ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 3

January 13, 2026

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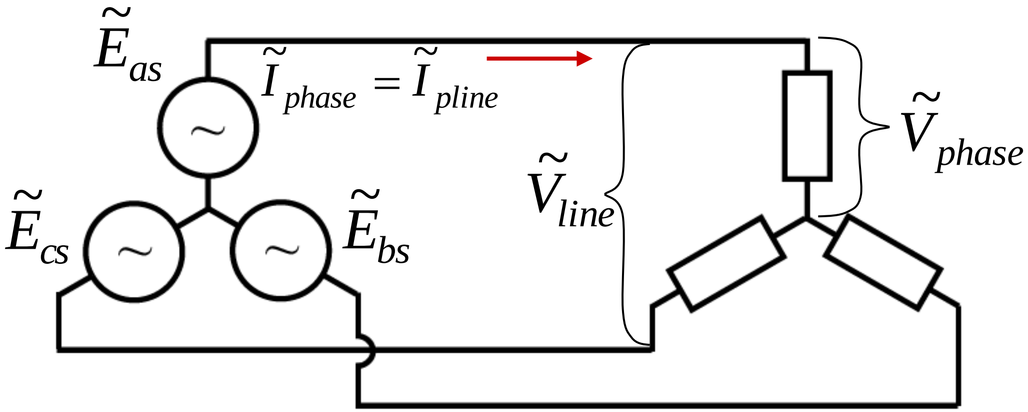
Course logistics

- TA Office Hour: Wednesdays 4 PM – 5 PM in KAIS 3047
- Tutorials start this Friday: We only use 12 PM – 1 PM portion
- Assignment 1 is posted, due one week later on Canvas
  - Assignments are for you to check your understanding in this course

Power in AC Circuits

- Active/real power: $P = V_{rms} I_{rms} \cos(\theta - \varphi)$ [W]
- Power factor angle: $\theta - \varphi$
- Power factor: $\mathrm{pf} = \cos(\theta - \varphi)$
- Reactive power: $Q = V_{rms} I_{rms} \sin(\theta - \varphi)$ [var]
- Complex power: $S = P + jQ = V_{rms} I_{rms} \cos(\theta - \varphi) + jV_{rms} I_{rms} \sin(\theta - \varphi)$ [VA] $= VI^*$
- Apparent power: $|S| = \sqrt{P^2 + Q^2} = V_{rms} I_{rms}$ [VA]

Power in Three-Phase Systems



For balanced three phase system

$$V_{line} = \sqrt{3} \cdot V_{phase}$$

$$S_{3\varphi} = 3V_{as}I_{as} = \sqrt{3}V_{line}I_{line}$$

$$P_{3\varphi} = 3V_{as}I_{as}\cos(\varphi)$$

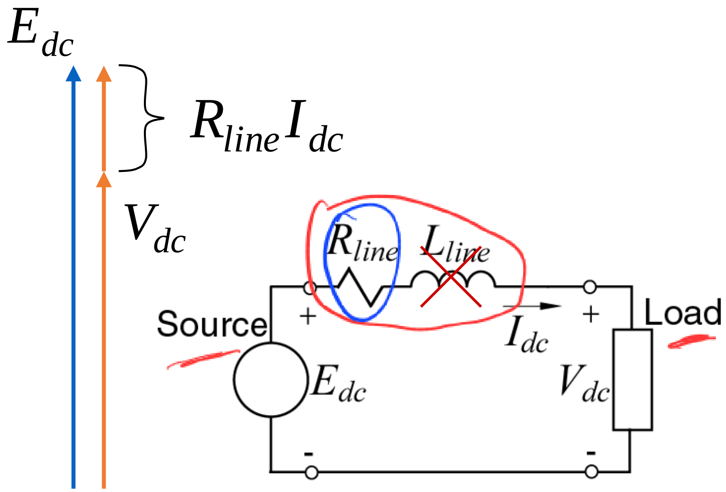
$$= \sqrt{3}V_{line}I_{line}\cos(\varphi)$$

$$Q_{3\varphi} = 3V_{as}I_{as}\sin(\varphi)$$

$$= \sqrt{3}V_{line}I_{line}\sin(\varphi)$$

How to transmit power

- **For DC System**
  - In steady state, the line is just a resistor, R



$$P_{source} = E_{dc}I_{dc}$$

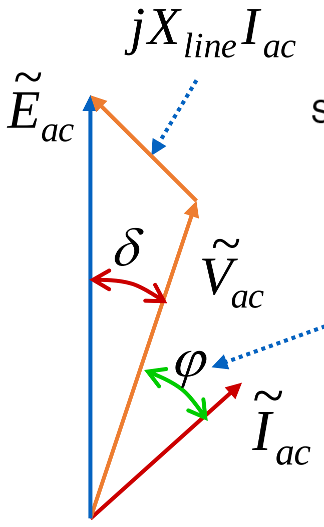
$$P_{load} = V_{dc}I_{dc}$$

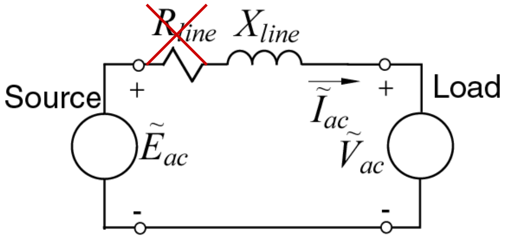
$$\boxed{\begin{aligned} P_{loss} &= (E_{dc} - V_{dc})I_{dc} \\ &= R_{line}I_{dc}^2 \end{aligned}}$$

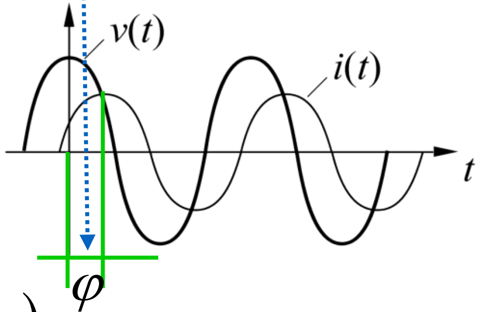
- **For AC System**
  - In steady state or transient, lines are mostly inductive

Simplification

$$X_{line} >> R_{line}$$







Apparent power $S_{load} = V_{ac}I_{ac}$

Real power $P_{load} = V_{ac}I_{ac}\cos(\varphi)$

Reactive power $Q_{load} = V_{ac}I_{ac}\sin(\varphi)$

Line loss $\boxed{P_{loss} = R_{line}I_{ac}^2}$

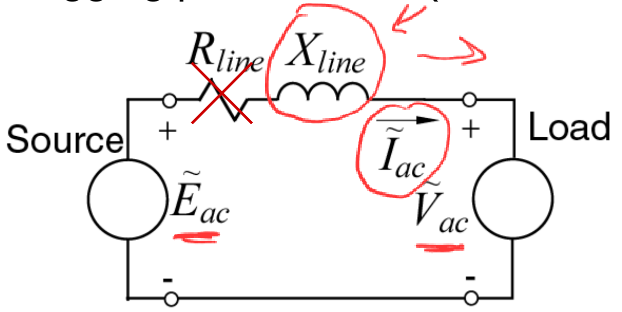
Note: Line losses are proportional to current squared

How to transmit power

$P = V_{ac} I_{ac} \cos(\varphi)$

$Q = V_{ac} I_{ac} \sin(\varphi)$

- **For AC System**
  - Lines are mostly inductive $X_{line} >> R_{line}$
  - Assume lagging power factor (inductive load)

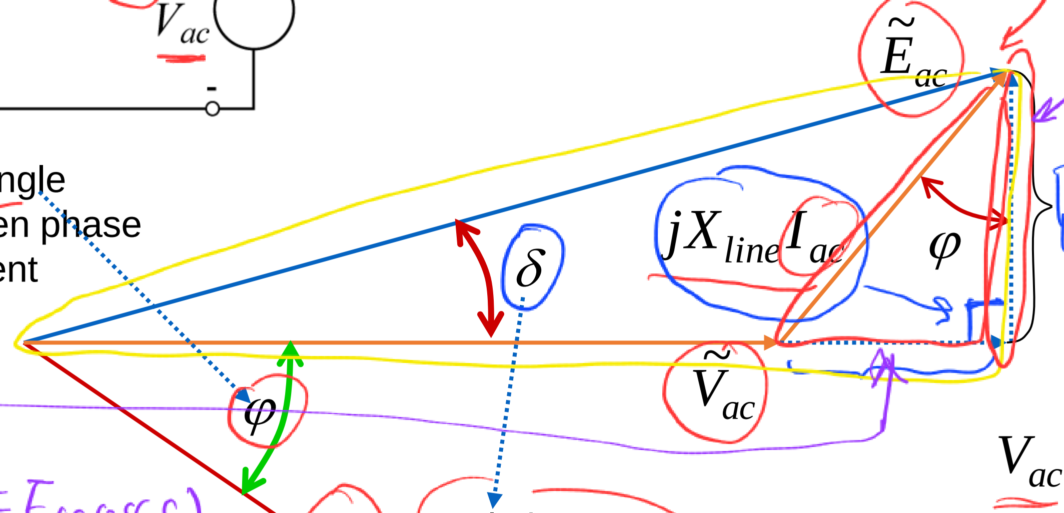


Voltage Equation & Phasor Diagram

$$E_{ac} \approx V_{ac} + jX_{line} I_{ac}$$ → KVL

Assume inductive load ←

Power factor angle = angle between phase voltage & current



Angle between source voltage & load voltage

$$E_{ac}\sin(\delta) = X_{line} I_{ac}\cos(\phi)$$

How do we get an expression of real power from this?

multiply both sides by V_{ac}

$$V_{ac}E_{ac}\sin(\delta) = V_{ac}X_{line}I_{ac}\cos(\varphi)$$

$$\frac{V_{ac}E_{ac}}{X_{line}}\sin(\delta) = V_{ac}I_{ac}\cos(\varphi) = P_{transfer}$$

$X_{line} I_{ac} \sin(\varphi) + V_{ac} = E_{ac}\cos(\delta)$

$V_{ac} I_{ac} \sin(\varphi) X_{line} + V_{ac}^2 = V_{ac}E_{ac}\cos(\delta)$

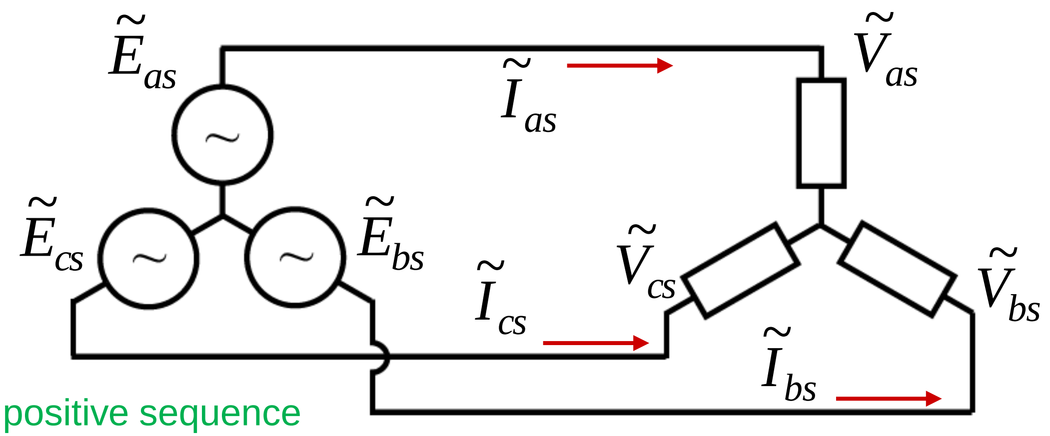
$Q_{transfer}$

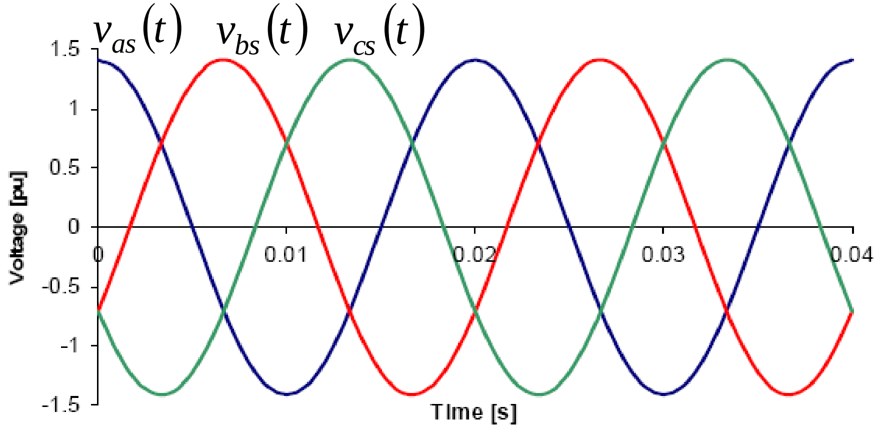
$$Q_{transfer} = \frac{V_{ac}E_{ac}\cos(\delta) - V_{ac}^2}{X_{line}}$$

We can reverse the flow of real power from "Load" to "Source" by making the Vac lead Eac

$$P_{transfer} = \frac{V_{ac}E_{ac}}{X_{line}}\sin(\delta)$$

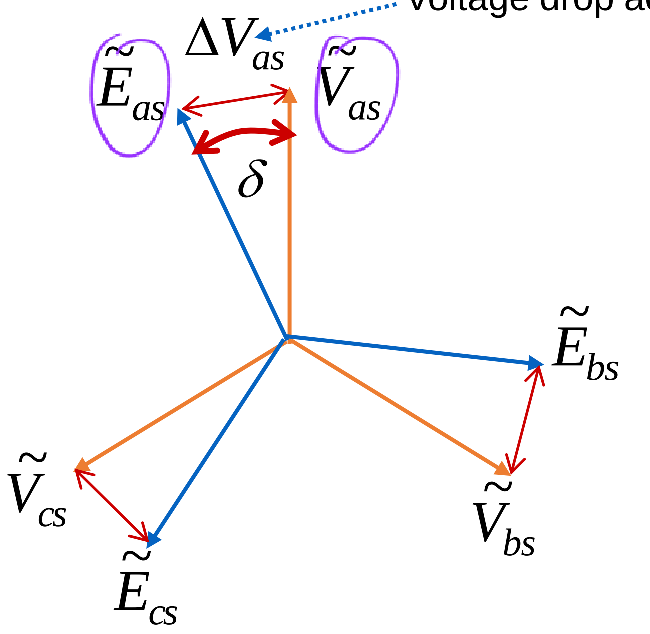
How to transmit power (**3-phase**)



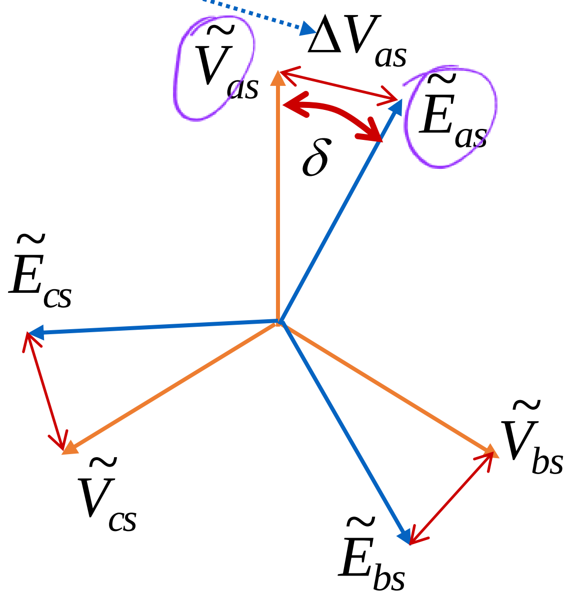


Assume positive sequence of phases, i.e., A, B, C…

Voltage drop across the line



Power transmitted from E to V



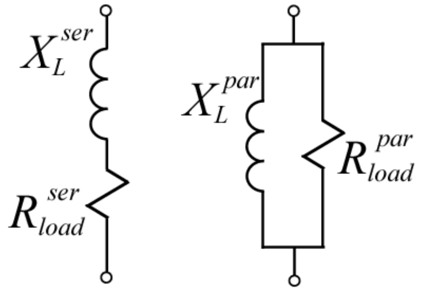
Power transmitted from V to E

$$P_{transfer} \approx 3\boxed{\frac{V_{as}E_{as}}{X_{line}}\sin(\delta)}$$

We can reverse the flow of real power from "Load" to "Source" by making the Vas lead Eas

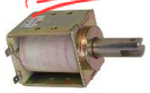
Types of loads in AC power system

Inductive Loads
(lagging power factor)
(current lags the voltage)

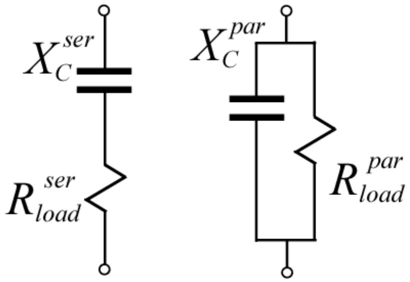


Per-phase, series or parallel

Most loads are of this type!



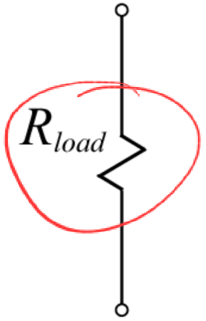

Capacitive Loads
(leading power factor)
(current leads the voltage)



Per-phase, series or parallel

Some synchronous motor and power-electronic loads could be of this type, but very few!


Resistive Loads
(unity power factor)



Heating loads are of this type!




Power converters can exhibit almost any behavior

Module 1, Part 2: Basic Magnetic Circuits

Objectives & important concepts

- Fundamentals of Electromagnetics, Maxwell’s Equations
- Sign & direction conventions
- Basic magnetic circuits, concepts, analogies, calculations
- Flux, flux linkage, inductance
- Magnetic materials, saturation, hysteresis loop
- Coil under ac excitation, type of core losses

- Read Chapter 1 of the textbook

Review of Basic Quantities and Units

E – electric field intensity $\left[\frac{V}{m}\right]$ $\rightarrow$ $\frac{\text{Force}}{\text{charge}}$ $\left[\frac{N}{C}\right]$

B – magnetic flux density $\left[Tesla = \frac{Weber}{meter^2}\right]$ $\left[T = \frac{Wb}{m^2}\right]$

H – magnetic field intensity $\left[\frac{A}{m}\right]$

Φ – magnetic flux $\left[Wb = T \cdot m^2\right]$

μ = constant
⇓
magnetic linear material
⇓
does not really exist

B-H Relation

- Current produces the H field (see Ampere's law)
- H is related to B

$$B = \mu H = \mu_0 \mu_r H$$

μ – permeability (characteristic of the medium) $\left[\frac{T \cdot m}{A} = \frac{Henry}{meter} = \frac{H}{m}\right]$

μ_0 – permeability of vacuum $= 4 \cdot \pi \cdot 10^{-7} [H/m]$

μ_r – relative permeability of material

magnetic materials $\mu_r = 100 \cdots 100{,}000$

Fundamentals

Summarized in Maxwell's Equations (1870s)

1) Gauss's Law for Electric Field

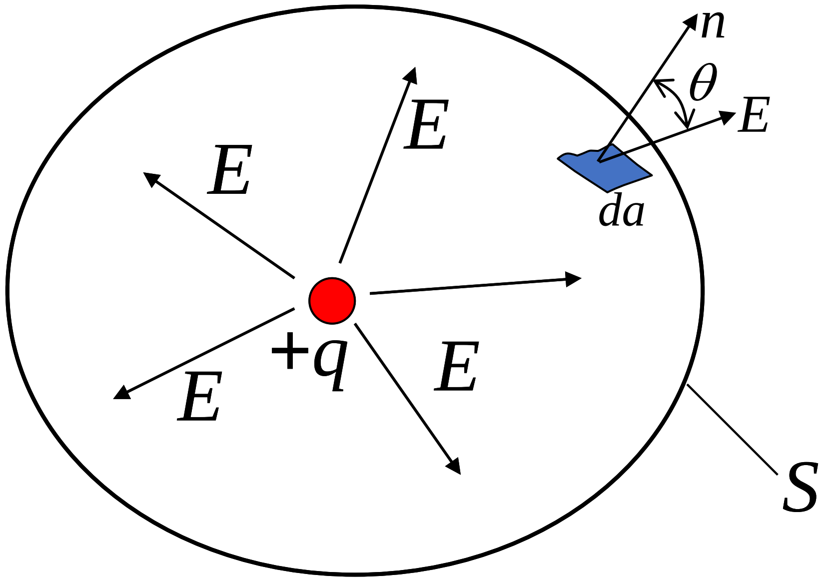
$$\oint_S \mathbf{E}\cdot d\mathbf{a} = \int E\cos\theta\, da = \Phi_e = \frac{q}{\varepsilon_0}$$

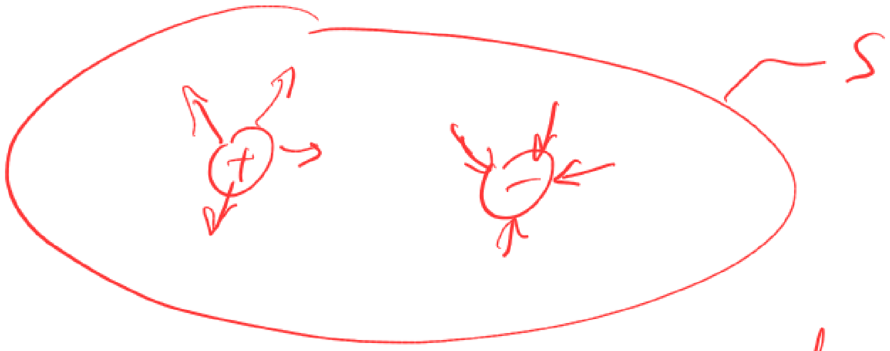
q → charge

ε_0 → permittivity of vacuum

Electric flux out of any closed surface is proportional to the total charge enclosed

closed surface → no edge





Electric field can be produced by point charges

Fundamentals

Summarized in Maxwell's Equations (1870s)

2) Gauss's Law for Magnetic Field

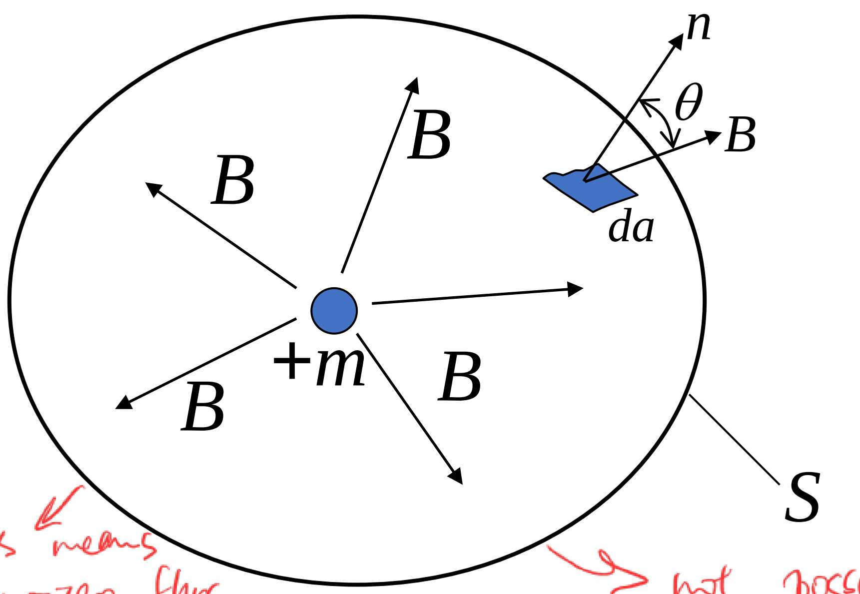
$$\oint_S \mathbf{B}\cdot d\mathbf{a} = \oint_S B\cos(\theta)da = \Phi_m = 0$$

Magnetic flux out of any closed surface is zero

There are no magnetic charges

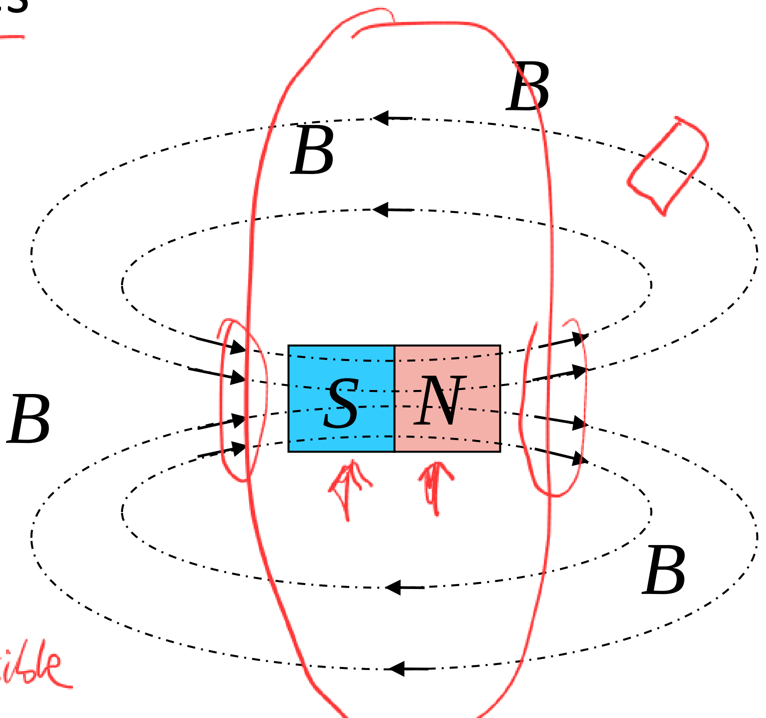
So magnetic field always forms closed-loop: no beginning or end.

Also, the magnetic field do not cross each



this means non-zero flux

not possible



Fundamentals

Summarized in Maxwell's Equations (1870s)

3) Faraday's Law

collect by this line around magnetic flux

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{a} = -\frac{d\Phi_m}{dt} = e = emf\,[Volts]$$

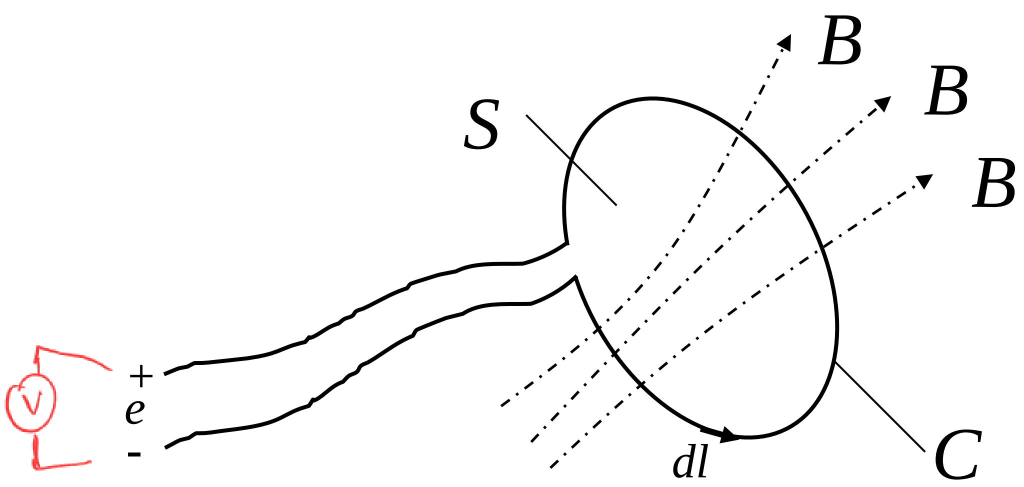
$$e = -\frac{d\Phi_m}{dt}$$

Electro**M**otive **F**orce (emf)

The line integral of the electric field around a closed loop/contour C is equal to the negative of the rate of change of the magnetic flux through that loop/contour

There is still emf if you don't have a wire

↳ If you have wire, you can have current flow



How can you generate voltage?

↳ By changing Φ_m

Fundamentals

Summarized in Maxwell's Equations (1870s)

4) Ampere's Law (for static electric field)

Net current enclosed

$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 \int_S J \cdot da = \mu_0 I_{net} + \cancel{\mu_0 \varepsilon_0 \frac{d\Phi_e}{dt}}$$

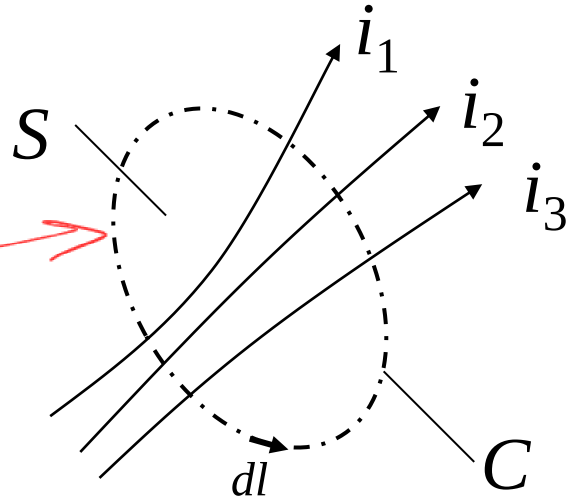
this term denotes the contribution of Φ_e to B

↳ In power, we work around DC – 60Hz or 100kHz for converters

↳ $\frac{d\Phi_e}{dt}$ is small so it is neglected

The line integral of the magnetic field B around a closed loop *C* is proportional to the net electric current flowing through that loop/contour *C*

closed loop to collect B



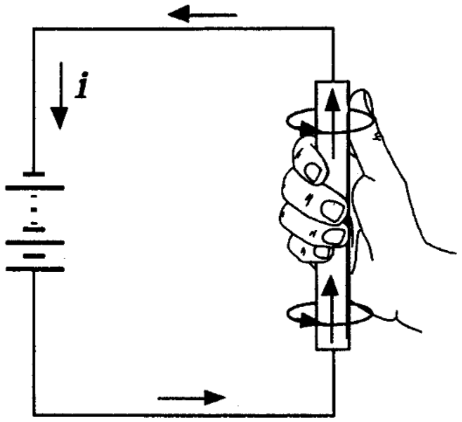
$$\oint_C \mathbf{H}\cdot d\mathbf{l} = \int_S J \cdot da = \sum i_i = I_{net}$$

How can you generate magnetic field?

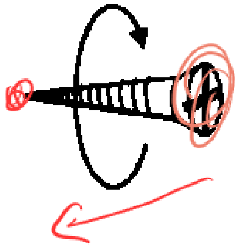
↳ Use current

Fundamentals and Conventions

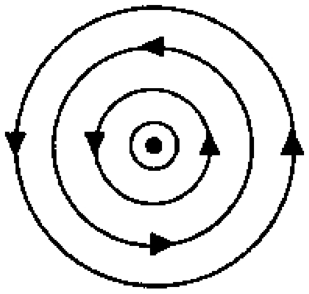
Right hand rule



Right-screw rule
Dot and cross notations



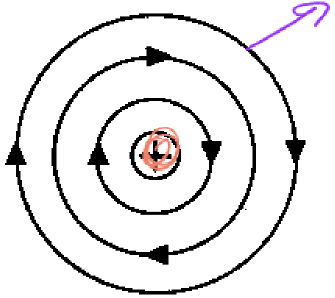
Counter-clockwise (CCW)



Out of the page

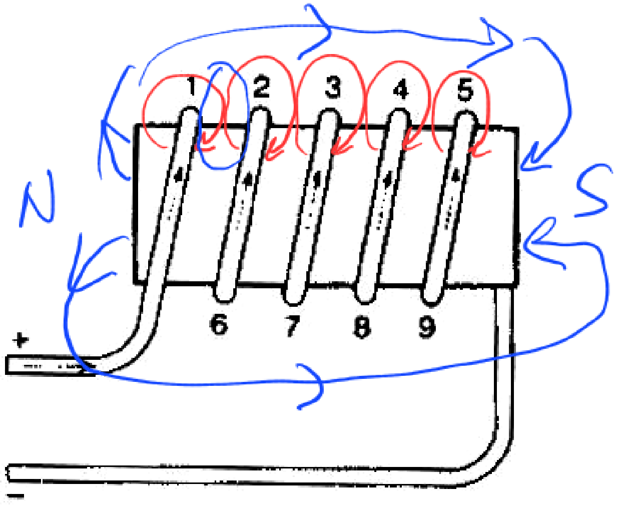
Clockwise (CW)

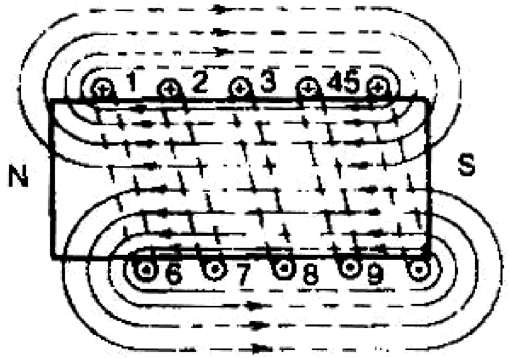
field lines



Into the page

Magnetic field produced by coil (solenoid)





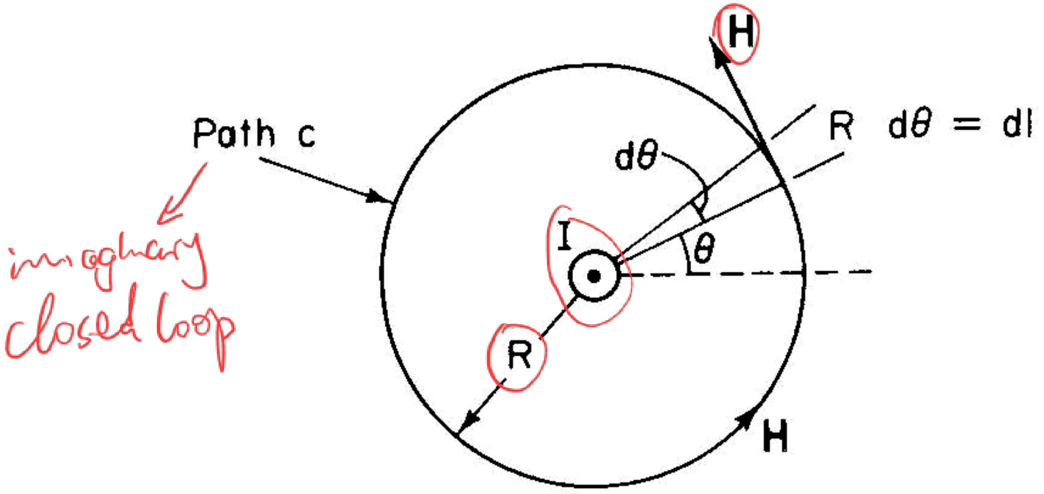
field goes out

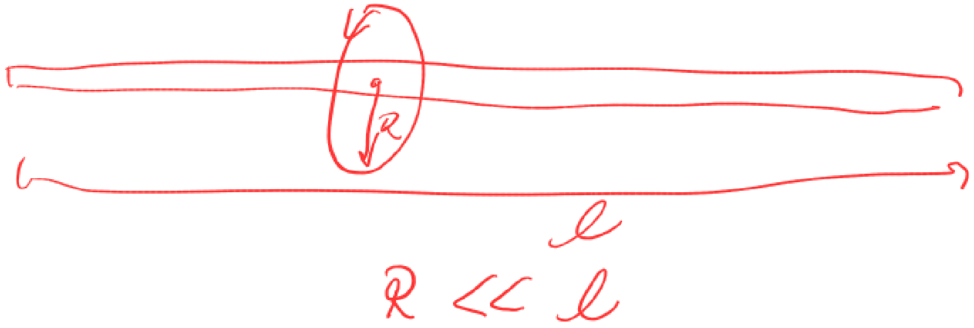
field enters

Flux Lines:

- Form a closed loop/path
- Lines do not cut across or merge
- Go from **North** to **South** magnetic poles

Magnetic Field of an Infinite Conductor





Apply Ampere's Law

$$\oint_C \mathbf{H} \cdot \mathbf{dl} = I_{enclosed}$$

Incremental length $\mathbf{dl} = R d\theta$

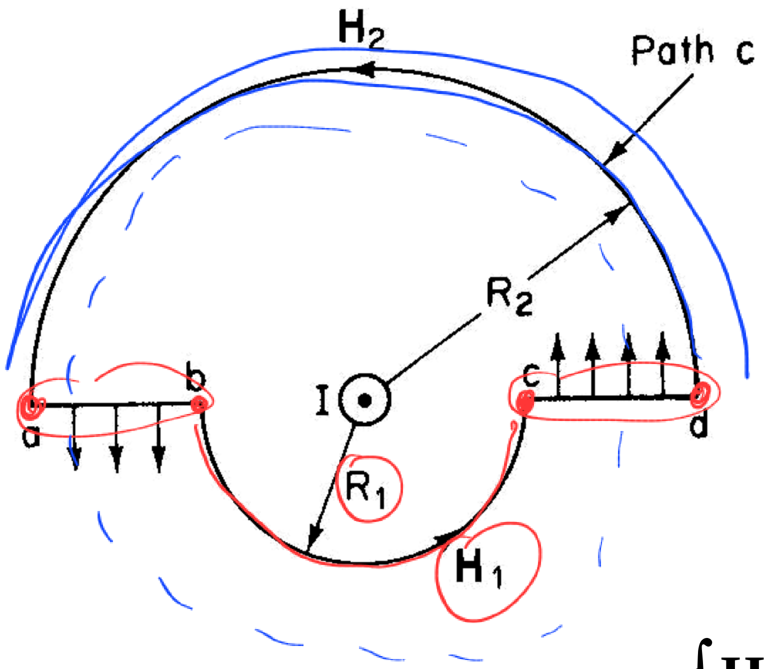
$$H \cdot 2\pi R = I$$

$$\boxed{H = \frac{I}{2\pi R}}$$

H and **dl** have the same direction

$$B = \mu H = \frac{\mu I}{2\pi R}$$

Magnetic Field of an Infinite Conductor



Apply Ampere's Law

$$\oint_C \mathbf{H}\cdot\mathbf{dl} = I_{enclosed}$$

$$\oint_C \mathbf{H}\cdot\mathbf{dl} = \underbrace{\int_a^b H\cdot dl}_{\emptyset} + \int_b^c H_1\cdot dl + \underbrace{\int_c^d H\cdot dl}_{\emptyset} + \int_d^a H_2\cdot dl = \frac{I}{2}+\frac{I}{2} = I$$

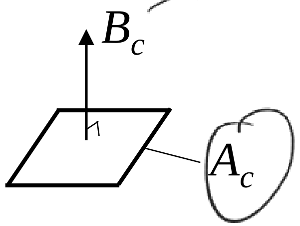
$$\int_b^c H_1\cdot dl = \int_{-\pi}^{0}\frac{I}{2\pi R_1}R_1 d\theta = \frac{I}{2}$$

Formula from previous slide

$$\int_d^a H_2\cdot dl = \int_0^{\pi}\frac{I}{2\pi R_2}R_2 d\theta = \frac{I}{2}$$

Some Definitions

assume B_c is uniform and perpendicular to A_c



open surface

Magnetic Flux

$$\Phi_m = \oint_S \mathbf{B} \cdot d\mathbf{a} = \oint_S B\cos(\theta) da = B_c A_c$$

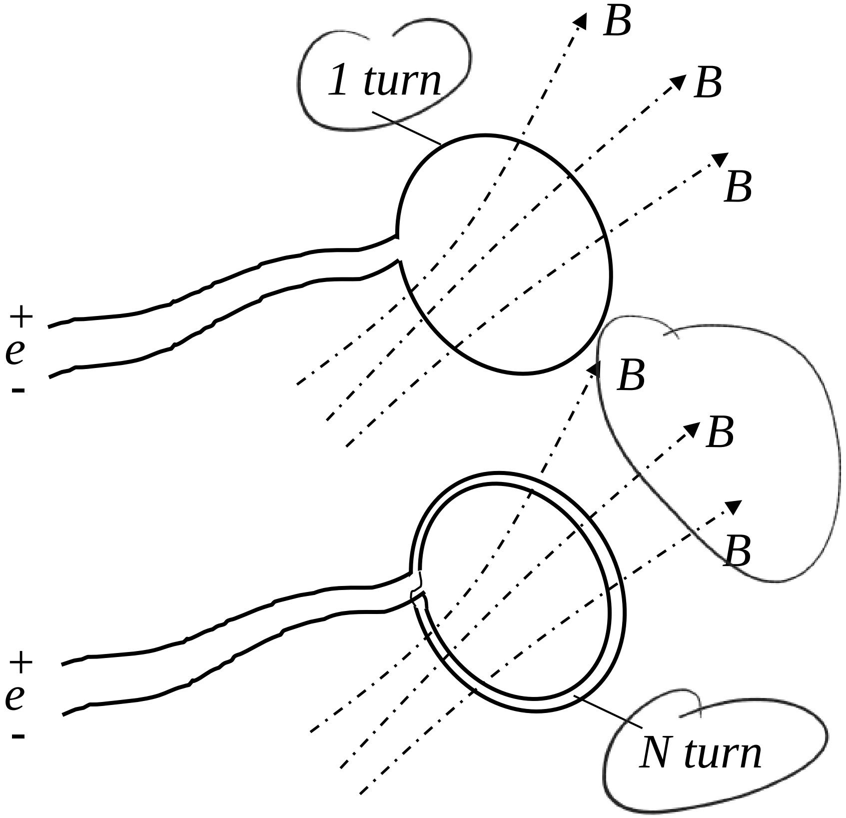
Flux is always continuous

Recall Faraday's Law - **E**lectro**m**otive **F**orce (emf)

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{a} = -\frac{d\Phi}{dt}$$

- voltage induced in one turn due to the changing magnetic flux

For coil with *N* turns $e = N \cdot \dfrac{d\Phi}{dt}$



Flux Linkage

$$\lambda = N \cdot \Phi \quad [Wb \cdot t]$$

flux scaled by the number of turns

Total induced emf $e = \dfrac{d\lambda}{dt}$ $[V]$

Flux Linkage is used very often to describe coils with many turns

Some Definitions

Ampere   Faraday

Recall important relationships $i \rightarrow H \rightarrow B \rightarrow \Phi_m \rightarrow \lambda \rightarrow e$

Inductance

Need a function that relates Flux Linkage to Current

we assume linear

Consider $\lambda = \underline{f(i)} = L(\cdot)\cdot i$ $L = \dfrac{\lambda}{i}$ $\left[\dfrac{Wb\cdot t}{A} = H\right]$

- voltage induced in a coil due to the changing magnetic flux

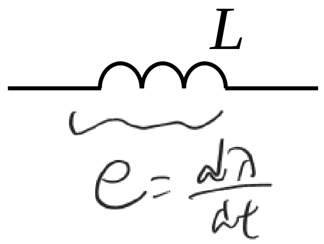
$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\cancel{\frac{dL}{dt}}$$

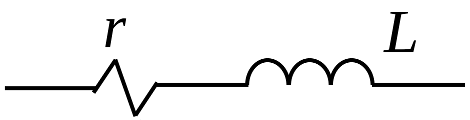
If L = constant => $\boxed{e = L\dfrac{di}{dt}}$

Recall

$$\boxed{L = \frac{\lambda}{i} = \frac{N\cdot\Phi}{i}}$$

Circuit Element



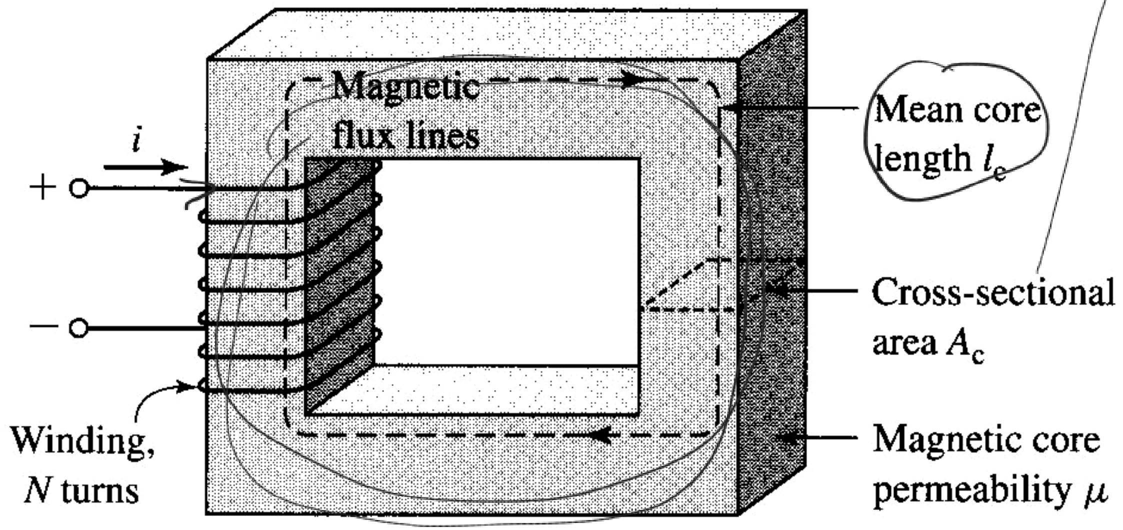


Equivalent series resistance (ESR) of the wire

Basic Magnetic Systems

we assume area is the same everywhere

- Consider basic magnetic circuit



Assume $\mu >> \mu_0$

⇒All magnetic field is concentrated inside the core

Recall Ampere's Law

$$\oint_{l_c} \mathbf{H}_c \cdot \mathbf{dl} = I_{net} = Ni$$

current enclosed

Increase this will increase B & H

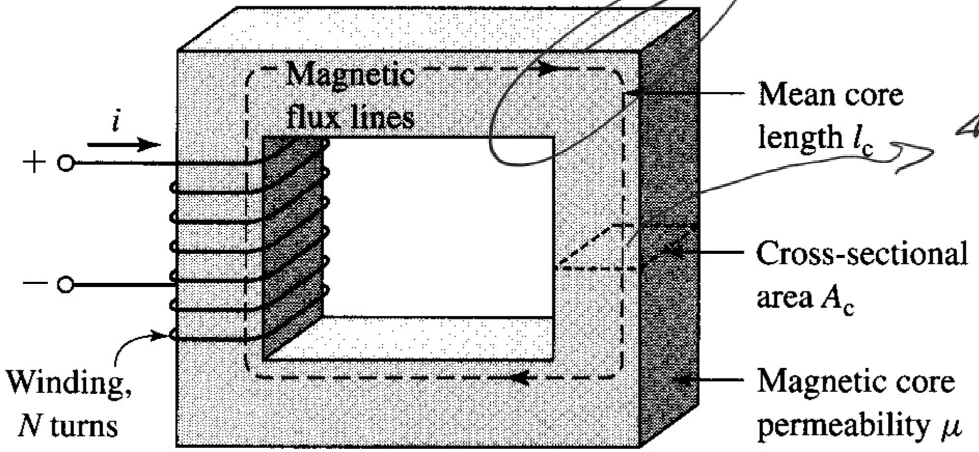
Magnetomotive force (mmf) $F = Ni$

Source of magnetic field is ampere-turn product

Assume uniform core $F = Ni = I_{net} = \oint_{l_c} H_c \cdot dl = H_c l_c$ $[Ampere\cdot turn]$

Magnetic Circuits

- Consider basic magnetic circuit



Assume all magnetic field is confined inside the core

Define Magnetic Flux

$$\Phi = \int_S \mathbf{B}\cdot\mathbf{da} = B_c A_c \quad [Wb]$$

Flux is always continuous

Consider mmf $F = Ni = H_c l_c = \dfrac{B_c l_c}{\mu} = \Phi \dfrac{l_c}{\mu A_c} = \Phi \Re_c$ → depends on core material + geometry

Define Reluctance (of the given magnetic path) $\boxed{\Re_c = \dfrac{l_c}{\mu A_c}}$ $\left[\dfrac{A}{Wb}\right]$

Defines properties of magnetic core (or path)

R_c doesn't contain coil information

Recall Inductance

$$\boxed{L = \frac{\lambda}{i} = \frac{N\cdot\Phi}{i} = \frac{N\cdot N\cdot i}{i\cdot\Re_C} = \frac{N^2}{\Re_C}}$$

Defines properties of magnetic core and the coils wound on it

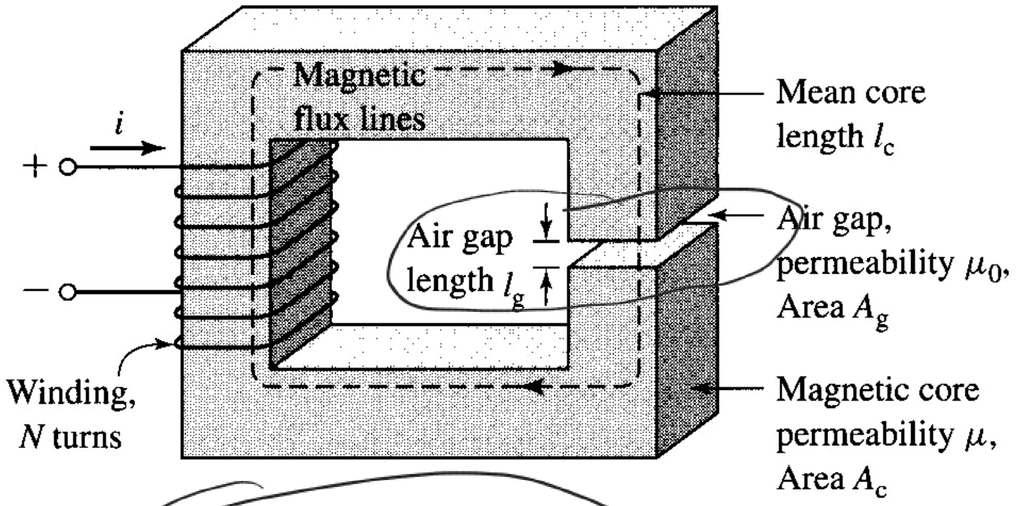
inversely related

L contains coil information

Magnetic Circuits

assume air gap is small so flux is still trapped inside core and uniform

- Magnetic circuit with air gap



$$B_c A_c = B_g A_g = \Phi_m$$

Assuming all magnetic flux is confined inside the core

Consider mmf

$$F = Ni = \oint_C \mathbf{H} \cdot \mathbf{dl}$$

$$= H_c l_c + H_g l_g$$

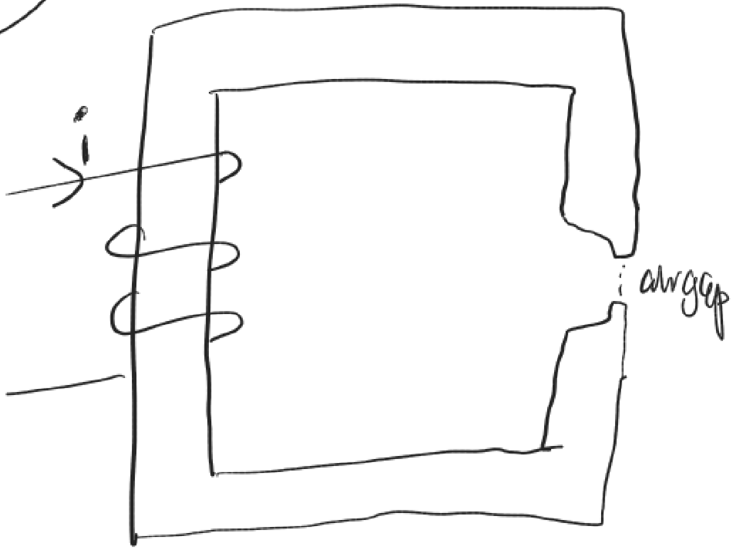
$$= \frac{B_c l_c}{\mu} + \frac{B_g l_g}{\mu_0}$$

$$B_c = \frac{\Phi}{A_c} \quad \text{and} \quad B_g = \frac{\Phi}{A_g}$$

core air gap

$$\boxed{F = \Phi\left(\frac{l_c}{\mu A_c} + \frac{l_g}{\mu_0 A_g}\right) = \Phi\left(\Re_c + \Re_g\right) = \Phi \sum \Re_i = \Phi \Re_{total}}$$

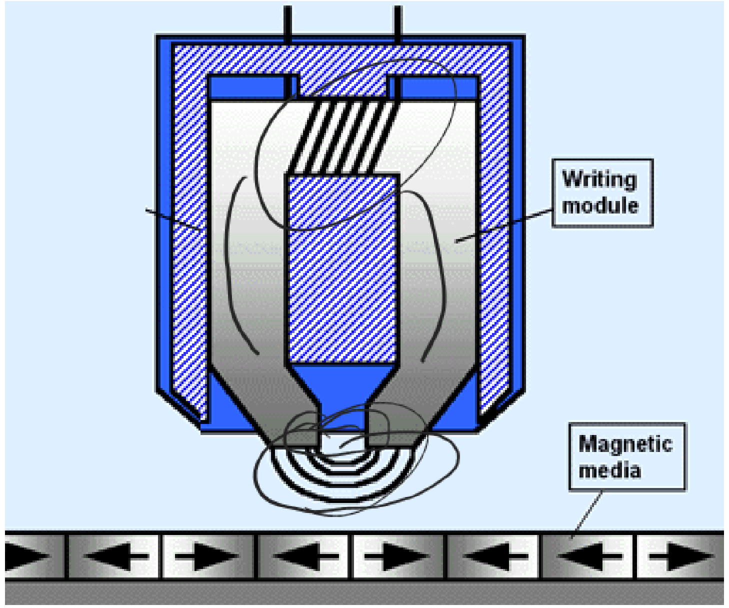
μ_0



$A_g \downarrow \to B_g \uparrow >> B_c$

Magnetic Circuits

- Applications of magnetic circuit with air gap
  - Magnetic head: A device with concentrated magnetic flux to change the magnetic media



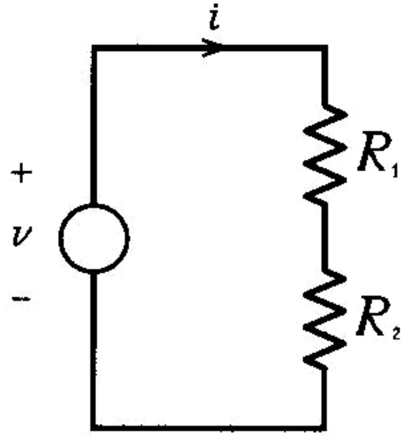
$$B_c A_c = B_g A_g$$

→ Hard drive
→ Credit card strip

Magnetic and Electric Circuits Analogy

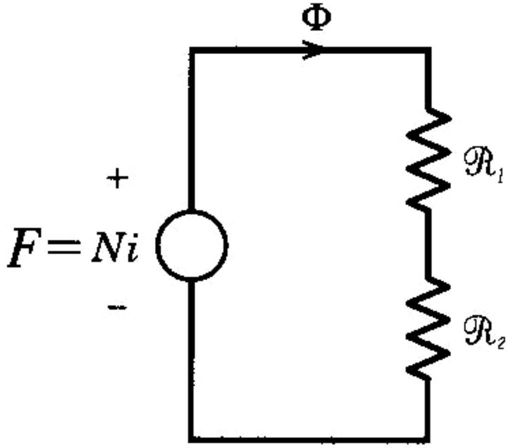
Electric Circuit

emf



Magnetic Circuit

mmf



$$i = \frac{v}{R_1 + R_2}$$

$$\Phi = \frac{F}{\Re_1 + \Re_2}$$

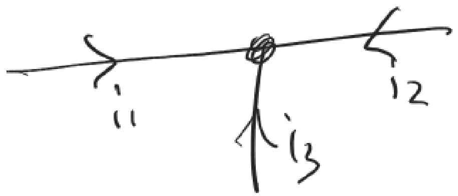
Magnetic and Electric Circuits Analogy

Electric Circuit

- Voltage (emf), $V, [Volt]$
- Current, $I, [Amps]$
- Resistance, $R = \dfrac{l}{\sigma A}, [\Omega]$
- Conductance, $G = \dfrac{1}{R}, [Siemens]$
- Conductivity, $\sigma, \left[\dfrac{Siemens}{m}\right]$

For loop $v = \sum R_n i_n$

For node $\sum_N i_n = 0$ KCL



Magnetic Circuit

- mmf, $F, [A \cdot t]$
- Flux $\Phi, [Wb]$
- Reluctance, $\Re = \dfrac{l}{\mu A}, \left[\dfrac{A}{Wb}\right]$
- Permeance, $\rho = \dfrac{1}{\Re}, \left[\dfrac{Wb}{A}\right]$
- Permeability, $\mu, \left[\dfrac{H}{m}\right]$

For loop $F = \sum H_n l_n$

For node $\sum_N \Phi_n = 0$

Example: Calculate Inductance

Consider the following electromagnetic system

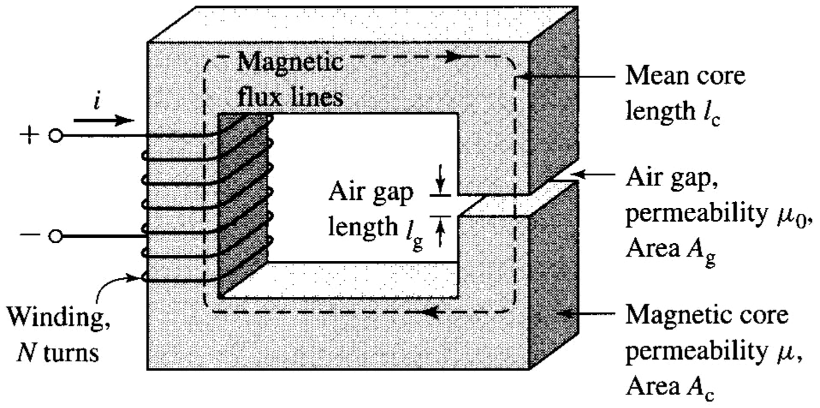
$i = 1\,A,\ N = 400$

$l_c = 50\,cm,\ l_g = 1\,mm$

$A_c = A_g = 15\,cm^2$

$\mu_r = 3000$

Find inductance $L = \dfrac{N^2}{\mathcal{R}_c + \mathcal{R}_g}$



$\mathcal{R}_c = \dfrac{l_c}{\mu_r \mu_0 A_c} = \dfrac{50e{-2}}{3000 \cdot 4\pi \cdot 1e{-7} \cdot 15e{-4}} \approx 88.42e{+3}\ A{\cdot}t/Wb$

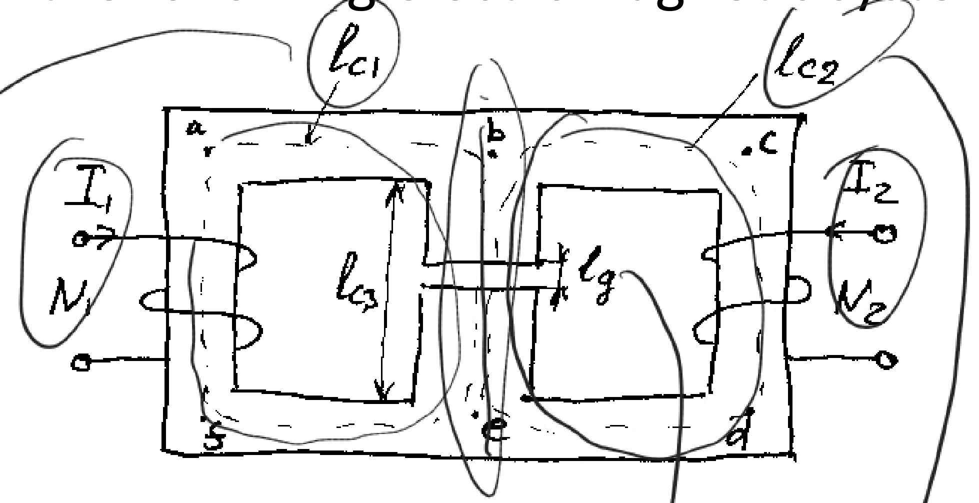
$\mathcal{R}_g = \dfrac{l_g}{\mu_0 A_g} = \dfrac{1e{-3}}{4\pi \cdot 1e{-7} \cdot 15e{-4}} \approx 530.515e{+3}\ At/Wb$

$L = \dfrac{400^2}{(88.42 + 530.515) \cdot e{+3}} = 258.52e{-3}\ H$

$= 258.52\ mH$

Example: Magnetic Circuits

Consider the following electromagnetic system



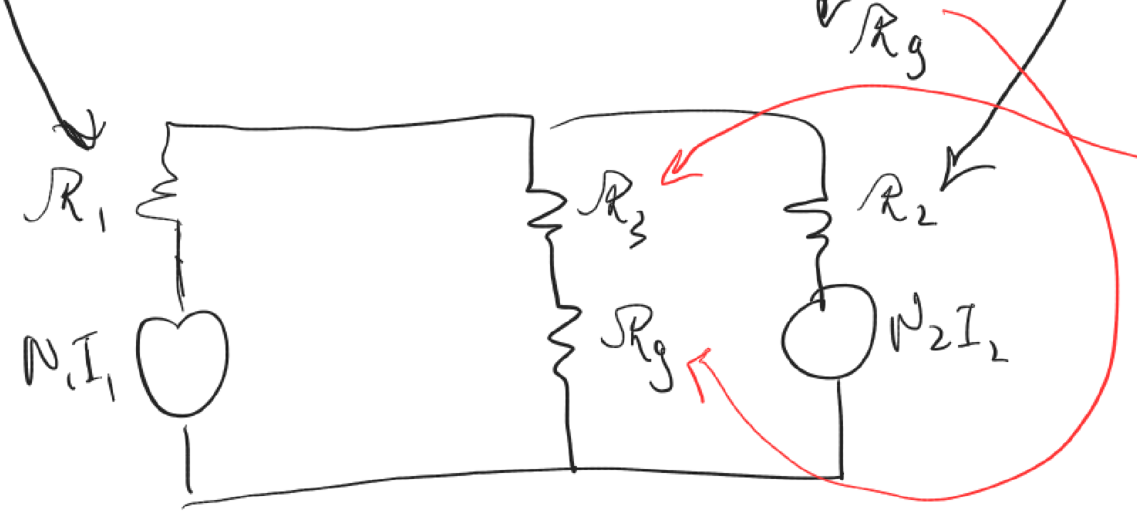
Path l_{c1} = bafe → $\mathcal{R}_1$

Path l_{c2} = bcde → $\mathcal{R}_2$

Path l_{c3} = be − l_g

l_{c3} includes only core → $\mathcal{R}_3$

Equivalent Magnetic Circuit



Looking ahead…

- Finish **Module 1**

- Start **Module 2**