a place of mind UBC

ELEC 341: Systems and Control

Lecture 20

Stability margin: Examples on frequency domain specifications

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Course roadmap

Modeling

- ✓ Laplace transform
- ✓ Transfer function
- Models for systems
  - ✓ • mechanical
  - ✓ • electrical
  - ✓ • electromechanical
- ✓ Linearization, delay

Analysis

- ✓ Stability
  - ✓ • Routh-Hurwitz
  - ✓ • Nyquist
- ✓ Time response
  - ✓ • Transient
  - ✓ • Steady state
- ✓ Frequency response
  - ✓ • Bode plot

Design

- ➤ Design specs
- ✓ Root locus
- ➤ Frequency domain
- ✓ PID & Lead-lag
- ➤ Design examples

Matlab simulations

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Closed-loop stability criterion (review)

- Closed-loop stability can be determined by the roots of the characteristic equation **(Ch. Eq.)**:

$$1 + L(s) = 0, \; L(s) = G(s)C(s)$$

- Closed-loop system is stable if the **Ch. Eq.** has all roots in the open left half plane.
- How to check the closed-loop stability?
  - **Computation of all the roots**
  - **Routh-Hurwitz stability criterion**
  - **Nyquist stability criterion**

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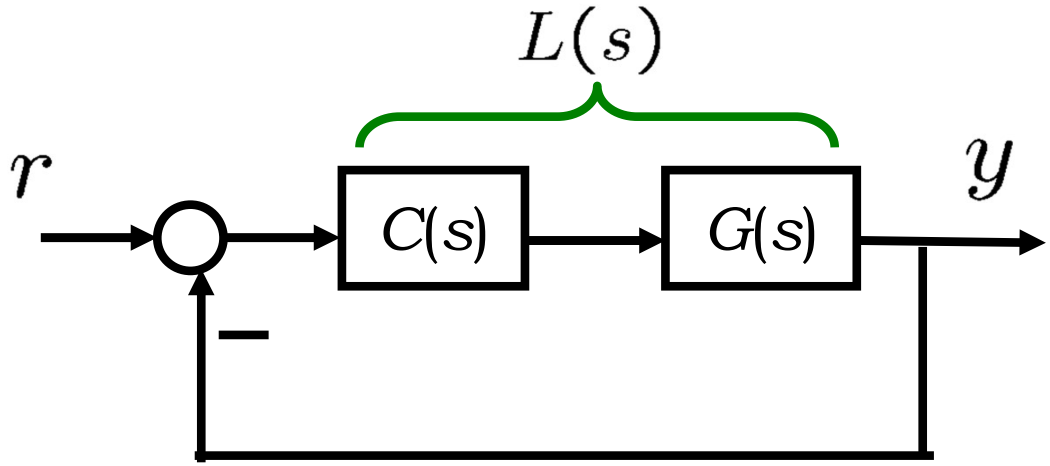
Nyquist stability criterion (review)



CL system is stable $\Leftrightarrow Z = P + N = 0$

Z : # of CL poles in open RHP

P : # of OL poles in open RHP (given)

N : # of clockwise/counterclockwise encirclement of **-1** by Nyquist plot of $L(s)$

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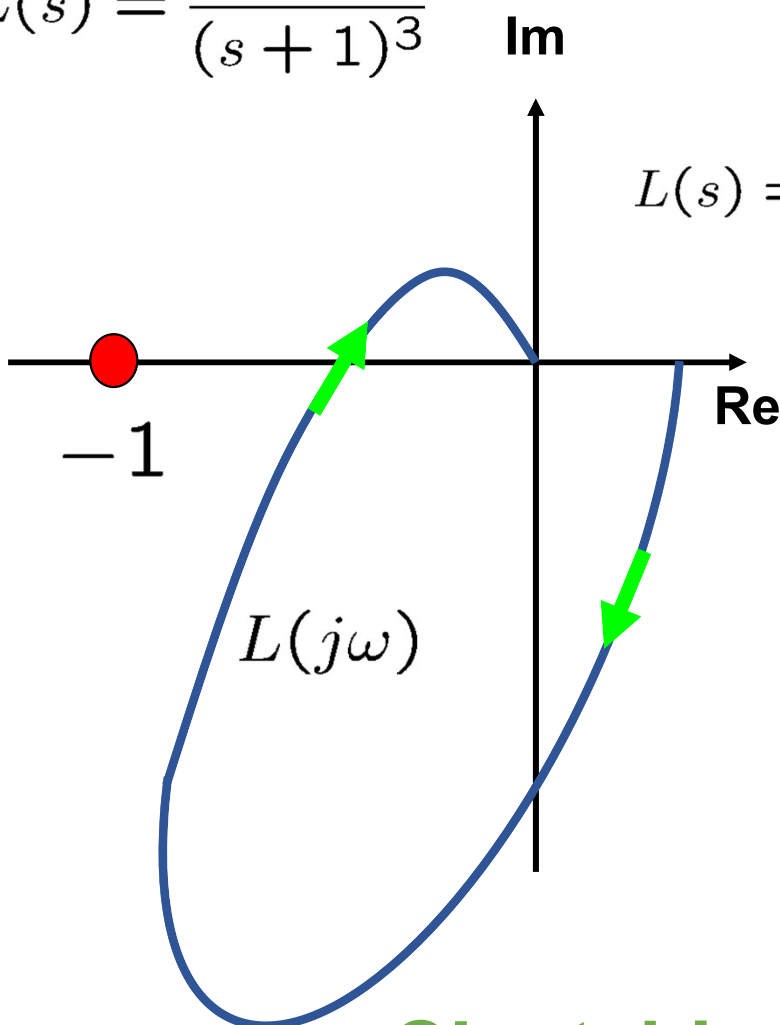
Examples when $P = 0$ (review)

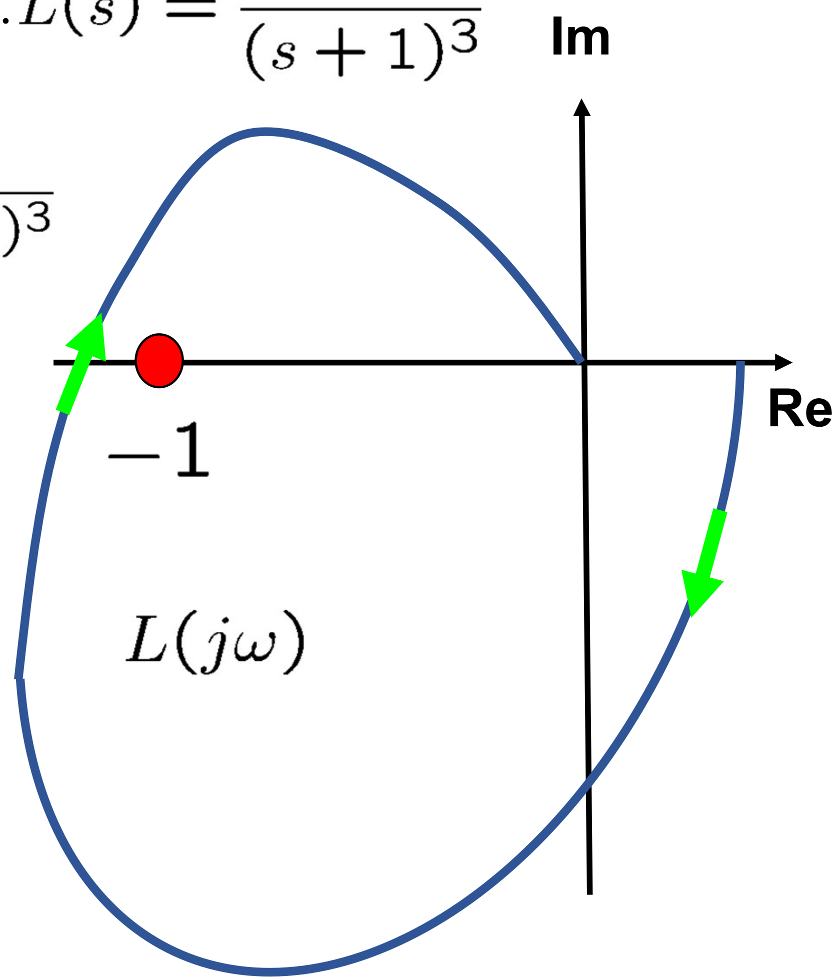
$$K\,.L(s) = \frac{1}{(s+1)^3}$$

$$K\,.L(s) = \frac{15}{(s+1)^3}$$

$$L(s) = \frac{1}{(s+1)^3}$$



CL stable



CL unstable

Gain margin (GM) (review)

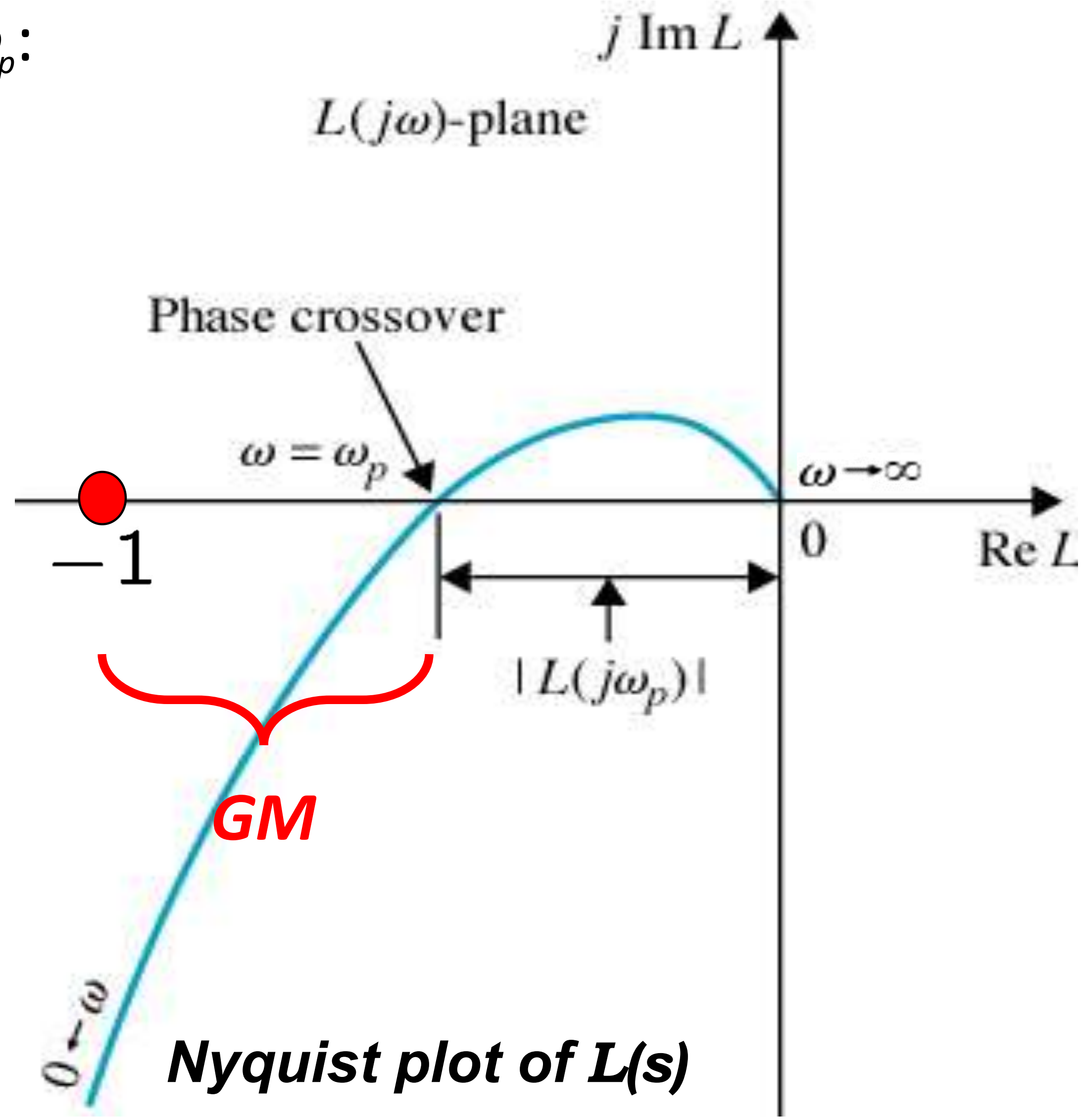
- **Phase crossover frequency**, ω_p:

$$\angle L(j\omega_p) = -180^\circ$$

- **Gain margin** (in dB), GM:

$$GM = 20\log_{10}\frac{1}{|L(j\omega_p)|}$$

Gain Margin: Additional gain that makes the system on the verge of instability.



Nyquist plot of L(s)

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Phase margin (PM) (review)

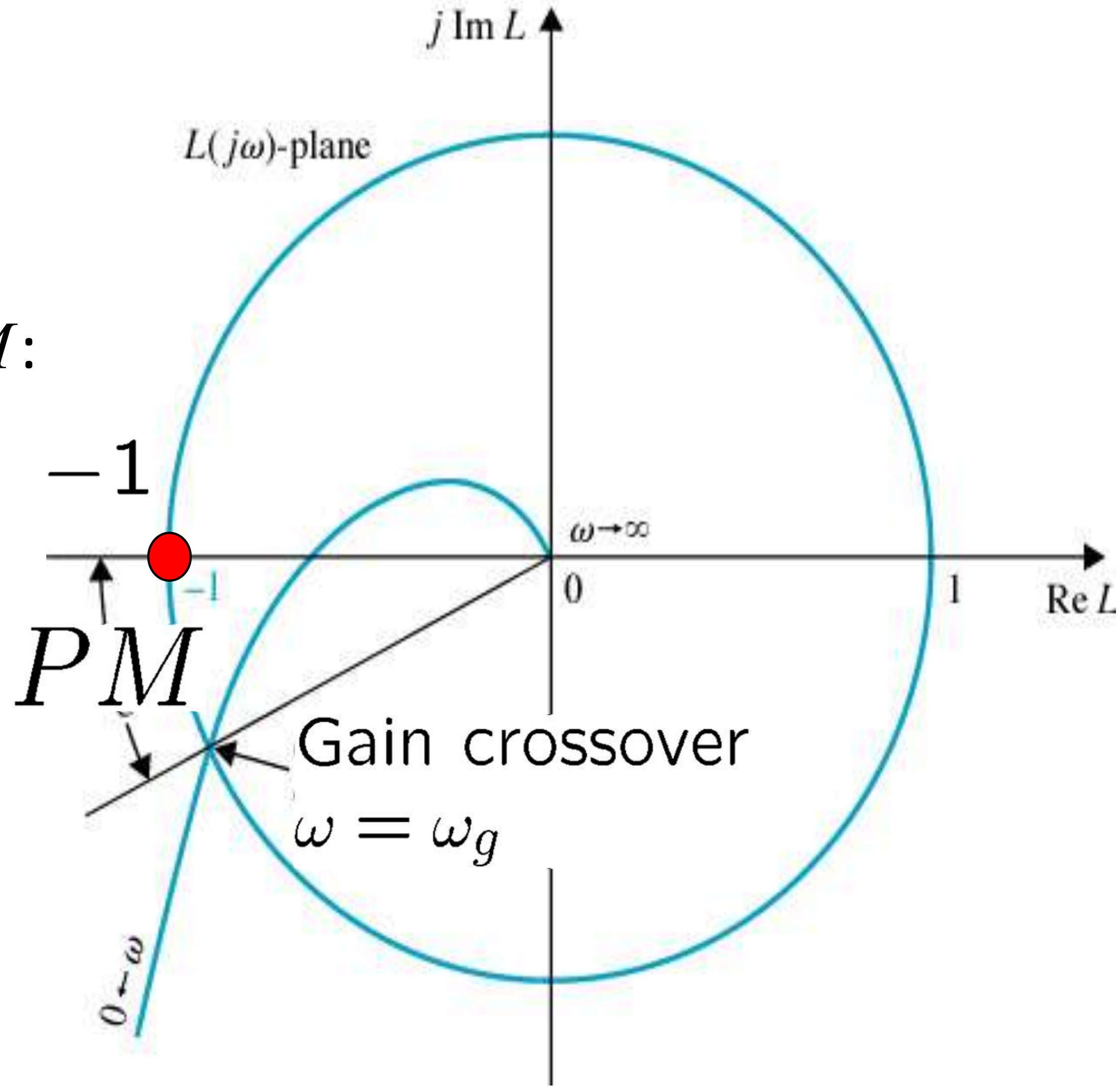
- **Gain crossover frequency**, ω_g:

$$|L(j\omega_g)| = 1$$

- **Phase margin** (in degrees), PM:

$$PM = \angle L(j\omega_g) + 180^\circ$$

Phase Margin: Additional phase lag that makes the system on the verge of instability.



Nyquist plot of L(s)

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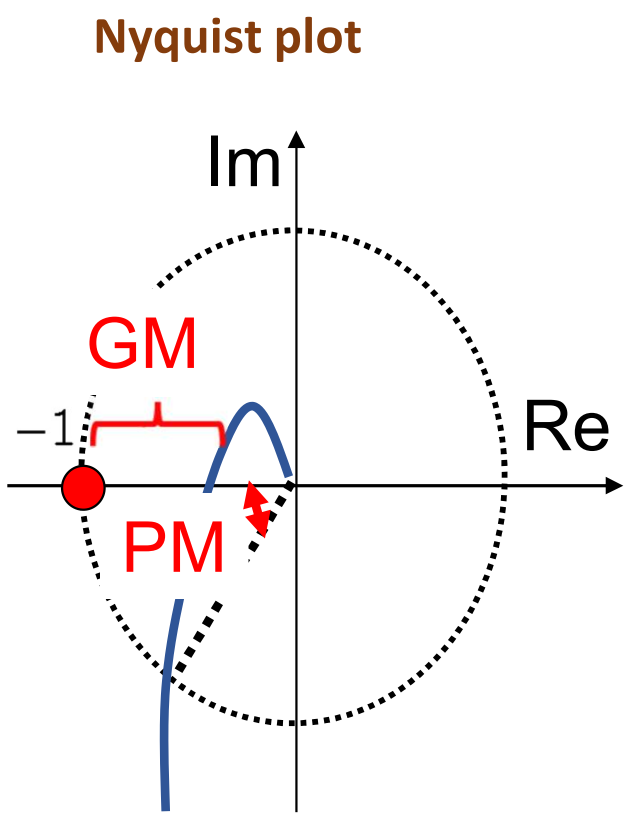
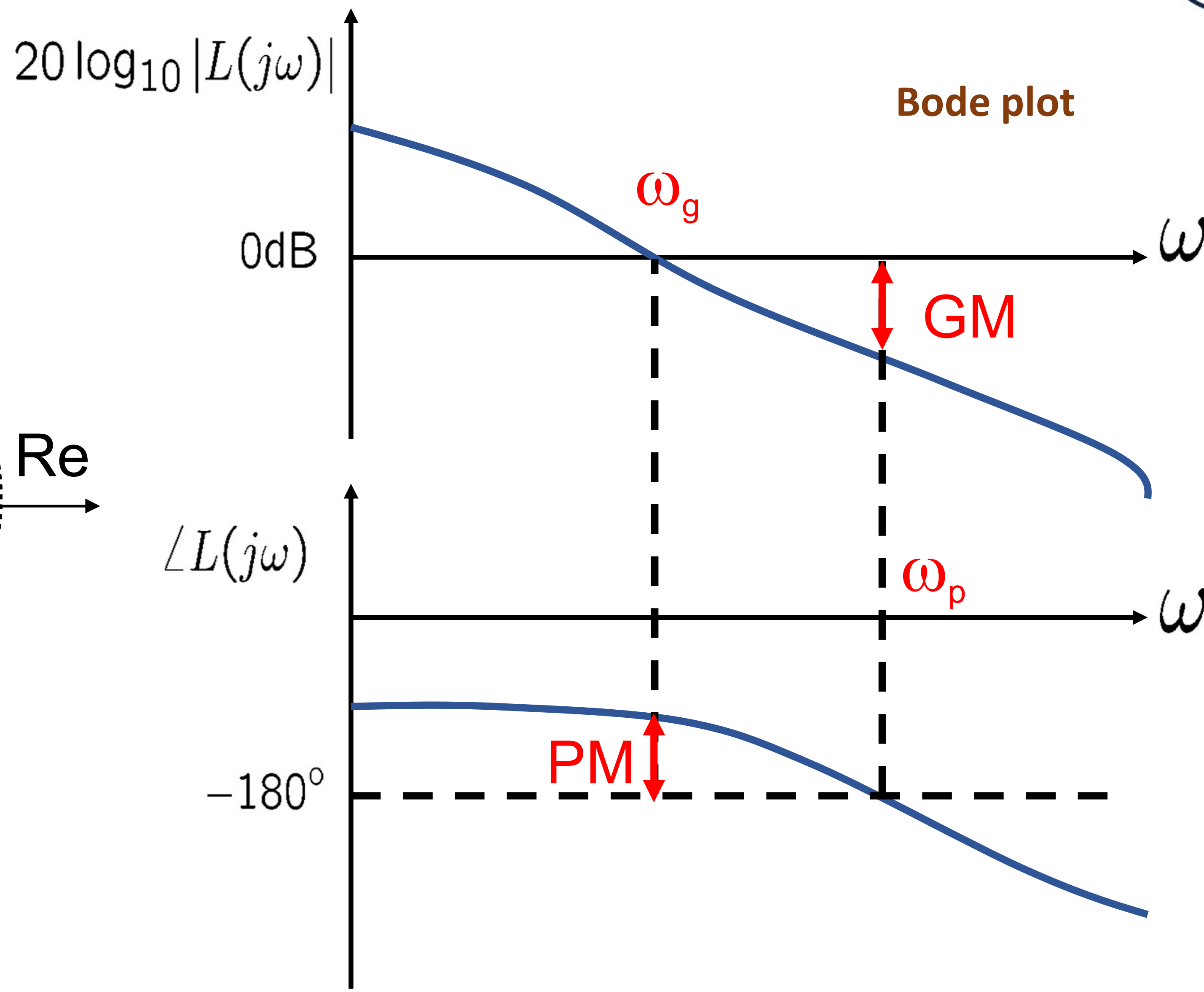
Relative stability on Nyquist plot and Bode plot





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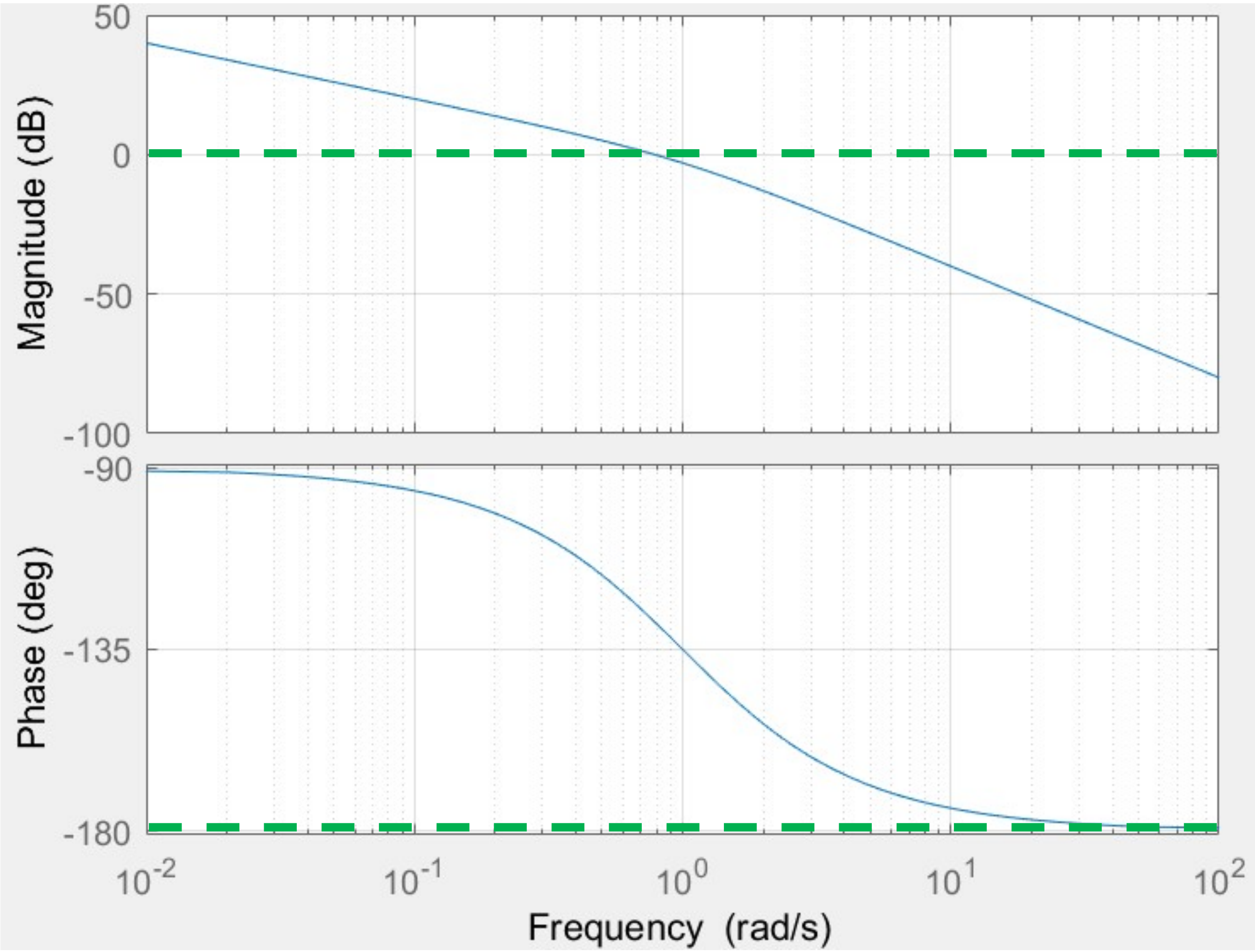
Example 1: (GM & PM)

$$L(s) = \frac{1}{s(s+1)}$$



ω_p : GM:

ω_g : PM:

Note 1: If the gain curve *asymptotically* approaches **0 dB line** when ω approaches 0, then we can show that ω_g will be equal to 0.

Note 2: If the phase curve *asymptotically* approaches **-180° line**, then we can show that ω_p will be equal to ∞ (and hence, GM will be ∞).

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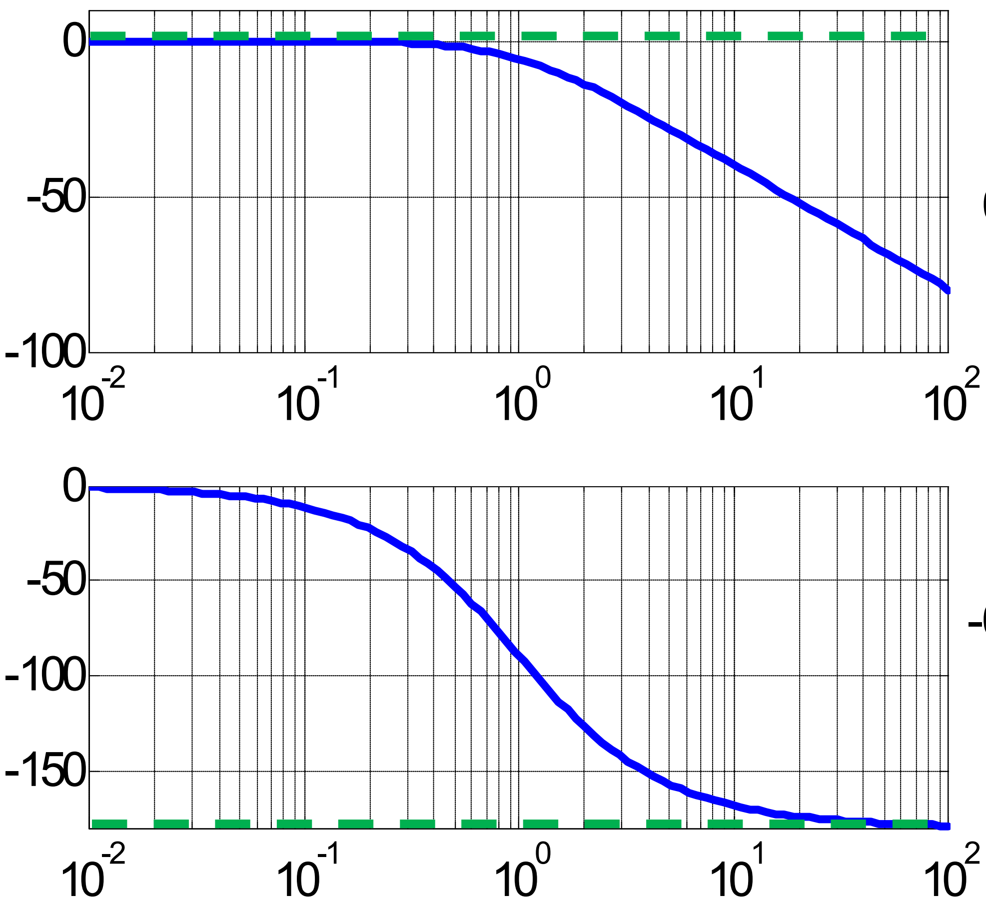
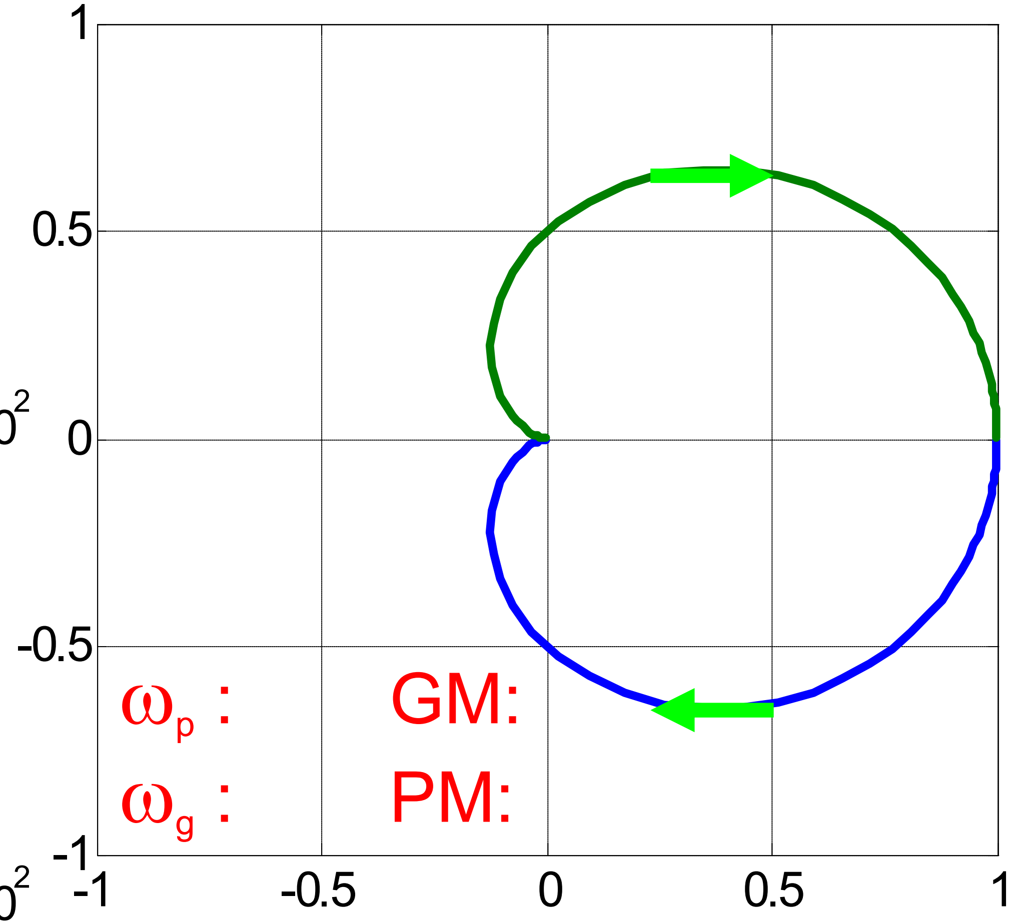
Example 2: (GM & PM)

$$L(s) = \frac{1}{(s+1)^2}$$

Bode plot



Nyquist plot



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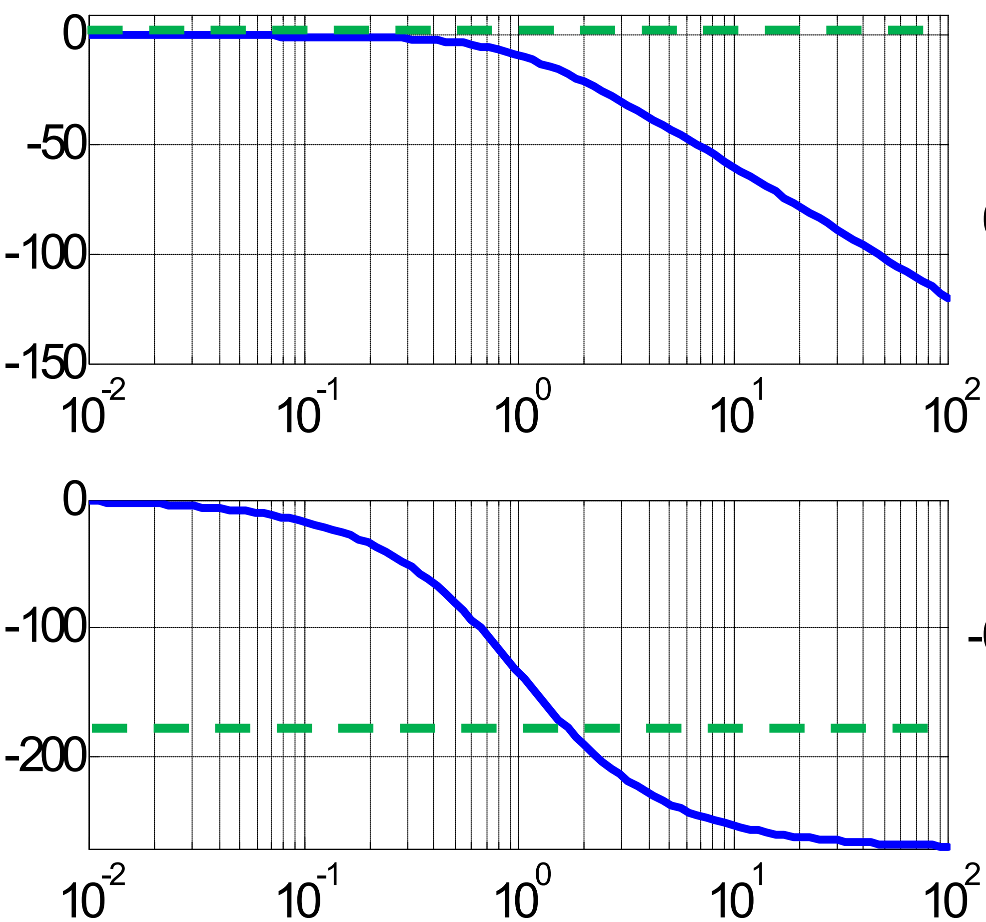
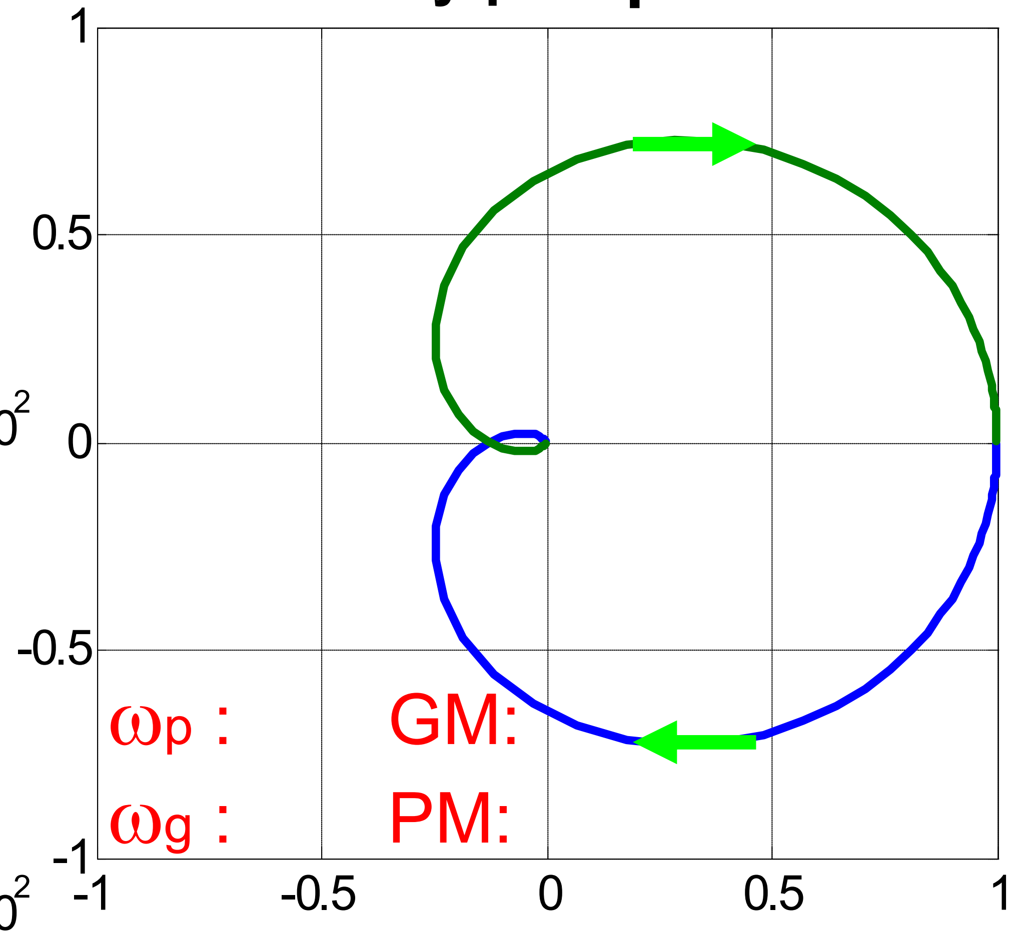
Example 3: (GM & PM)

$$L(s) = \frac{1}{(s+1)^3}$$

Bode plot



Nyquist plot



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Example 3 (cont'd): How to compute GM using $a+bj$ format?

- Frequency response function

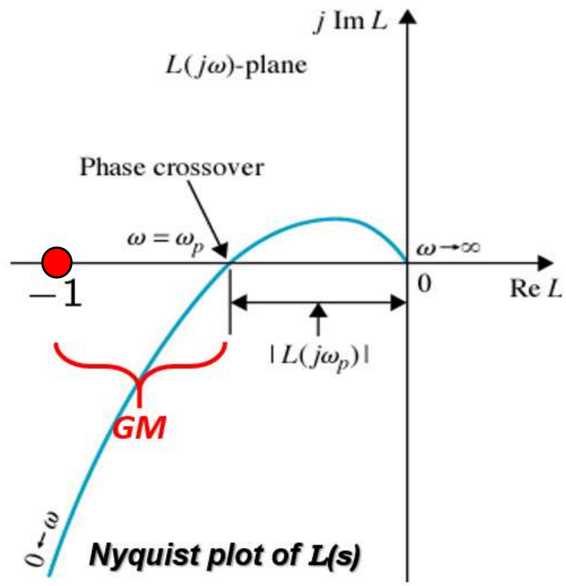
$$L(j\omega) = \frac{1}{(j\omega+1)^3}$$

$$\left[(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\right]$$

$$= \frac{1}{(j\omega)^3 + 3(j\omega)^2 + 3j\omega + 1}$$

$$= \frac{1}{1 - 3\omega^2 + j\omega(3-\omega^2)}$$

$$= \frac{1 - 3\omega^2 - j\omega(3-\omega^2)}{(1-3\omega^2)^2 + \omega^2(3-\omega^2)^2}$$



$$L(j\omega) = \frac{1-3\omega^2}{1+\omega^6+3\omega^2+3\omega^4} + j\,\frac{\omega^3-3\omega}{1+\omega^6+3\omega^2+3\omega^4}$$

$$\operatorname{Im}\{L(j\omega)\}\Big|_{@\ \omega_p} = 0 \Rightarrow \omega_p = \sqrt{3} \Rightarrow L(j\sqrt{3}) = -\frac{1}{8}$$

$$GM = 20\log_{10}\frac{1}{|L(j\omega_p)|} \Rightarrow \boxed{GM = 18.06}$$

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Controller design comparison

Design specifications in time domain
(rise time, settling time, overshoot, steady state error, etc.)

Approximate translation

Desired closed-loop pole location in s-domain

Desired open-loop frequency response in ω-domain

Root locus shaping

Frequency response shaping (loop shaping)

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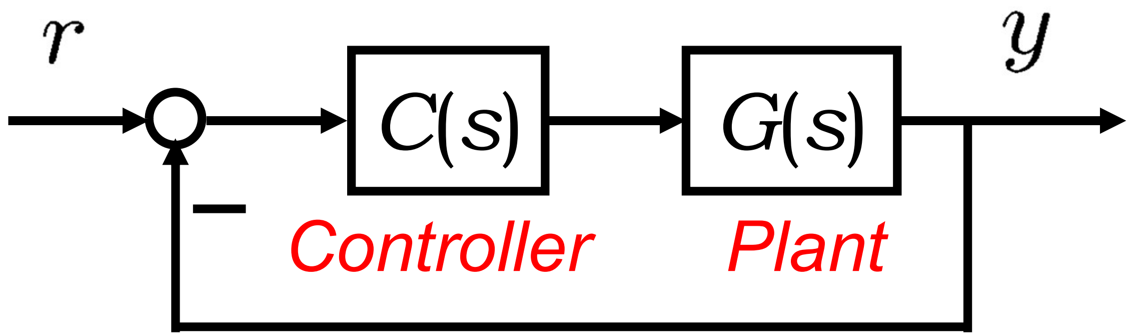
Feedback control system design



OL: $L(s) = G(s)C(s)$

CL: $T(s) = \dfrac{L(s)}{1+L(s)}$

$T(s) = CLTF(s)$

- **Goal of Design:** Given $G(s)$, design $C(s)$ that satisfies CL stability and time-domain specs, i.e., transient and steady-state responses.
- We will learn about typical modifications of OL Bode plot and their effect on closed-loop properties such as stability and time-domain responses.
- Many studies show that **PM ∝ 1/PO** (note that $\zeta \propto 1/\text{PO}$).
  - ➢ We can use this conclusion as a bridge between step response and frequency response analyses.

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How to design C (or K) to yield a specific PO in step response using frequency response?

Here is a summary of steps to take:

- **Step 1:** Use the given PO, find ζ, $$\zeta = \frac{\left|\ln\frac{PO}{100}\right|}{\sqrt{\pi^2+\left(\ln\frac{PO}{100}\right)^2}}$$

- **Step 2:** Use ζ and find a specific PM as shown below. Call this $\mathrm{PM}_{\text{compensator}}$.
  - There are two methods to calculate $\mathrm{PM}_{\text{compensator}}$:

$$\mathrm{PM}_{\text{compensator}} = 100\zeta$$

or

$$\mathrm{PM}_{\text{compensator}} = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}\right)$$

Note: Use this relation in your tests/assignments.

- **Step 3:** Find ω^* at which the following equation is satisfied:

$$\angle L(j\omega^*) = -180^\circ + \mathrm{PM}_{\text{compensator}}$$

- **Step 4:** Find the requested gain, i.e., C,

$$C = \frac{1}{|L(j\omega^*)|}$$

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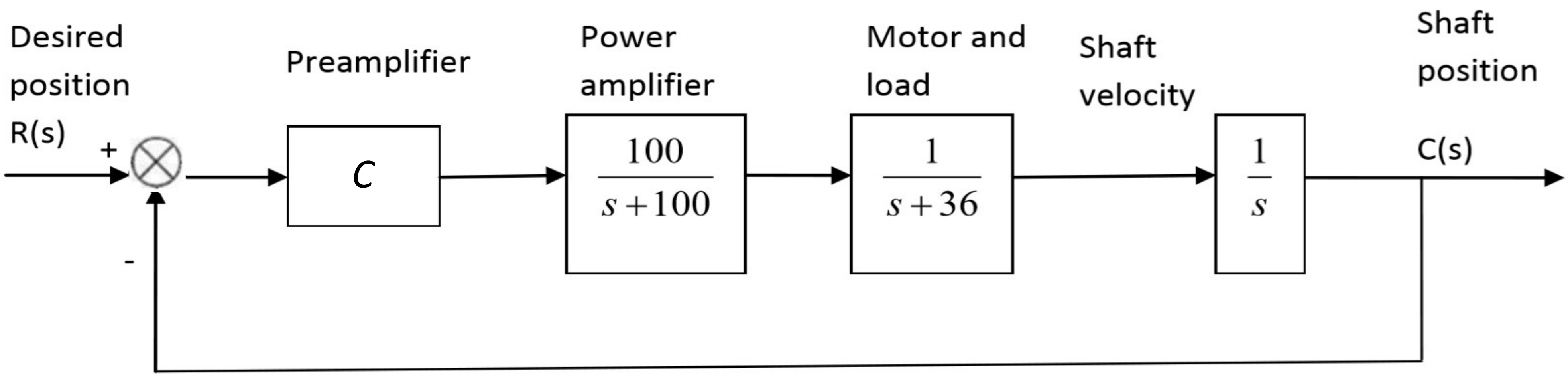
Example 4: A bridge between step response and frequency response analyses

For the position control system shown below, find the value of preamplifier gain, C, to yield a 9.5% overshoot in the transient response for a unit step input. Use frequency response methods.



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Example 4 (cont'd)

Solution:

$$L(s) = \frac{100}{s(s+36)(s+100)} \rightarrow \text{Find } C \text{ so that PO} = 9.5\%:$$

$$\zeta = \frac{\left|\ln\frac{PO}{100}\right|}{\sqrt{\pi^2+\left(\ln\frac{PO}{100}\right)^2}} = \frac{\left|\ln\frac{9.5}{100}\right|}{\sqrt{\pi^2+\left(\ln\frac{9.5}{100}\right)^2}} \rightarrow \zeta \approx 0.6$$

$$\text{PM}_{\text{compensator}} = 100\zeta \rightarrow \text{PM}_{\text{compensator}} = 60°$$

$$\angle L(j\omega^*) = -180° + \text{PM}_{\text{compensator}} = -180° + 60° = -120° \quad \rightarrow \angle L(j\omega^*) = -120°$$

Find ω^* at which $\angle L(j\omega^*) = -120^\circ$:

$$\angle L(j\omega^*) = 0° - \{90^\circ + \tan^{-1}\left(\frac{\omega^*}{36}\right) + \tan^{-1}\left(\frac{\omega^*}{100}\right)\} = -120^\circ \qquad \rightarrow \omega^* = 14.45$$

At $\omega^* = 14.45 \rightarrow C = ?$

$$|L(j \times 14.45)| = \frac{100}{14.45\sqrt{14.45^2+36^2}\sqrt{14.45^2+100^2}} \rightarrow |L(j \times 14.45)| = 1.7656 \times 10^{-3}$$

$$C = \frac{1}{|L(j\omega^*)|} = \frac{1}{1.7656\times10^{-3}} \rightarrow \boxed{C = 566}$$

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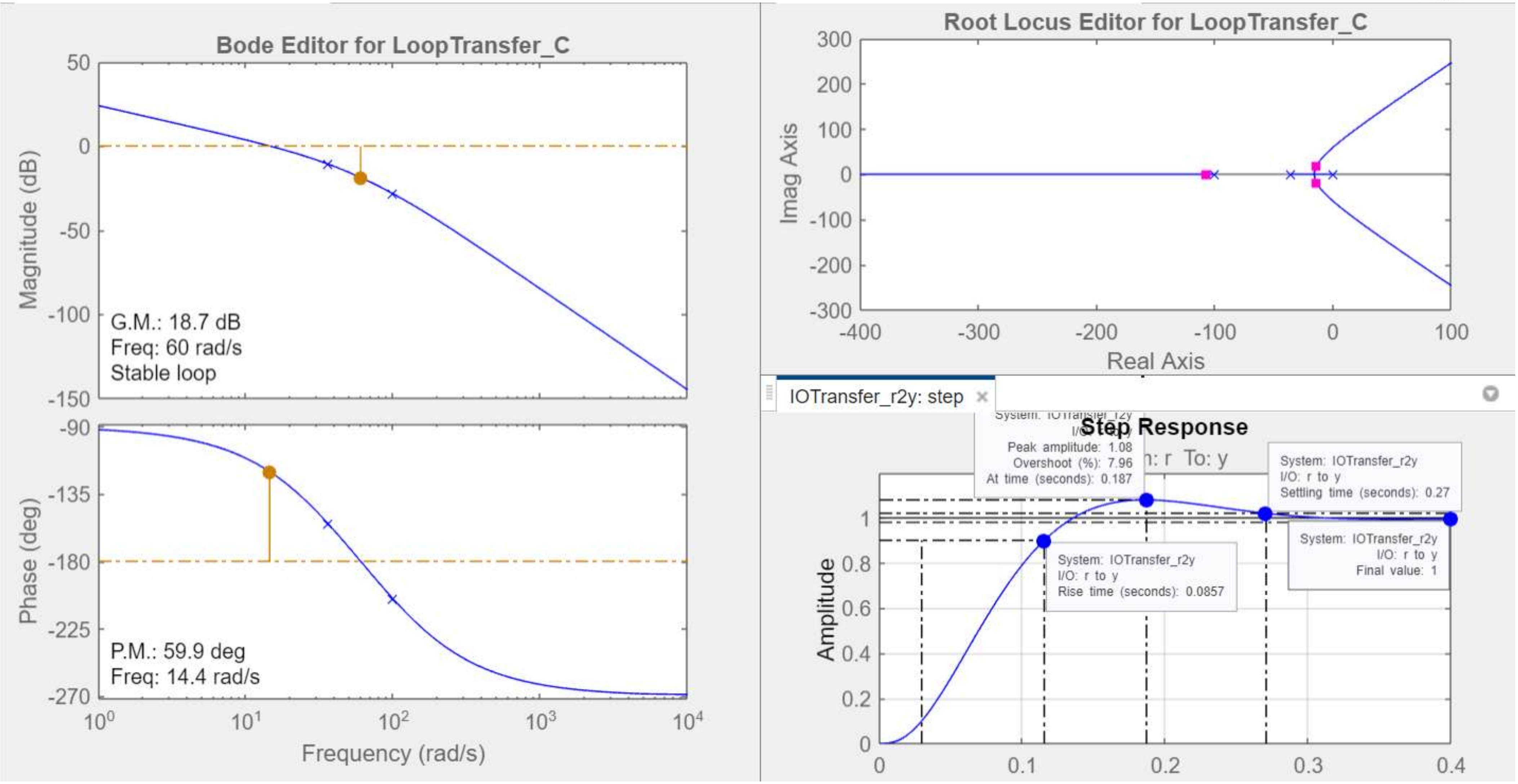
Example 4 (cont'd)

$$C.L(s) = 566\frac{100}{s(s+36)(s+100)} = \frac{56600}{s(s+36)(s+100)}$$



The enlarged version of this plot is presented on the next slide.

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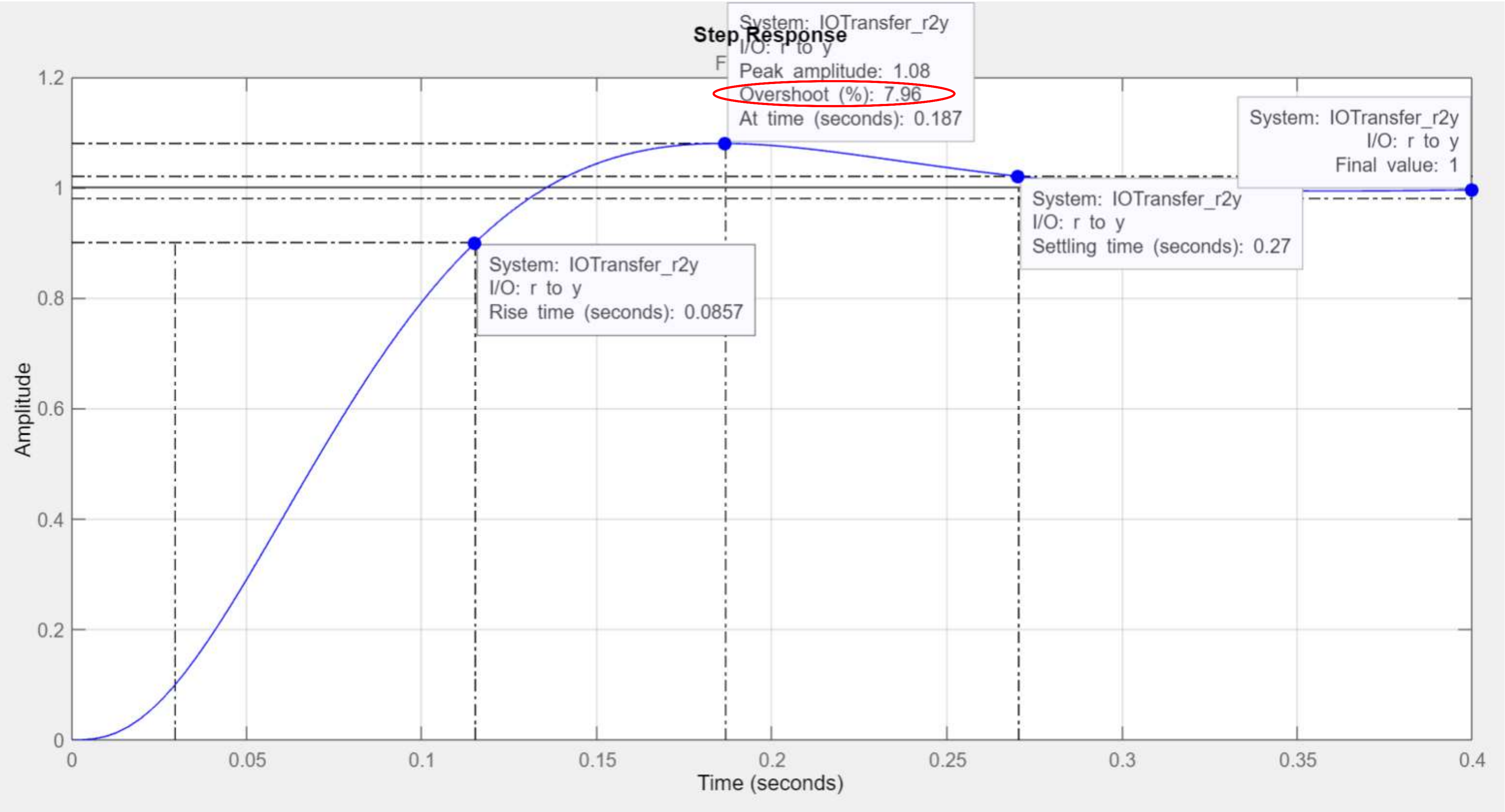
Example 4 (cont'd)

$$C.L(s) = 566\frac{100}{s(s+36)(s+100)} = \frac{56600}{s(s+36)(s+100)}$$



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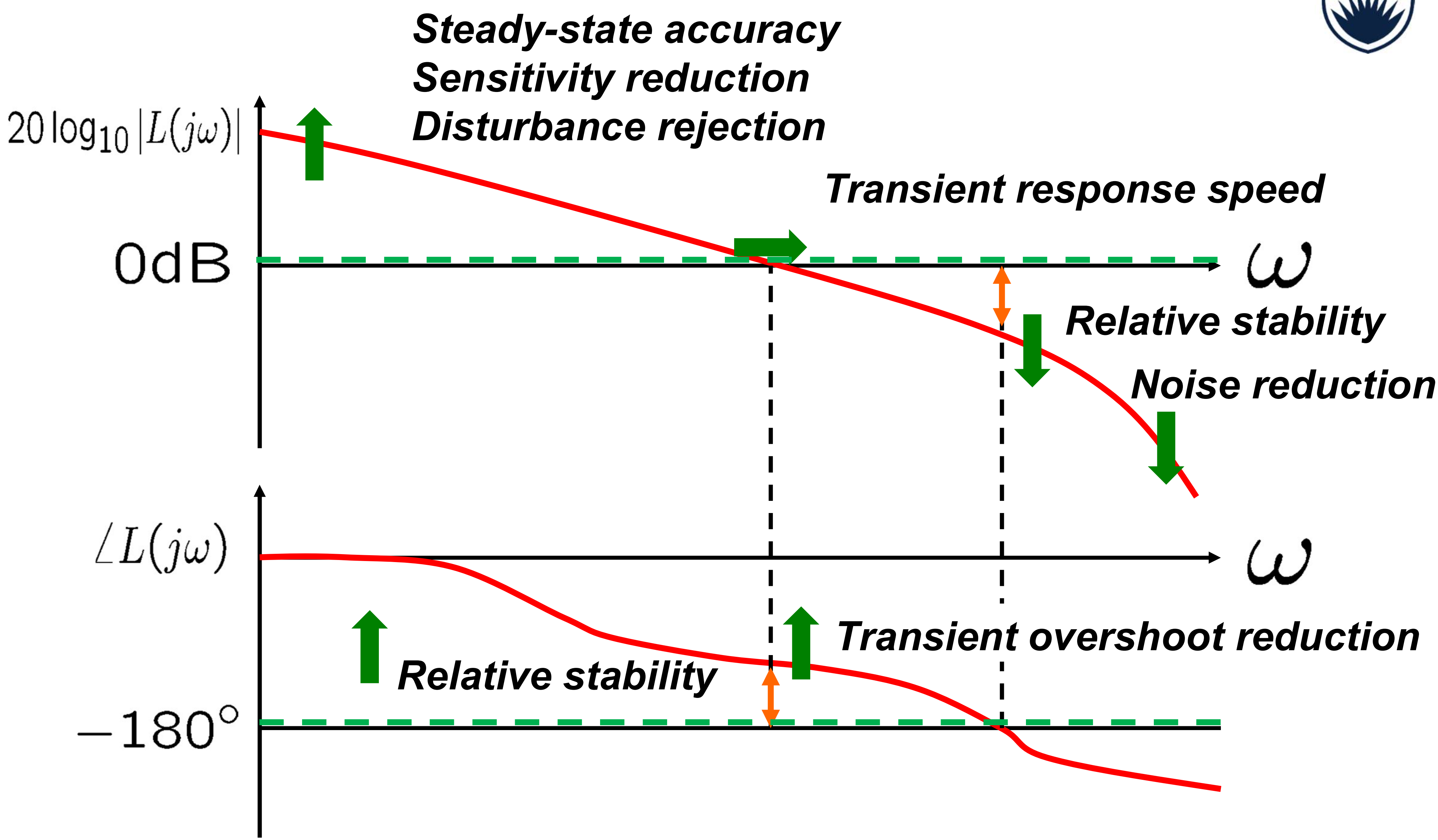
Typical modification of OL Bode plot



- Using the above is called **frequency response shaping (loop shaping) design.**

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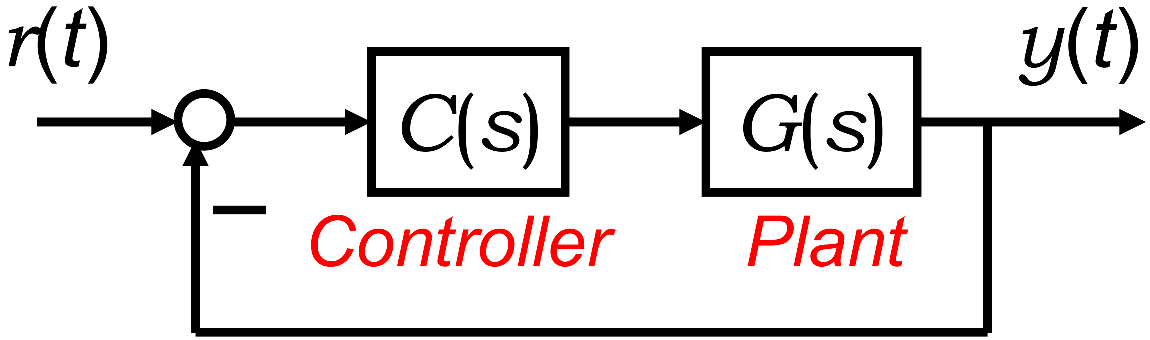
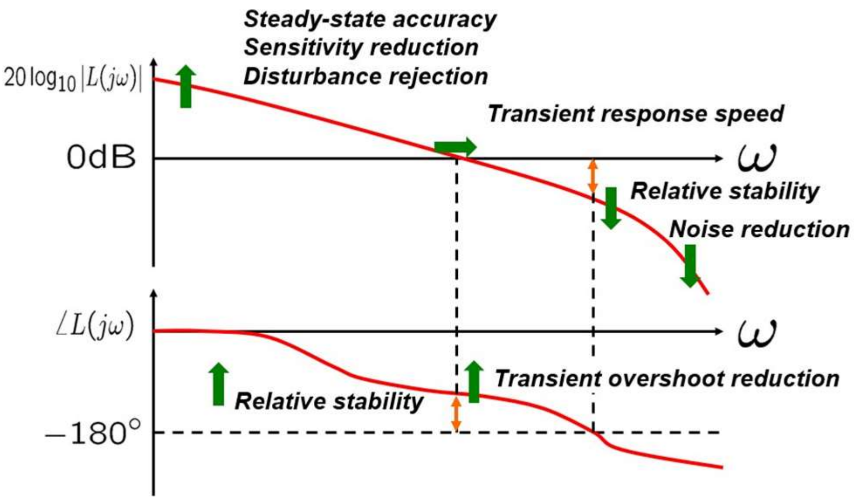
Steady-state accuracy



$$L(s) = G(s)C(s)$$

$$T(s) = \frac{L(s)}{1 + L(s)}$$

$y(t)$ tracks $r(t)$ very well at large t (or e_{ss} will decrease).



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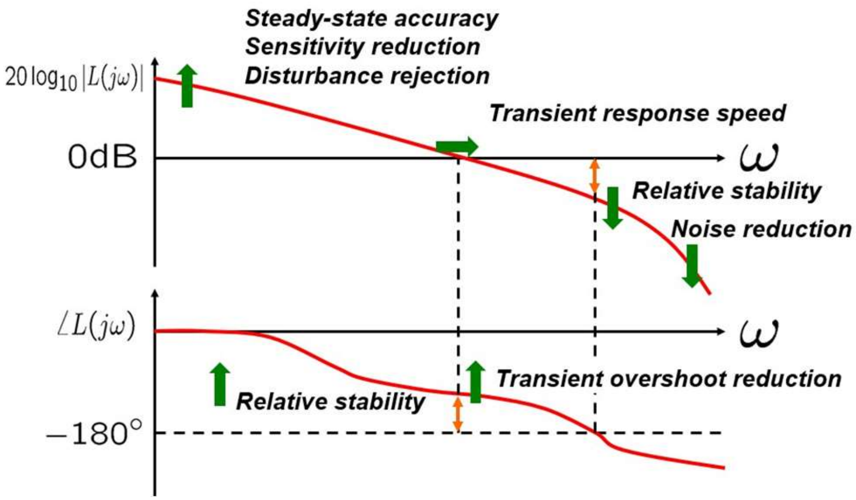
Sensitivity reduction

- Sensitivity indicates the influence of plant variations (due to temperature, humidity, age, etc.) on closed-loop performance.
- Sensitivity function:

$$S(s) = \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{1}{1+G(s)C(s)} = \frac{1}{1+L(s)}$$



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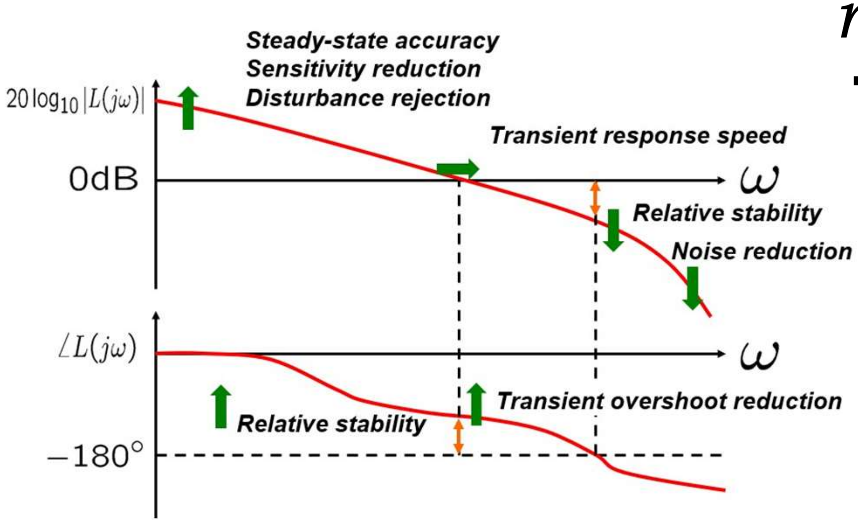
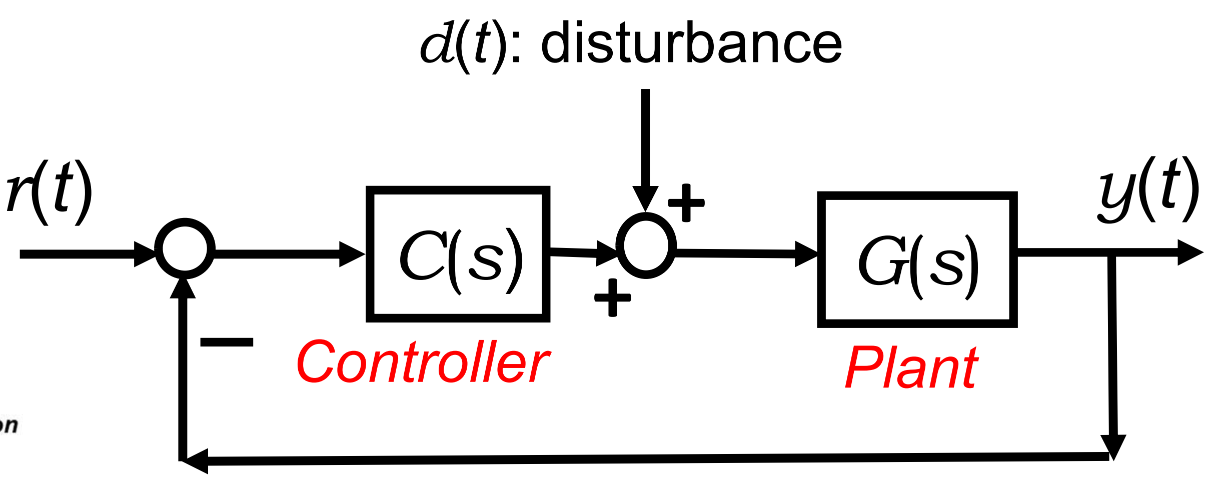
Disturbance rejection

- Unwanted signal
- Examples
  - Wind turbulence in airplane altitude control
  - Wave in ship direction control
  - Sudden temperature change outside the temperature-controlled room
  - Bumpy road in cruise control
- Often, disturbance is neither easily measurable nor easily predictable. Use feedback to compensate for it!





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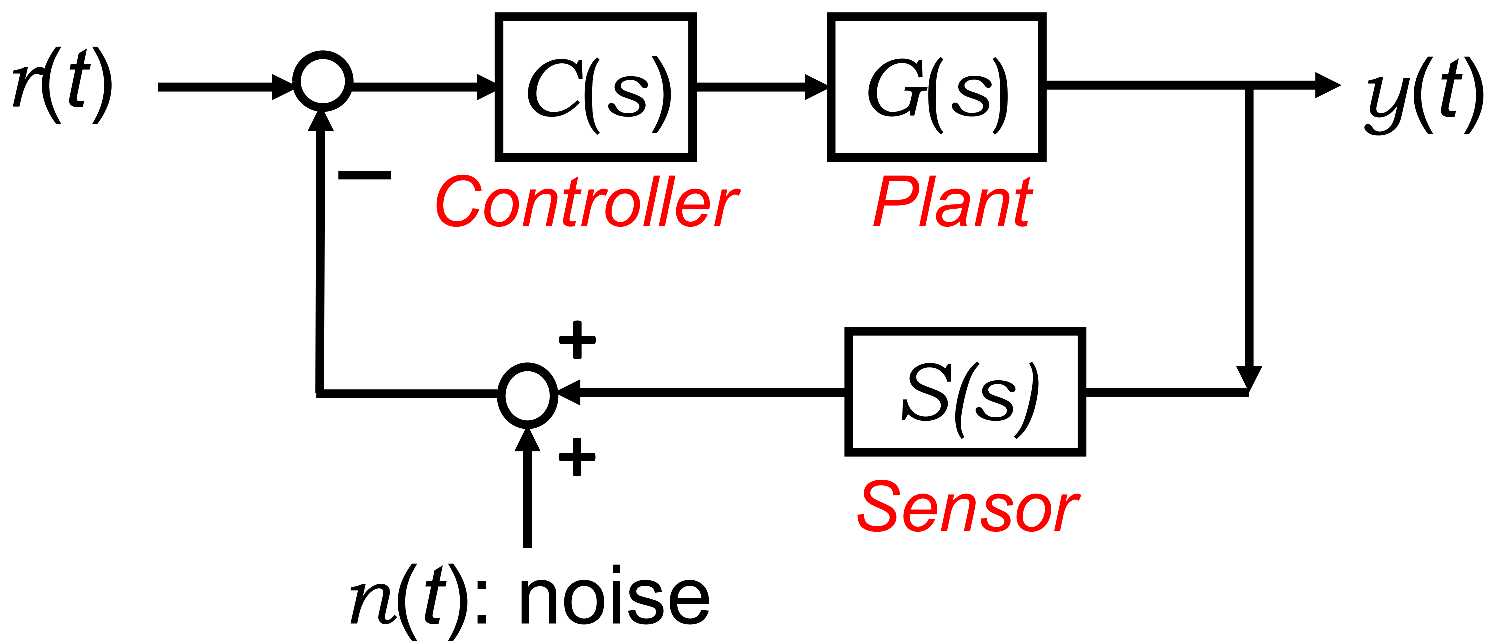
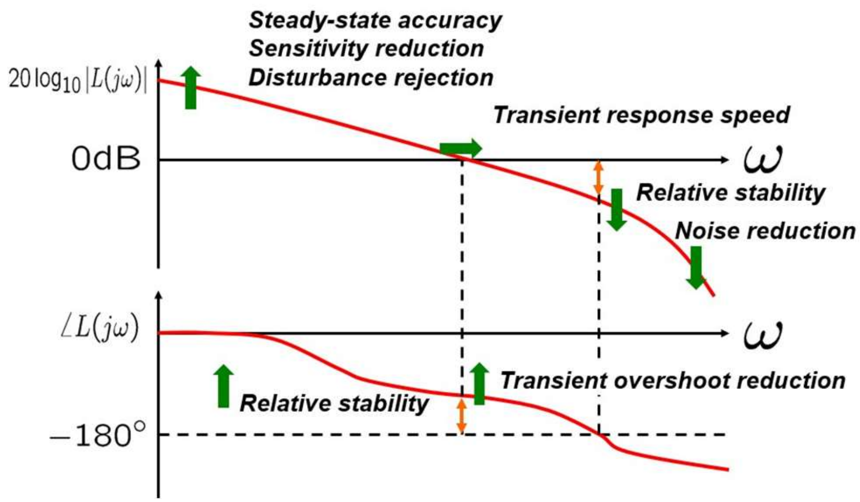
Noise reduction





The ideal goal of the noise reduction efforts: $y(t)$ is not affected by $n(t)$.

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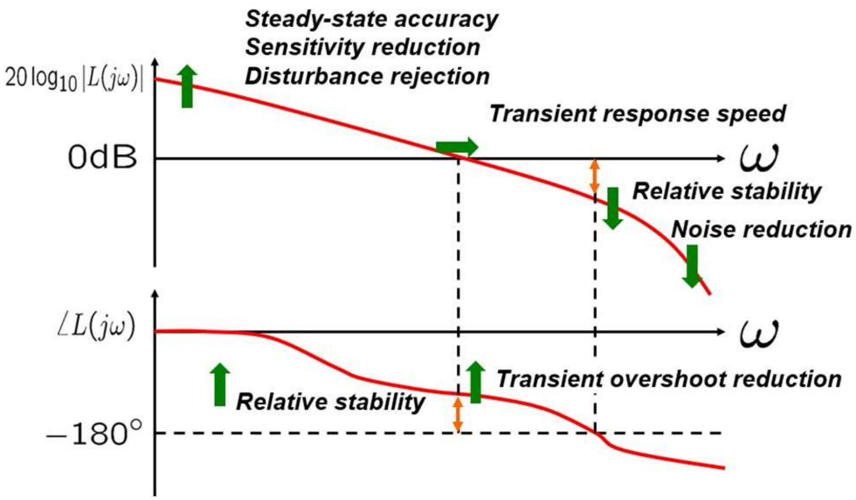
Relative stability

- We require adequate GM and PM for:
  - safety against inaccuracies in modeling
  - reasonable transient response (low overshoot)
- It is difficult to give reasonable numbers of GM and PM for general cases, but usually,
  - **PM** should be at least **45 deg**
  - **GM** should be at least **6 dB**

  (These values are not absolute but approximate!)
- In controller design, we are especially interested in PM (which typically leads to good GM).



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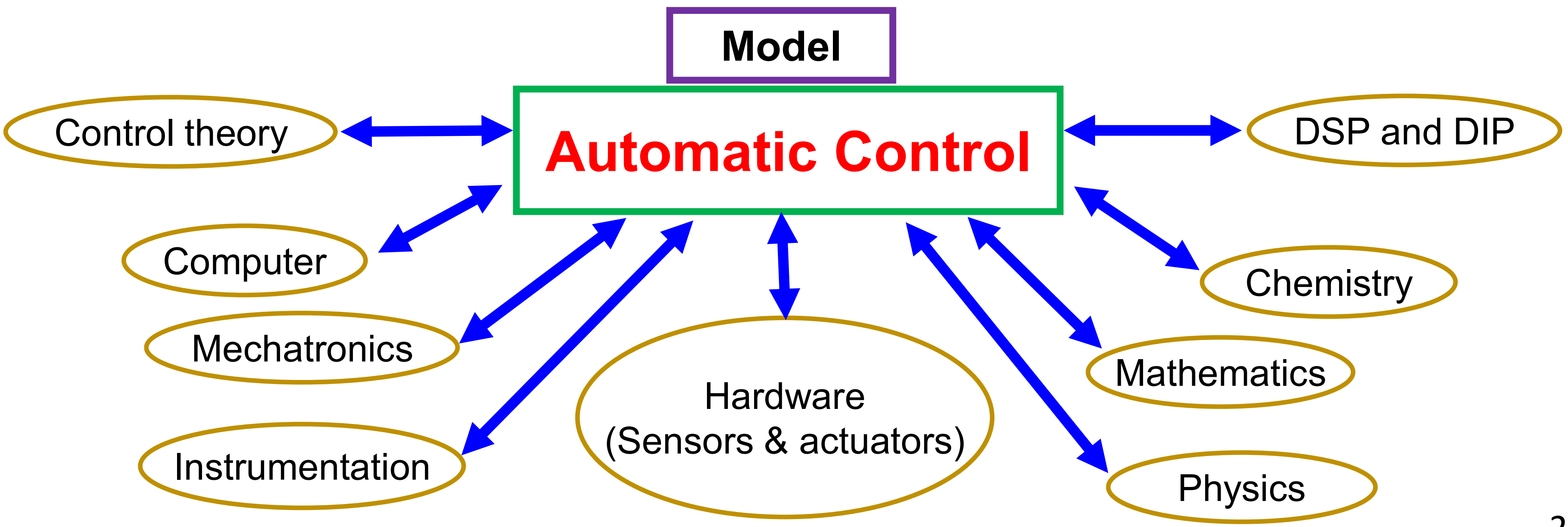
Application of systems and control in various disciplines

Automatic control supports various disciplines!
*(often called "**hidden technology**")*

Electrical engineering
Biomedical engineering
Mechanical engineering
Chemical engineering
Civil engineering
Aerospace engineering
Environmental engineering

Integrated engineering
Engineering physics
Computer engineering
Mechatronics engineering
MEMS, Nanotechnology
Medicine, Economics, Biology



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Control in automobiles

Automobile is a collection of control technologies.

- **Vehicle stability control**
  - Antilock Breaking System (ABS)
  - Traction control
  - Active suspension
- **Energy efficiency and emission reduction**
  - Engine control (fuel injection amount and timing, spark timing)
  - Transmission control
  - Energy management of hybrid vehicles or **fuel cell electric vehicles**
- **Driver-assist system and autonomous car**
  - Adaptive cruise control
  - Automated parallel parking
  - Automatic lane following
  - Collision prevention
  - Driverless car (Tesla, Google, Mercedes-Benz, Toyota, etc.)
  - Flying vehicle (AeroMobil)

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Wind turbines

- Multi-disciplinary
  - Electrical, civil, mechanical, materials, integrated, and environmental engineering
  - Dynamics (modeling)
    - Fluid dynamics (wind flow)
    - Hydro dynamics (offshore)
    - Aerodynamics (blade design)
  - Vibration (fatigue)
  - Control
    - Yaw control
    - Blade pitch control
    - Generator torque control

The Eye of the Wind
(60m hub height, 1.5MW)
Grouse Mountain

Onshore and offshore wind turbines
(Taken from NREL report)

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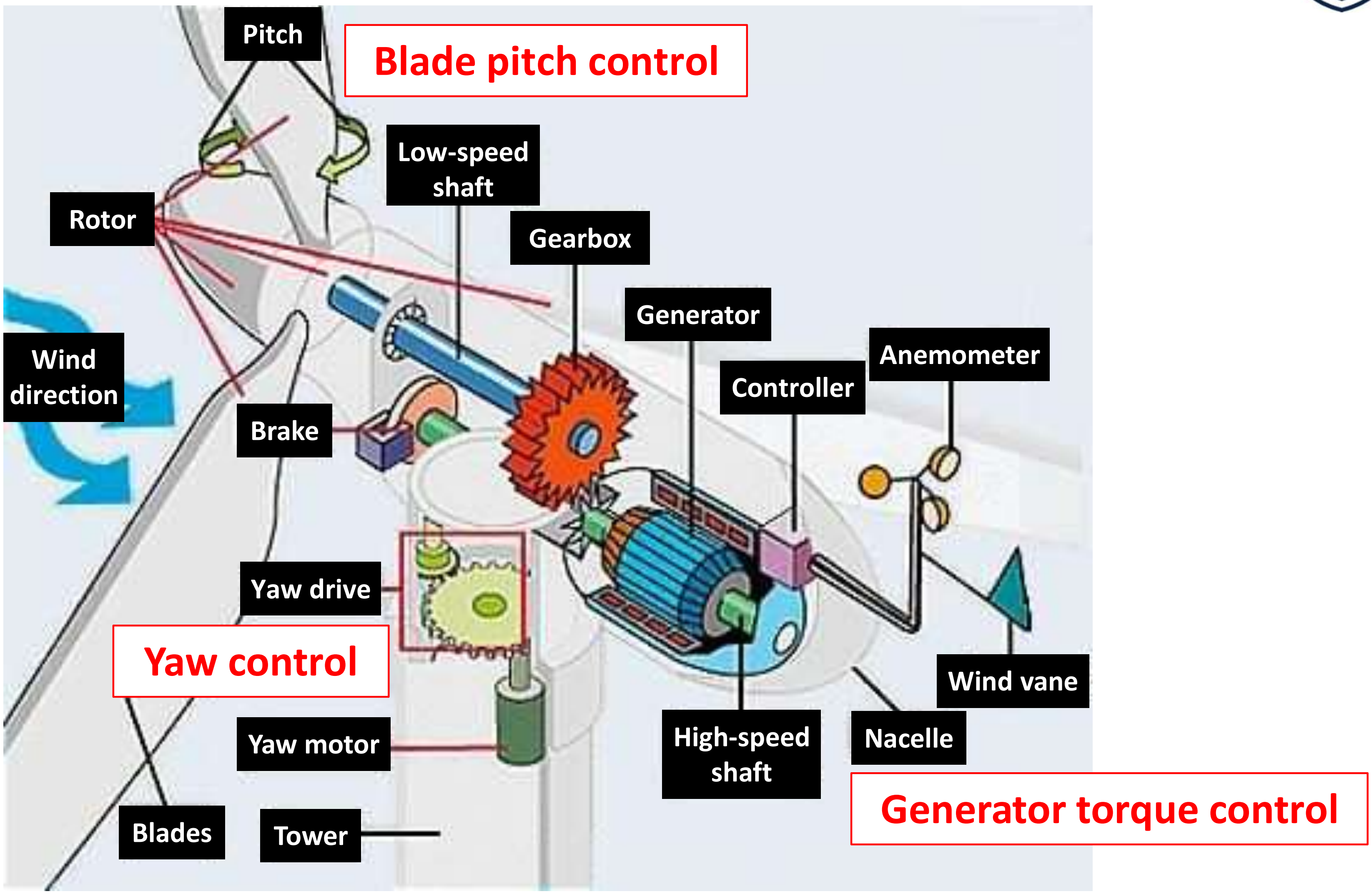
Inside the nacelle of a wind turbine



(Taken from NREL report)

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Control in biomedical engineering

- **Rehabilitation engineering**
  - Powered prostheses (use body signals to control external assistive devices)
  - Functional electrical stimulation (use external signal to control body)
    - Stroke
    - Spinal cord injuries
    - Parkinson’s disease
- **Drug delivery and administration**
  - Blood pressure regulation
  - Blood glucose control
  - Anesthesia control
- **Medical instruments**
  - Smart surgical tools
  - Artificial tactile sensing and haptics feedback
  - Temperature and humidity regulation
  - Robotically-assisted surgery
  - Artificial organs (such as heart, lung, liver, pancreas, etc.)

Note: I cover these subjects extensively in another course that I teach at UBC. The course is **ELEC 371 (Biomedical Engineering Instrumentation)**.

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Summary

- Examples of gain margin and phase margin
- Frequency domain design specifications
- Application of control engineering in various disciplines
- Next
  - Frequency response shaping (loop shaping) on Bode plot using Matlab for design of the following:
    - gain compensators
    - lead, lag, and lead-lag compensators

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