ and resistance $R$ often vary with other physical quantities such as displacement or temperature.
- For example, in **mechanical systems**, a spring might follow the relation $F(t) = K(x) \cdot x(t)$, where $K$ is not constant but changes with the position $x$.
- Similarly, in **electrical systems**, resistance can depend on temperature, as in $V(t) = R(T) \cdot i(t)$, where $R$ increases with temperature $T$.
- Many control analysis/design techniques are available for linear systems.
- Nonlinear systems are difficult to deal with mathematically.
- Often, ***we linearize nonlinear systems*** before analysis and design. How?

# Linearization

## Linear Systems:

- Easier to understand and obtain solutions.
- We can use superposition.
- Linear ordinary differential equations (ODEs):
  - **Homogeneous solution** and **particular solution**
  - **Transient solution** and **steady state solution**
  - **Solution caused by initial conditions (ICs)** and **forced solution**
  - Easy to check the **stability** (after we apply Laplace Transform to ODEs).
- As part of the control theory, we will look at a systematic method called **Generalized Method for Linearization** for handling these types of control problems.

# Taylor series expansion

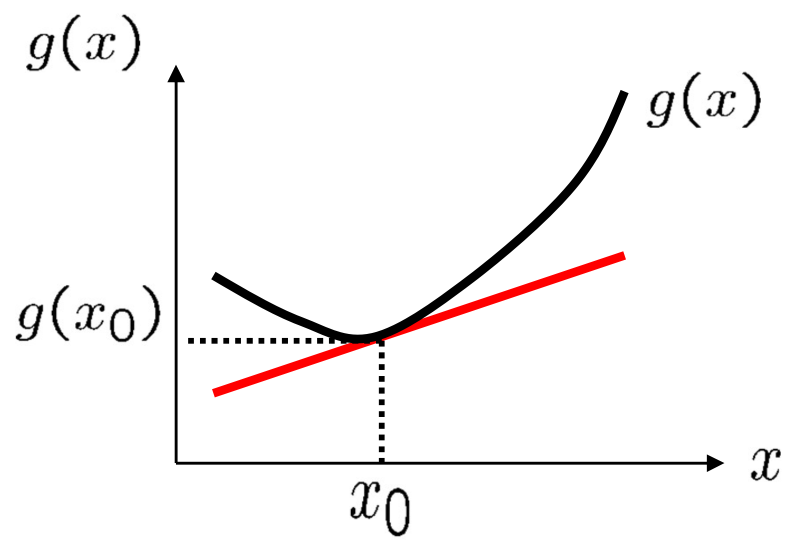
- Taylor series expansion of a smooth function (i.e., infinitely differentiable), $g(x)$, around $x = x_0$ can be obtained as shown below:

$$g(x) = \underline{g(x_0) + \left.\frac{dg(x)}{dx}\right|_{x=x_0}(x - x_0)}$$

$$\underline{+ \left.\frac{d^2g(x)}{dx^2}\right|_{x=x_0}\frac{(x - x_0)^2}{2} + \cdots}$$

$(\approx 0 \text{ if } x \approx x_0)$

$$\boxed{g(x) \approx g(x_0) + \left.\frac{dg(x)}{dx}\right|_{x=x_0}(x - x_0)}$$

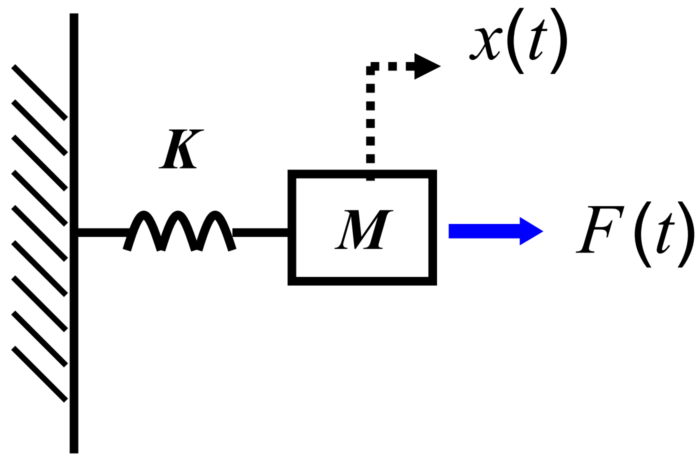


# Example 1: Nonlinear spring

- Linear spring:

$$M\ddot{x}(t) = F(t) - Kx(t)$$

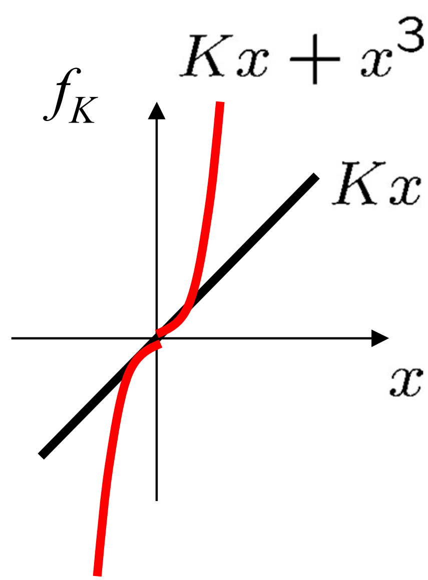
$$\Rightarrow \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + K}$$




- Example of nonlinear spring:

Nonlinear term!

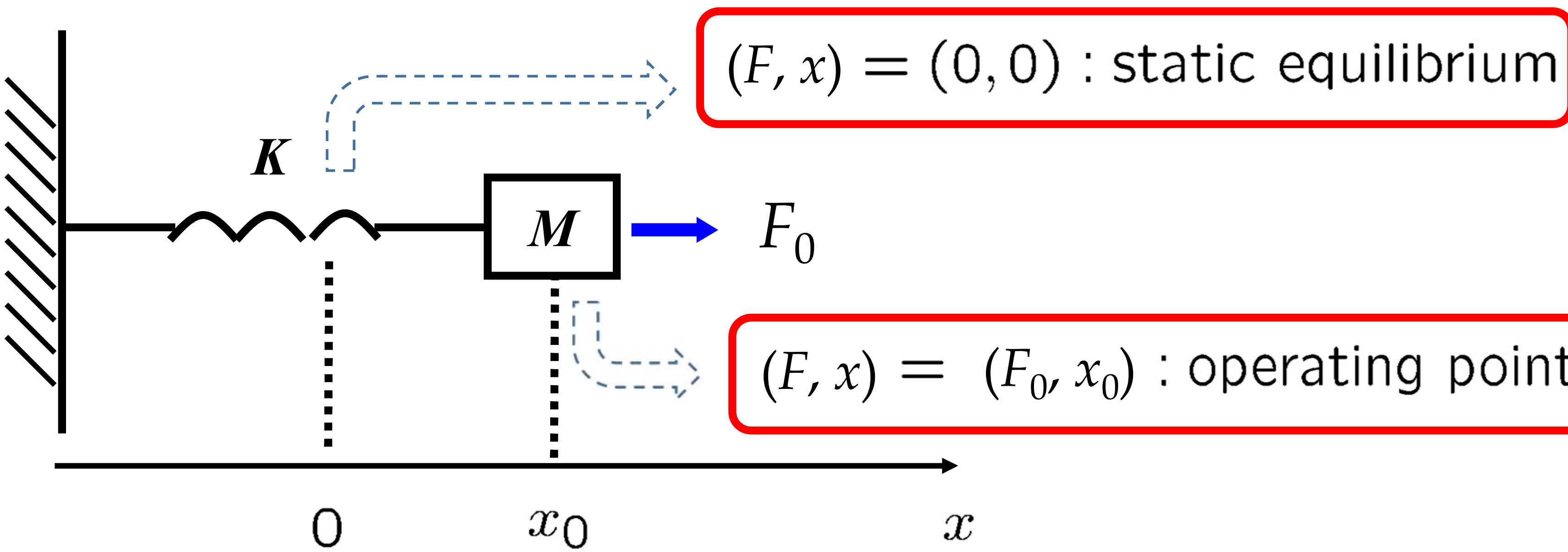
$$M\ddot{x}(t) = F(t) - Kx(t) - \underline{x^3(t)}$$




- We cannot represent $\mathcal{L}\left\{x^3(t)\right\}$ in terms of $X(s)$.

# Example 1 (cont'd): Specifying an operating point

- Operating point: The point around which the system is assumed to be operating.




- We linearize the nonlinear system around a specific operating point.
    - Note that sometimes the **operating point** is also called ***equilibrium operating point*** or even ***equilibrium point***.

# Example 1 (cont'd): Linearization Procedure

**Step 1:** Identify input and output variables:

$$\begin{cases} F(t) = \text{input} \\ x(t) = \text{output} \end{cases}$$

**Step 2:** Express non-linear ODE in the form of $f(\ddot{x}, \dot{x}, x, F) = 0$.

**Step 3:** Find the **Operating Point** (**OP**) of $(x_0, F_0)$. That is, find $F_0$ at the given $x_0$.

**Step 4:** Write the Taylor series expansion at $(x_0, F_0)$, i.e., at the OP.

**Step 5:** Change variables to perturbation variables in the Taylor series expansion.

**Step 6:** Re-write the Taylor series expansion as a linear ODE.

The linearized model is valid only around the specified operating point!

# Example 1 (cont'd)

Our aim is to linearize the ODE at $x = x_0$.

$$M\ddot{x}(t) = F(t) - Kx(t) - x^3(t)$$

**Step 1:**

Identify input and output variables:

$$\boxed{\begin{cases} F(t) = \text{input} \\ x(t) = \text{output} \end{cases}}$$

**Step 2:**

Express non-linear ODE in the form of $f(\ddot{x}, \dot{x}, x, F) = 0$, (in this case, $f(\ddot{x}, x, F) = 0$):

$$M\ddot{x}(t) = F(t) - K \cdot x(t) - x^3(t) \Longrightarrow f(\ddot{x}, x, F) = M\ddot{x} + K \cdot x + x^3 - F(t) = 0$$

$$\Longrightarrow \boxed{M\ddot{x} + K \cdot x + x^3 - F(t) = 0}$$

**Step 3:**

Find the operating point $(x_0, F_0)$. That is, find $F_0$, which is the operating value of $F$ at $x = x_0$:

$$x(t) = x_0,\ \ \dot{x}(t) = 0,\ \ \ddot{x}(t) = 0 \Longrightarrow M\underbrace{\cancel{\ddot{x}_0}}_{0} + K.x_0 + x_0^3 - F_0 = 0$$

$$\Longrightarrow \boxed{F_0 = K.x_0 + x_0^3}$$

**Note:** $x_0$ is a given value. If it is explicitly given, you can substitute it in the $F_0$ equation and find the value of $F_0$.

# Example 1 (cont'd)

**Step 4:**

Write the Taylor series expansion at $(x_0, F_0)$, i.e., at the Operating Point (OP):

$$\boxed{f(\ddot{x}, x, F) = f(\ddot{x}_0, x_0, F_0) + \left.\frac{\partial f}{\partial \ddot{x}}\right|_{OP} (\ddot{x} - \ddot{x}_0) + \left.\frac{\partial f}{\partial x}\right|_{OP} (x - x_0) + \left.\frac{\partial f}{\partial F}\right|_{OP} (F - F_0)}$$

**Step 5:**

Change variables to perturbation variables in the Taylor series expansion:

$$\begin{cases} \delta x = x - x_0 \\ \delta F = F - F_0 \\ \delta \ddot{x} = \ddot{x} - \ddot{x}_0 \end{cases}$$

$$\boxed{f(\ddot{x}, x, F) = f(\ddot{x}_0, x_0, F_0) + \left.\frac{\partial f}{\partial \ddot{x}}\right|_{OP} \delta\ddot{x} + \left.\frac{\partial f}{\partial x}\right|_{OP} \delta x + \left.\frac{\partial f}{\partial F}\right|_{OP} \delta F} \qquad \text{Eqn. 1}$$

# Example 1 (cont'd)

**Step 6:**

Re-write the Taylor series expansion as a linear ODE:

$$f(\ddot{x}, x, F) = M\ddot{x} + K \cdot x + x^3 - F(t)$$

$$\left.\frac{\partial f}{\partial \ddot{x}}\right|_{OP} = M \quad ; \quad \left.\frac{\partial f}{\partial x}\right|_{OP} = K + 3x_0^2 = K^* \quad ; \quad \left.\frac{\partial f}{\partial F}\right|_{OP} = -1$$

Substitute in Eqn. 1

$$f(\ddot{x}, x, F) = \underbrace{\cancel{f(\ddot{x}_0, x_0, F_0)}}_{0} + M\delta\ddot{x} + K^* \cdot \delta x + (-1)\delta F$$

We know that $f(\ddot{x}, x, F)$ is also equal to 0: $\boxed{M\delta\ddot{x} + K^* \cdot \delta x = \delta F}$

*Linear!*

Note that if we take the Laplace transform of the above linear ODE, we obtain (use $\delta x = \tilde{x}$):

$$\boxed{\frac{\tilde{X}(s)}{\tilde{F}(s)} = \frac{1}{Ms^2 + K^*}}$$

**Note:** "$\sim$" is the same as "$\delta$"

where $\mathcal{L}\{\delta x(t)\} = \mathcal{L}\{\tilde{x}(t)\} = \tilde{X}(s)$ and $\mathcal{L}\{\delta F(t)\} = \mathcal{L}\{\tilde{F}(t)\} = \tilde{F}(s)$.

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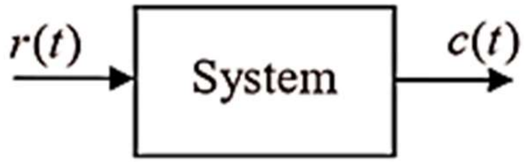
# Linearization (General Method)

## The Six Steps of Linearization




1) Identify the system model's input $r(t)$ and output $c(t)$.

2) Express Model in the form $f(r,\dot{r},\ddot{r},\ldots,c,\dot{c},\ddot{c},\ldots)=0$.

3) Define an equilibrium operating point $(r,c)=(r_o,c_o)$.

4) Perform a Taylor Series expansion about the operating point $(r_o,c_o)$ retaining only 1st derivative terms.

$$f(r,\dot{r},\ddot{r},\ldots,c,\dot{c},\ddot{c},\ldots) \cong f(r_o,0,0,\ldots,c_o,0,0,\ldots)$$
$$+\left.\frac{\partial f}{\partial r}\right|_{(r_o,c_o)}(r-r_o)+\left.\frac{\partial f}{\partial \dot{r}}\right|_{(r_o,c_o)}(\dot{r}-\dot{r}_o)+\left.\frac{\partial f}{\partial \ddot{r}}\right|_{(r_o,c_o)}(\ddot{r}-\ddot{r}_o)+\ldots$$
$$+\left.\frac{\partial f}{\partial c}\right|_{(r_o,c_o)}(c-c_o)+\left.\frac{\partial f}{\partial \dot{c}}\right|_{(r_o,c_o)}(\dot{c}-\dot{c}_o)+\left.\frac{\partial f}{\partial \ddot{c}}\right|_{(r_o,c_o)}(\ddot{c}-\ddot{c}_o)+\ldots$$

# Linearization (General Method)

5) Change variables from original input $r(t)$ and output $c(t)$ to deviations about the defined operating point. These new variables are the differences required in the Taylor expansion.

**Note:**
"$\sim$" is the same as "$\delta$"

$$\tilde{r} = (r - r_o),\ \dot{\tilde{r}} = (\dot{r} - \dot{r}_o),\ \ddot{\tilde{r}} = (\ddot{r} - \ddot{r}_o),\ etc.$$

$$\tilde{c} = (c - c_o),\ \dot{\tilde{c}} = (\dot{c} - \dot{c}_o),\ \ddot{\tilde{c}} = (\ddot{c} - \ddot{c}_o),\ etc.$$

with $f(r_o,0,0,\ldots,c_o,0,0,\ldots) = 0$ from step 3 yields

$$f(\tilde{r},\dot{\tilde{r}},\ddot{\tilde{r}},\ldots,\tilde{c},\dot{\tilde{c}},\ddot{\tilde{c}},\ldots) \cong 0 + \left[\frac{\partial f}{\partial r}\bigg|_{(r_o,c_o)}\right]\tilde{r} + \left[\frac{\partial f}{\partial \dot{r}}\bigg|_{(r_o,c_o)}\right]\dot{\tilde{r}} + \left[\frac{\partial f}{\partial \ddot{r}}\bigg|_{(r_o,c_o)}\right]\ddot{\tilde{r}} + \ldots$$

$$+ \left[\frac{\partial f}{\partial c}\bigg|_{(r_o,c_o)}\right]\tilde{c} + \left[\frac{\partial f}{\partial \dot{c}}\bigg|_{(r_o,c_o)}\right]\dot{\tilde{c}} + \left[\frac{\partial f}{\partial \ddot{c}}\bigg|_{(r_o,c_o)}\right]\ddot{\tilde{c}} + \ldots$$

Note: Each of the terms in square brackets evaluates as a constant.

6) Rewrite the function defined in 5) in the standard ordinary differential equation form.

$$\boxed{\left[\frac{\partial f}{\partial \ddot{c}}\bigg|_{(r_o,c_o)}\right]\ddot{\tilde{c}} + \left[\frac{\partial f}{\partial \dot{c}}\bigg|_{(r_o,c_o)}\right]\dot{\tilde{c}} + \left[\frac{\partial f}{\partial c}\bigg|_{(r_o,c_o)}\right]\tilde{c} = -\left[\frac{\partial f}{\partial \ddot{r}}\bigg|_{(r_o,c_o)}\right]\ddot{\tilde{r}} - \left[\frac{\partial f}{\partial \dot{r}}\bigg|_{(r_o,c_o)}\right]\dot{\tilde{r}} - \left[\frac{\partial f}{\partial r}\bigg|_{(r_o,c_o)}\right]\tilde{r}}$$

# Linearization (General Method)

**The Six Linearization Steps Summarized:**

1) Identify input and output variables.
2) Express non-linear differential equation in the form $f(r,\dot{r},\ddot{r},\ldots,c,\dot{c},\ddot{c},\ldots)=0$
3) Find the equilibrium operating point $(r,c)=(r_o,c_o)$, i.e., find $r_0$ at the given $c_0$.
4) Perform Taylor expansion neglect derivatives above first order.
5) Change variables: $\tilde{r}=(r-r_o), \dot{\tilde{r}}=(\dot{r}-\dot{r}_o),\ldots,\tilde{c}=(c-c_o), \dot{\tilde{c}}=(\dot{c}-\dot{c}_o),\ldots$
6) Rewrite result as a linear ODE in standard form.

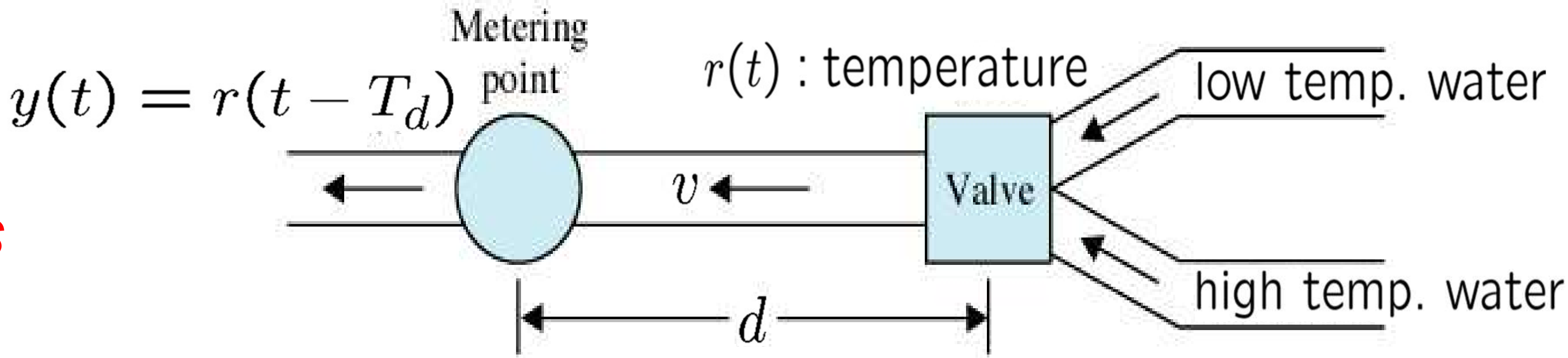
**Note:**

The **standard form of a linear ODE** means that all the terms related to the input are placed on the right-hand side of the ODE and all the remaining terms on the left-hand side.

# Time-delay

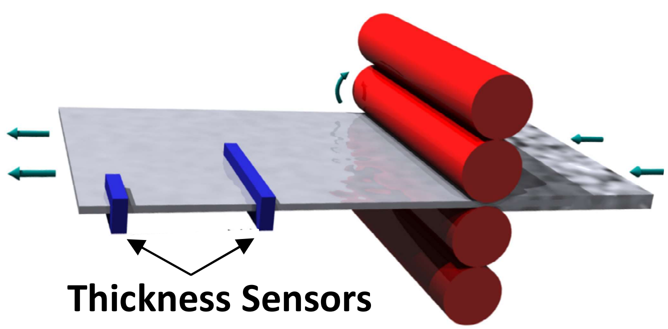
A few applied examples are given below:

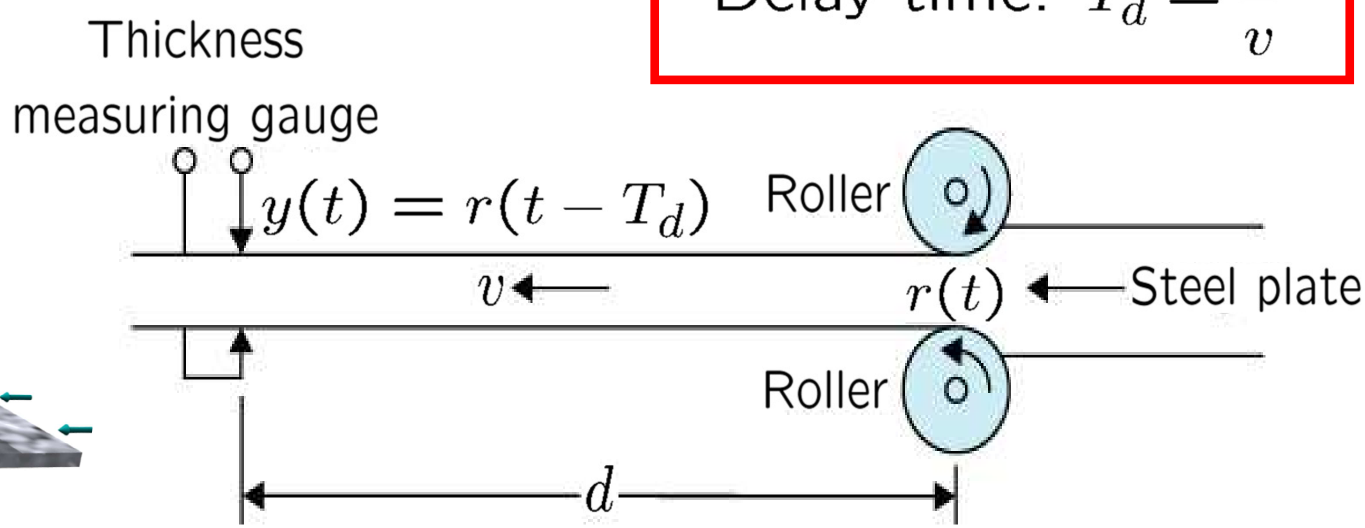
*Mixing fluids*




$$\text{Delay time: } T_d = \frac{d}{v}$$

*Steel thickness control*



# Time-delay transfer function

- TF derivation

$$y(t) = r(t - T_d) \ (T_d\text{: delay time})$$

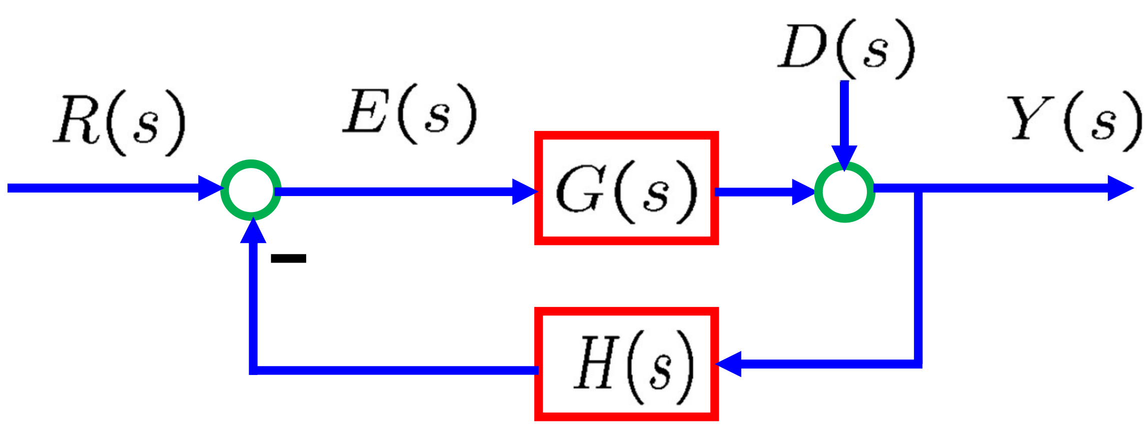
$$\xrightarrow{\mathcal{L}} \quad Y(s) = e^{-T_d s} R(s) \quad \Rightarrow \quad \frac{Y(s)}{R(s)} = e^{-T_d s}$$

- The greater the time delay, the harder it becomes to control the system! (You'll learn the theoretical reasoning behind this later.)

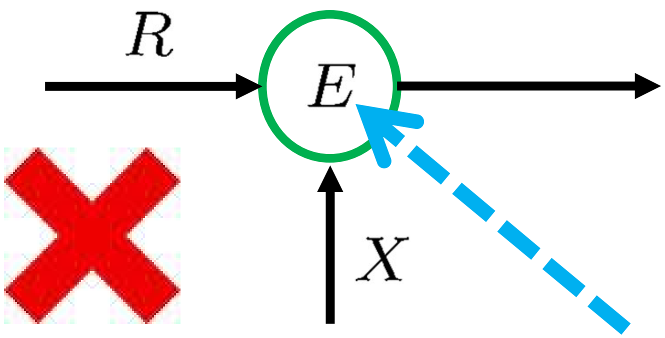
- Imagine you are trying to control the flow rate of water at the end of a very long pipe by adjusting a valve at the beginning. When you open or close the valve, the change in flow does not reach the other end instantly—it takes time for the pressure and flow rate to adjust along the length of the pipe. If you react too quickly before seeing the actual result, you might keep adjusting unnecessarily, causing unstable or oscillating flow at the outlet.

# Block diagram

- Represents relations among ***signals*** and ***systems***
- Very useful in representing control systems
- Also useful in computer simulations (**Simulink**)
- Elements
  - **Block**: transfer function ("gain" block)
  - **Arrow**: signal
  - **Node**: summation (or subtraction) of signals

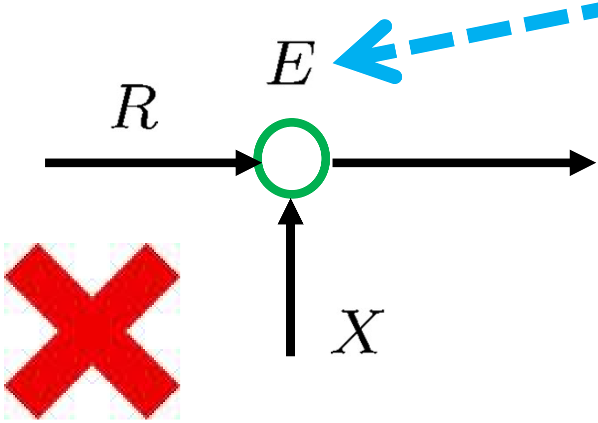


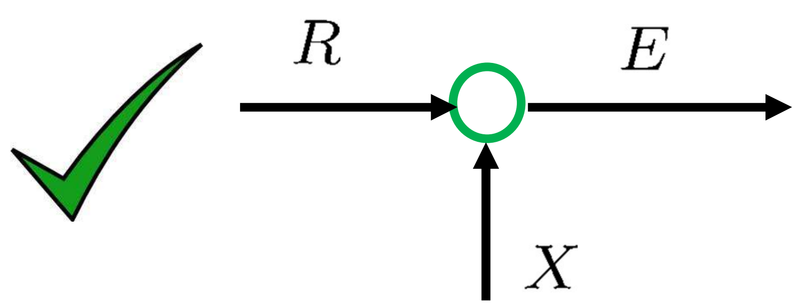
# Typical mistakes



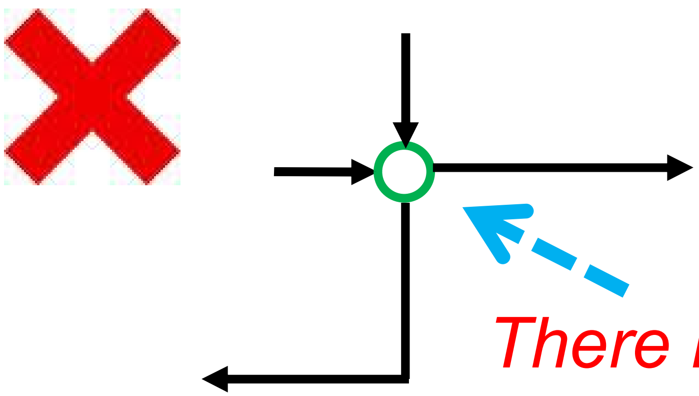

*Unclear which signal is "E"*

*Signal must be indicated* ***on*** *an arrow.*



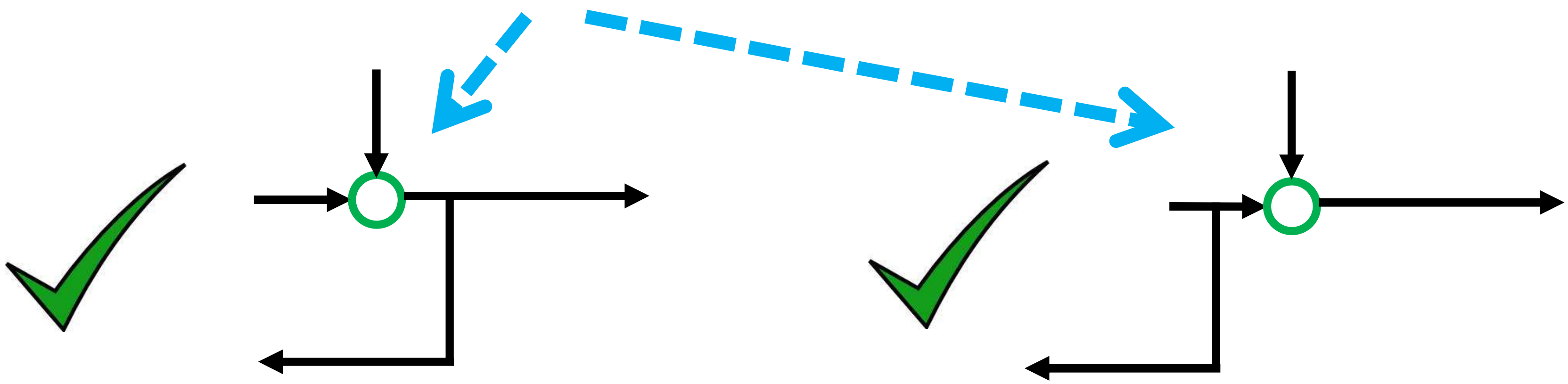



# Typical mistakes (cont'd)

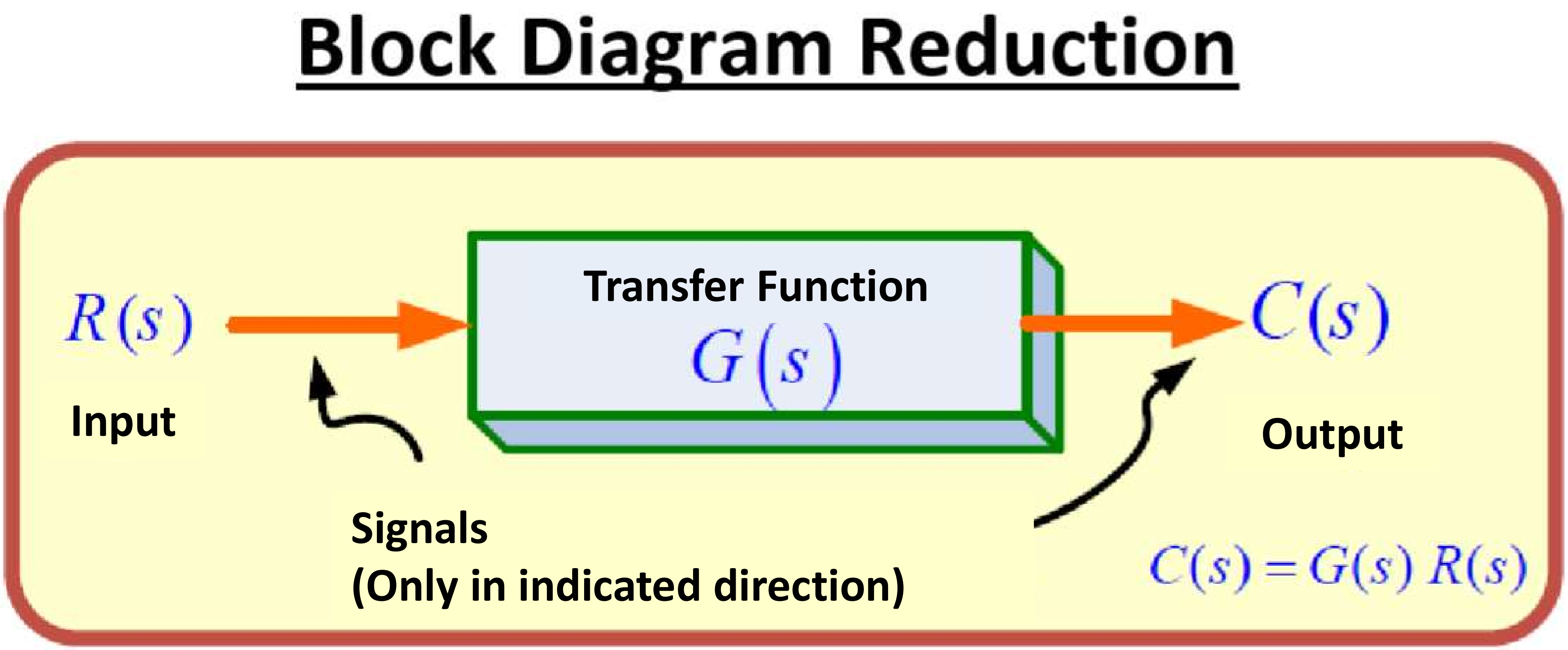


*There must be **only one output** from a node.*

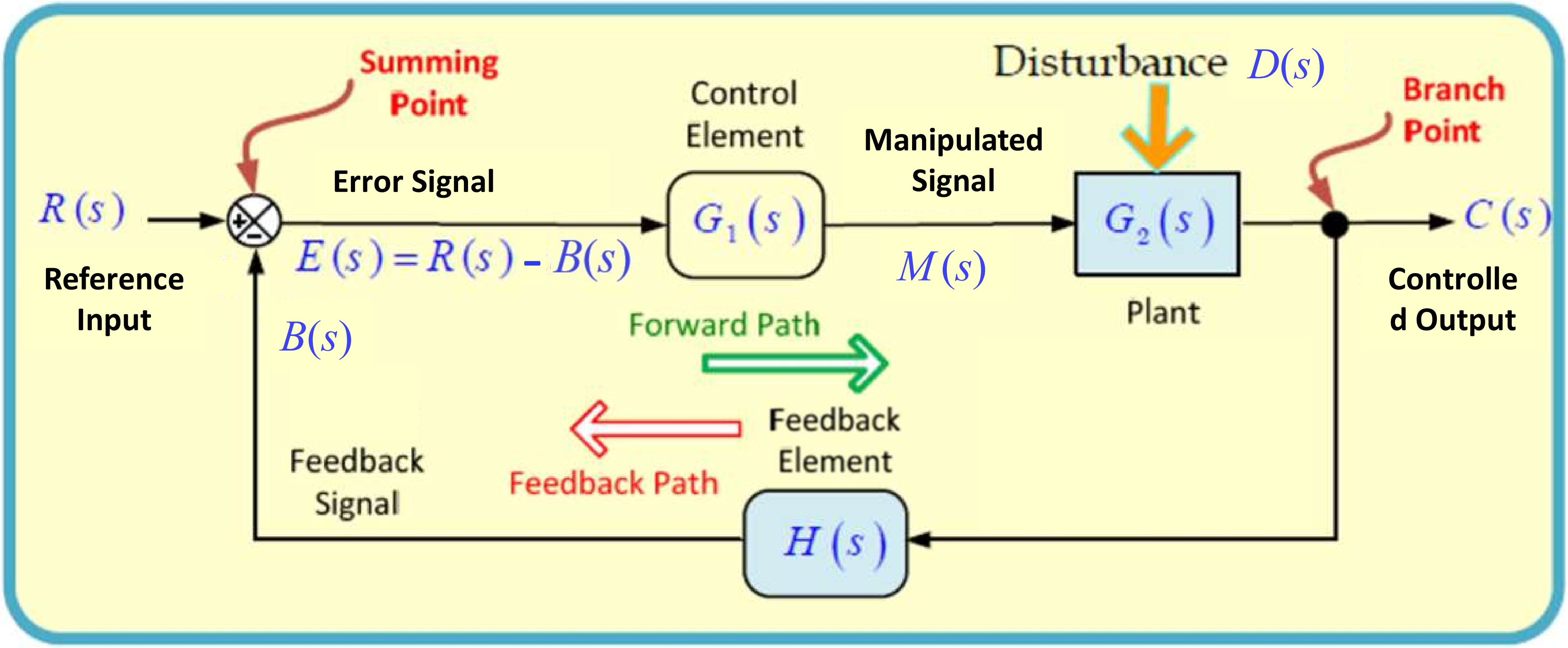
*Both are fine, but they have different meanings!*



# Block diagram



# Block diagram



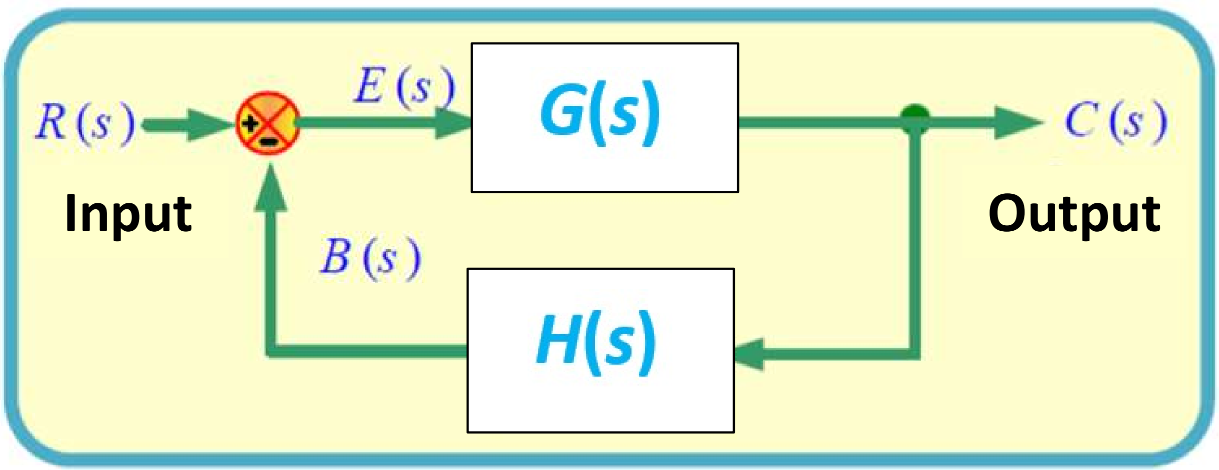

- In the block diagram above, we assume that the Control Element consists of both the Controller and the Actuator, combined into a single unit.

# Block diagram

## Definitions

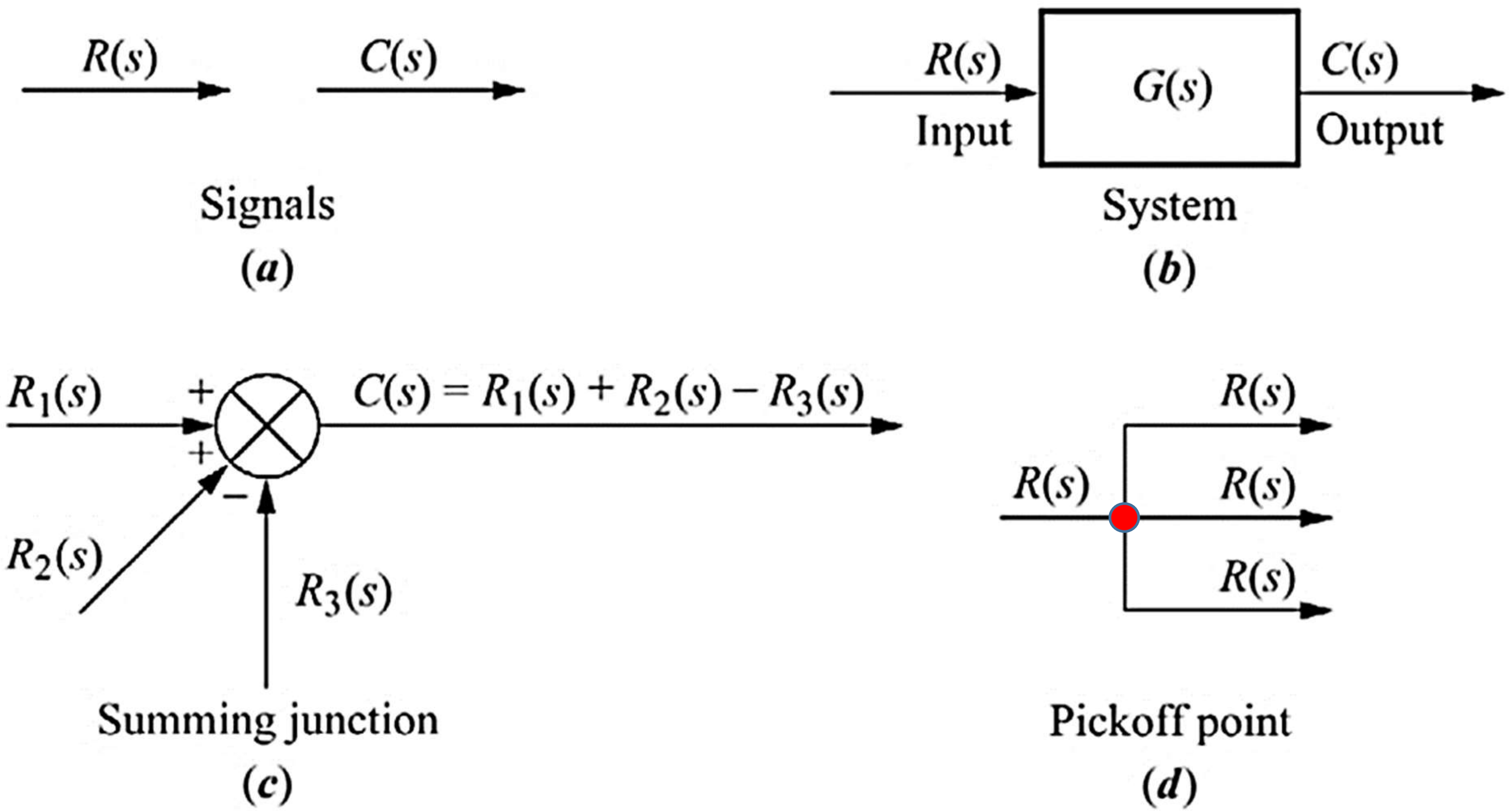
- $G(s) \equiv$ Direct transfer function = Forward transfer function = Feed-forward transfer function.
- $H(s) \equiv$ Feedback transfer function.
- $G(s)H(s) \equiv$ Open-loop transfer function.
- $C(s)/R(s) \equiv$ Closed-loop transfer function = Control ratio
- $C(s)/E(s) \equiv$ Feed-forward transfer function.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$




Block diagram of a closed-loop system with a feedback element

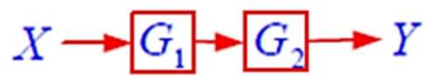
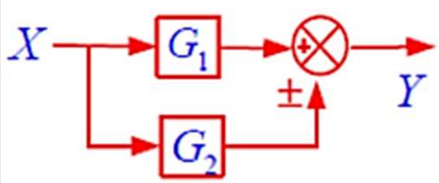
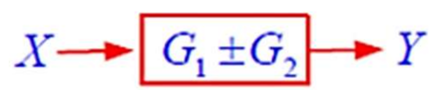
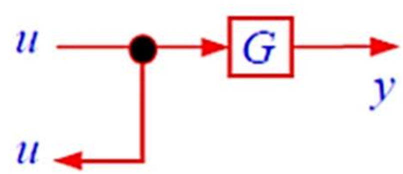
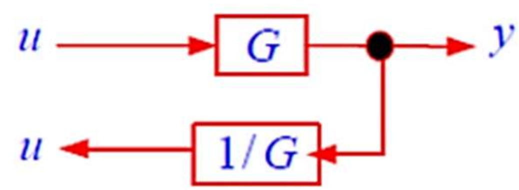
# Block diagram



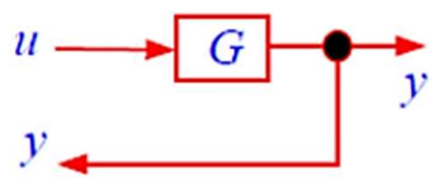
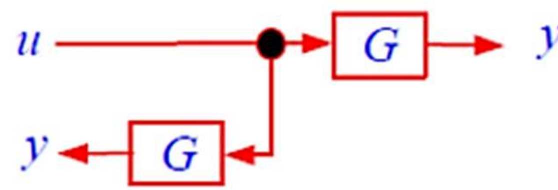
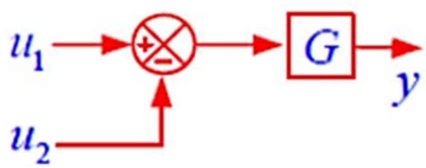
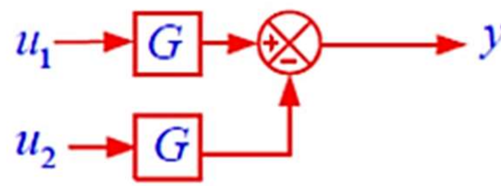
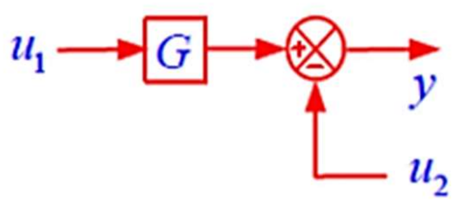
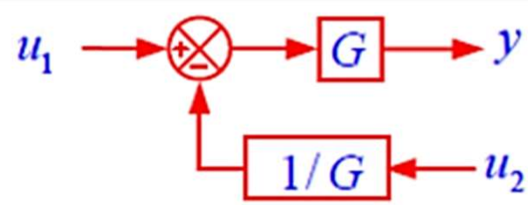
# Block diagram

a place of mind
UBC

## Basic rules with block diagram transformation

| | Manipulation | Original Block Diagram | Equivalent Block Diagram | Equation |
|---|---|---|---|---|
| 1 | Combining blocks in cascade |  |  | $Y = (G_1G_2)X$ |
| 2 | Combining blocks in parallel; or eliminating a forward loop |  |  | $Y = (G_1 \pm G_2)X$ |
| 3 | Moving a pickoff point after a block |  |  | $y = G\,u$<br>$u = \dfrac{1}{G}\,y$ |

# Block diagram

| | Manipulation | Original Block Diagram | Equivalent Block Diagram | Equation |
|---|---|---|---|---|
| 4 | Moving a pickoff point before a block |  |  | $y = Gu$ |
| 5 | Moving a summing point after a block |  |  | $y = G(u_1 - u_2)$ |
| 6 | Moving a summing point before a block |  |  | $y = Gu_1 - u_2$ |

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# Summary

- Modeling of DC motor, nonlinear systems, delay time.
- Main message up to this point: *"Many systems can be represented as transfer functions!"*
- Next
  - *Stability* of linear control systems, which is one of the most important topics in feedback control.