ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 2

January 8, 2026

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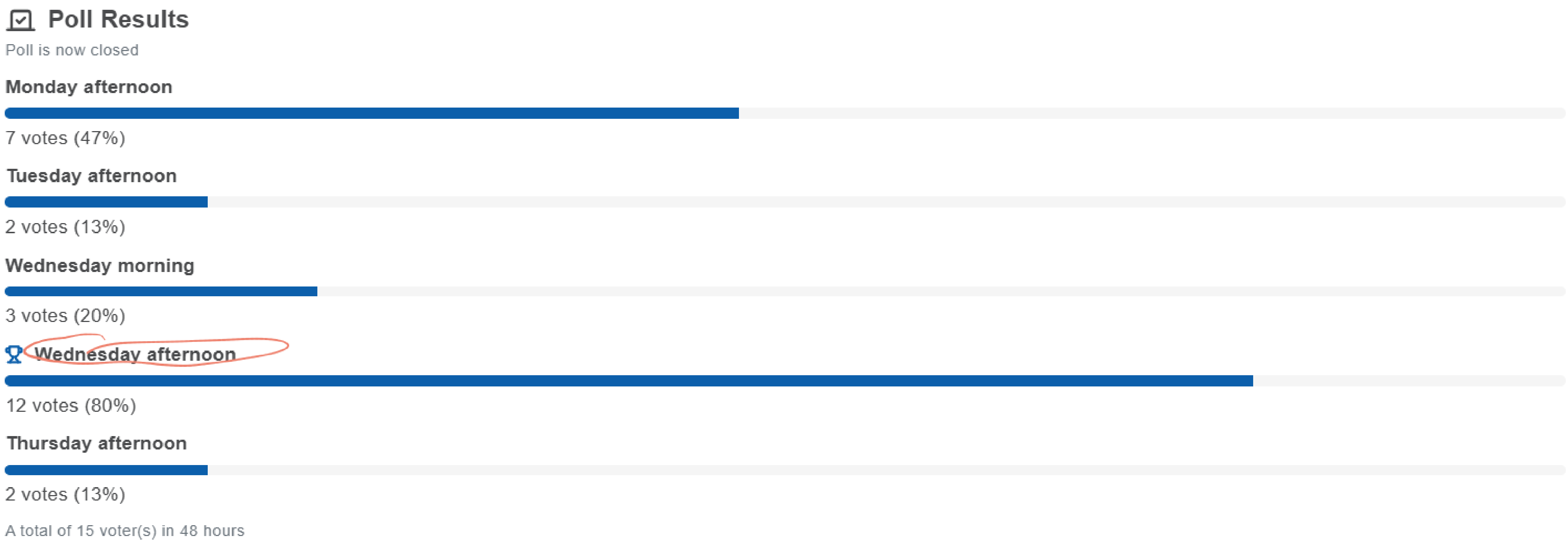
Course logistics

- Labs: Need to work in pairs, find your partner now

- TA office hour poll:



Module 1, Part 1: Review of AC Circuits & Phasors

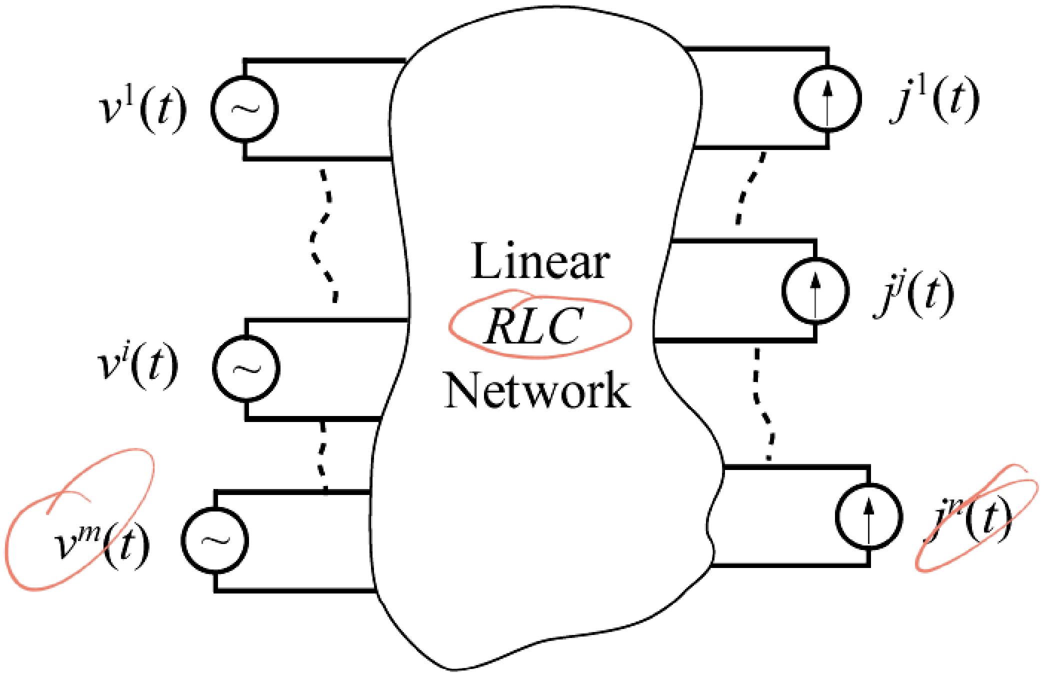
Objectives & important concepts

- Concept of phasors & notations
- RMS value
- Phasor diagrams for basic RLC circuits
- Balanced 3-phase system, source, load, Y / D connection, line-to-line and phase voltages and currents, phase shifts
- Real, reactive, and apparent power in 1-phase and 3-phase systems, power factor

- Read Appendix B, get ready for Lab 1

AC circuits

- Consider arbitrary network
  - With voltage and current sources



$$v^i(t) = V^i \cos\left(\omega t + \varphi^i\right)$$

$$j^j(t) = J^j \cos\left(\omega t + \varphi^j\right)$$

Steady state solution?

For all circuit branches the currents & voltages are sinusoidal

$$i_k(t) = I_k \cos\left(\omega t + \theta_i^k\right)$$

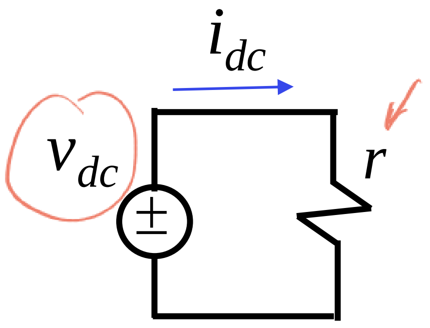
$$v_k(t) = V_k \cos\left(\omega t + \theta_v^k\right)$$

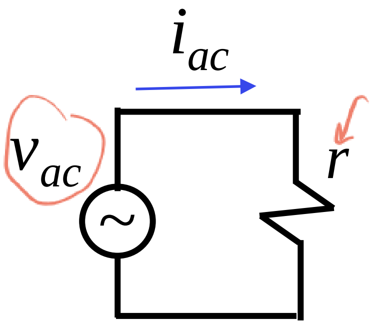
Note:

For Steady State analysis we need to know only amplitudes J^k, V^k & phases θ_i^k, θ_v^k

Root-Mean-Square (RMS) Value

* Value equivalent to the DC voltage (current) that, when applied to a resistor, dissipates the same average power as the given AC value





Instantaneous power in terms of voltage

$$P(t) = vi = v\frac{v}{r} = \frac{1}{r}v_{dc}^2$$

Average power over period T

$$P_{ave} = \frac{1}{T}\int_0^T P(t)dt = \frac{1}{r}\cdot\boxed{\frac{1}{T}\int_0^T v^2(t)dt}$$

Instantaneous power in terms of current

$$P(t) = vi = rii = ri_{dc}^2$$

Average power over period T

$$P_{ave} = \frac{1}{T}\int_0^T P(t)dt = r\cdot\boxed{\frac{1}{T}\int_0^T i^2(t)dt}$$

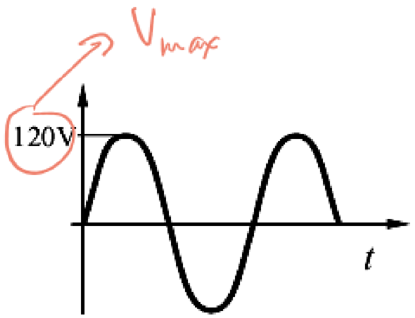
Therefore, RMS values are defined as

$$V_{(rms)} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

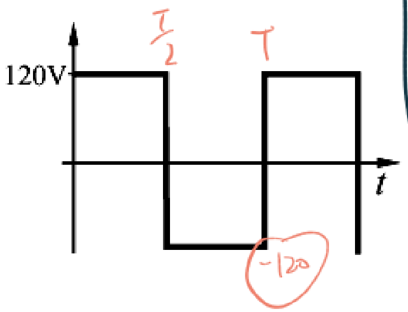
$$I_{(rms)} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

Root-Mean-Square (RMS) Value

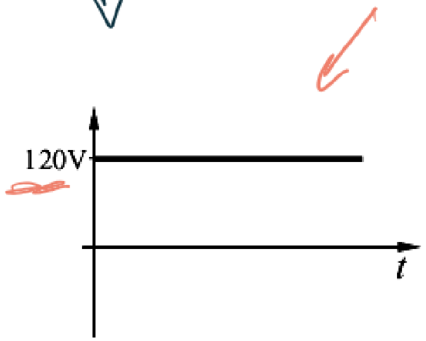
$$\boxed{\begin{aligned} V_{(rms)} &= \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \\ I_{(rms)} &= \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} \end{aligned}}$$



$V_{rms} = \frac{120}{\sqrt{2}}\ V$



$V_{rms} = 120\ V$

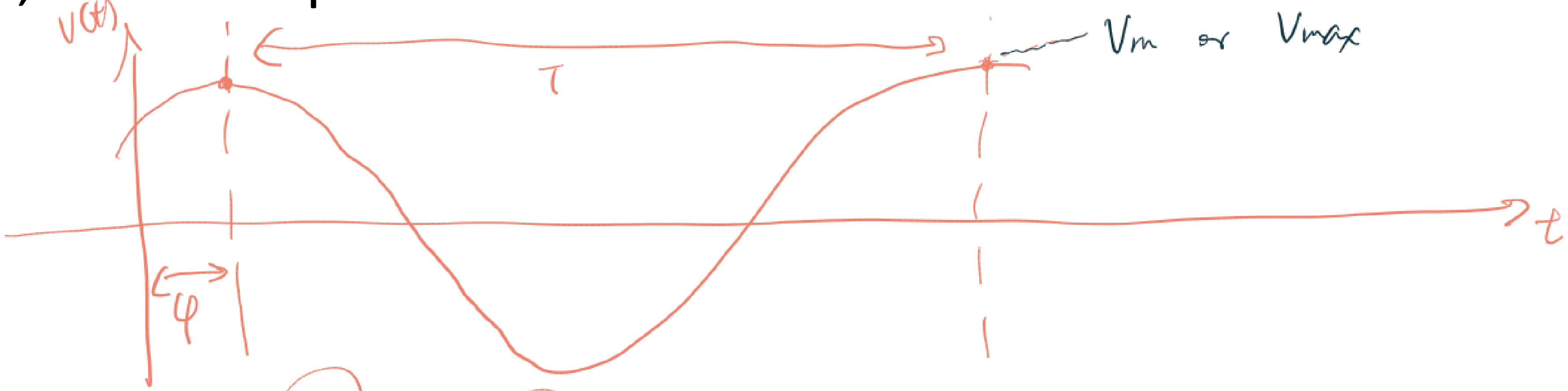


$V_{rms} = 120\ V$

$\sqrt{\frac{1}{T}\int_0^T 120^2\,dt} = 120V$

Root-Mean-Square (RMS) Value

Therefore, we can represent a sinusoidal waveform as:



$$v(t) = V_m \cos(\omega t + \varphi) = \sqrt{2} V_{rms} \cos(\omega t + \varphi)$$

Period: T [s]

Frequency: $f = \frac{1}{T}$ [Hz]

Angular frequency: $\omega = 2\pi f$ [rad/s]

Phase: φ [rad]

Amplitude: $V_m = \sqrt{2}\, V_{rms}$

Phasor representation

Euler's Identity

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Only for steady-state ⇒ Amplitude + phase

lower case to represent time-domain signal

- It is like an abbreviation:

$$v(t) = \sqrt{2}V_{rms}\cos(\omega t + \varphi)$$

$$= \sqrt{2}V_{rms}Re[e^{j(\omega t+\varphi)}]$$

$$= \sqrt{2}V_{rms}Re[e^{j\omega t}e^{j\varphi}]$$

Always common in all sinusoidal signals

Upper case

- Phasor notation: $V = V_{rms}e^{j\varphi} = V_{rms}\angle\varphi$ ⇒ complex number
  - Therefore, you can also express the phasor as $V = V_{rms}\cos(\varphi) + jV_{rms}\sin(\varphi)$

Phasor representation

- You can use phasor to express both current and voltage:

$$v(t) = \sqrt{2} \cdot V_{rms} \cos(\omega t + \varphi_v)$$

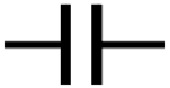
$$i(t) = \sqrt{2} \cdot I_{rms} \cos(\omega t + \varphi_i)$$

$$\boxed{\begin{aligned} \tilde{V} &= V_{rms} \angle \varphi_v \\ \tilde{I} &= I_{rms} \angle \varphi_i \end{aligned}}$$

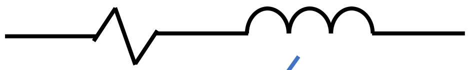
What is the advantage of using phasors?

(1) Phasor algebra
(2) Visualization

Advantage of phasor analysis

Linear Passive Elements		Complex Impedance
	R	$Z = R$ Resistance
	L	Reactance ωL $Z = j\omega L = (\omega L)\angle 90^\circ = jX_L$
	C	$Z = \dfrac{1}{j\omega C} = \left(\dfrac{1}{\omega C}\right)\angle -90^\circ = -jX_C$

$$Z = \frac{V}{I} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i = Z_m \angle \theta_z$$

RL Circuit 

$$Z = R + jX, \quad |Z| = \sqrt{R^2 + X^2}, \quad \theta_z = \arctan\left(\frac{X}{R}\right)$$

(Note: Z is a complex number but not a phasor)

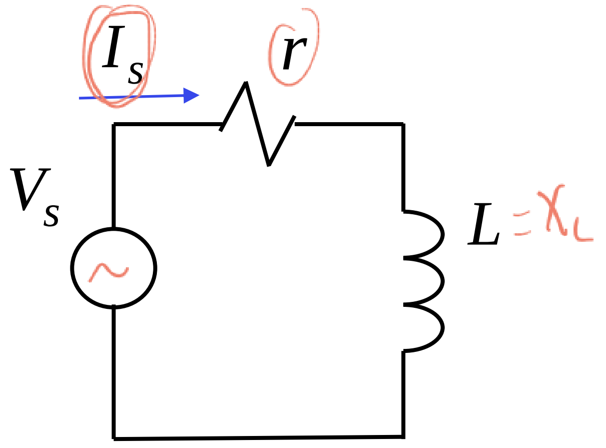
Qualitative Phasor Diagrams

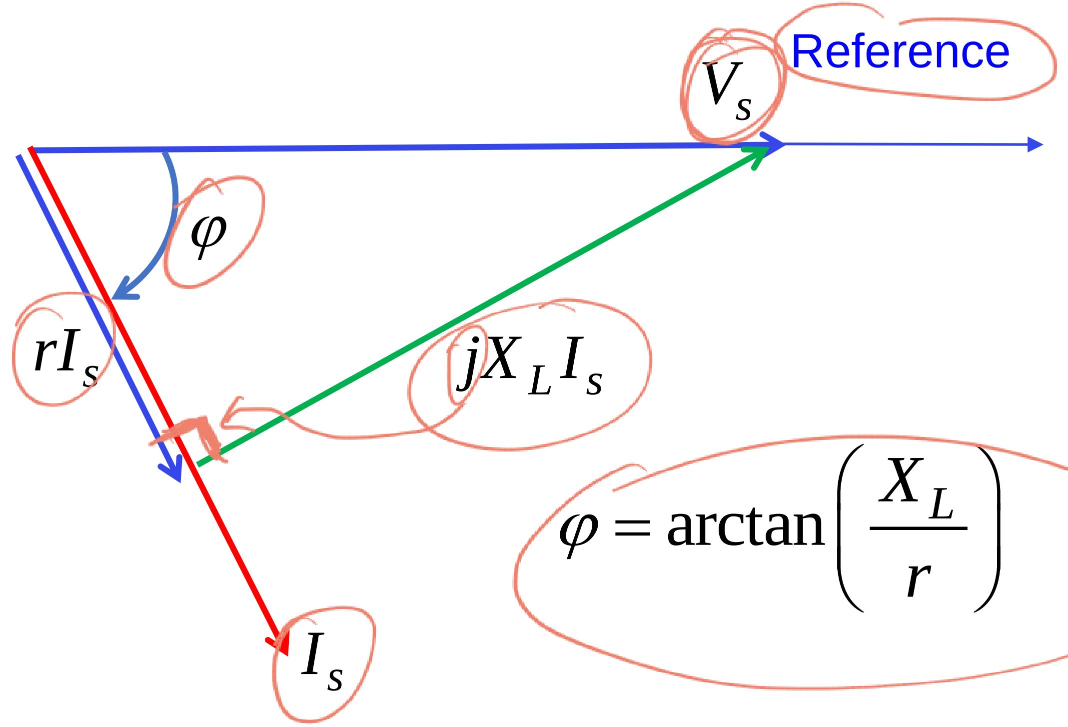
Use phasors (vectors) to represent voltage and/or current equations (relationship)

perpendicular

Consider *RL* Circuit

$$V_s = rI_s + j\omega L I_s = rI_s + jX_L I_s$$



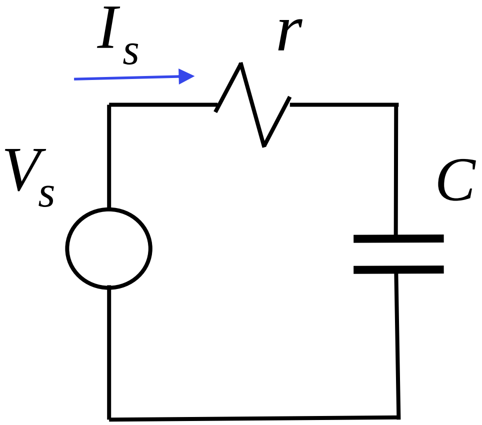


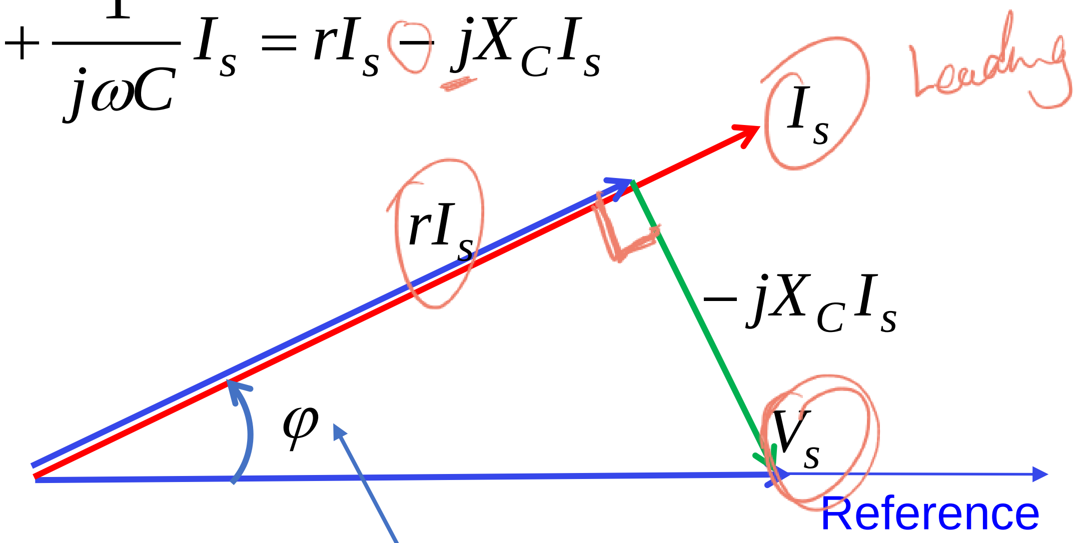
$$\varphi = \arctan\left(\frac{X_L}{r}\right)$$

Qualitative Phasor Diagrams

Use phasors (vectors) to represent voltage and/or current equations (relationship)

Consider *RC* Circuit $V_s = rI_s + \frac{1}{j\omega C} I_s = rI_s - jX_C I_s$





$$\varphi = \arctan\left(\frac{X_C}{r}\right)$$

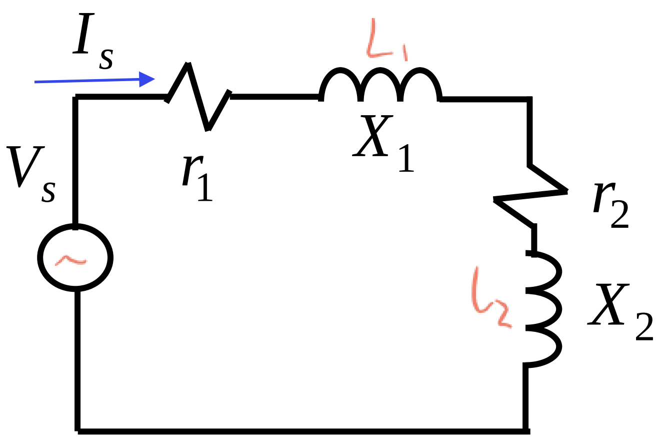
Qualitative Phasor Diagram Problem

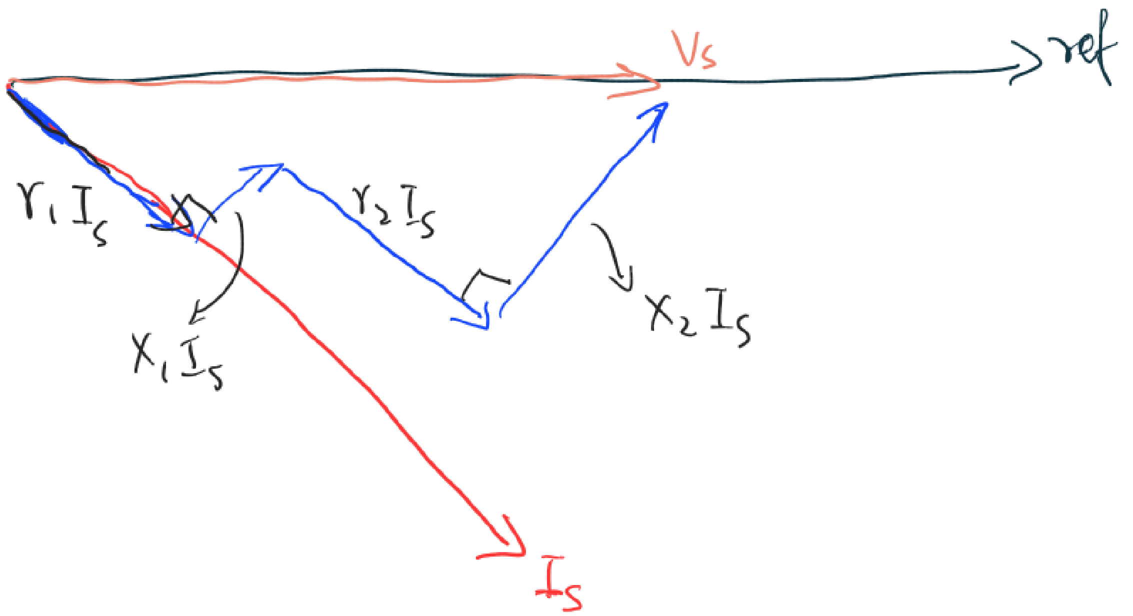
Use phasors (vectors) to represent voltage and/or current equations (relationship)

$X_1 = \omega L_1$, $X_2 = \omega L_2$

Consider RLC Circuit

$V_s = r_1 I_s + jX_1 I_s + r_2 I_s + jX_2 I_s$ (KVL)



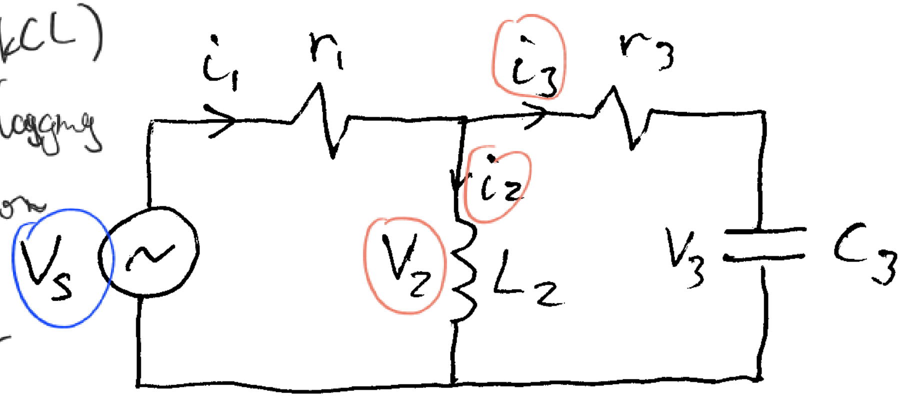


Qualitative Phasor Diagram Problem

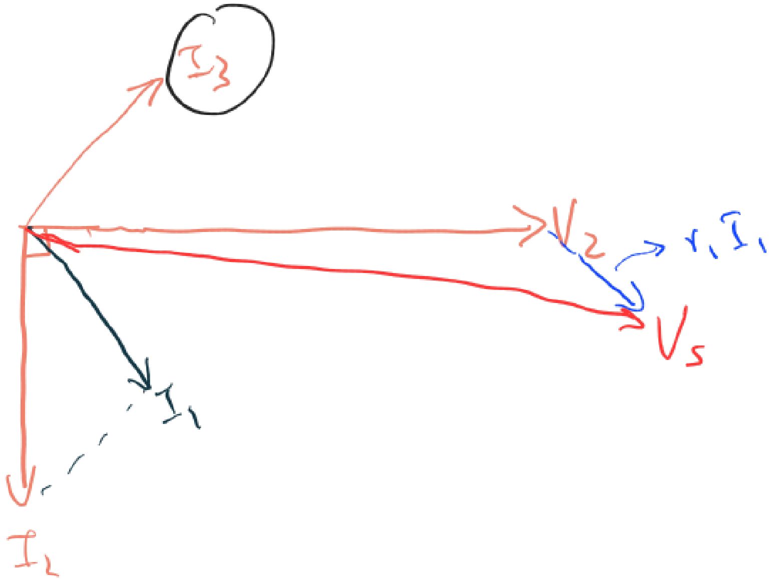
Use phasors (vectors) to represent voltage and/or current equations (relationship)

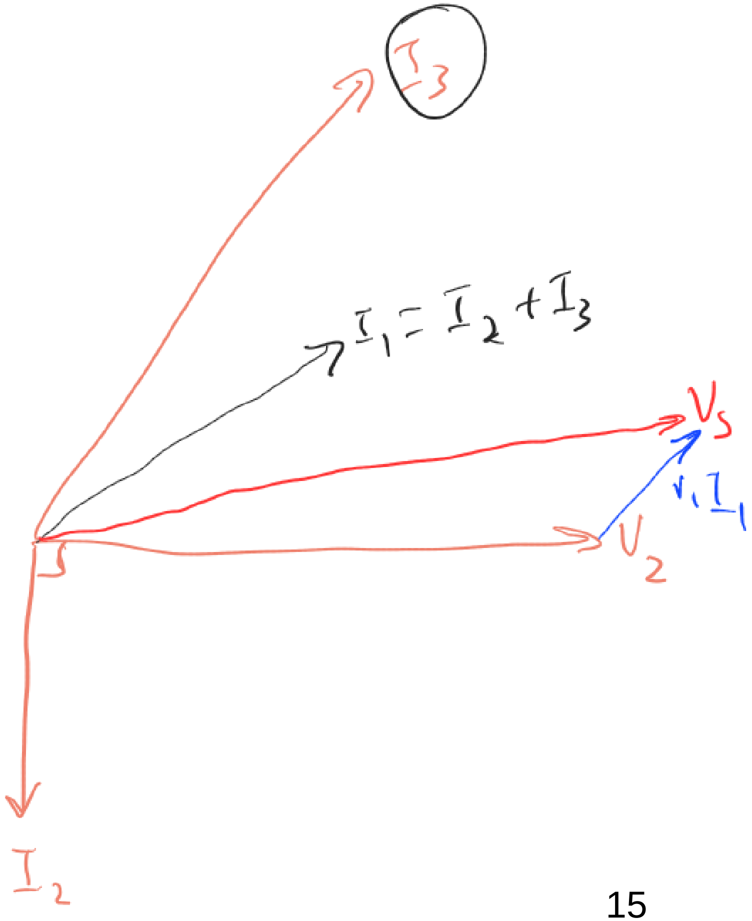
Consider RLC Circuit

- $i_1 = i_2 + i_3$ (KCL)
- i_1 leading or lagging depends on v_s values of L and C



$V_s = V_2 + r_1 I_1$

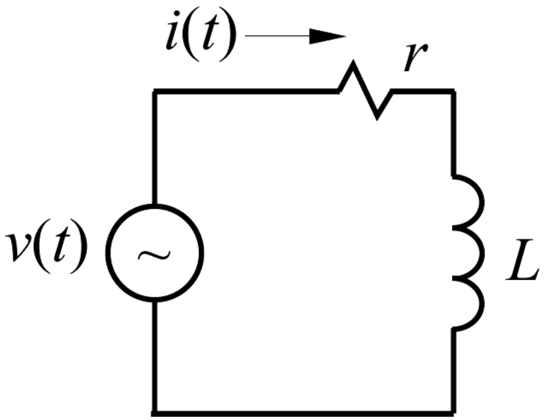




Power in AC Circuits

rms is dropped

Assume inductive load $v(t) = \sqrt{2} \cdot V \cos(\omega t)$ $\quad i(t) = \sqrt{2} \cdot I \cos(\omega t + \varphi)$



Instantaneous power is always voltage times current

$$p(t) = vi = 2VI\cos(\omega t)\cos(\omega t + \varphi)$$

Recall $$\boxed{\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)}$$

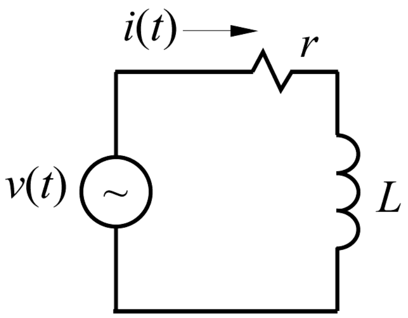
$$p(t) = VI\cos(\varphi) + VI\cos(2\omega t + \varphi)$$

Average ≠ 0

Average power = 0

Power in AC Circuits

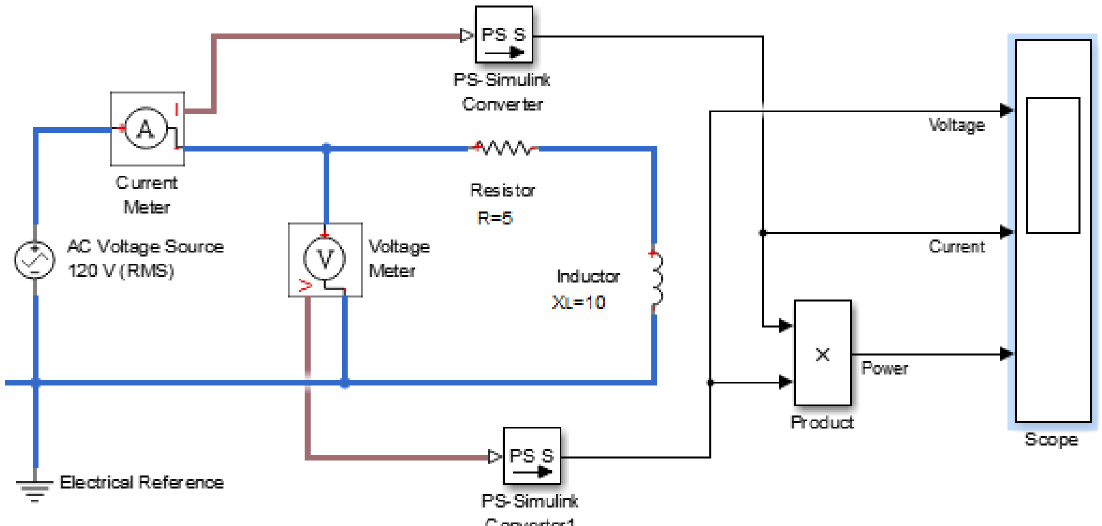
Given inductive load

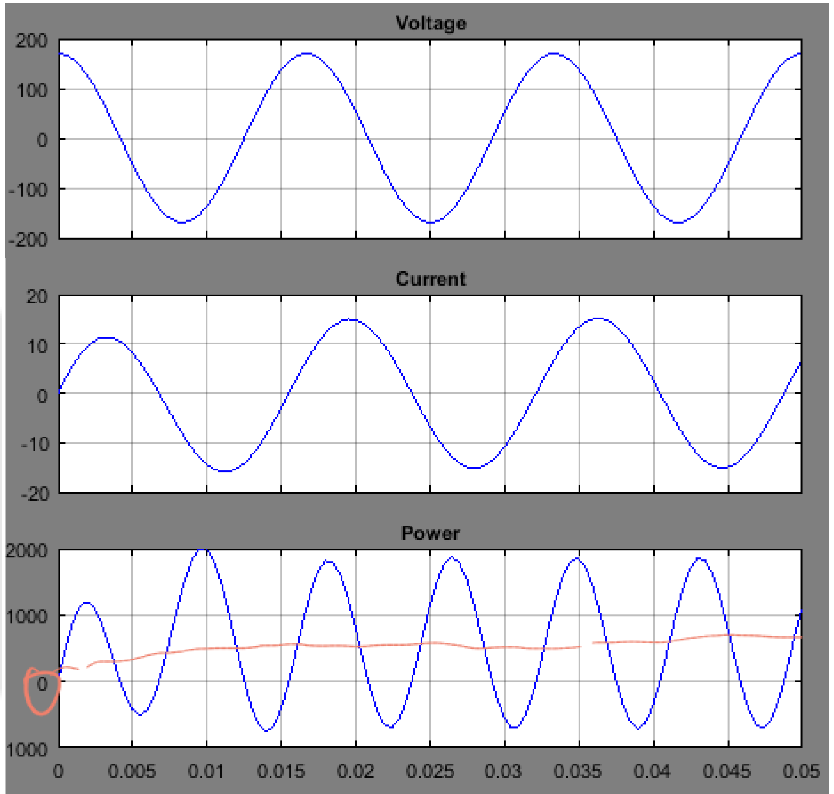


$$v(t)=\sqrt{2}\cdot V\cos(\omega t)\qquad i(t)=\sqrt{2}\cdot I\cos(\omega t+\varphi)$$

Instantaneous power is always this

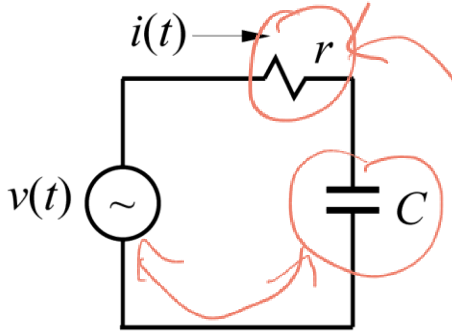
$$p(t)=vi=2VI\cos(\omega t)\cos(\omega t+\varphi)$$
$$=VI\cos(\varphi)+VI\cos(2\omega t+\varphi)$$





Power in AC Circuits

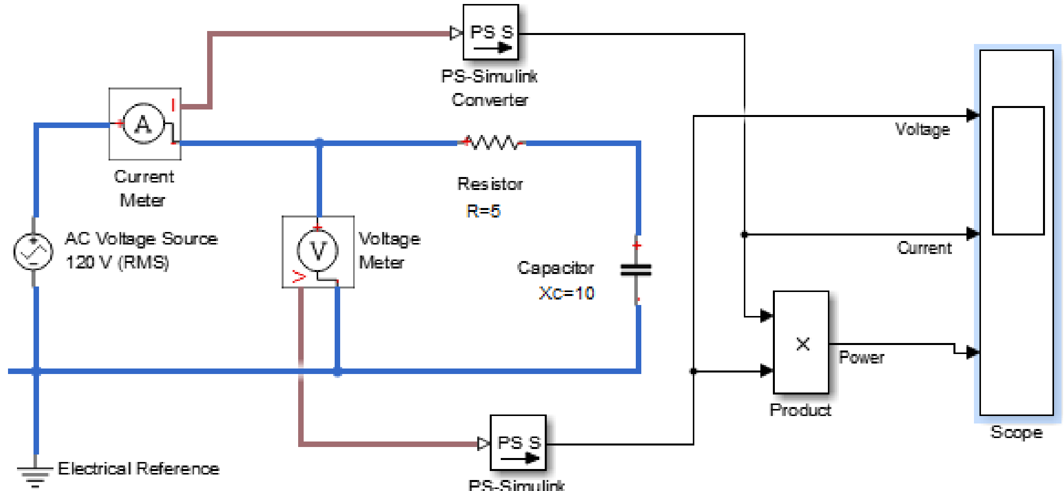
Given capacitive load

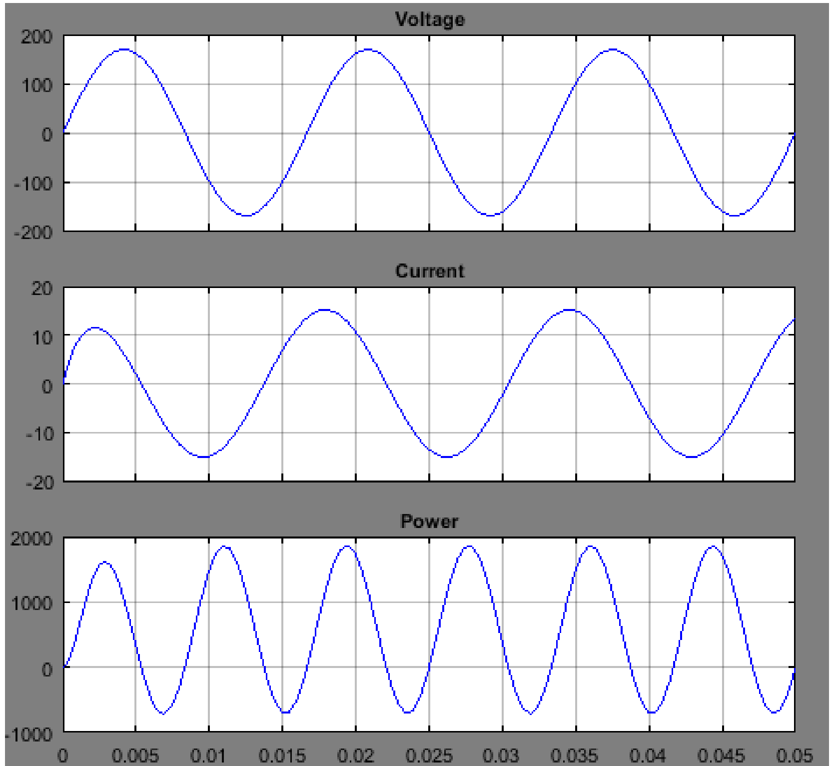


$$v(t) = \sqrt{2} \cdot V \cos(\omega t) \qquad i(t) = \sqrt{2} \cdot I \cos(\omega t + \varphi)$$

Instantaneous power is always this

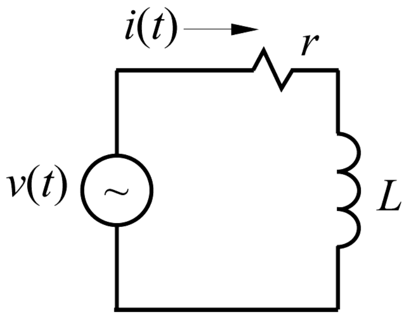
$$p(t) = vi = 2VI\cos(\omega t)\cos(\omega t + \varphi)$$
$$= \underline{VI\cos(\varphi)} + \underline{VI\cos(2\omega t + \varphi)}$$





Power in AC Circuits

Given inductive load



$$v(t) = \sqrt{2} \cdot V\cos(\omega t) \qquad i(t) = \sqrt{2} \cdot I\cos(\omega t + \varphi)$$

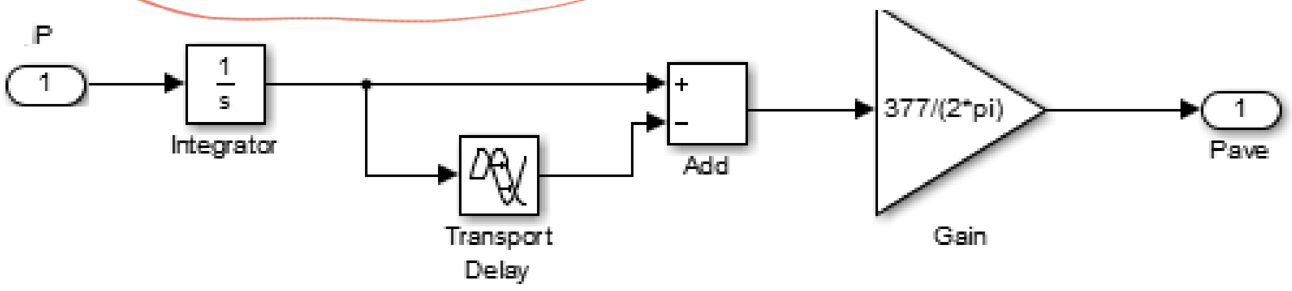
Instantaneous power is always this

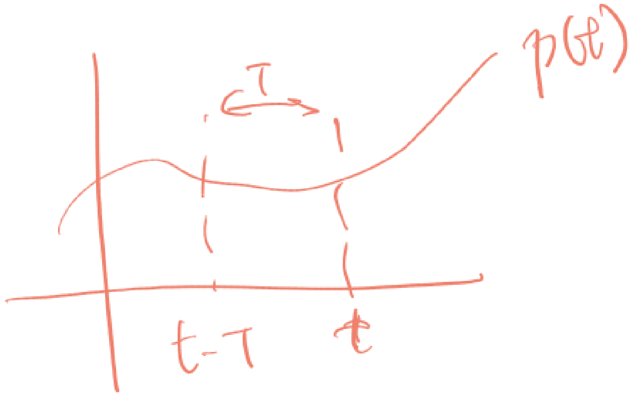
$$p(t) = vi = 2VI\cos(\omega t)\cos(\omega t + \varphi)$$
$$= \underbrace{VI\cos(\varphi)}_{\text{Constant part – average power over cycle}} + \underbrace{VI\cos(2\omega t + \varphi)}_{\text{Pulsating part - oscillating with double frequency}}$$

Note: Average power depends on V, I, and φ

Average power over cycle

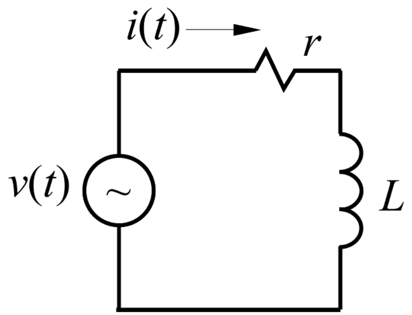
$$P = \frac{1}{T}\int_{t-T}^{t} p(t)\,d\tau$$



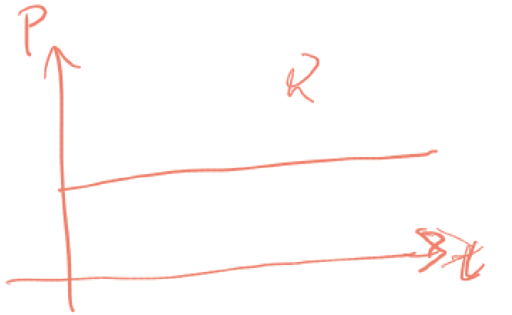


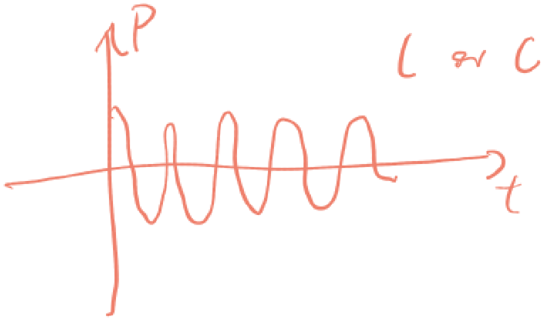
Power in AC Circuits

Given inductive load



$$v(t) = \sqrt{2} \cdot V \cos(\omega t) \qquad i(t) = \sqrt{2} \cdot I \cos(\omega t + \varphi)$$

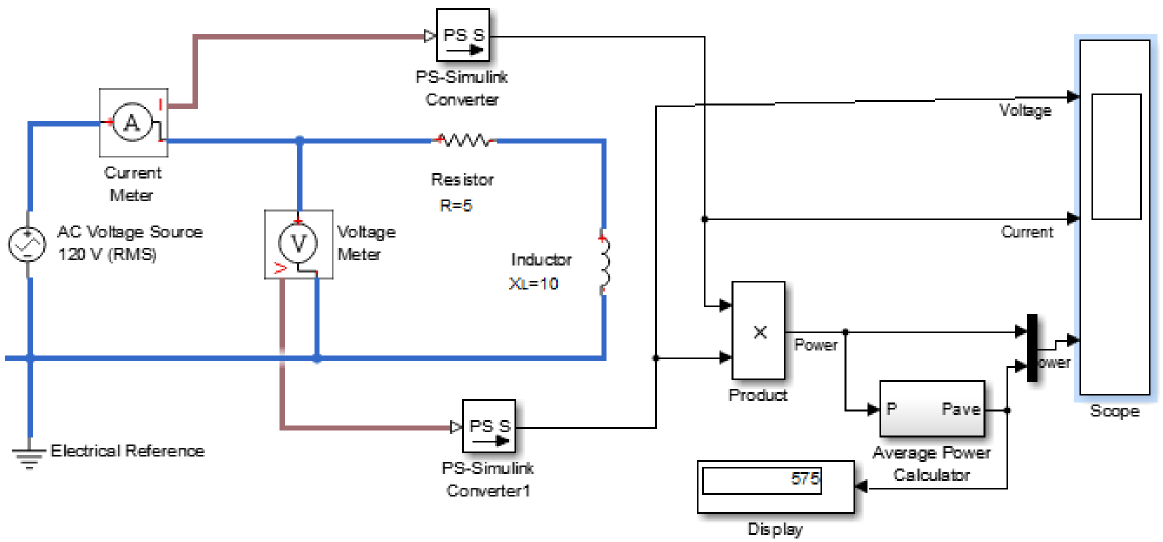


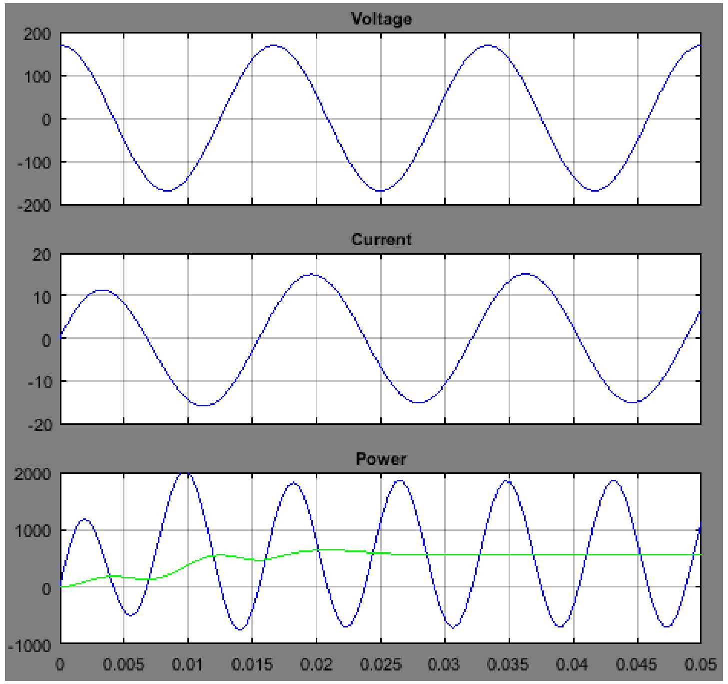


Instantaneous power is always this

$$p(t) = vi = 2VI\cos(\omega t)\cos(\omega t + \varphi)$$

$$= \underbrace{VI\cos(\varphi)}_{\text{Constant part – average power over cycle}} + \underbrace{VI\cos(2\omega t + \varphi)}_{\text{Pulsating part - oscillating with double frequency}}$$





Power in AC Circuits

trig identity

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$
$$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

Define general voltage and current expressions:

$$v(t) = \sqrt{2}V_{rms}\cos(\omega t + \theta)$$
$$i(t) = \sqrt{2}I_{rms}\cos(\omega t + \varphi)$$

Then instantaneous power is:

$$p(t) = V_{rms}I_{rms}[\cos(\theta-\varphi) + \cos(2\omega t + \theta + \varphi)]$$

$$= V_{rms}I_{rms}\cos(\theta-\varphi) + V_{rms}I_{rms}\cos(2\omega t + \theta + \varphi)$$

$$= V_{rms}I_{rms}\cos(\theta-\varphi) + V_{rms}I_{rms}\cos\big(2(\omega t + \theta) - (\theta - \varphi)\big)$$

$$= V_{rms}I_{rms}\cos(\theta-\varphi) + V_{rms}I_{rms}\cos(\theta-\varphi)\cos\big(2(\omega t + \theta)\big) + V_{rms}I_{rms}\sin(\theta-\varphi)\sin\big(2(\omega t + \varphi)\big)$$

$$= \boxed{V_{rms}I_{rms}\cos(\theta-\varphi)\{1 + \cos[2(\omega t + \varphi)]\}} + \boxed{V_{rms}I_{rms}\sin(\theta-\varphi)\sin[2(\omega t + \varphi)]}$$

Power in AC Circuits

The instantaneous power is:

$$p(t) = \underbrace{V_{rms}I_{rms}\cos(\theta-\varphi)\{1+\cos[2(\omega t+\varphi)]\}}_{\text{Non-zero average}} + \underbrace{V_{rms}I_{rms}\sin(\theta-\varphi)\sin[2(\omega t+\varphi)]}_{\text{zero average}}$$

Based on this expression, we define:

- Active/real power: $P = V_{rms}I_{rms}\boxed{\cos(\theta-\varphi)}$ [W]
- Power factor angle: $\theta - \varphi$
  - Leading (current leads voltage): $\theta - \varphi \leq 0$
  - Lagging (current lags voltage): $\theta - \varphi \geq 0$
- Power factor: $\mathrm{pf} = \cos(\theta - \varphi)$
  - Note that power factor is positive by convention
  - So if power factor angle is larger than 90^o, then reverse current direction
- Reactive power: $Q = V_{rms}I_{rms}\sin(\theta-\varphi)$ [var] (although [VAR] and [VAr] are also commonly used)
- Complex power: $S = P + jQ = V_{rms}I_{rms}\cos(\theta-\varphi) + jV_{rms}I_{rms}\sin(\theta-\varphi)$ [VA] not Watt

Power in AC Circuits

Based on this expression, we define:

- Active/real power: $P = V_{rms} I_{rms} \cos(\theta - \varphi)$ [W]
- Power factor angle: $\theta - \varphi$
  - Leading (current leads voltage): $\theta - \varphi < 0$
  - Lagging (current lags voltage): $\theta - \varphi > 0$
- Power factor: $\mathrm{pf} = \cos(\theta - \varphi)$
  - Note that power factor is positive by convention
  - So if power factor angle is larger than 90^o, then reverse current direction
- Reactive power: $Q = V_{rms} I_{rms} \sin(\theta - \varphi)$ [var] (although [VAR] and [VAr] are also commonly used)
- Complex power: $S = P + jQ = V_{rms} I_{rms} \cos(\theta - \varphi) + jV_{rms} I_{rms} \sin(\theta - \varphi)$ [VA]
- Apparent power: $|S| = \sqrt{P^2 + Q^2} = V_{rms} I_{rms}$ [VA]

magnitude of S

Power in AC Circuits

In phasor domain:

- $V = V_{rms}\angle\theta,\ I = I_{rms}\angle\varphi$
- We can get the same complex power by defining:

$$S = VI^* = VI\angle(\theta - \varphi)$$

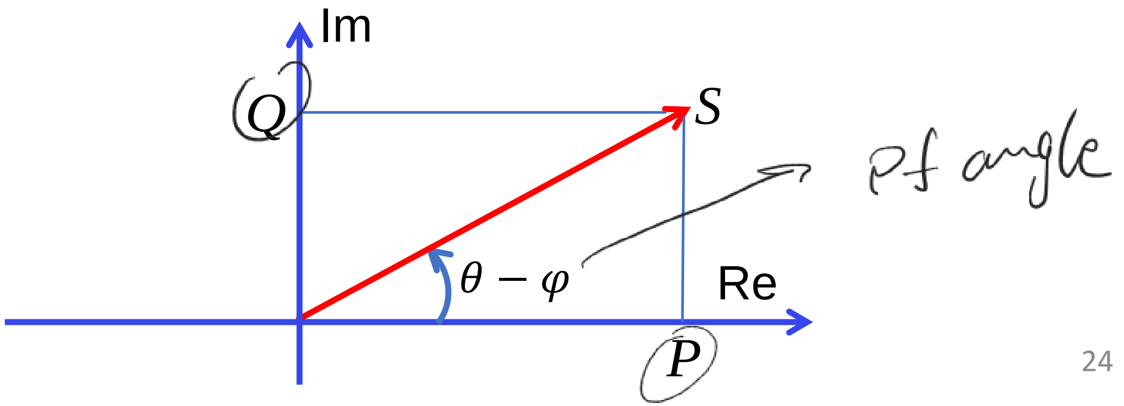
(note that the subscript *rms* is now dropped hereafter)

$$P = Re(S) = |S|\cos(\theta - \varphi)$$

(this means that only the current component in phase with voltage produces real power)

$$Q = Im(S) = |S|\sin(\theta - \varphi)$$

- We have the famous power triangle:



Power in resistor, inductor, capacitor

Component	Power	Comments
Resistor	$p(t) = VI\{1 + \cos[2(\omega t + \delta)]\}$ $P = VI$	Absorbs positive real power, zero reactive power
Inductor	$p(t) = VI \sin[2(\omega t + \delta)]$ $Q = VI$	Absorbs zero real power, positive reactive power, lagging power factor
Capacitor	$p(t) = -VI \sin[2(\omega t + \delta)]$ $Q = -VI$	Absorbs zero real power, negative reactive power (delivers positive reactive power), leading power factor

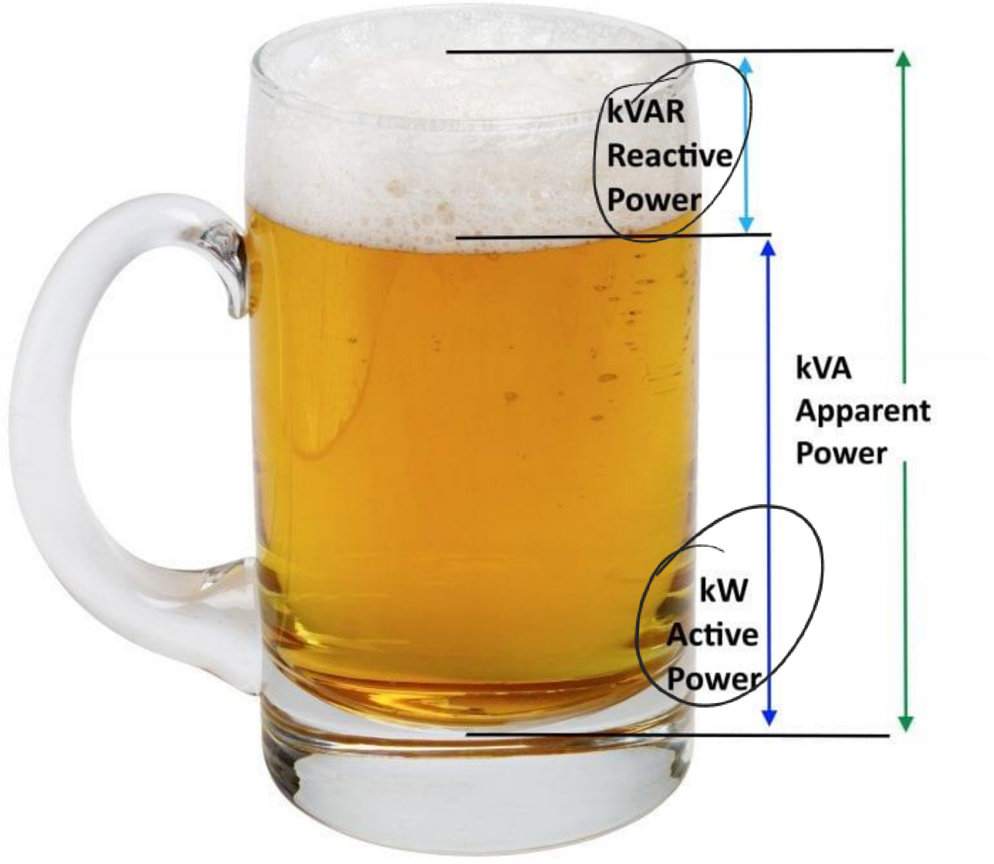
What is reactive power?

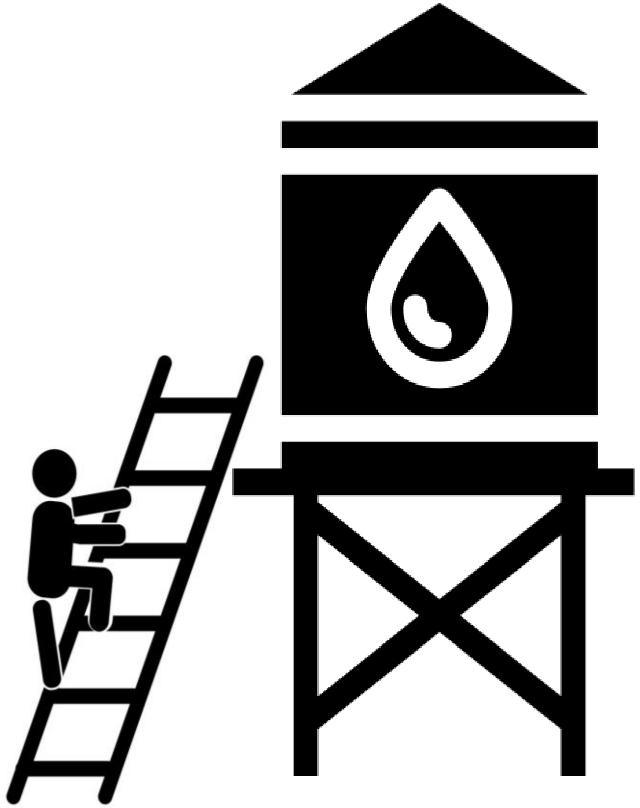
- We know that reactive power does not contribute to useful work – why it is called power?

- In case of purely resistive loads, electrical energy is directly converted into useful work (e.g., light and heat), no intermediate electric or magnetic field is required in between
  - Hence no power is used to create these fields – total power is entirely active power

- In case of inductive or capacitive loads, electrical energy can't directly be converted into useful work (e.g., rotation of a motor)
  - This is because these loads require electric or magnetic field
  - The work going into creating the field is called reactive power
  - From efficiency point of view, reactive power is seen as power loss because it contributes to the creation of the field and does not contribute to driving the load

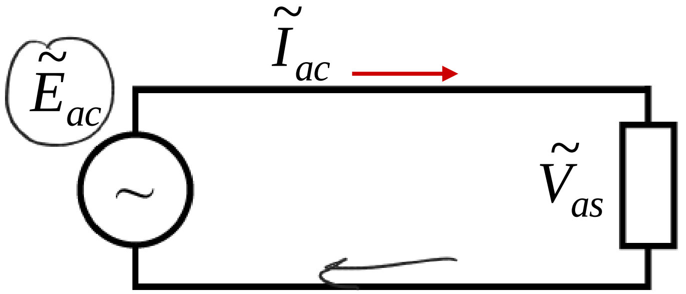
What is reactive power?

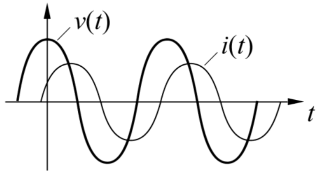




Single vs. Three-Phase Systems

$$e_{ac}(t) = \sqrt{2}\cdot V_{rms}\cos(\omega_e t)$$





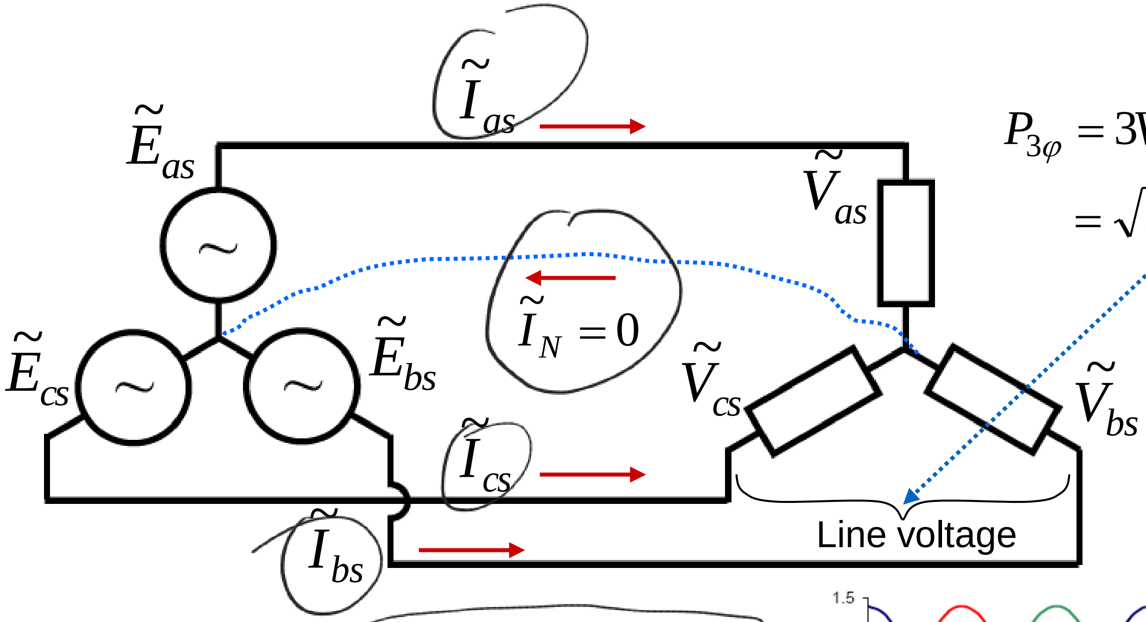
$$P_{load} = V_{ac} I_{ac}\cos(\varphi)$$

Need to have 2 conductors!

1 unit of power / 2 cables/wires

$$V_{LL} = \sqrt{3}V_{LN}$$

$$e_{as}(t) = \sqrt{2}\cdot V_{rms}\cos(\omega_e t)$$
$$e_{bs}(t) = \sqrt{2}\cdot V_{rms}\cos(\omega_e t - 120^\circ)$$
$$e_{cs}(t) = \sqrt{2}\cdot V_{rms}\cos(\omega_e t + 120^\circ)$$



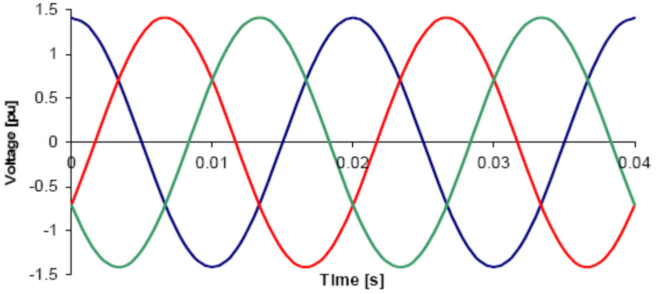
$$P_{3\varphi} = 3V_{as}I_{as}\cos(\varphi)$$
$$= \sqrt{3}V_{line}I_{line}\cos(\varphi)$$

Need to have 3 conductors, but can transmit 3-times the power!

3 units of power / 3 cables/wires

For balanced three phase system

$$e_{as} + e_{bs} + e_{cs} = 0$$
$$v_{as} + v_{bs} + v_{cs} = 0$$
$$i_{as} + i_{bs} + i_{cs} = 0$$



Balanced Three-Phase (ϕ) Systems

- A balanced three-phase (ϕ) system has
  - Three voltage sources with equal magnitude, but with an angle shift of 120°
  - Equal loads on each phase
  - Equal impedance on the lines connecting the generators to the loads

- Can transmit more power for same amount of wire (twice as much as single phase)

- Bulk power systems are almost exclusively 3ϕ

- Single-phase is used primarily only in low voltage, low power settings, such as residential and some commercial

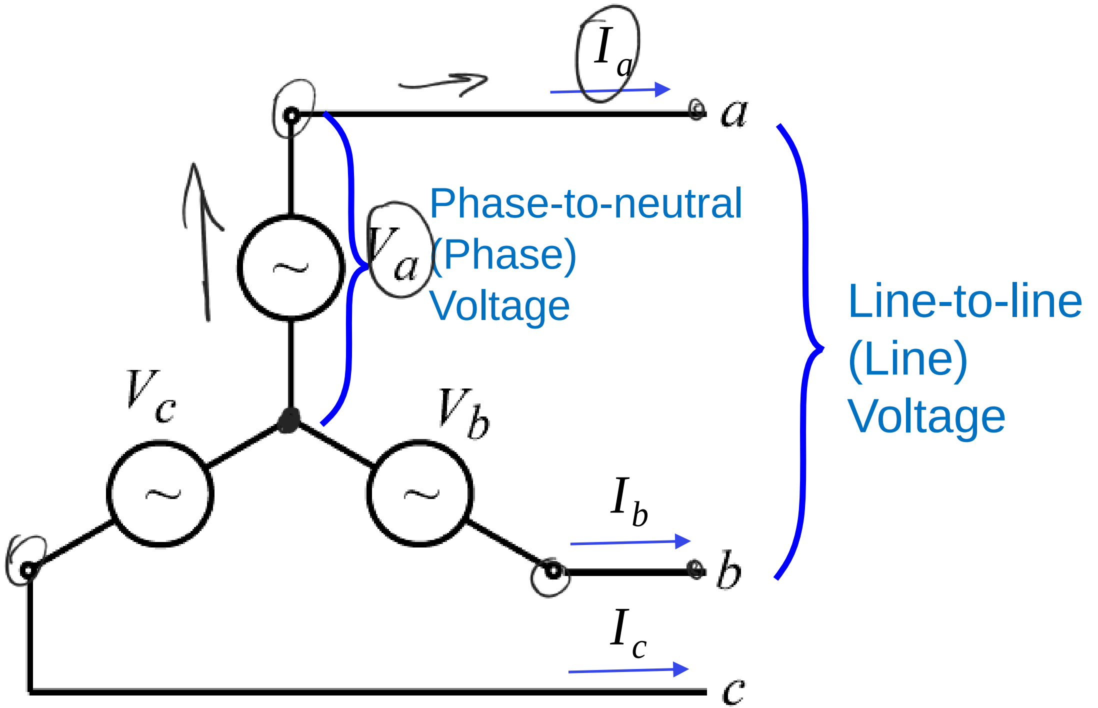
Balanced Three-Phase (ϕ) Systems

- There are two ways to connect 3ϕ systems
  - Wye (Y)
  - Delta (Δ)

- Terminology:
  - Define voltage/current across/through device to be **phase** voltage/current
  - Define voltage/current across/through lines to be **line** voltage/current

line to line

Wye (Y) - Connection



$$|V_{line}| = \sqrt{3}|V_{phase}|$$

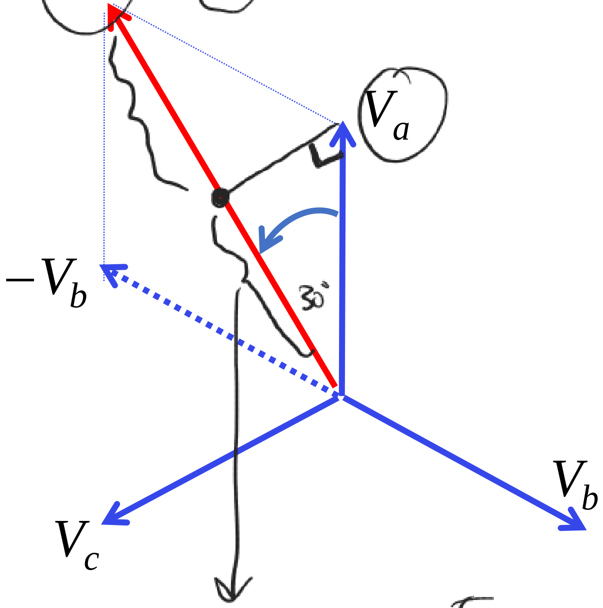
Line-to-line (Line) voltage is larger

$$\boxed{|I_{line}| = |I_{phase}|}$$

Line Voltages

$$V_{ab} = V_a - V_b$$

$$V_{ab} = \sqrt{3}V_a\angle+30^\circ$$



cos 30° = √3/2

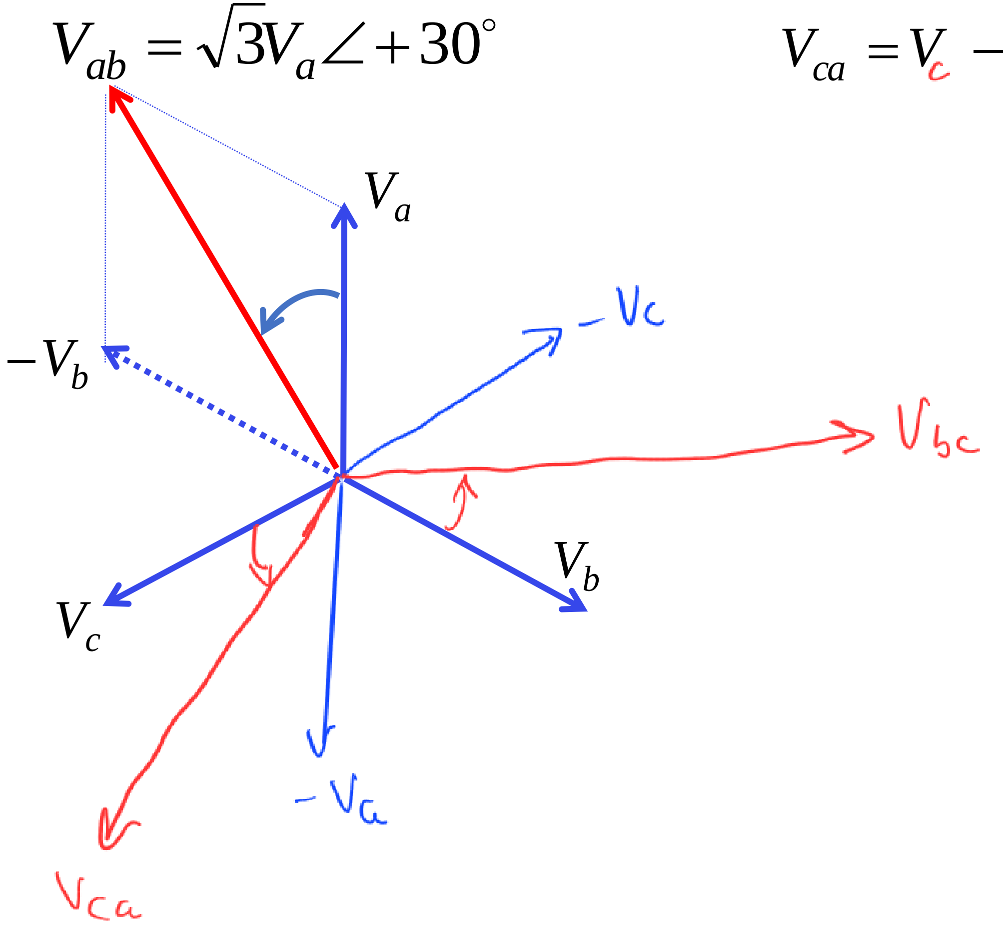
Wye (Y) - Connection

Line Voltages

$$V_{ab} = V_a - V_b$$

$$V_{bc} = V_b - V_c$$

$$V_{ca} = V_c - V_a$$

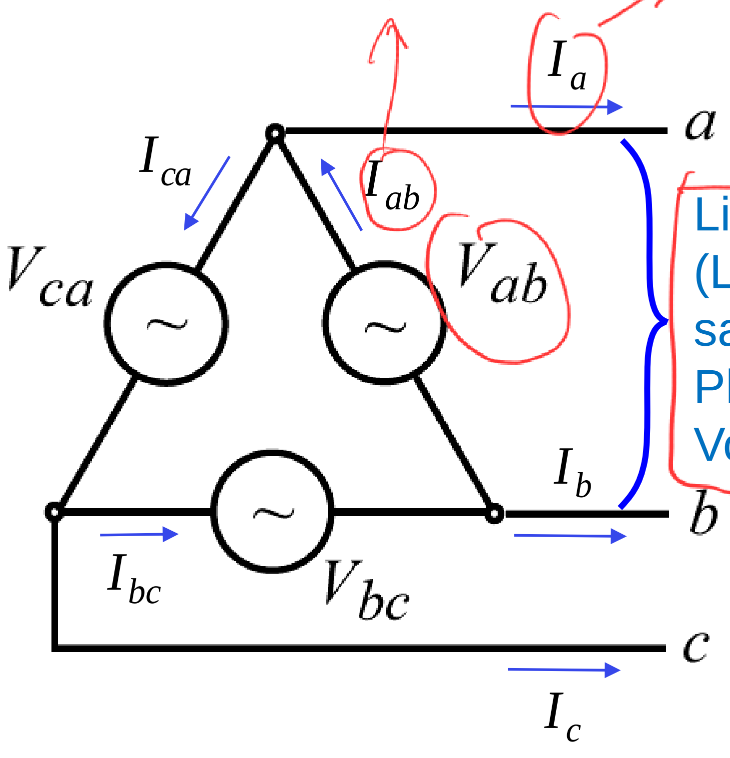


Delta (Δ) - Connection

Better tolerance for larger line currents

Phase current

line current



Line-to-line (Line) is the same as Phase Voltage

$$|I_{line}| = \sqrt{3}|I_{phase}|$$

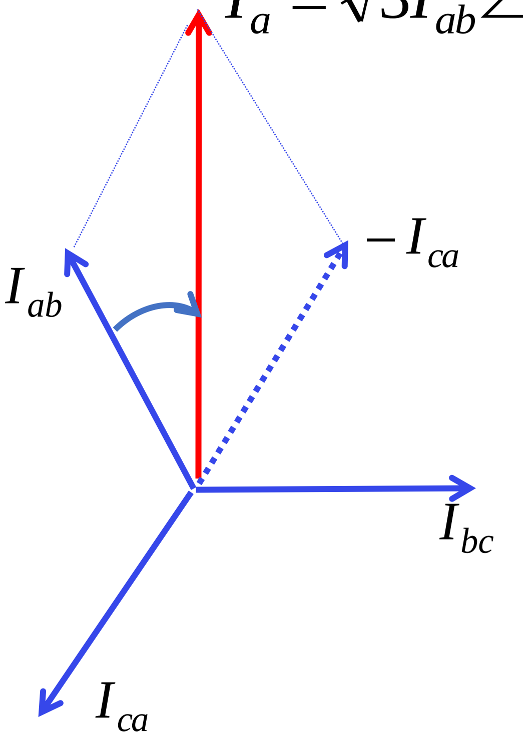
Line current is larger

$$|V_{line}| = |V_{phase}|$$

Line Currents

$$I_a = I_{ab} - I_{ca}$$

$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$



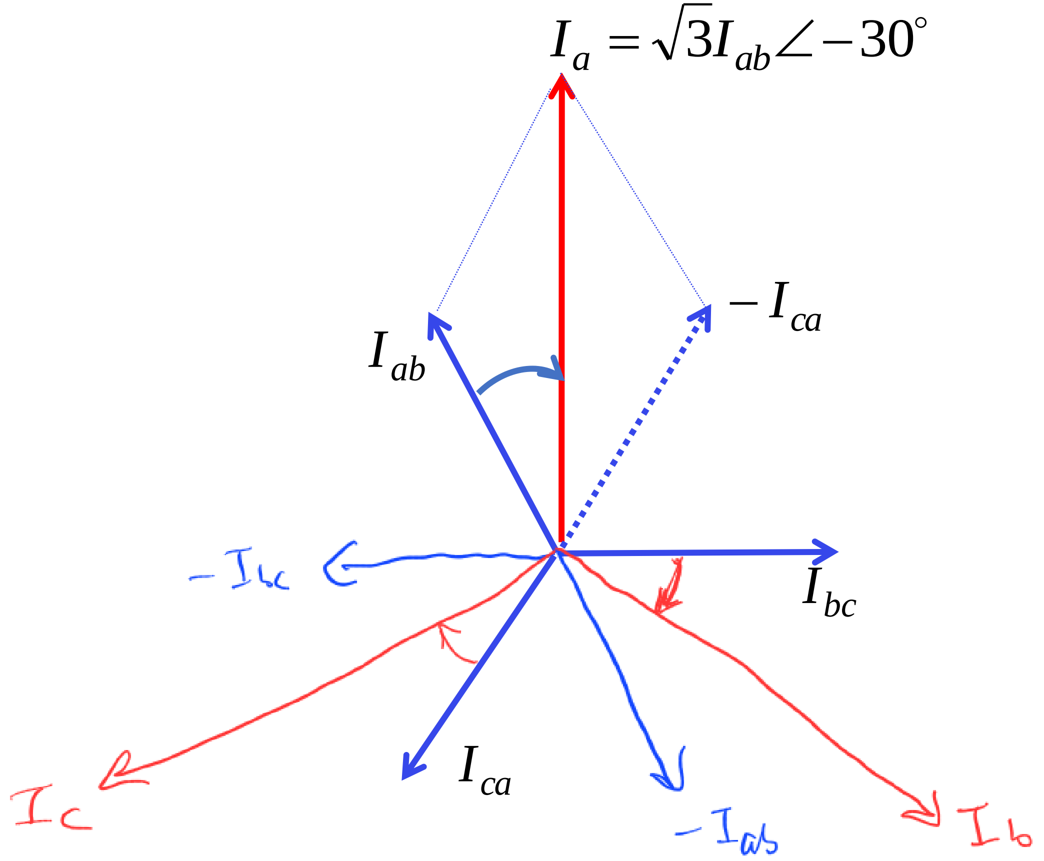
Delta (Δ) - Connection

Line Currents

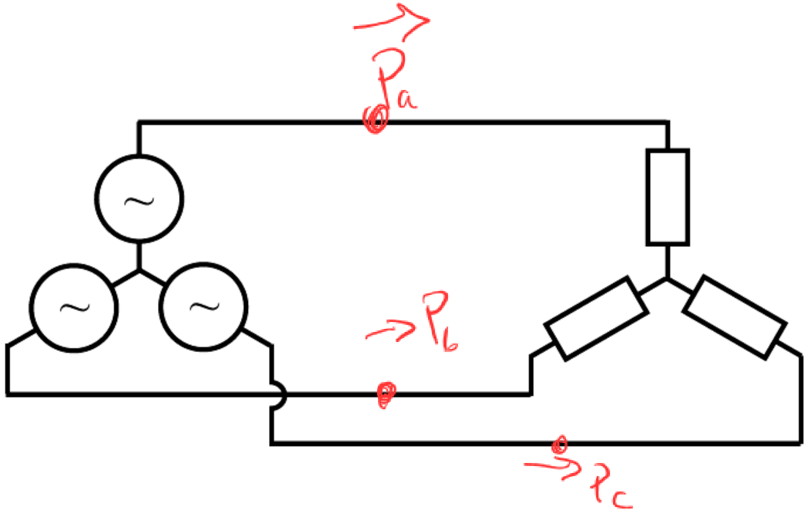
$$I_a = I_{ab} - I_{ca}$$

$$I_b = I_{bc} - I_{ab}$$

$$I_c = I_{ca} - I_{bc}$$



Power in Three-Phase Systems



Instantaneous Power can always be calculated as

$$P_{3\phi}(t) = P_a + P_b + P_c$$
$$= i_a v_a + i_b v_b + i_c v_c$$

For balanced system we have

$$v_a(t) = \sqrt{2}V_{ph}\cos(\omega t)$$
$$v_b(t) = \sqrt{2}V_{ph}\cos(\omega t - 120^\circ)$$
$$v_c(t) = \sqrt{2}V_{ph}\cos(\omega t + 120^\circ)$$

$$i_a(t) = \sqrt{2}I_{ph}\cos(\omega t - \varphi)$$
$$i_b(t) = \sqrt{2}I_{ph}\cos(\omega t - 120^\circ - \varphi)$$
$$i_c(t) = \sqrt{2}I_{ph}\cos(\omega t + 120^\circ - \varphi)$$

Power in Three-Phase Systems

- In terms of phase quantities

$$P_{3\phi}(t) = 3P_{ph} = 3V_{ph}I_{ph}\cos(\varphi_{ph})$$

$$Q_{3\phi}(t) = 3Q_{ph} = 3V_{ph}I_{ph}\sin(\varphi_{ph})$$

$\cancel{V_{ph} I_L}$

$\cancel{V_L I_{ph}}$

Y - connection

$$I_{ph} = I_L,\ V_{ph} = V_L/\sqrt{3}$$

Δ - connection

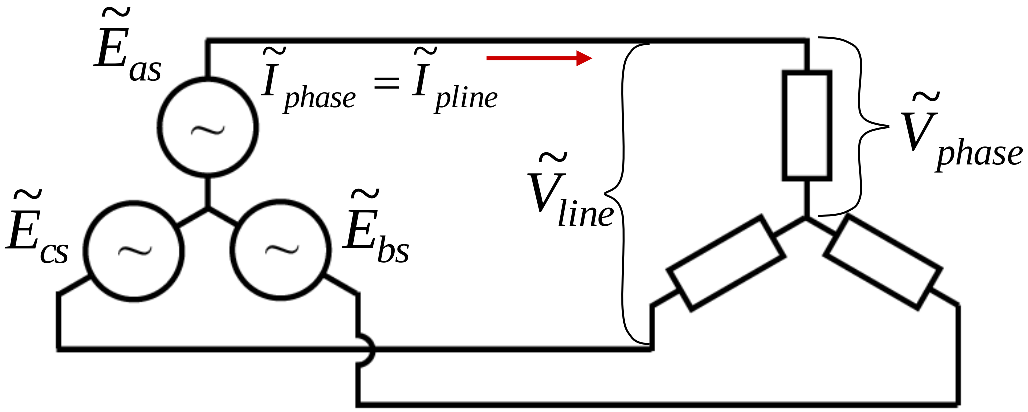
$$I_{ph} = I_L/\sqrt{3},\ V_{ph} = V_L$$

- In terms of line-to-line quantities

$$P_{3\phi} = \sqrt{3}V_L I_L\cos(\varphi_{ph})$$

$$Q_{3\phi} = \sqrt{3}V_L I_L\sin(\varphi_{ph})$$

Power in Three-Phase Systems



For balanced three phase system

$$V_{line} = \sqrt{3} \cdot V_{phase}$$

$$S_{3\varphi} = 3V_{as}I_{as} = \sqrt{3}V_{line}I_{line}$$

$$P_{3\varphi} = 3V_{as}I_{as}\cos(\varphi)$$

$$= \sqrt{3}V_{line}I_{line}\cos(\varphi)$$

$$Q_{3\varphi} = 3V_{as}I_{as}\sin(\varphi)$$

$$= \sqrt{3}V_{line}I_{line}\sin(\varphi)$$

$$|S| = 3V_{ph}I_{ph} = \sqrt{3}V_L I_L$$

Looking ahead…

- **Module 1**, part 2