a place of mind

ELEC 341: Systems and Control

Lecture 10

Time response: Examples

Course roadmap

Modeling

- ✓ Laplace transform
- ✓ Transfer function
- Models for systems
  - ✓ Electrical
  - ✓ Electromechanical
  - ✓ Mechanical
- ✓ Linearization, delay

Analysis

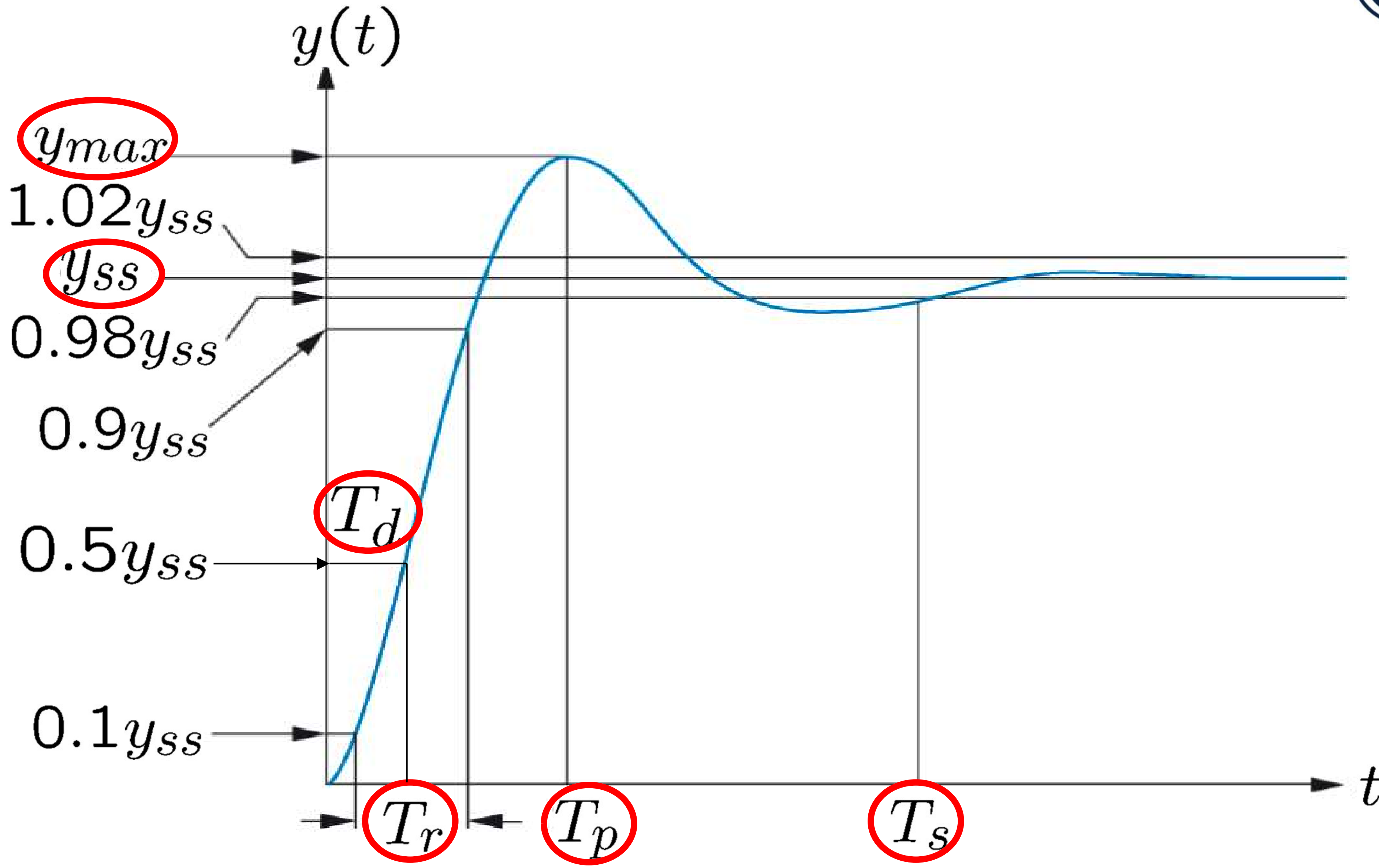
- ✓ Stability
  - ✓ Routh-Hurwitz
  - Nyquist
- ✓ Time response
  - ➤ Transient
  - ✓ Steady state
- Frequency response
  - Bode plot

Design

- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

Matlab simulations

Typical step response (review)



Performance measures

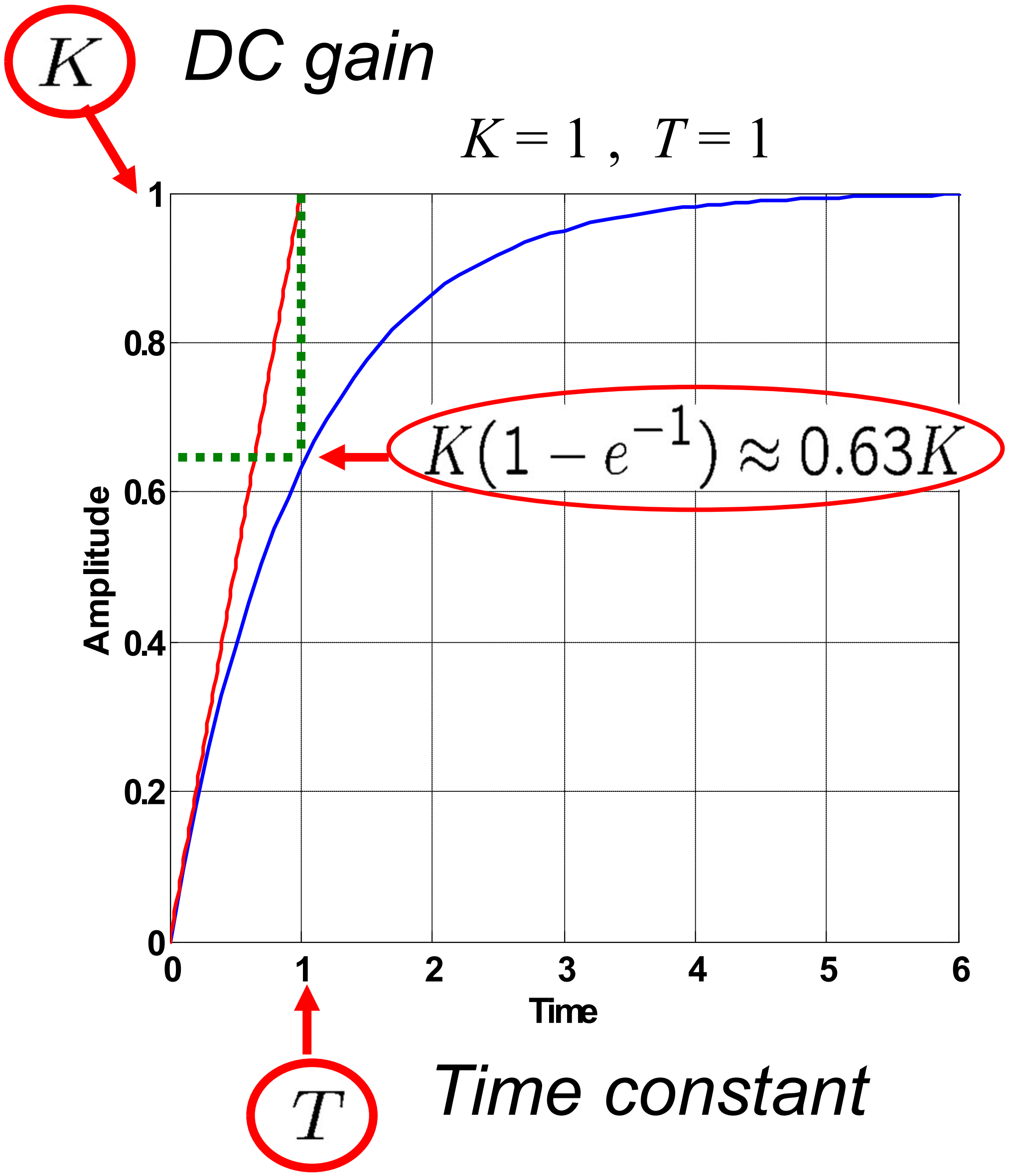
- Transient response
    - Peak value
    - Peak time
    - Percent overshoot
    - Delay time
    - Rise time
    - Settling time
- Steady state response
    - Steady state error

(Done for 1st & 2nd order systems) ✓

We connected these performance measures with s-domain. ✓

(Done) ✓

Step response for 1st-order system (review)



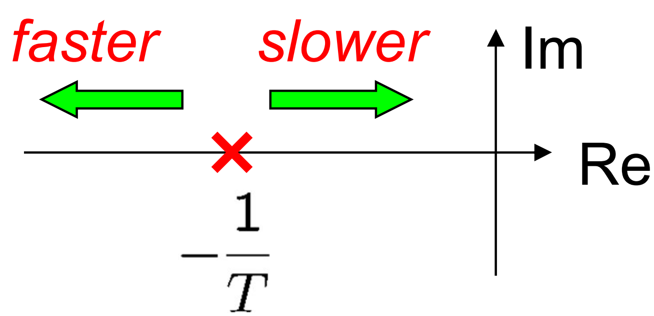
$$G(s) = \frac{K}{Ts + 1}$$

Settling time

$$5\% T_s : 3T = \frac{3}{|\text{Re}|}$$

$$2\% T_s : 4T = \frac{4}{|\text{Re}|}$$

× is used to show **poles**



Step response of 2nd-order system for various damping ratios (review)

- Undamped

$$\zeta = 0$$

- Underdamped

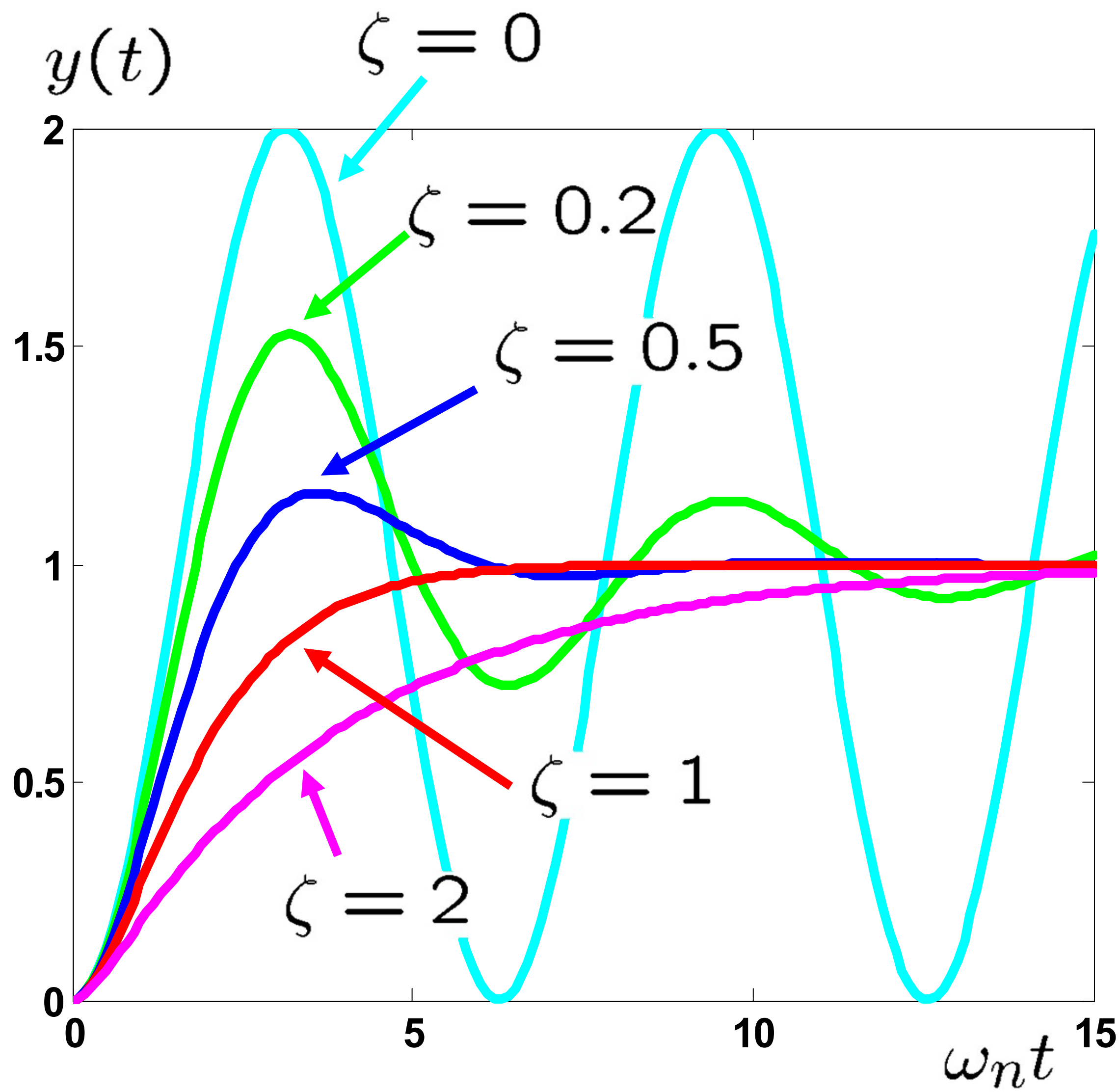
$$0 < \zeta < 1$$

- Critically damped

$$\zeta = 1$$

- Overdamped

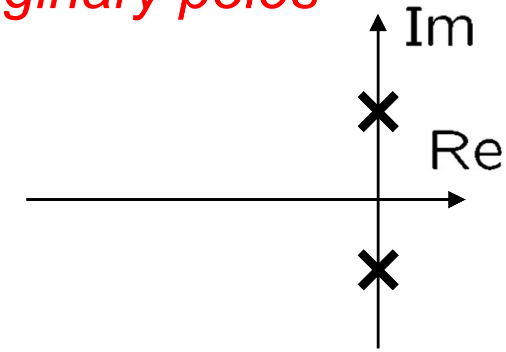
$$\zeta > 1$$



Pole locations & damping (2nd order Systems)

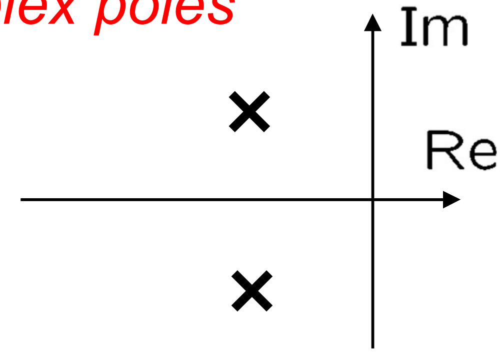
- Undamped

Imaginary poles



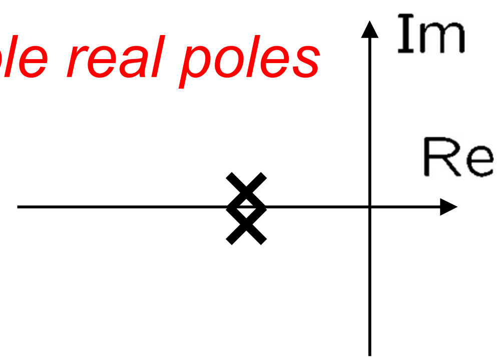
- Underdamped

Complex poles



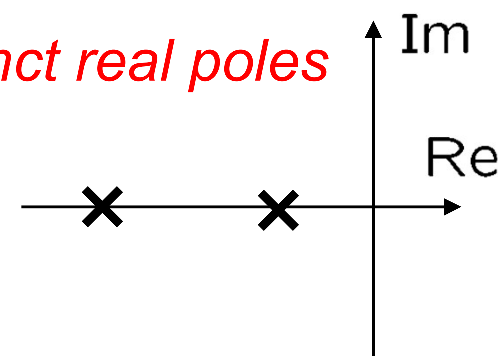
- Critically damped

Double real poles



- Overdamped

Distinct real poles



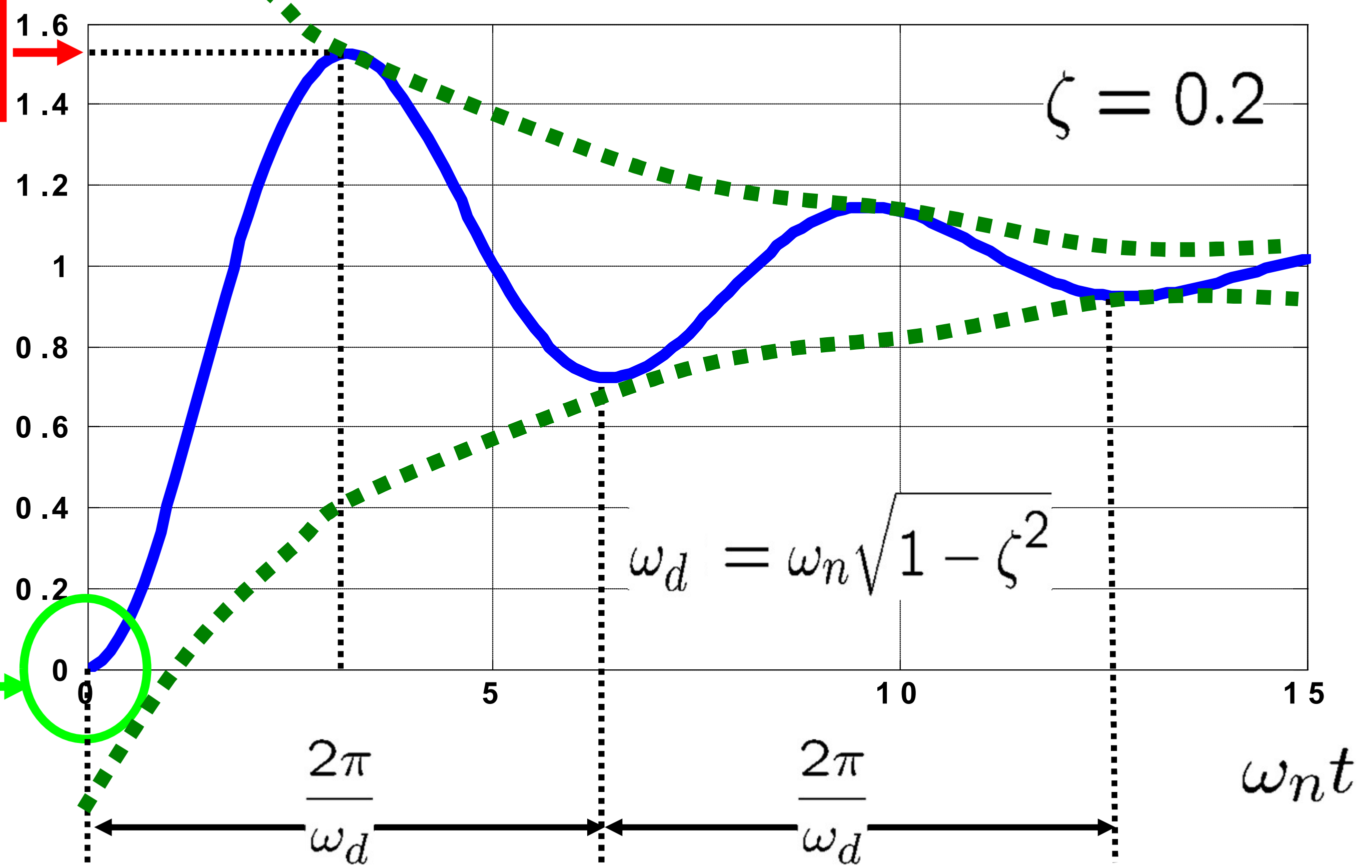
Step response for 2nd order system: Underdamped case (review)

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad 0 < \zeta < 1$$

$$y_{max} = 1 + e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$T_p = \frac{\pi}{\omega_d}$$

$$y(0) = 0$$
$$y'(0) = 0$$



Properties of underdamped 2nd order system in terms of ζ and ω_n (review)

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad 0 < \zeta < 1$$

	(5%) (2%)
Settling time, T_s	$\approx \frac{3}{\zeta\omega_n}$ or $\frac{4}{\zeta\omega_n}$
Peak time, T_P	$\frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
Peak value, y_{max}	$1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}$
Percent overshoot, PO or $\%OS$	$100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

Time constant = $T = \frac{1}{\zeta\omega_n}$

$PO = 100e^{-\pi/\tan\theta}$

Delay time = $T_d = \frac{1 + 0.7\zeta}{\omega_n}$

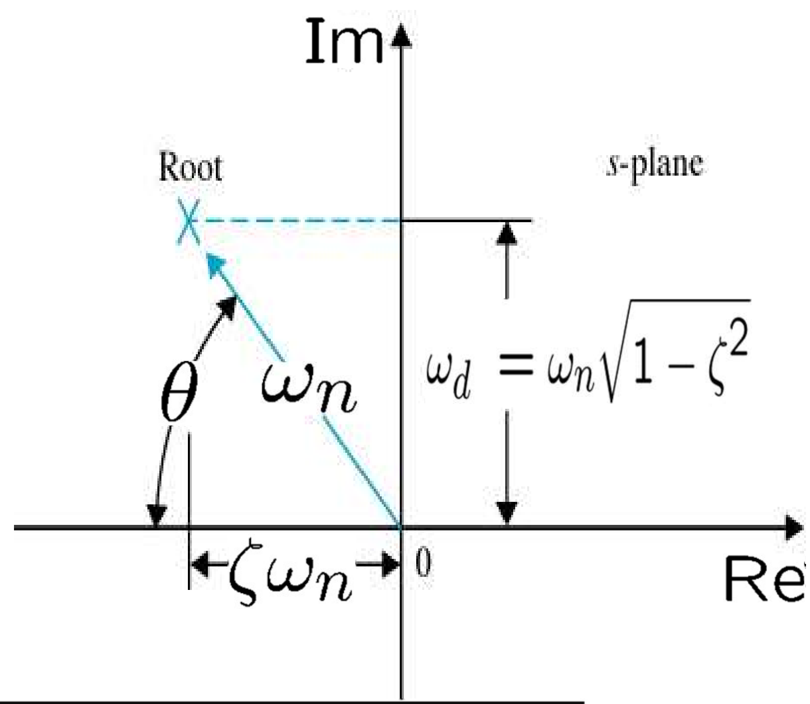
Step response properties of underdamped 2nd order system in terms of pole locations (review)

- Poles ($0 < \zeta < 1$)

$$s = -\zeta\omega_n \pm j\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}$$

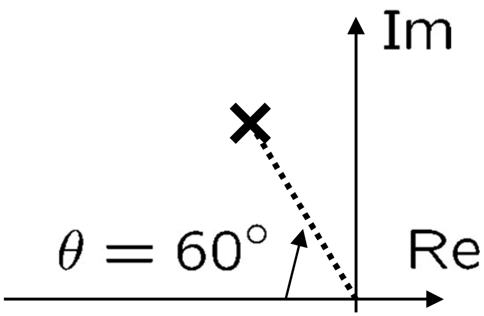
$$\zeta = \cos\theta$$

$$T = \text{Time Constant} = 1/\zeta\omega_n$$

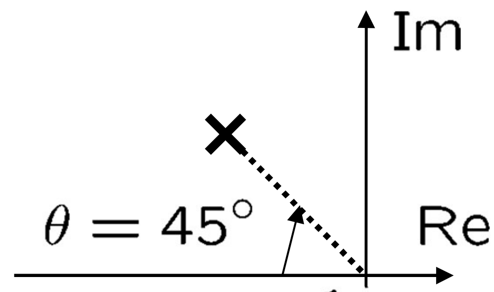


Pole			Performance
Real part	$\zeta\omega_n$	determines	$T_s = \frac{3}{\zeta\omega_n}, \frac{4}{\zeta\omega_n}$ $= \frac{3}{\|\text{Re}\|}, \frac{4}{\|\text{Re}\|}$
Imag. part	ω_d	determines	$T_p = \frac{\pi}{\omega_d}$ $= \frac{\pi}{\|\text{Im}\|}$
Angle	θ	determines	overshoot

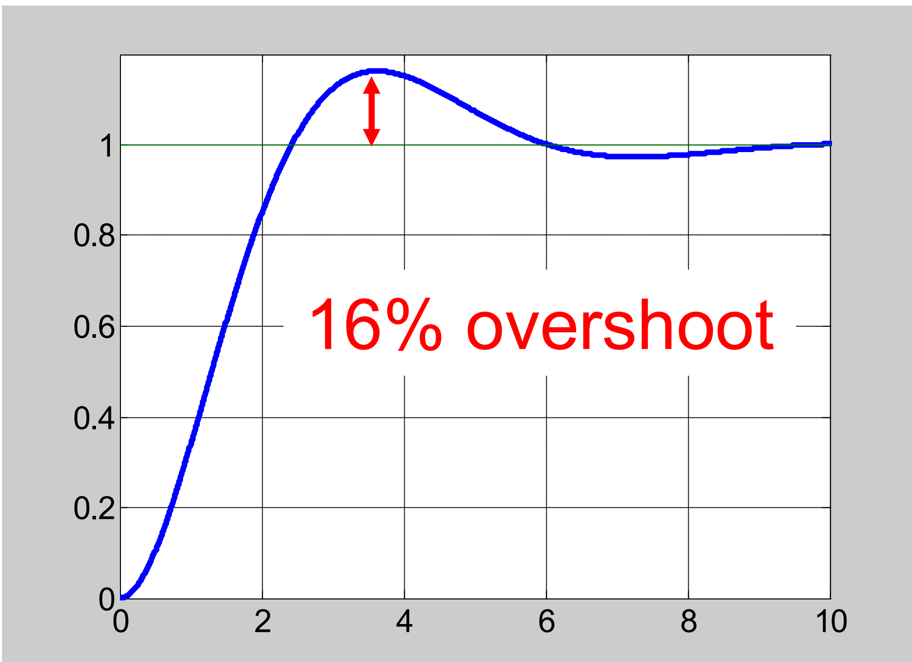
Angle θ and the overshoot (review)

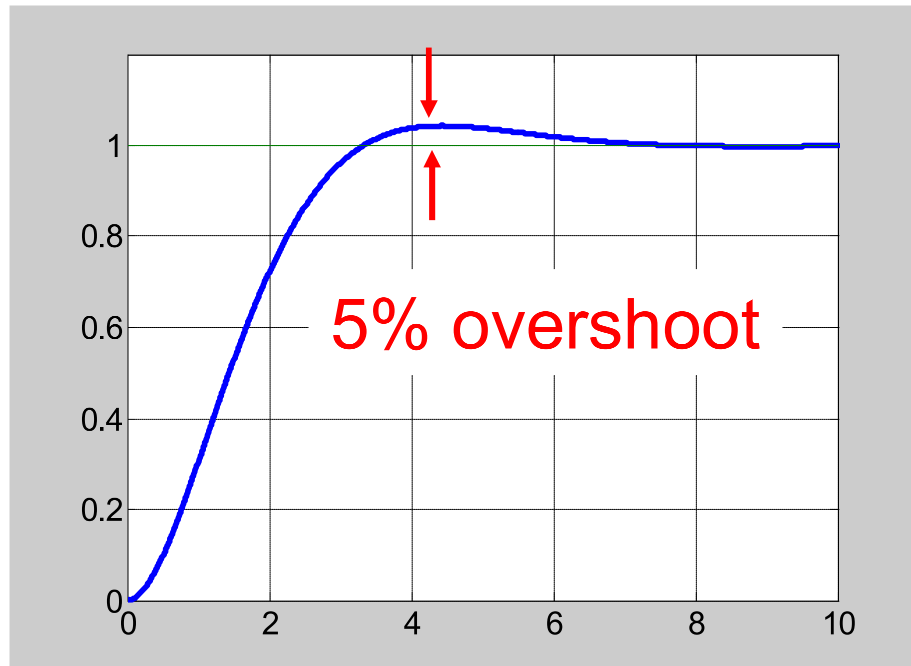


$$\zeta = \cos\theta = 0.5$$

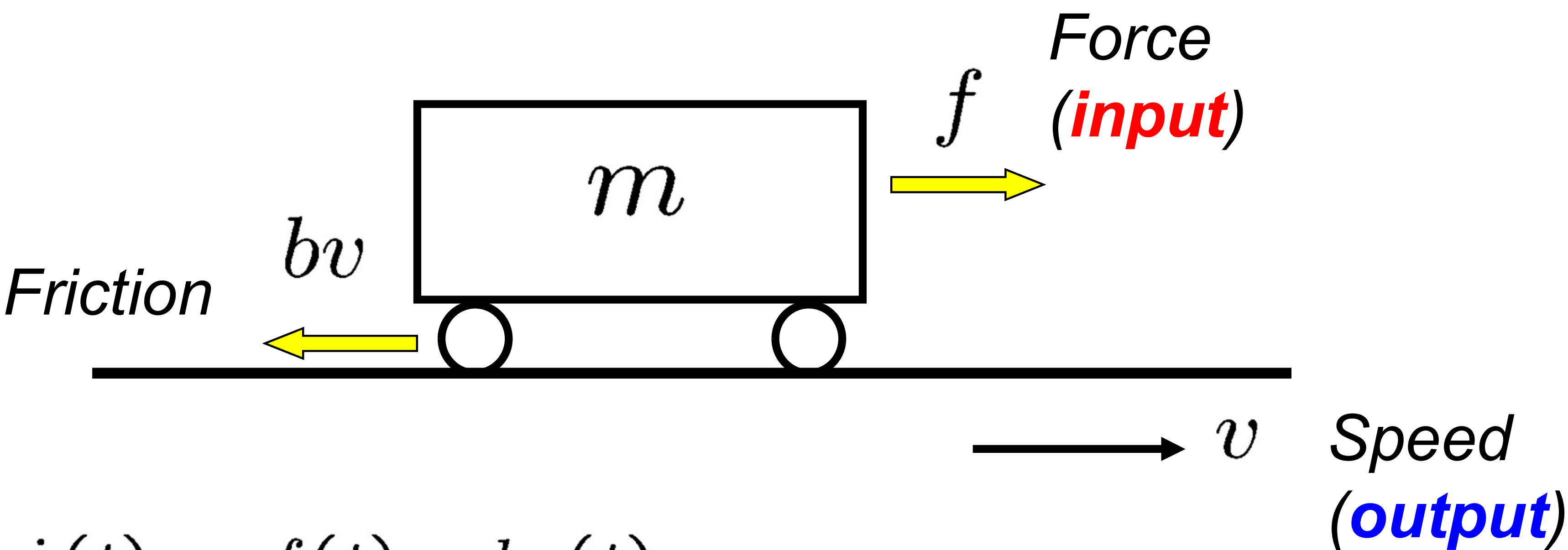


$$\zeta = \cos\theta = \frac{1}{\sqrt{2}} \approx 0.707$$





Example 1: Cruise control



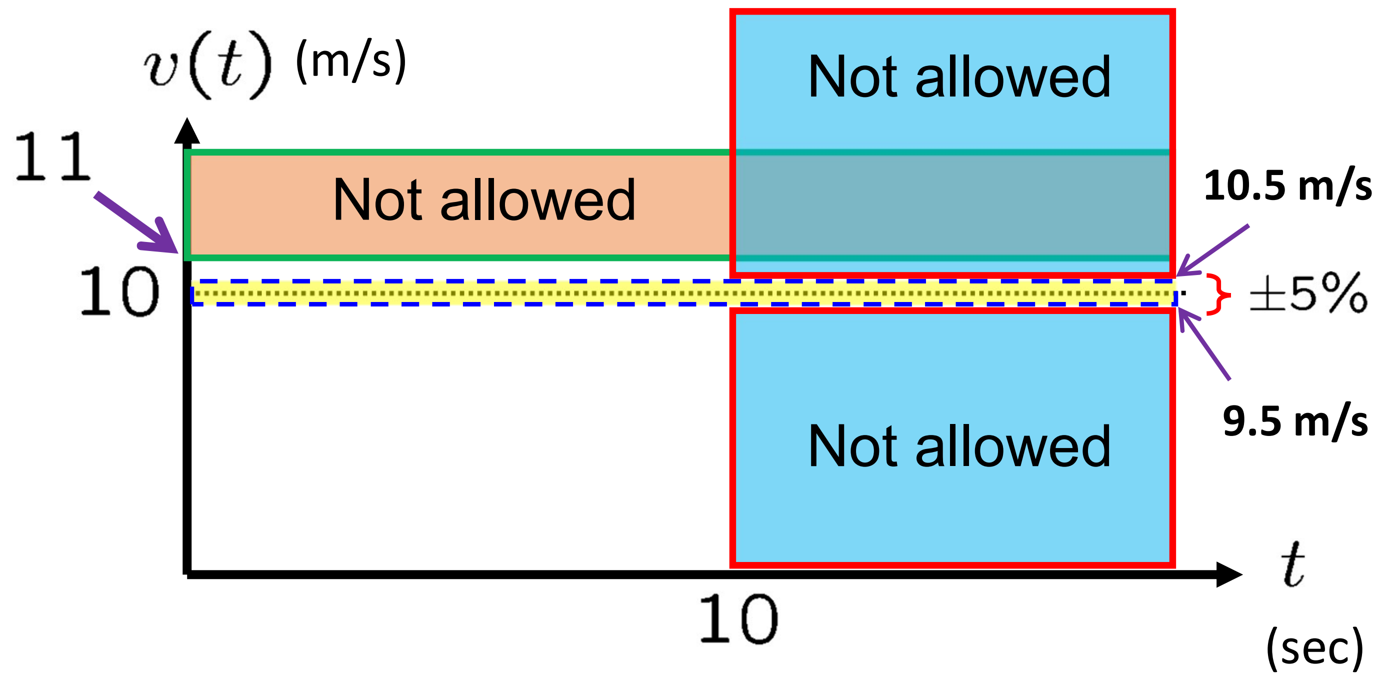
$$m\dot{v}(t) = f(t) - bv(t)$$

$$\Rightarrow \frac{V(s)}{F(s)} = \frac{1}{ms+b}$$

$$m = 1000 \text{ (kg)}$$
$$b = 50 \text{ (N·sec/m)}$$

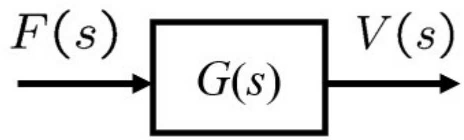
Example 1 (cont'd)

- Control goal: SS (steady-state) speed 10 m/s (= 36 km/h).
  - Stable
  - Zero steady-state error
  - Percent overshoot of 10 % or less
  - 5% T_s < 10 sec

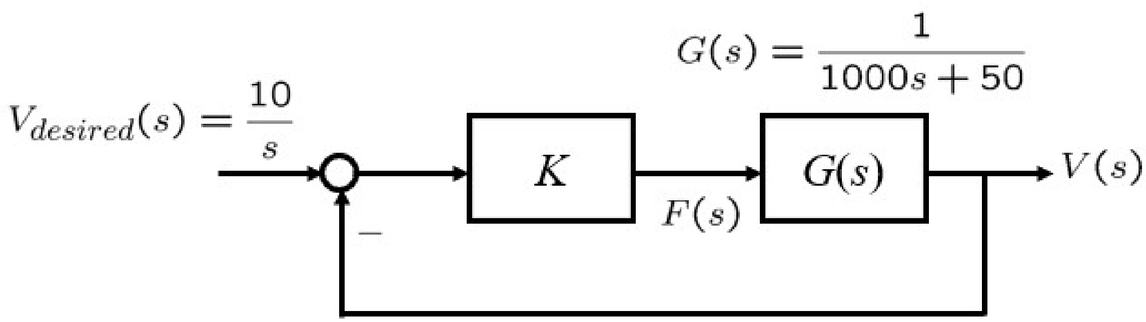


Example 1 (cont'd)

(a) The following open-loop control system is given. **(a1)** Obtain the amplitude of the step force (R) necessary to get the steady-state speed of 10 m/s. **(a2)** What is $5\%T_s$ for this system?



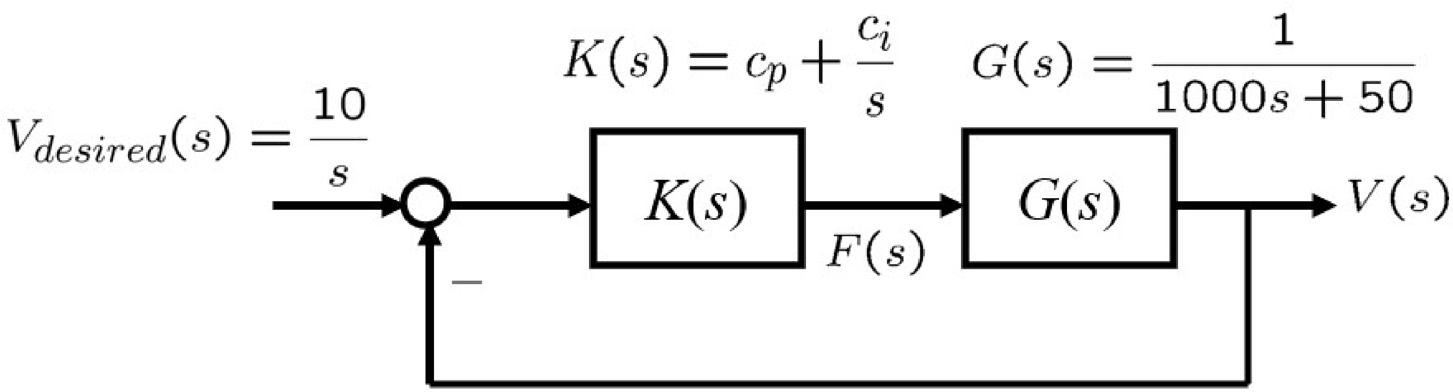
(b) If we decide to use a proportional controller and a negative feedback system, we will have the following block diagram:



Now apply the design requirement of $5\%T_s < 10$ sec to the above system. **(b1)** What will be the range of K? **(b2)** For the same block diagram, what will be the value of K if we want to satisfy the design requirement of zero steady-state error?

Example 1 (cont'd)

(c) If we decide to use a proportional-integral controller and a negative feedback system, we will have the following block diagram:



(c1) Is this system stable? **(c2)** What is the value of K_p (i.e., position error constant)? **(c3)** What is its steady-state error? **(c4)** Without finding c_p and c_i, discuss how one might compute the numerical values of these parameters?

Example 1: Open-loop control (cont'd)

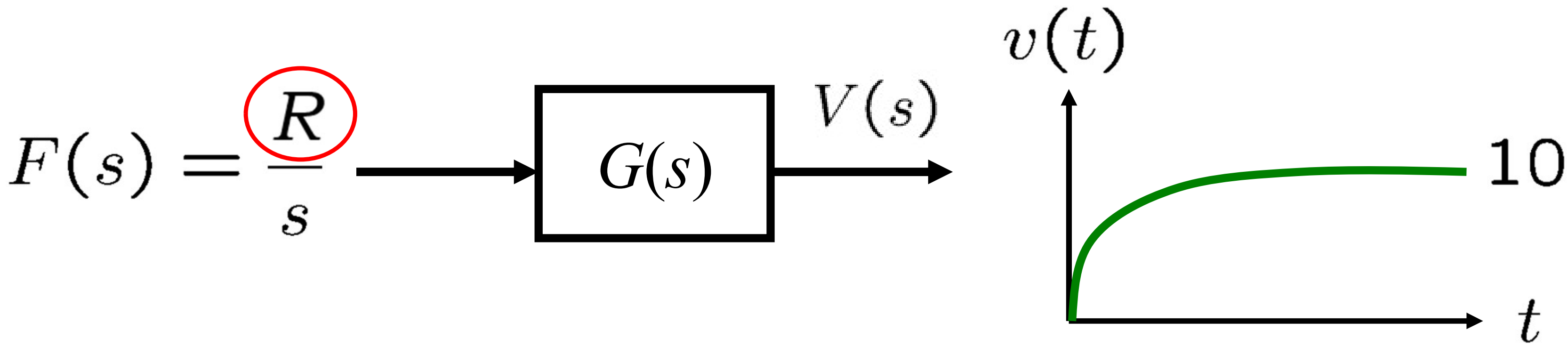
Solution:

(a1)

$$G(s) = \frac{V(s)}{F(s)} = \frac{1}{1000s + 50} = \frac{1/50}{20s + 1}$$

$T = 20$

- Obtain the amplitude of the step force necessary to get the steady-state speed of 10 m/s. This is shown by R.



$$v_{ss} = RG(0) = 10 \Rightarrow R = \frac{10}{G(0)} = 500(N) \Rightarrow R = 500$$

(a2)

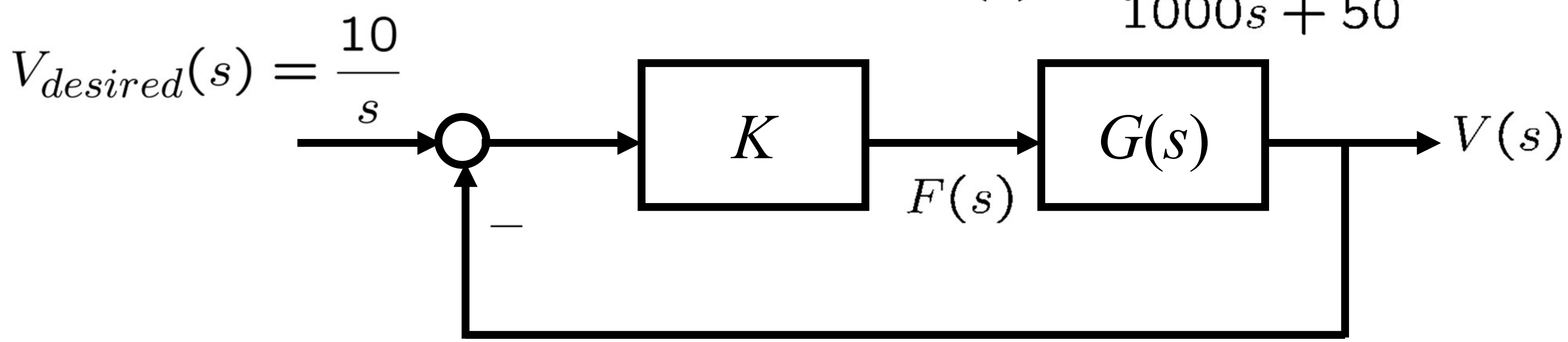
- Note that $5\% T_s = 3T$ or $3 \times 20 = 60$ sec. However, we need it to be less than 10 sec.
- So, by open-loop control, $5\% T_s = 60$ sec! and thus Too Slow!

Example 1: P control (cont'd)

(b1)

- Block diagram:

$$G(s) = \frac{1}{1000s+50}$$



$$\frac{V(s)}{V_{desired}(s)} = \frac{KG(s)}{1+KG(s)} = \frac{K}{1000s+50+K} = \frac{K/(50+K)}{[1000/(50+K)]\,s+1}$$

$[1000/(50+K)]$ → T

No overshoot

- $5\%T_s < 10$ sec ⇨ $3T < 10$ sec

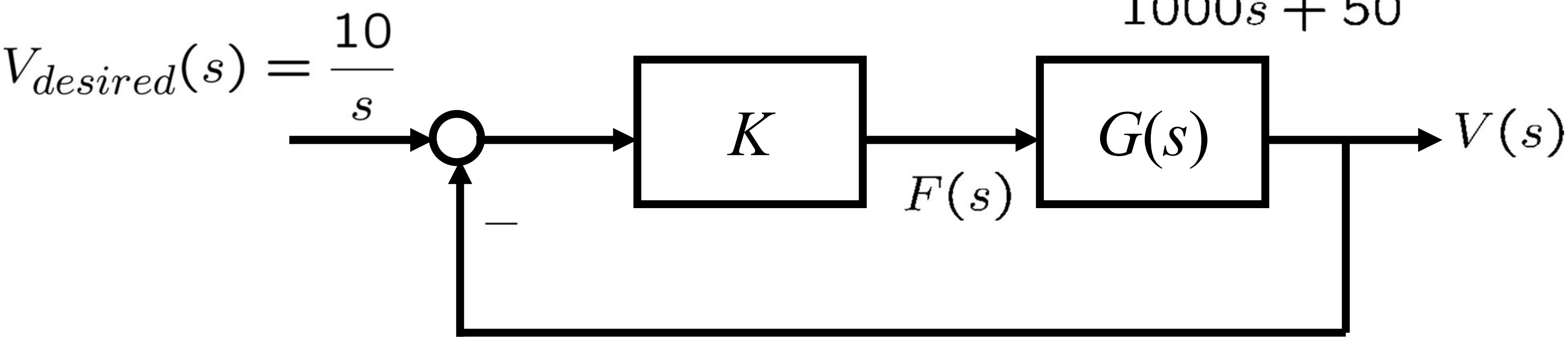
⇨ $3 \times \frac{1000}{50+K} < 10$ ⇨ $300 < 50+K$ ⇨ $\boxed{K > 250}$

Example 1: P control (cont'd)

(b2)

- $K > 250$

$$G(s) = \frac{1}{1000s + 50}$$



- To have zero SS error:

$$e_{ss} = \frac{10}{1 + K_p} = \frac{10}{1 + L(0)} = \frac{10}{1 + KG(0)} = \frac{10}{1 + K/50}$$

$$e_{ss} = 0 \Rightarrow \boxed{K = \infty}$$

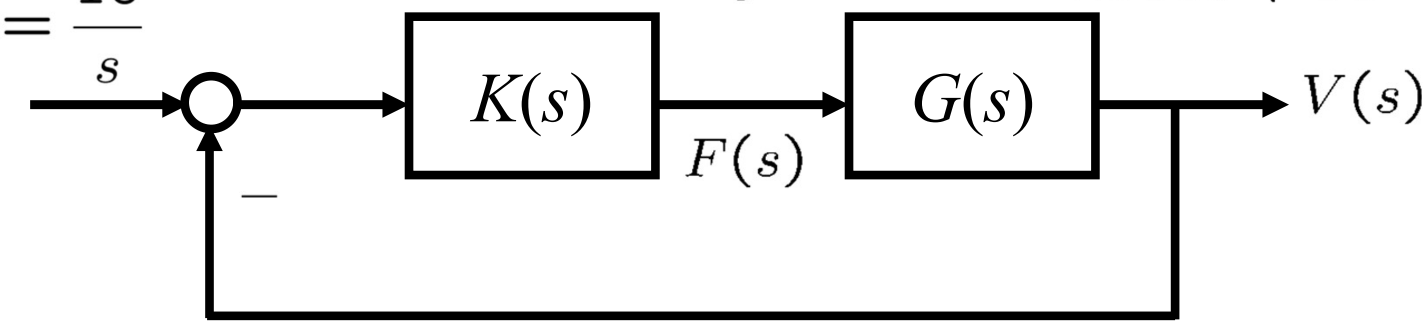
Example 1: PI control (cont'd)

(c1)

- Block diagram:

$$K(s) = c_p + \frac{c_i}{s} \qquad G(s) = \frac{1}{1000s + 50}$$

$$V_{desired}(s) = \frac{10}{s}$$



$$\frac{V(s)}{V_{desired}(s)} = \frac{K(s)G(s)}{1 + K(s)G(s)} = \frac{c_p s + c_i}{1000s^2 + (c_p + 50)s + c_i}$$

- Stable: **OK**

(c2)

$$K_p = L(0) = K(0)G(0) = \infty \quad \Rightarrow \quad K_p = \infty$$

(c3)

- Zero SS error: **OK**

Note: $e_{ss} = R/(1+K_p) = 10/(1+\infty) = 0$

Example 1: PI control (cont'd)

(c4)

- How to satisfy transient requirements?
  - **Tune controller parameters** to place poles of the closed-loop transfer function at the "right places".

$$\frac{V(s)}{V_{desired}(s)} = \frac{c_p s + c_i}{1000s^2 + (c_p + 50)s + c_i}$$

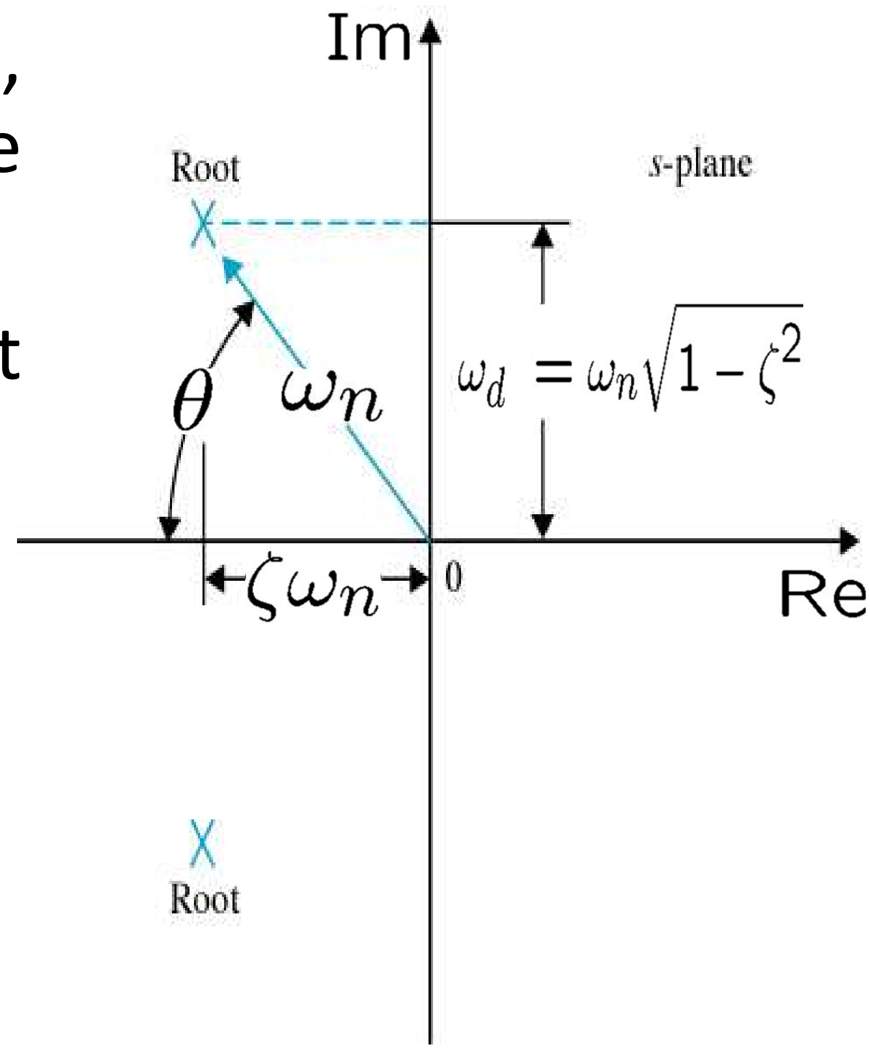
- We will learn
  - where the "right places" are (today's lecture: Example 2)
  - how to tune controller parameters (from next lecture)
- We can show (by trial and error and computer simulation) that the following controller is one of the controllers that can satisfy all the required conditions (not the topic of this lecture):

$$K(s) = 1050 + \frac{274}{s}$$

Example 2

- For the following **design specs**, find the allowable region of the second-order poles:

  PO at most 5% and T_s (5%) at most 7 sec.



Note:
Design Specs (i.e., Design Specifications) is just another term for **Performance Measures**.

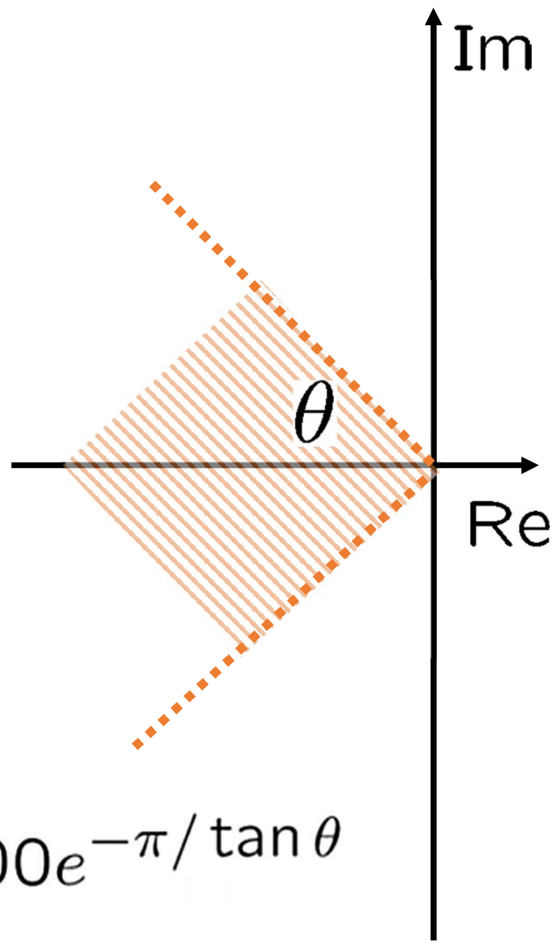
Example 2 (cont'd)

Solution:

- From the given percent overshoot constraint, ***PO* constraint**, we can obtain **allowable angles θ**.
- From the given settling time constraint, **T_s constraint**, we can obtain **allowable real part of poles**.

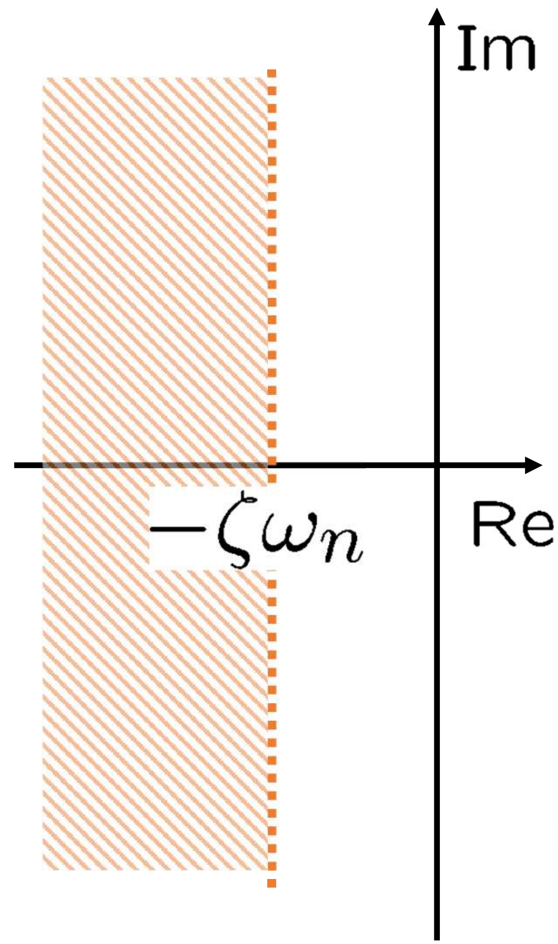
Example 2 (cont'd)

PO constraint



$$PO = 100e^{-\pi/\tan\theta}$$

T_s constraint

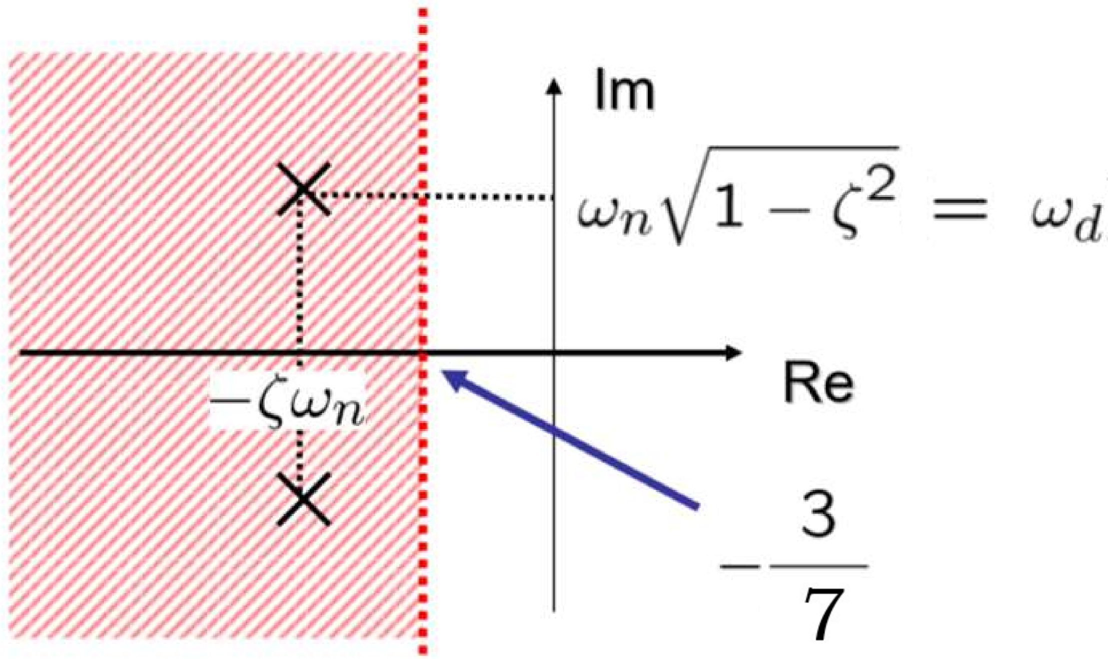


$$T_s = \frac{3}{\zeta\omega_n}$$

Example 2 (cont'd)

- Require 5% settling time $T_s < 7$

$$T_s = \frac{3}{\zeta\omega_n} \rightarrow \frac{3}{\zeta\omega_n} < 7 \Rightarrow \boxed{\zeta\omega_n > \frac{3}{7}}$$

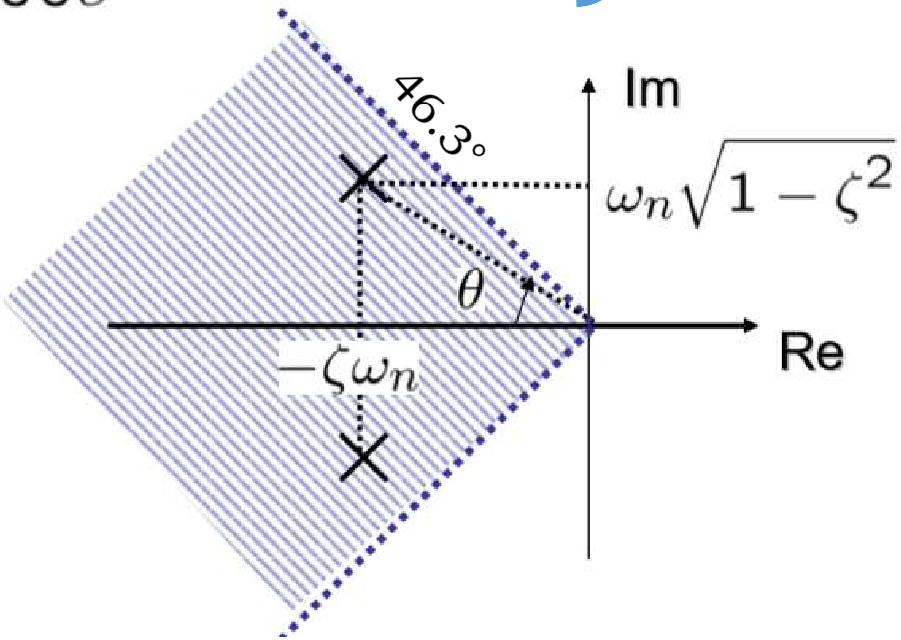


Example 2 (cont'd)

- Require $PO <$ 5%

$$PO = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} < 5\%$$
$$= 100e^{-\pi/\tan\theta}$$

$$\theta < 46.3°$$

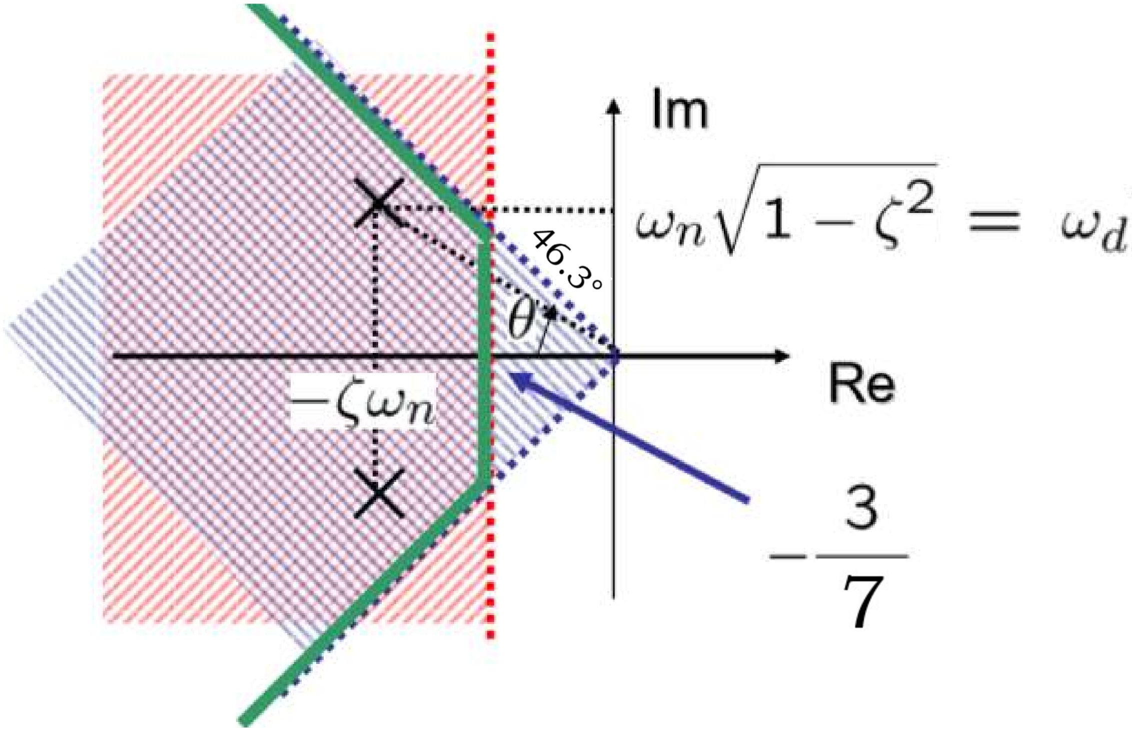


PO_{max}	θ_{max}
5%	46.3°

Example 2 (cont'd)

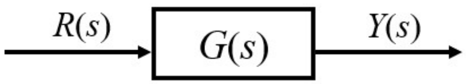
- Combination of two requirements

$$\zeta\omega_n > \frac{3}{7} \quad \& \quad \theta < 46.3°$$



Example 3

- Consider a 2nd order overdamped system



$$G(s) = \frac{10}{s^2 + 11s + 10} = \frac{10}{(s+1)(s+10)}$$

For unit step input, obtain (or estimate)

- steady state value,
- percent overshoot, and
- 2% settling time.

Important Note: The formulas for 2nd order system that were summarized in a table can only be used when $0 < \zeta < 1$. That is, if ζ is greater than 1, we cannot use the formulas in the table.

Solution:

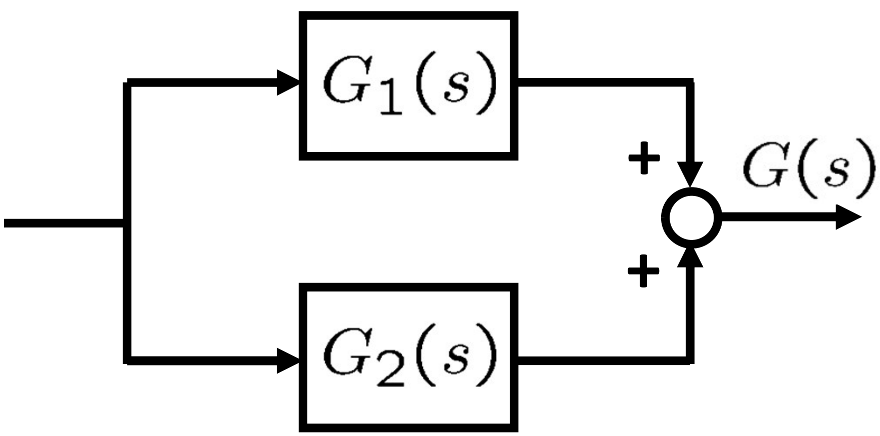
- Steady state value is the DC gain $G(0) = 1$.
- No overshoot (overdamped!)

Example 3 (cont'd)

- Poles are -1 and -10.

$$G(s) = \frac{10}{s^2 + 11s + 10} = \underbrace{\frac{A}{s+1}}_{G_1(s)} + \underbrace{\frac{B}{s+10}}_{G_2(s)}$$

Time constant = 1



Time constant = 0.1

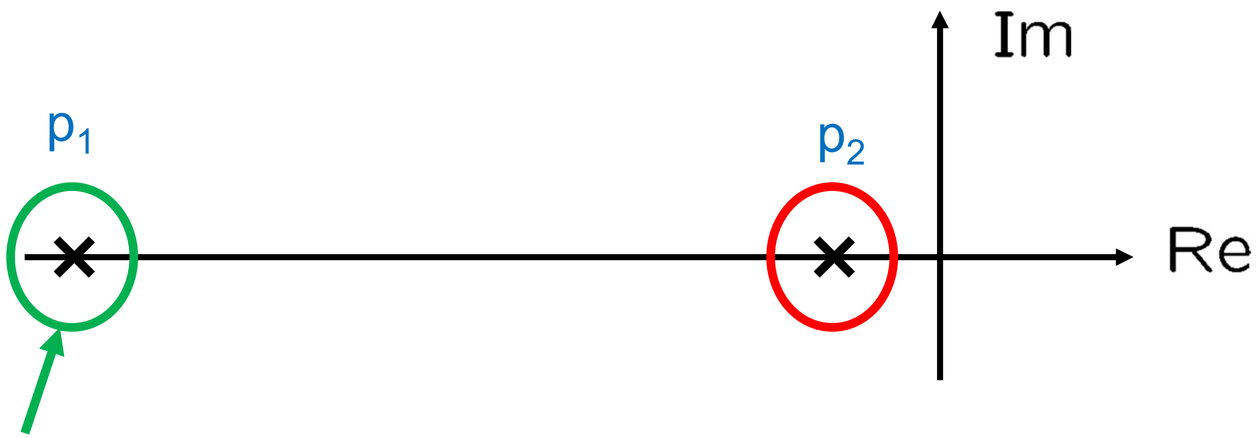
*Time constant of $G(s)$ is estimated by the slower subsystem $G_1(s)$, i.e., the **larger** T.*

$2\%T_s = 4T$. In this case ($T = 1$). So,
$2\%T_s \approx 4 \times 1 \approx 4$ sec

2% settling time: ***about*** 4 seconds

Example 3 (cont'd)

- Dominant poles: Poles closest to the imaginary axis and far away from remaining poles dominate the behavior of responses. We only compare the real parts of the roots when determining dominance because it is the real part that determines how fast the response decreases.

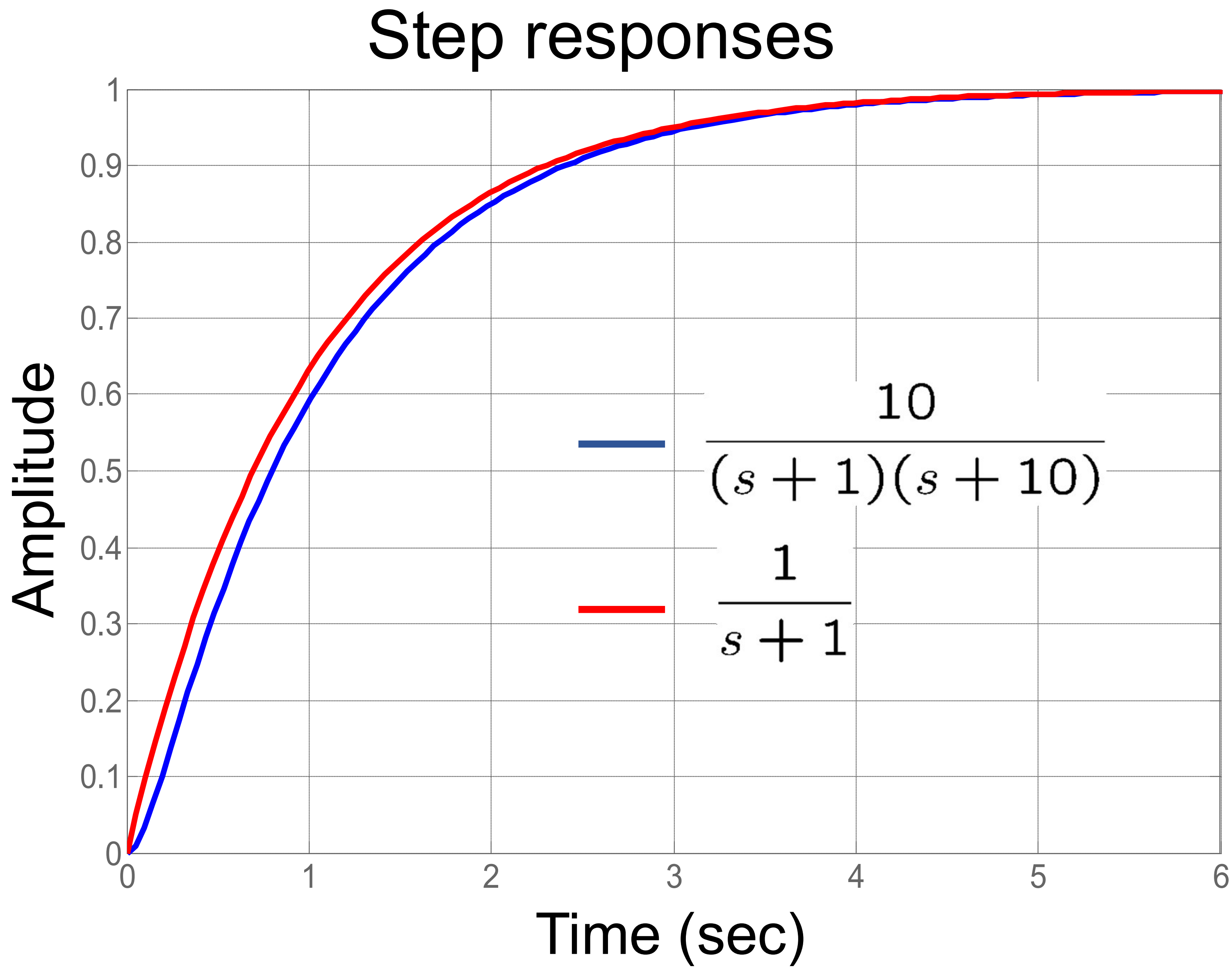


Poles far left (5-10 times) from dominant poles may be ignored. That is, if $p_1 > 5p_2$ or $10p_2$, we can ignore the effect of p_1.

$$G(s) = \frac{10}{(s+1)(s+10)} \approx \frac{1}{s+1}$$

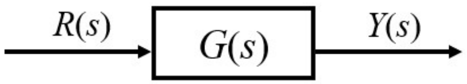
Same DC gain

Example 3 (cont'd)



Example 4

- For the unit step response of a stable 3rd-order system,



$$G(s) = \frac{8}{(s+2.5)(s^2+2s+4)}$$

obtain (or estimate)

- steady state value,
- 2% settling time, and
- percent overshoot.

Solution:

- Steady state value is the DC gain $\boxed{G(0) = 0.8}$.

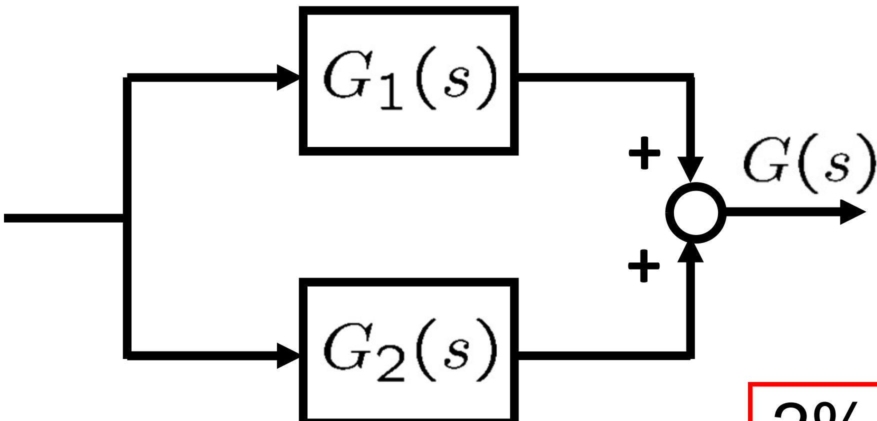
Example 4 (cont'd)

- Partial fraction expansion

$$G(s) = \frac{8}{(s+2.5)(s^2+2s+4)} = \underbrace{\frac{A}{s+2.5}}_{G_1(s)} + \underbrace{\frac{Bs+C}{s^2+2s+4}}_{G_2(s)}$$

$\omega_n = 2,\ \zeta = 0.5$

Time constant = 1/2.5 = 0.4 s



Time constant = $1/(\zeta\omega_n)$ = 1 s

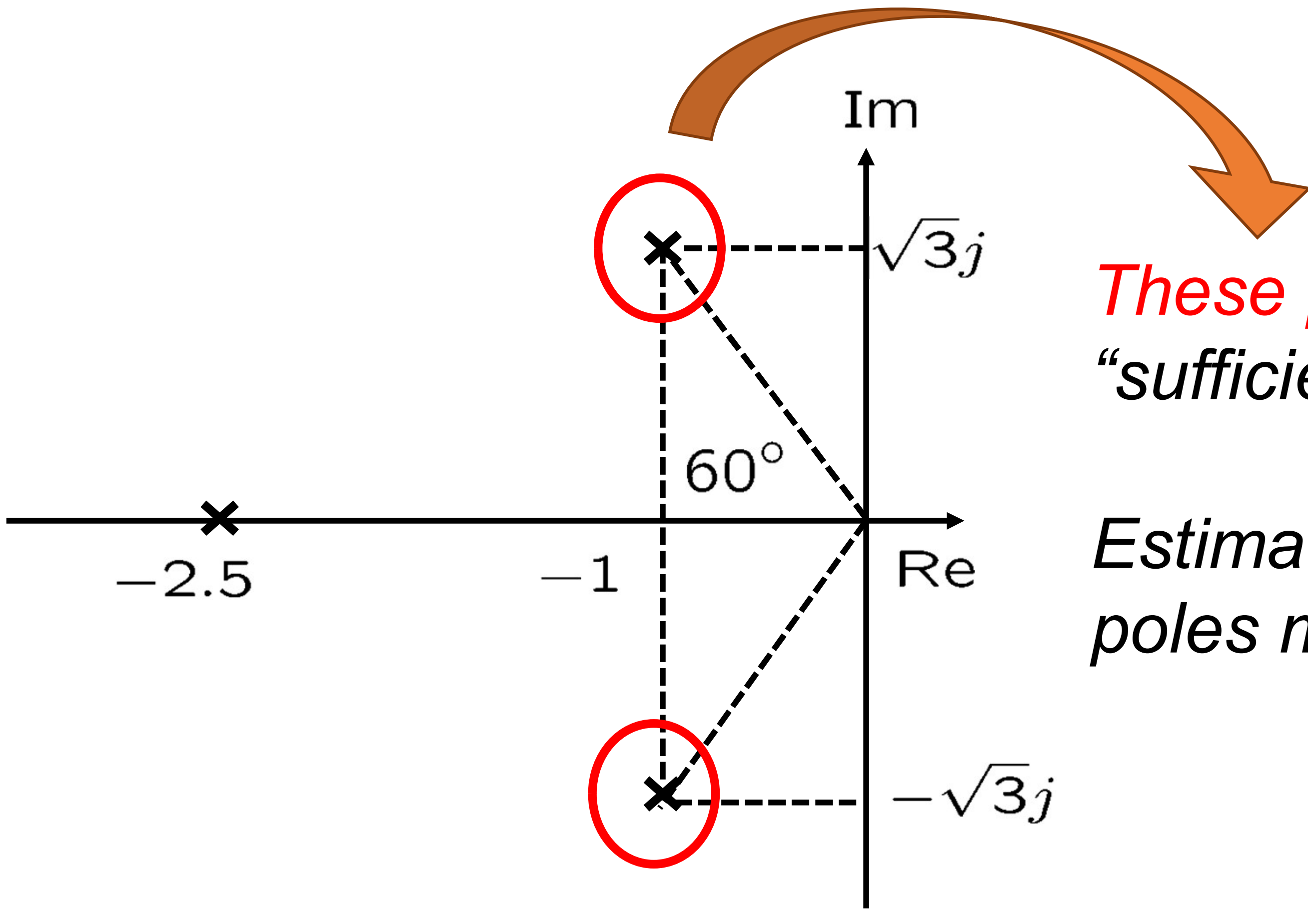
*Time constant of $G(s)$ is estimated by the slower subsystem $G_2(s)$, i.e., the **larger** T.*

2% settling time: ***about*** 4 seconds

$$PO = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} \xrightarrow{\zeta = 0.5} PO = 16.3\%$$

Example 4 (cont'd)

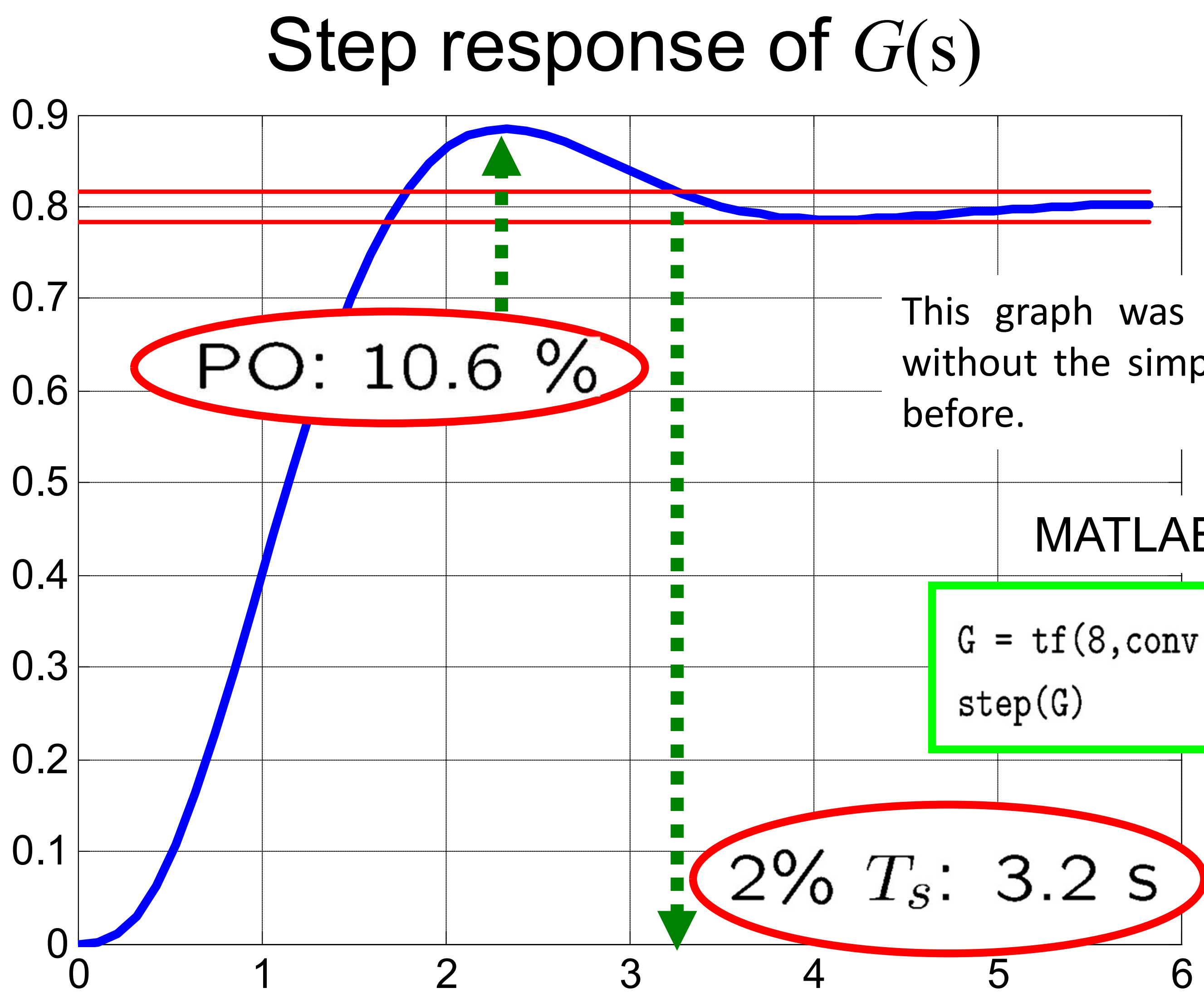
- Poles $s = -2.5,\ -1 \pm \sqrt{3}j$



These poles are not "sufficiently dominant"!

Estimation based on these poles may be inaccurate.

Example 4 (cont'd)



This graph was obtained accurately without the simplification mentioned before.

MATLAB command

```
G = tf(8,conv([1 2.5],[1 2 4]));
step(G)
```

Example 5

- For the standard 2nd order systems, derive the relationship for damping ratio as a function of percent overshoot, from the following formula:

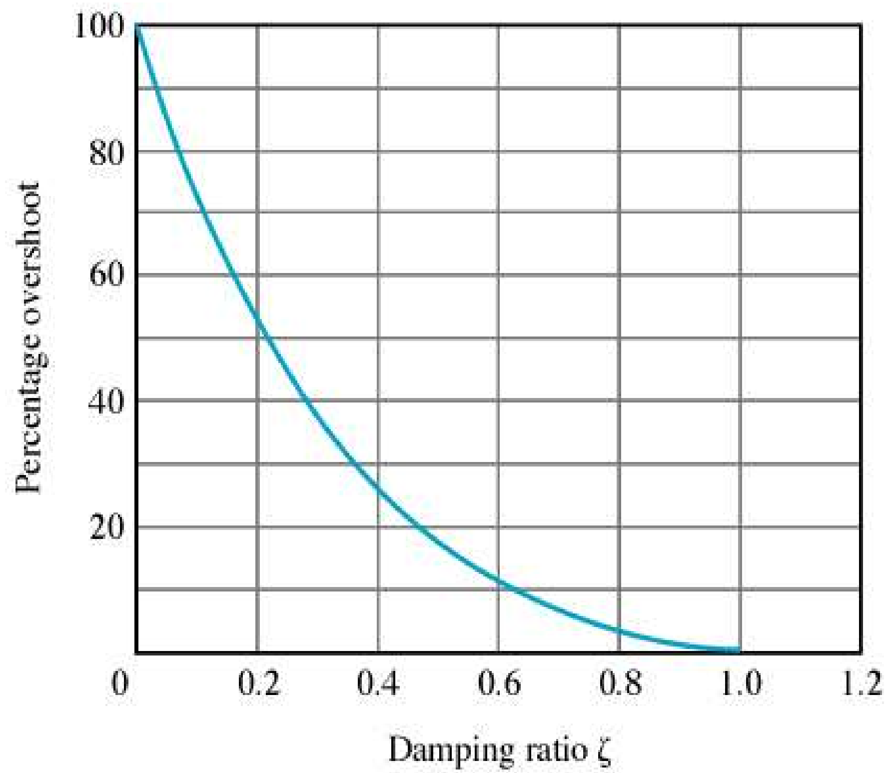
$$PO = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

Solution:

$$\left(\ln \frac{PO}{100}\right)^2 = \frac{\zeta^2\pi^2}{1-\zeta^2}$$

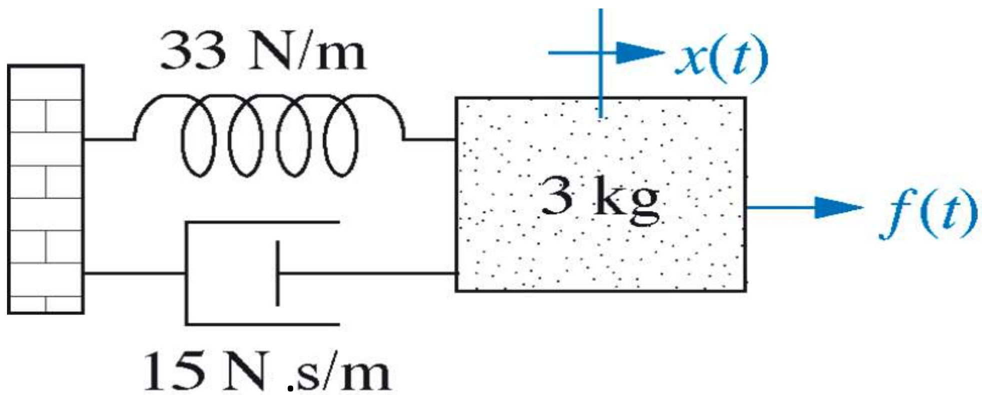
$$\Leftrightarrow \zeta^2 = \frac{\left(\ln \frac{PO}{100}\right)^2}{\pi^2 + \left(\ln \frac{PO}{100}\right)^2}$$

$$\Leftrightarrow \zeta = \frac{\left|\ln \frac{PO}{100}\right|}{\sqrt{\pi^2 + \left(\ln \frac{PO}{100}\right)^2}}$$



Example 6

- For the system below,
  - Find the transfer function from $F(s)$ to $X(s)$, i.e., $F(s)$ is the input and $X(s)$ is the output.
  - Find DC gain, ζ, ω_n, ω_d, T_s (2%), T_p, and PO.



Solution:

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{3s^2 + 15s + 33}$$

Example 6 (cont'd)

- Find DC gain $G(0)$, ζ, ω_n, ω_d, T_s (2%), T_p, and PO.

$$G(s) = \frac{1}{3s^2 + 15s + 33} = \frac{1}{33} \cdot \frac{11}{s^2 + 5s + 11} \qquad 2\zeta\omega_n = 5,\ \omega_n^2 = 11$$

$G(0) = 1/33 = 0.03$

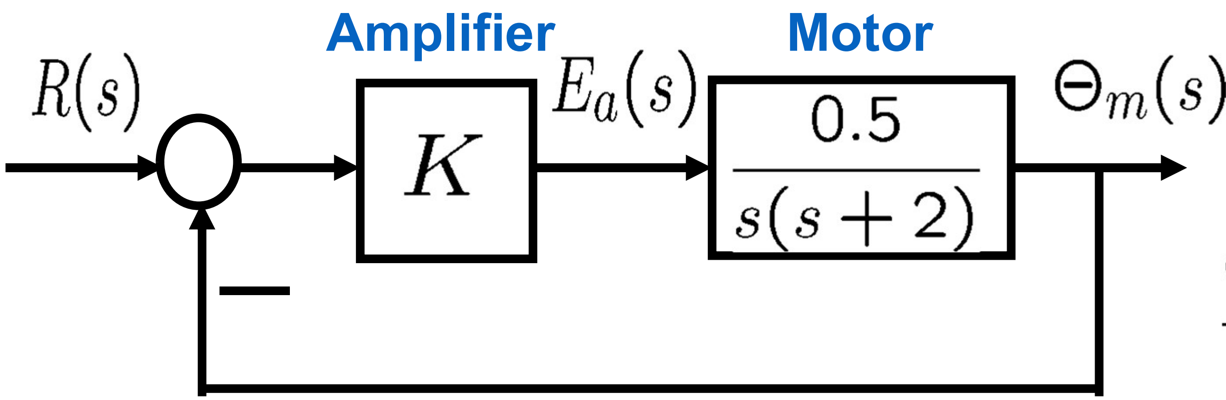
$$\omega_n = \sqrt{11} = 3.31 \quad ; \quad \zeta = \frac{5}{2\sqrt{11}} = 0.75 \quad ; \quad \omega_d = \omega_n\sqrt{1-\zeta^2} = \frac{\sqrt{19}}{2} = 2.17$$

$$T_s = \frac{4}{\zeta\omega_n} = 1.6 \quad ; \quad T_p = \frac{\pi}{\omega_d} = \frac{2\pi}{\sqrt{19}} = 1.44 \quad ; \quad PO = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 2.72\%$$

$$\begin{aligned} G(0) &= 0.03 \\ \zeta &= 0.75 \\ \omega_n &= 3.31 \\ \omega_d &= 2.17 \\ T_s &= 1.6 \\ T_p &= 1.44 \\ PO &= 2.72\% \end{aligned}$$

Example 7

- DC motor position control



Closed-loop TF

$$\frac{\Theta_m(s)}{R(s)} = \frac{0.5K}{s^2 + 2s + 0.5K}$$

- Design the amplifier gain K so that
  - Percent overshoot is 5%.

$$\begin{cases} 2\zeta\omega_n = 2 \\ \omega_n^2 = 0.5K \end{cases}$$

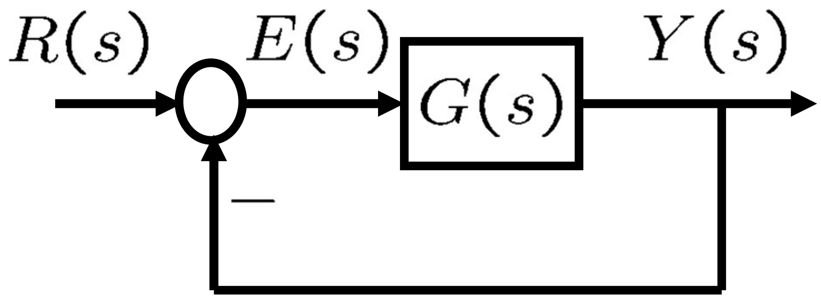
Solution:

$$\zeta = \frac{\left| \ln \frac{PO}{100} \right|}{\sqrt{\pi^2 + \left(\ln \frac{PO}{100} \right)^2}} = \frac{1}{\sqrt{2}} \rightarrow \omega_n = \sqrt{2} \rightarrow K = \frac{\omega_n^2}{0.5}$$

$$\boxed{K = 4}$$

Example 8

- For the unity feedback system, check and compute the followings:



$$G(s) = \frac{4900}{s(s+70)}$$

- **(a)** Stability
- **(b)** 2% settling time for $r(t) = u(t)$ (unit step input)
- **(c)** Steady-state errors:
  - **(c1)** For $r(t) = 5u(t)$ (step input)
  - **(c2)** For $r(t) = 5tu(t)$ (ramp input)
  - **(c3)** For $r(t) = 5t^2u(t)$ (parabolic input)

Example 8 (cont'd)

Solution:

- **(a) Stability**
  - Characteristic equation

$$1 + G(s) = 0 \Leftrightarrow s^2 + 70s + 4900 = 0$$

$$\zeta = \frac{1}{2}, \ \omega_n = 70$$

Since all the coefficients in the first column of Routh array have the same sign, the CL system is **stable**.

- **(b) 2% settling time** $(\zeta < 1 : \text{underdamped case})$
  - Time constant is $T = \frac{1}{|\text{Re(pole)}|} = \frac{1}{\zeta\omega_n} = \frac{1}{35}$
  - 2% settling time is $4T = \frac{4}{35}$ ➡ 2% settling time = 0.114

Example 8 (cont'd)

- **(c) Steady-state errors**

$$G(s) = \frac{4900}{s(s+70)}$$

  - **(c1)** For $5u(t)$ (step)

$$K_p = \lim_{s\to 0} G(s) = \infty \Rightarrow e_{ss} = \frac{5}{1+K_p} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

  - **(c2)** For $5tu(t)$ (ramp)

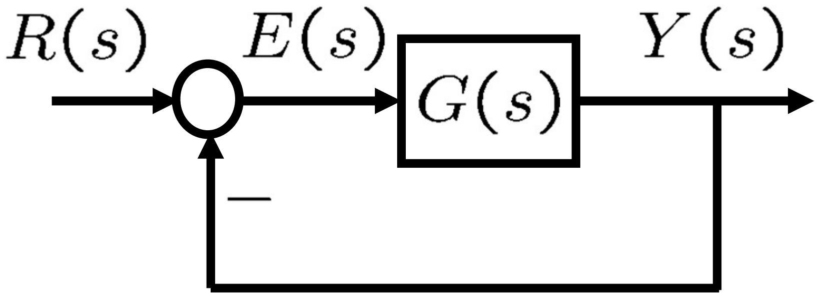
$$K_v = \lim_{s\to 0} sG(s) = 70 \Rightarrow e_{ss} = \frac{5}{K_v} = \frac{1}{14} \Rightarrow \boxed{e_{ss} = 0.071}$$

  - **(c3)** For $5t^2u(t)$ (parabolic)

$$K_a = \lim_{s\to 0} s^2G(s) = 0 \Rightarrow e_{ss} = \frac{10}{K_a} = \infty \Rightarrow \boxed{e_{ss} = \infty}$$

Example 9

- Design the unity feedback system such that:



$$G(s) = \frac{K(s+\alpha)}{s(s+\beta)}$$

  - **(a)** Steady-state error for unit ramp input is 1/10 and
  - **(b)** Closed-loop poles are at $-1 \pm j$

Example 9

Solution:

Unit ramp input: $R \cdot t \cdot u(t), 1 \cdot t \cdot u(t) \Rightarrow R = 1$

$$K_v = \lim_{s \to 0} sG(s) = \lim_{s \to 0} \frac{sK(s+\alpha)}{s(s+\beta)} = \frac{K\alpha}{\beta} \qquad \Rightarrow K_v = \frac{K\alpha}{\beta}$$

$$e_{ss} = \frac{R}{K_v} = \frac{1}{K_v} \overset{e_{ss}=\frac{1}{10}}{\Rightarrow} \frac{K\alpha}{\beta} = 10$$

Find ζ and ω_n:

Given pole locations:

$$s_{1,2} = -1 \pm j$$

Comparing with standard form:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

Equating real and imaginary parts:

$$-1 = -\zeta\omega_n \Rightarrow \omega_n = \frac{1}{\zeta}$$

$$\omega_n\sqrt{1-\zeta^2} = 1 \Rightarrow \frac{1}{\zeta}\sqrt{1-\zeta^2} = 1 \Rightarrow \zeta = \frac{1}{\sqrt{2}} \approx 0.707 \Rightarrow \omega_n = \frac{1}{\zeta} = \sqrt{2} \approx 1.414$$

Example 9

Find α, β, and K:

From the closed-loop transfer function:

$$1 + G(s) = 1 + \frac{K(s+\alpha)}{s(s+\beta)} = 0 \Rightarrow s(s+\beta) + K(s+\alpha) = 0 \Rightarrow s^2 + (\beta + K)s + K\alpha = 0$$

Compare with the standard second-order form:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Matching coefficients:

$$\begin{cases} s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \\ s^2 + (\beta + K)s + K\alpha = 0 \end{cases} \Rightarrow \begin{cases} \beta + K = 2\zeta\omega_n \\ K\alpha = \omega_n^2 \end{cases}$$

$$\beta + K = 2 \cdot \frac{1}{\sqrt{2}} \sqrt{2} = 2 \Rightarrow \beta + K = 2$$

$$K\alpha = \omega_n^2 = 2$$

$$\frac{K\alpha}{\beta} = 10 \Rightarrow K\alpha = 10\beta \Rightarrow 10\beta = 2 \Rightarrow \beta = 0.2$$

$$\beta + K = 2 \Rightarrow 0.2 + K = 2 \Rightarrow K = 1.8$$

$$K\alpha = 2 \Rightarrow 1.8\alpha = 2 \Rightarrow \alpha = \frac{2}{1.8} \approx 1.111$$

Example 9

a place of mind
UBC

Summary:

- **(a)**

$$e_{ss} = \frac{1}{10}$$

$$K_v = \lim_{s\to 0} sG(s) = \frac{K\alpha}{\beta} \Rightarrow K_v = \frac{K\alpha}{\beta} \;;\; e_{ss} = \frac{R}{K_v} \Rightarrow K_v = 10 \Rightarrow \frac{K\alpha}{\beta} = 10$$

- **(b)** Closed-loop poles are at $-1 \pm j$

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

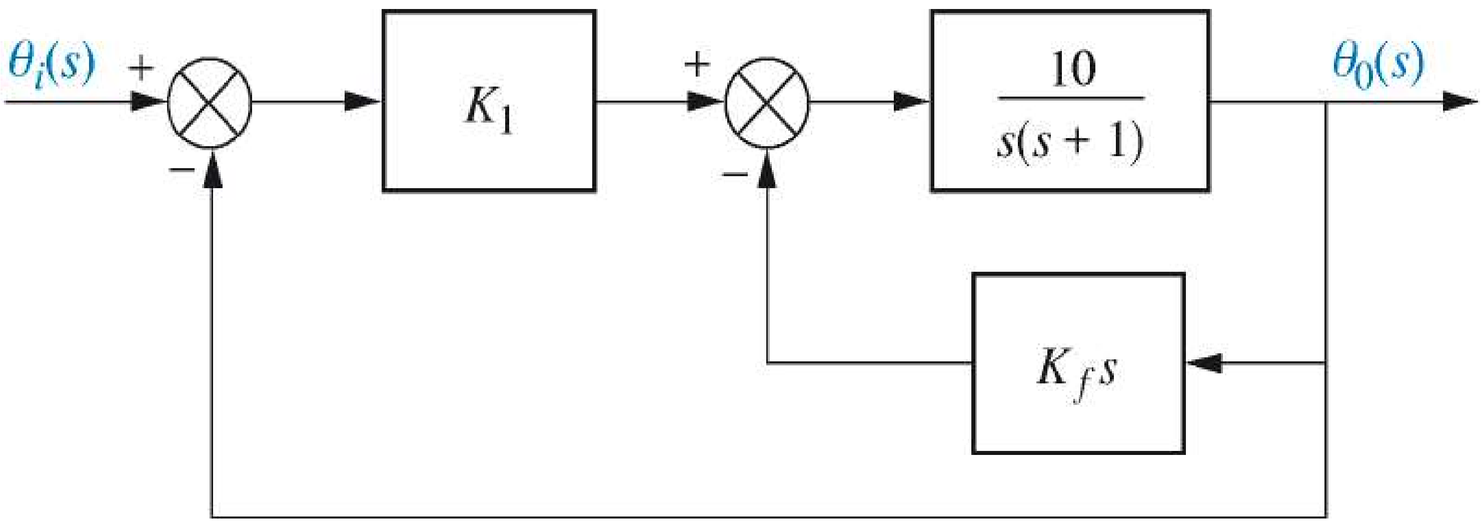
characteristic polynomial = $s(s+\beta) + K(s+\alpha) = s^2 + 2\zeta\omega_n s + \omega_n^2 \Rightarrow$

$$\alpha = 1.111, \beta = 0.2, K = 1.8$$

Example 10

- Design the parameters K_1 & K_f so that:

$$K_v = 10,\ \zeta = 0.5$$



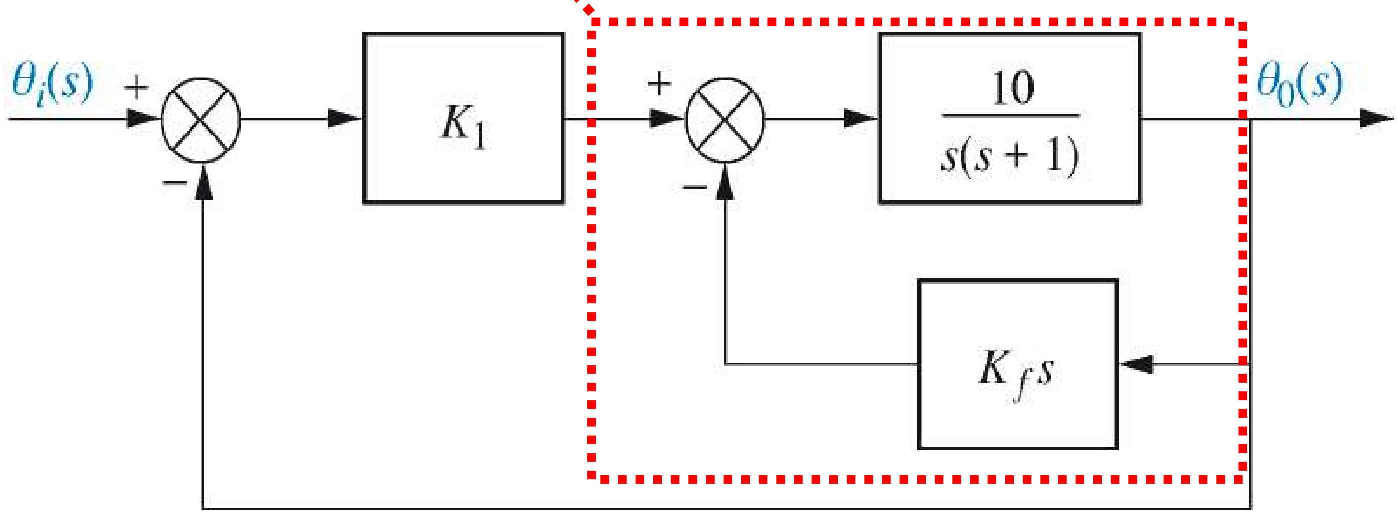
Example 10 (cont'd)

Solution:

- Closed-loop transfer function:

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{K_1 F(s)}{1 + K_1 F(s)} = \frac{10K_1}{s^2 + (1 + 10K_f)s + 10K_1}$$

$$F(s) = \frac{\frac{10}{s(s+1)}}{1 + \frac{10}{s(s+1)} K_f s} = \frac{10}{s(s + 1 + 10K_f)}$$



Example 10 (cont'd)

- From the two requirements,

$$K_v = \lim_{s\to 0} sK_1F(s) = \frac{10K_1}{1+10K_f} = 10 \Rightarrow K_1 = 1 + 10K_f$$

$$s^2 + (1+10K_f)s + 10K_1 = s^2 + 2\zeta\omega_n s + \omega_n^2 \Rightarrow \begin{cases} 2\zeta\omega_n = 1+10K_f = K_1 \\ \omega_n^2 = 10K_1 \end{cases}$$

$$2\zeta\omega_n = 1 + 10K_f = K_1 \xrightarrow{\zeta = 0.5} \omega_n = K_1 \Rightarrow$$ Substitute for ω_n in $\omega_n^2 = 10K_1$

$$\Rightarrow K_1^2 - 10K_1 = 0 \Rightarrow \boxed{K_1 = 10,\ K_f = 0.9}$$

Summary

- Cruise control example
- Allowable pole locations
- Step responses
  - 2nd order overdamped systems
  - Higher order systems
- Design of some feedback systems
- Next
  - Root locus