ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 4

January 15, 2026

Zhi Jin (Justin) Zhang

Course Logistics

- Lab materials will be posted this weekend, please start reading them

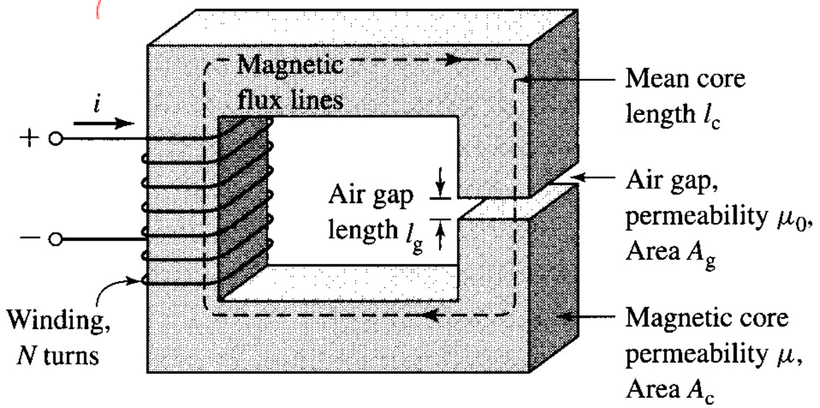
- Quiz 1
  - Coverage: up to and including end of Module 1
  - Date: January 30 (Friday) at the beginning of the tutorial
  - Closed book and notes
  - Calculators allowed

Module 1, Part 2: Basic Magnetic Circuits Continued

Example: Calculate Inductance

Consider the following electromagnetic system

inductor



Given:

$i = 1\,A$, $N = 400$

$l_c = 50\,cm$, $l_g = 1\,mm$ } core geometry

$A_c = A_g = 15\,cm^2$

$\mu_r = 3000$ } $\mu = \mu_r \mu_0$

Find inductance $L = \dfrac{N^2}{\mathcal{R}_c + \mathcal{R}_g}$

$$\Rightarrow \mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{50e{-2}}{3000 \cdot 4\pi \cdot 1e{-7} \cdot 15e{-4}} \approx 88.42\,e{+3}\ \text{A·t/Wb}$$

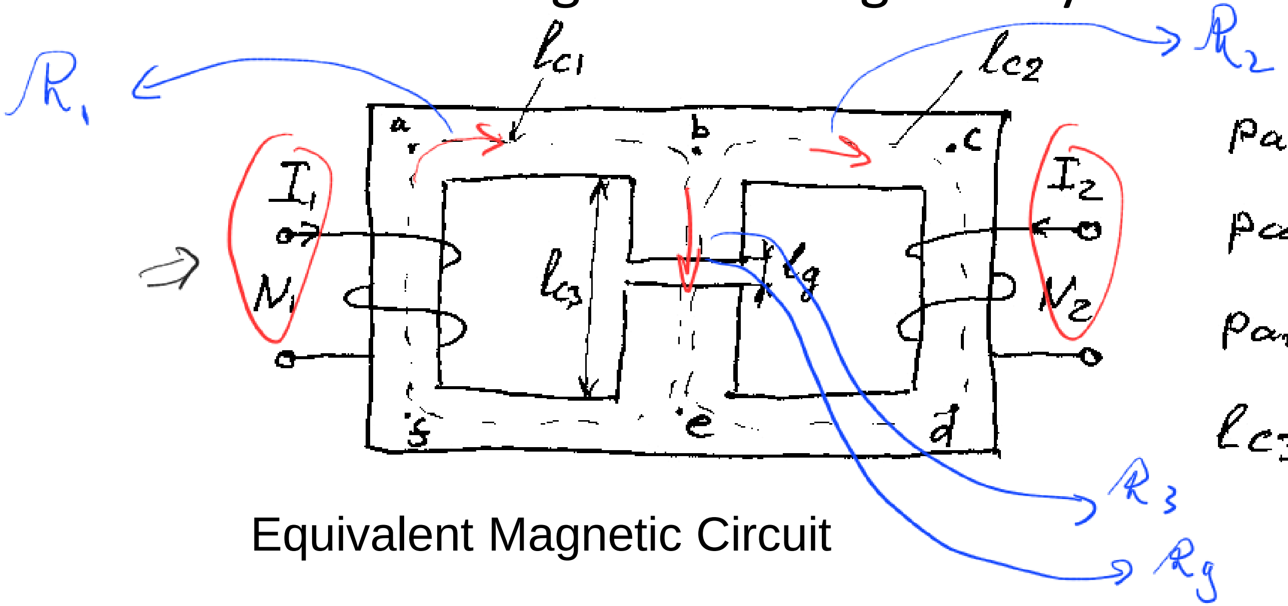
$$\Rightarrow \mathcal{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{1e{-3}}{4\pi \cdot 1e{-7} \cdot 15e{-4}} \approx 530.515\,e{+3}\ \text{At/Wb}$$

$$\Rightarrow L = \frac{400^2}{(88.42 + 530.515) \cdot e{+3}} = 258.52\,e{-3}\ H$$

$$= 258.52\ mH$$

Example: Magnetic Circuits → Assignment 2

Consider the following electromagnetic system



Path l_{c1} = bafe

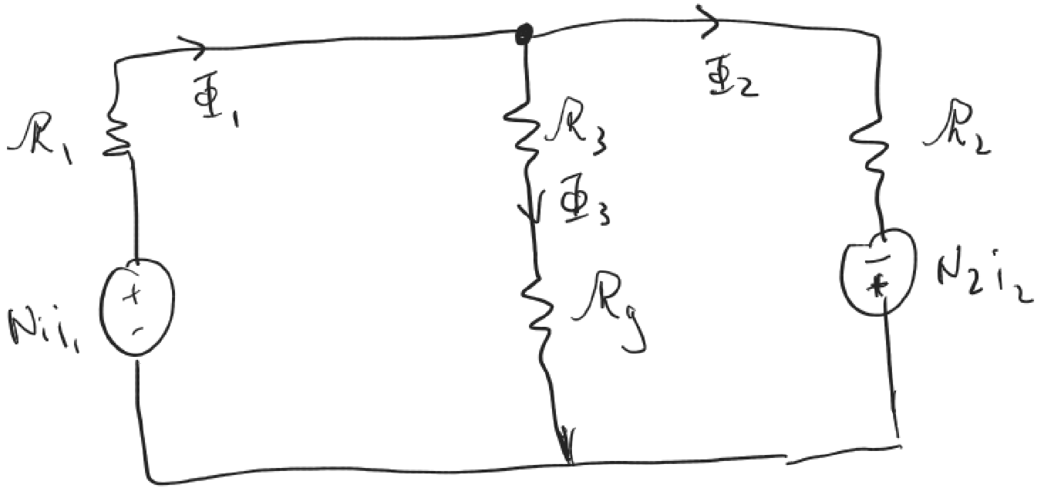
Path l_{c2} = bcde

Path l_{c3} = be − l_g

l_{c3} includes only core

l_g

Equivalent Magnetic Circuit



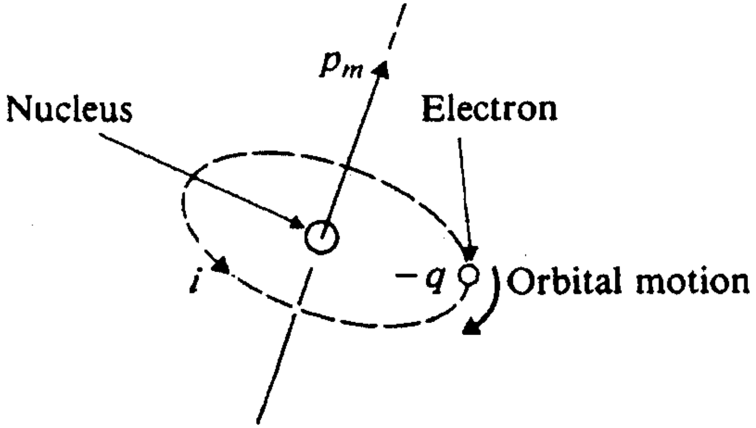
$$\mathcal{R} = \frac{l}{\mu A}$$

Magnetic Materials

Preferred orientation of atom's orbit

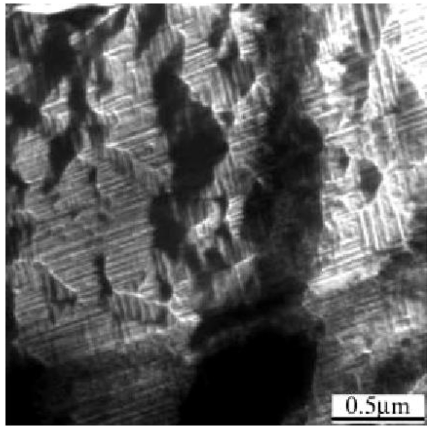
Magnetic moment of an atom

Unpaired electrons (e^-)



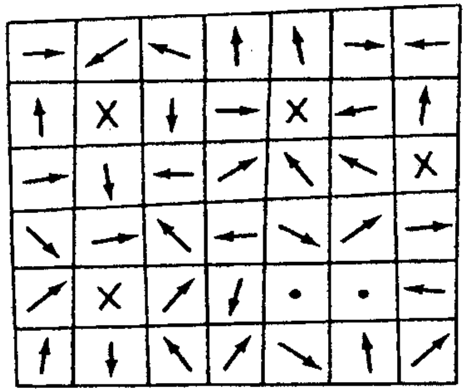
Crystal → graining

Magnetic Domain Structure



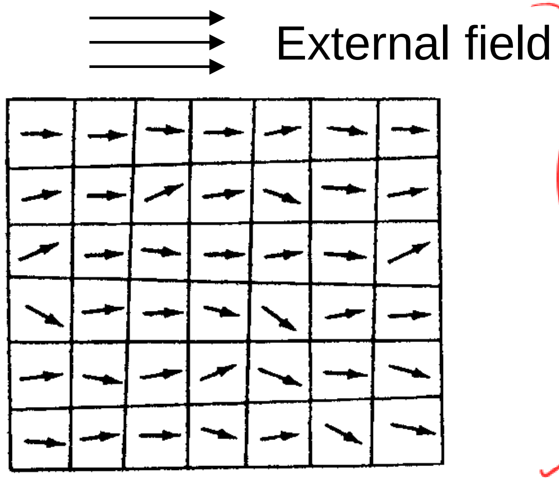
Not all metals are magnetic:
① Paramagnetic
② Ferromagnetic
③ Dimagnetic

Magnetic Materials - Domain Model



demagnetized

Apply

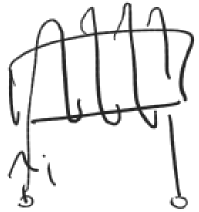


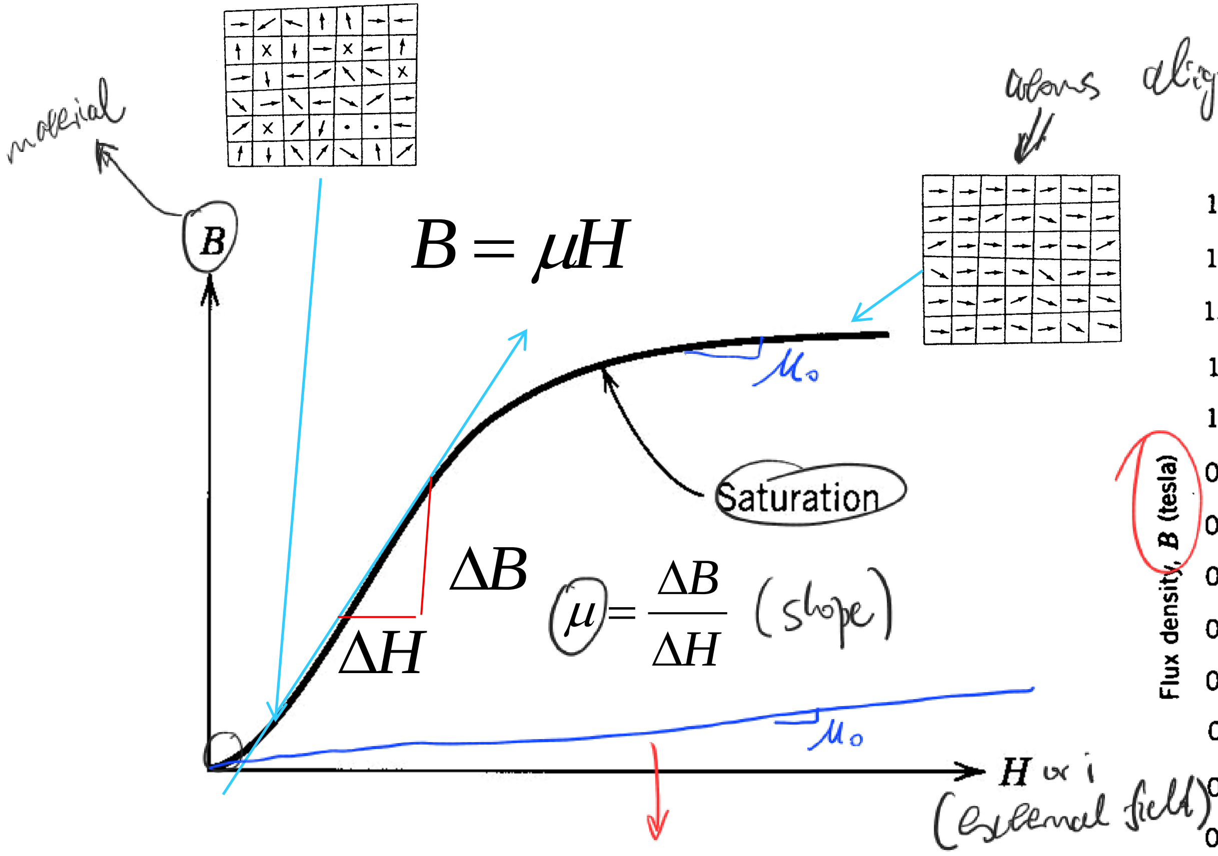
(H, or i)

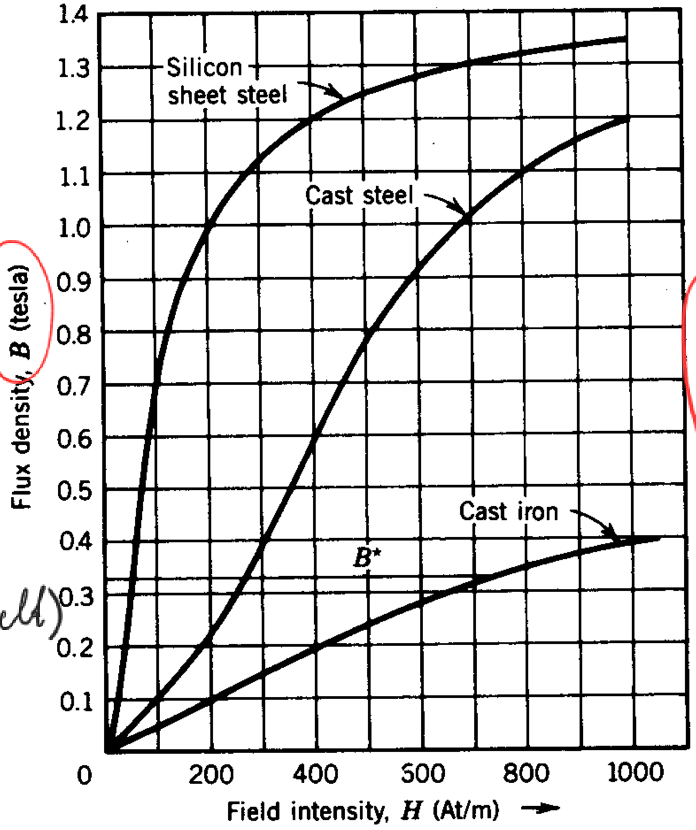
B

magnetized

Magnetic Saturation

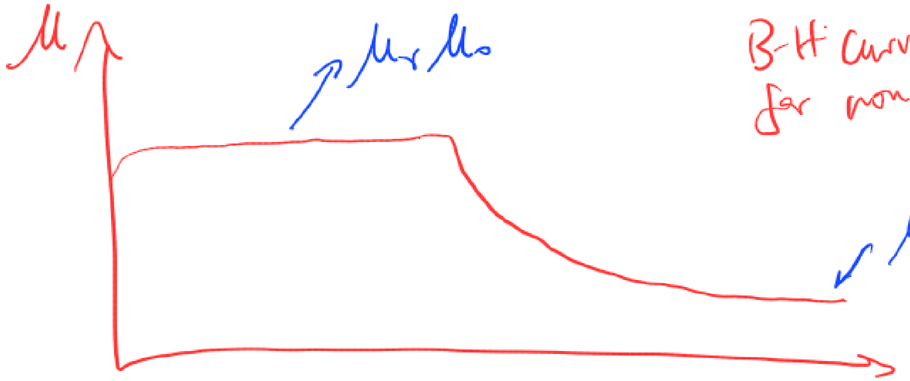






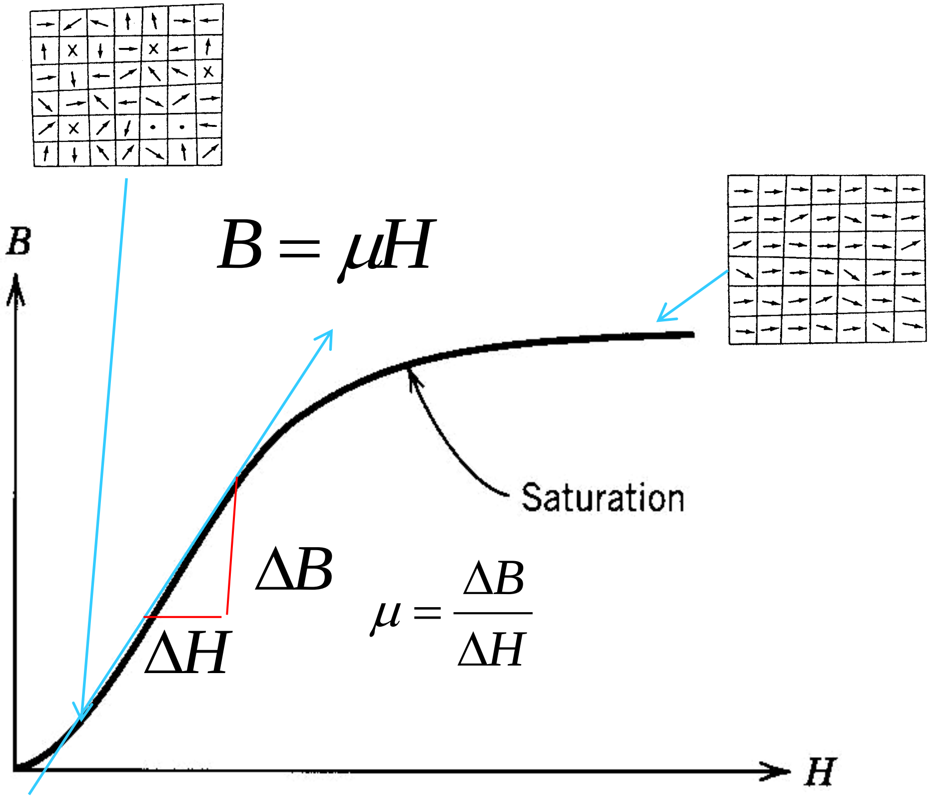
$\approx 1 \ldots 1.5T$

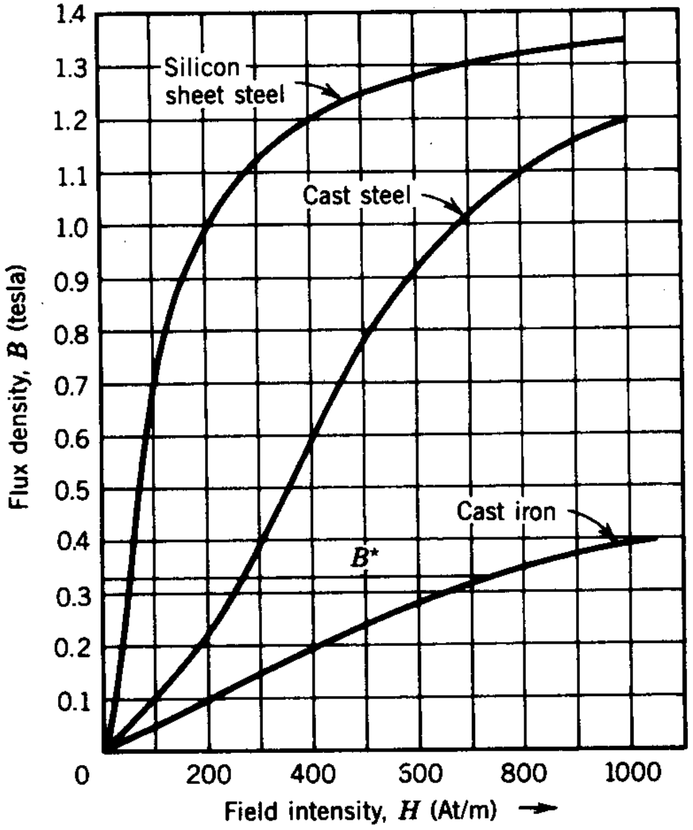
All practical materials saturates after about 1 to 1.5 T



As you saturates, slope → μ_0

Magnetic Saturation





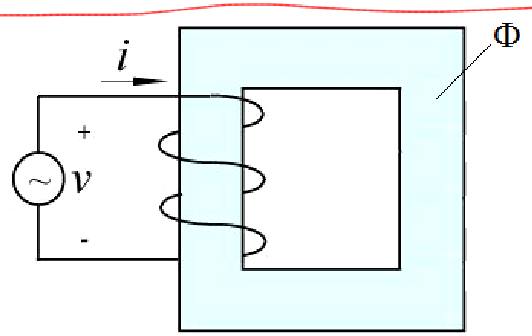
$\approx 1\ldots1.5T$

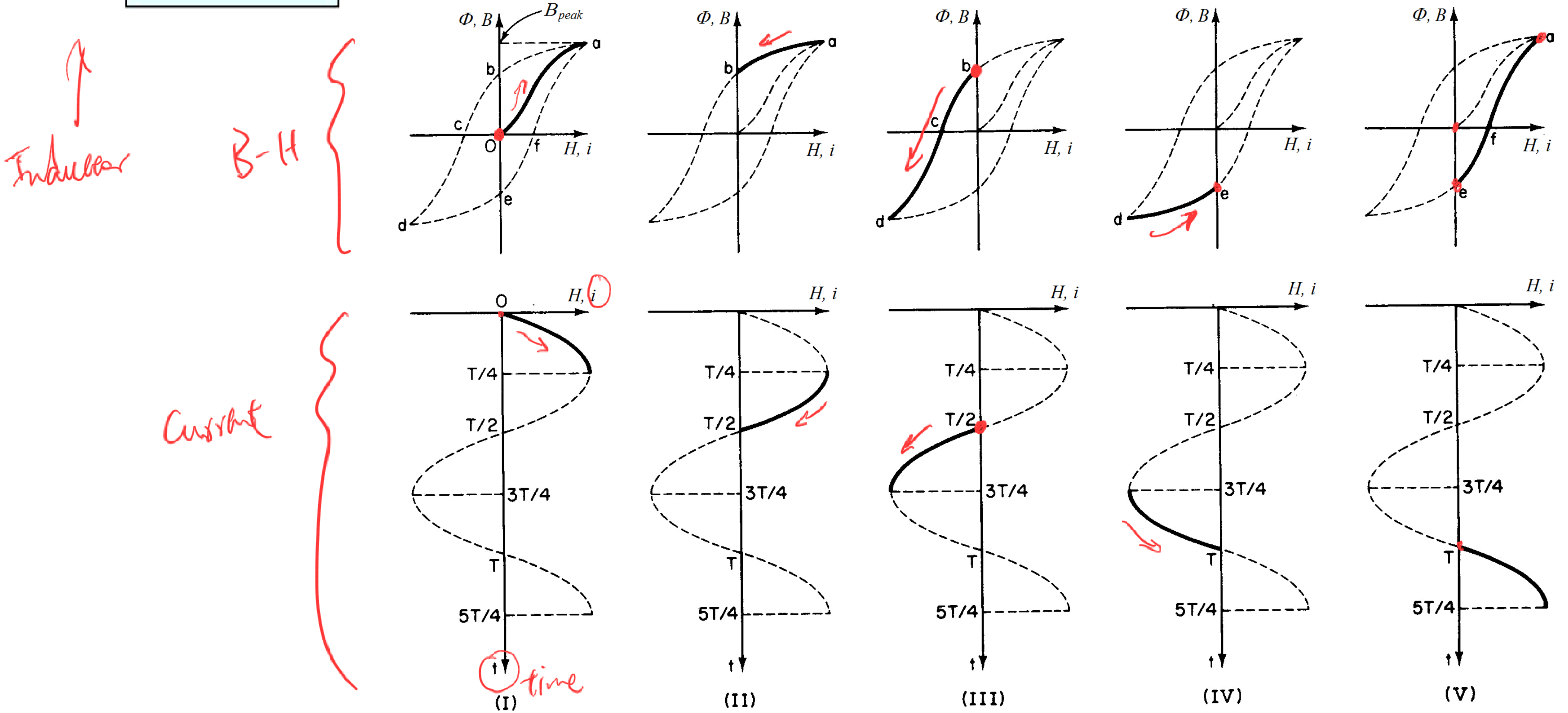
All practical materials saturates after about 1 to 1.5 T

Generally Yes

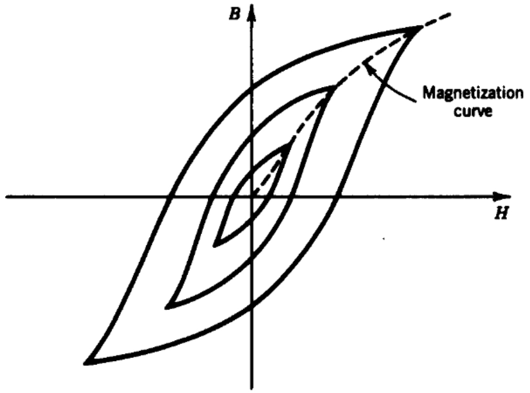
- Does it take energy to magnetize material and aligned all internal domains? Yes
- Would the material demagnetize after external field is removed? Depends on the material

Hysteresis Loop

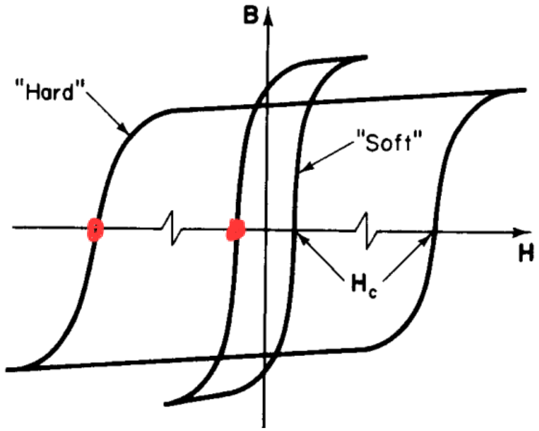


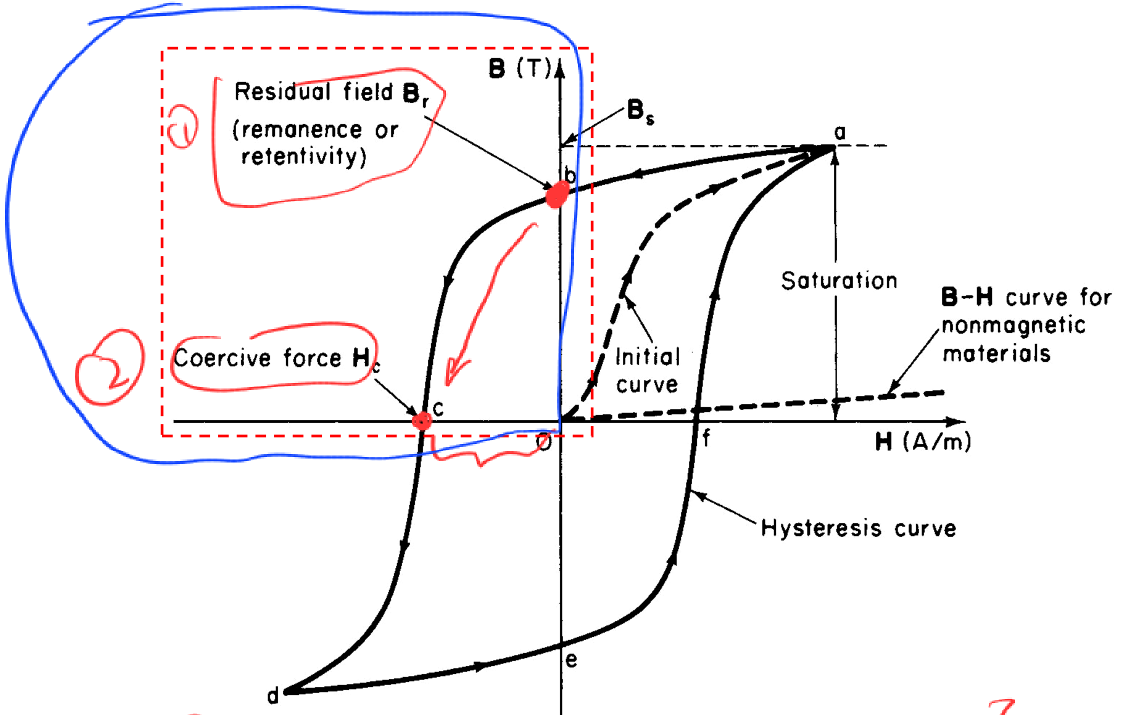


Hysteresis Loop



Hysteresis loops for different excitation current levels





B_r – residual magnetism
H_c – coercivity force, external field required to demagnetize the material

Soft mag. materials $H_c \sim 0.1 \cdots 100$

Hard mag. materials $H_c > 100$

Permanent magnets (PM) $H_c \sim 10^4 \cdots 10^6$

Types of PMs

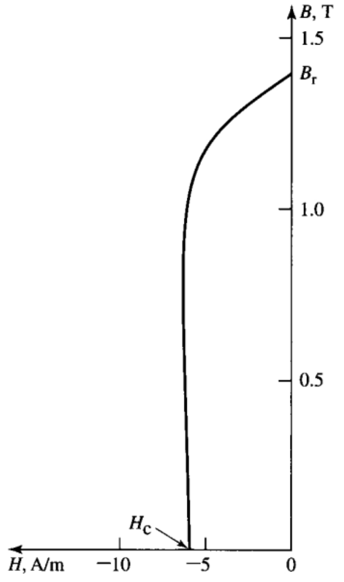
- Neodymium Iron Boron (NdFeB or NIB)
- Samarium Cobalt (SmCo)
- Aluminum Nickel Cobalt (Alnico)
- Ceramic or Ferrite, Iron-oxide, compressed powder

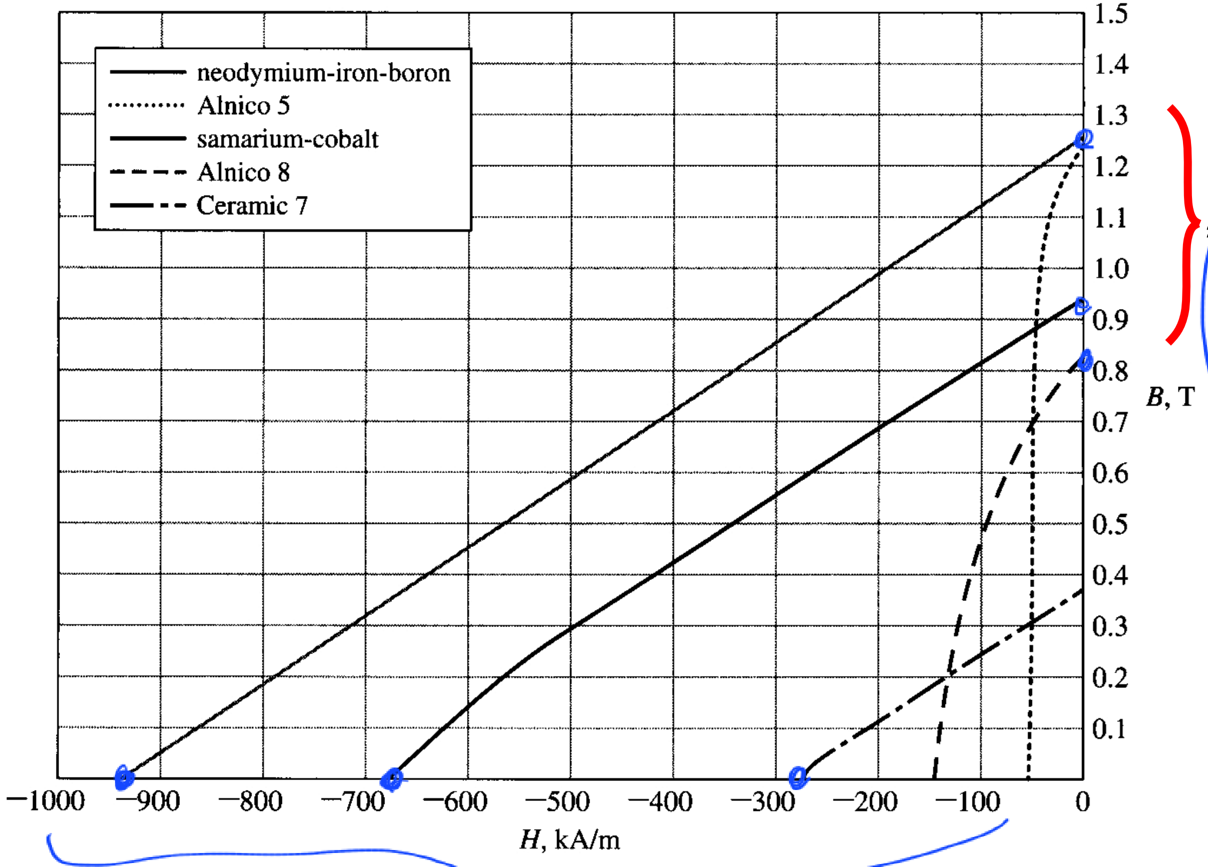
Magnetic Materials → Datasheets show second quadrant

Second quadrant

Some common PM materials

Second quadrant hysteresis curve for M-5 steel





$\approx 1 \ldots 1.5T$

All practical materials saturates after about 1 to 1.5 T

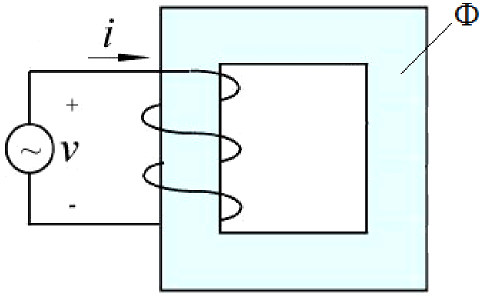
positive B axis

Core Losses in Practical Devices

We ignore resistivity of wires

Hysteresis Losses

Consider AC excitation



$$e = \frac{d\lambda}{dt}$$

Power & Energy going into the magnetic core

$$P_{in}(t) = ie = i\frac{d\lambda}{dt}$$

$$\Delta W_{cycle} = \oint P_{in}(t)dt = \oint i\frac{d\lambda}{dt}dt = \int i\, d\lambda$$

Recall MMF and Flux Linkage

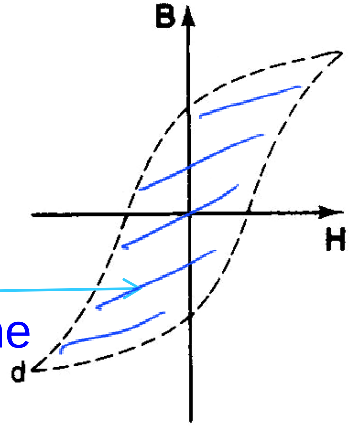
$$F = Ni = H_c l_c \qquad \lambda = N\Phi = NB_c A_c$$

Volume of the core

$$\Delta W_{h,cycle} = \oint i\, d\lambda = \oint\left(\frac{H_c l_c}{N}\right)(NA_c dB_c) = l_c A_c \oint H_c dB_c$$

E per cycle

Area of the loop



Power loss can be approximated as

$$P_h = K_h \cdot f \cdot (B_{c,\max})^n \qquad n \sim 1.5 \cdots 2.5$$

Where the constants K_h and n determined experimentally

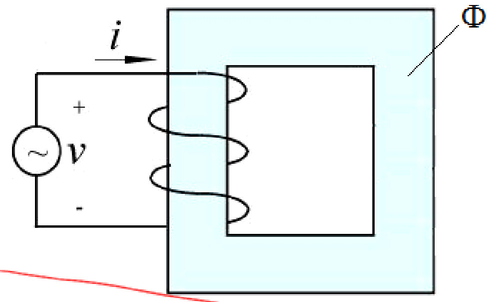
Power = Energy × frequency

$P_{loss} = \Delta W_{cycle} \times f$

Core Losses in Practical Devices

Eddy Current Losses

Consider AC excitation



Faraday's law

$$\oint_C \overline{E} \cdot dl = -\frac{d}{dt}\int_S B \cdot da = \frac{d\Phi}{dt} = e$$

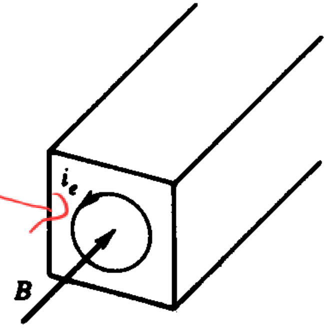
Power loss can be approximated as

$$P_e = K_e \cdot f^2 \cdot (B_{c,\max})^2$$

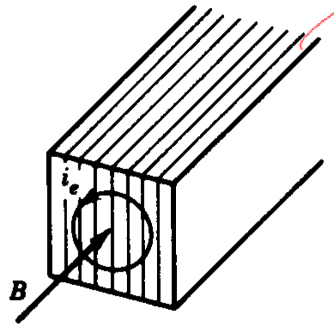
Where the constant K_e depends on lamination thickness and is determined experimentally

Cross-section area of magnetic core

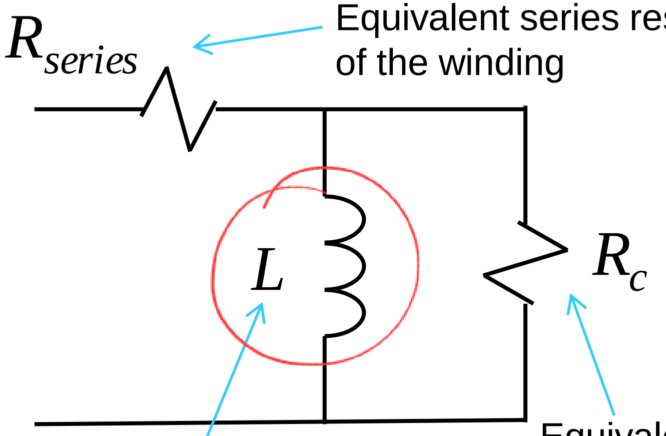
Solid-iron core



Laminated core

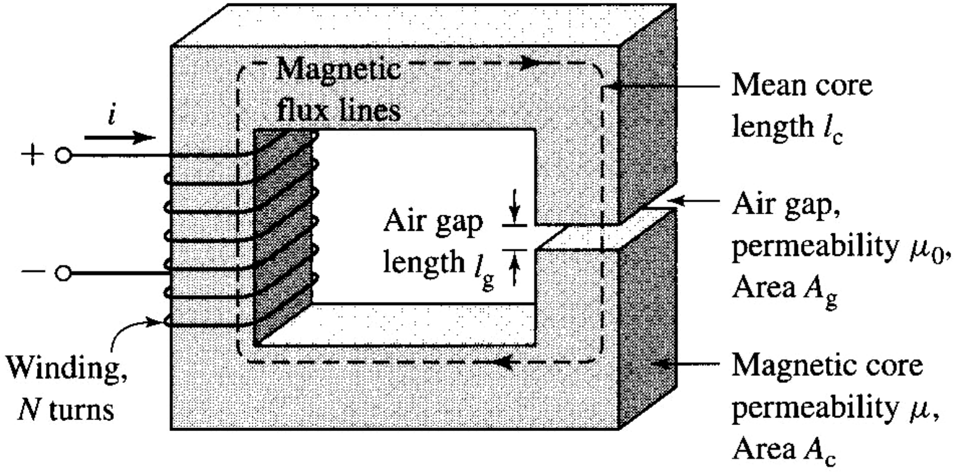


Equivalent circuit including core losses

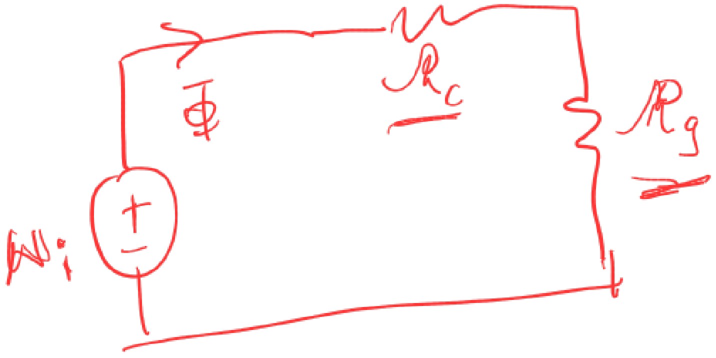


Magnetic & Electric Equivalent Circuit

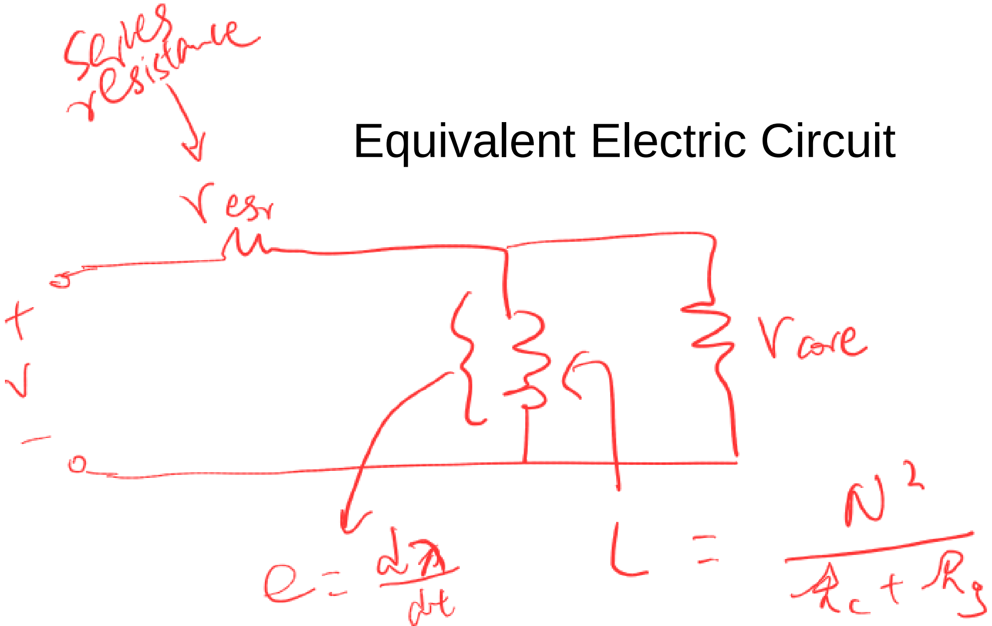
Consider the following electromagnetic system (device)



Equivalent Magnetic Circuit



Equivalent Electric Circuit



Cannot be combined

End M1
Coverage of Quiz 1

Module 2: Transformers

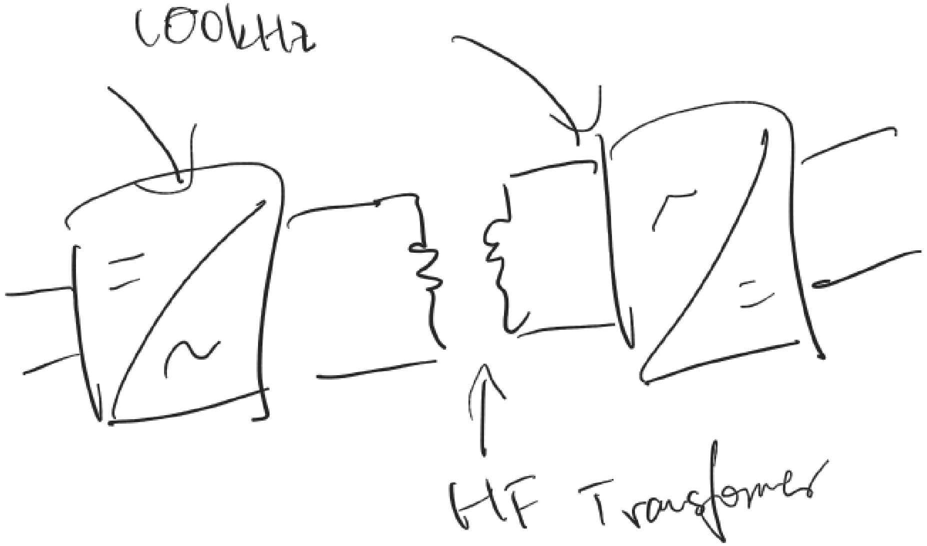
Objectives & important concepts

- Types and construction of typical transformers
- Terminology, winding polarity
- Ideal transformer, turns ratio, referring quantities
- Non-ideal transformer, equivalent magnetic and electric circuits, approximate equivalent circuits
- Tests on transformers for determination of equivalent circuit parameters
- Power flow, efficiency, voltage regulation, phasor diagrams
- Autotransformers
- Inrush current, harmonics in transformers
- 3-phase transformers, winding connections, phase shifting
- Per-unit system, calculations in pu

- Must read Chapter 2, get ready for Lab 2

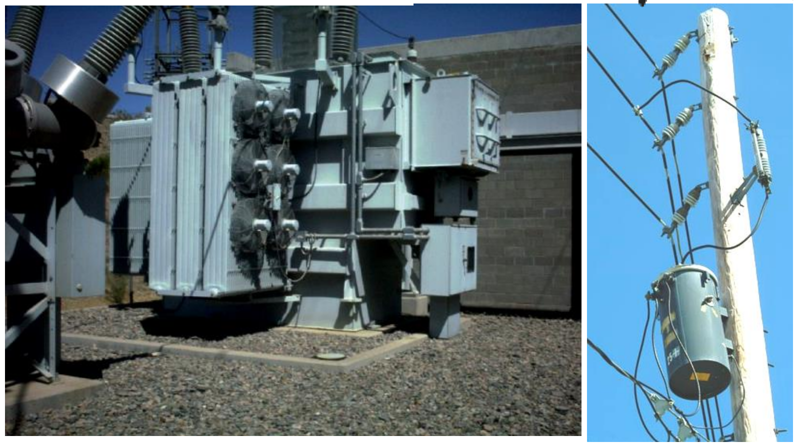
Transformer Applications

- Stationary electromagnetic device
  - Electrical energy conversion device
  - It allows to scale voltage/current levels
  - Inverting polarity of signals
  - Galvanic decoupling
  - A transformer may have multiple windings for different voltage and current levels



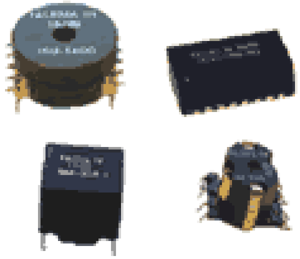
Transformer Applications

Power systems 50 Hz / 60 Hz



100 kHz

Small power supplies



communication

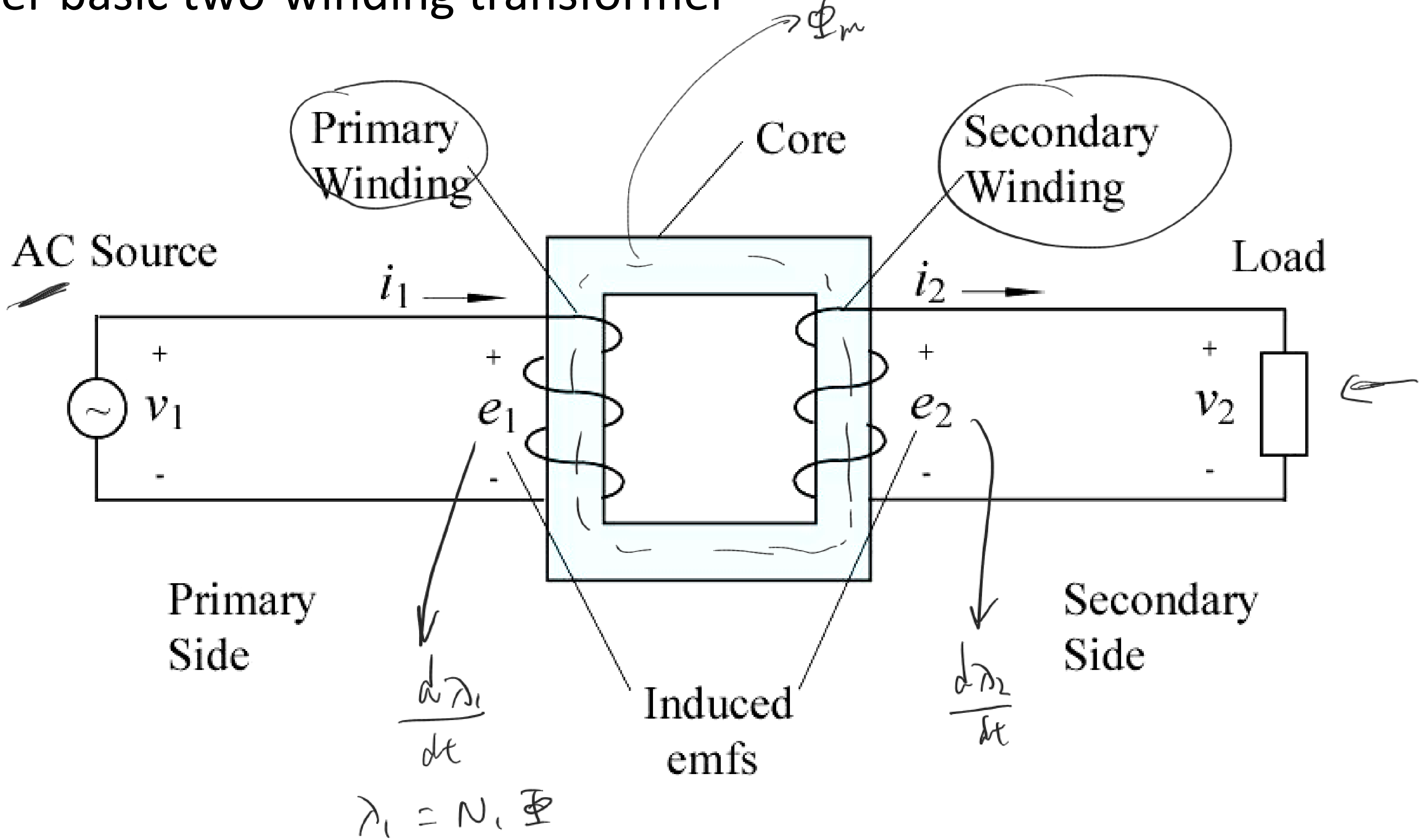
instrumentation

Variacs

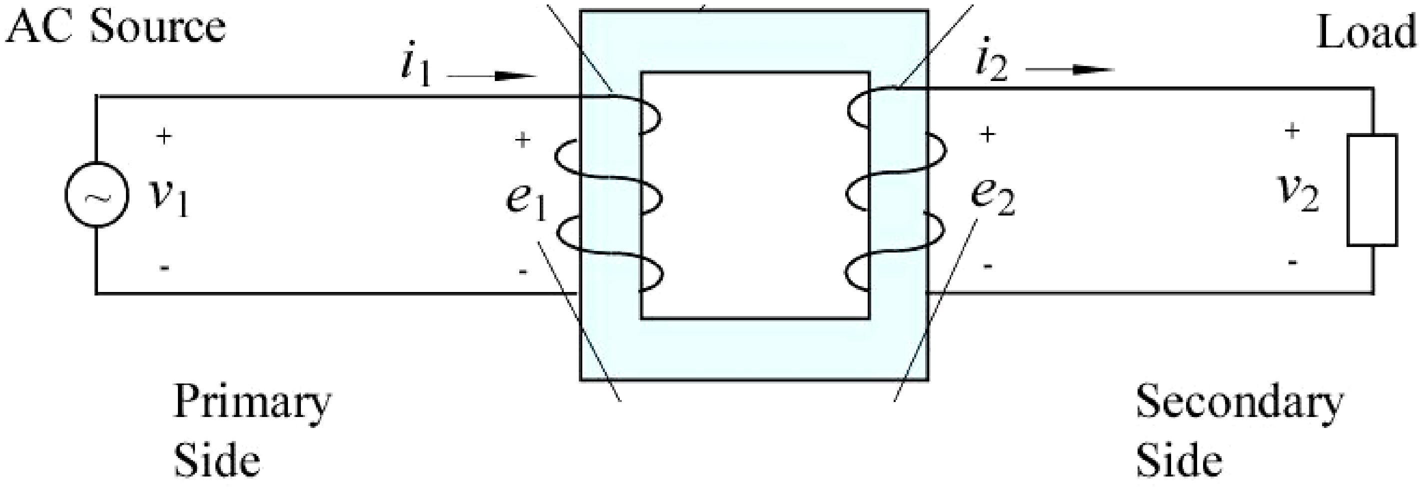
Toroids

Transformer Terminology

- Consider basic two-winding transformer

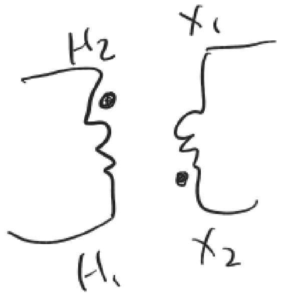


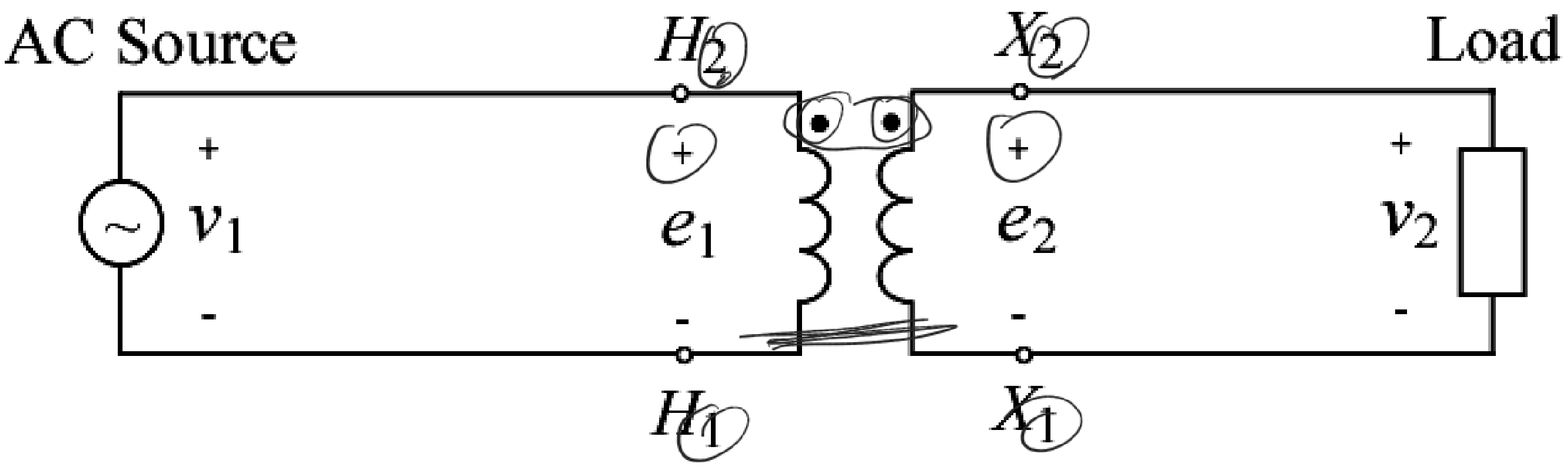
Transformer Terminology



- Step-down
  - Primary (Source) – high voltage / number of turns
  - Secondary (Load) – low voltage / number of turns
- Step-up
  - Primary (Source) – low voltage / number of turns
  - Secondary (Load) –high voltage / number of turns

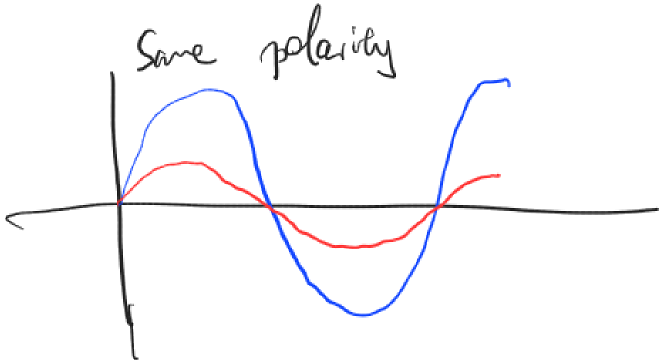
Transformer Schematic Diagram

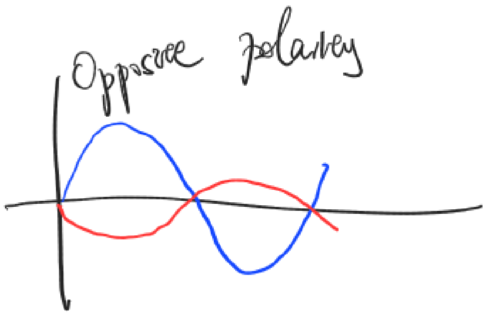




means they are in phase

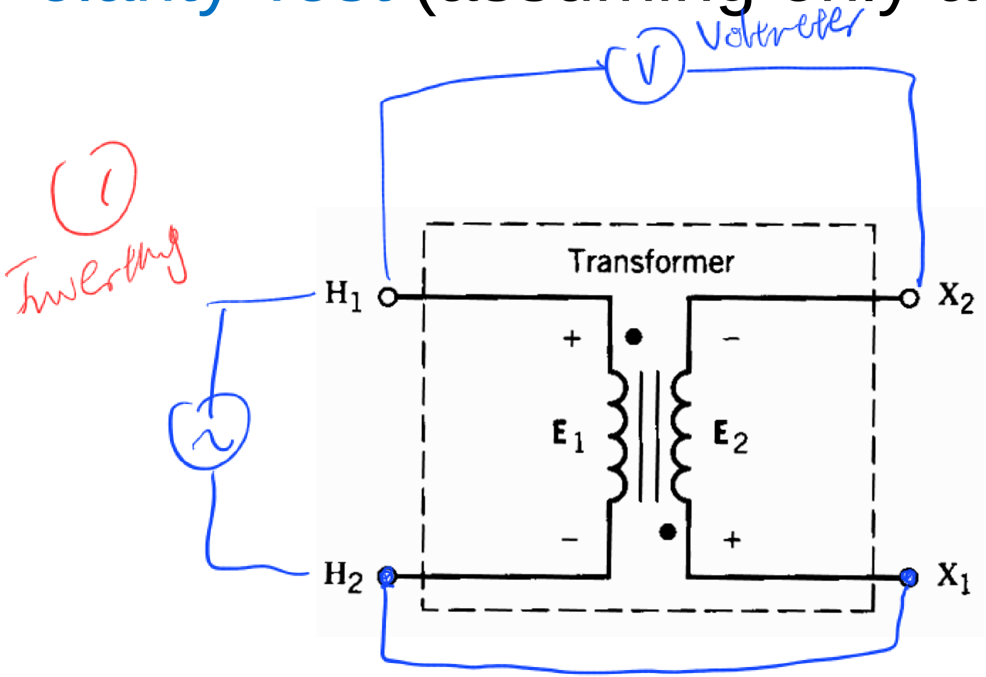
- Dot convention (most common) – mark same polarity terminals
- *H-X* convention (*H* – high voltage, *X* – low voltage)

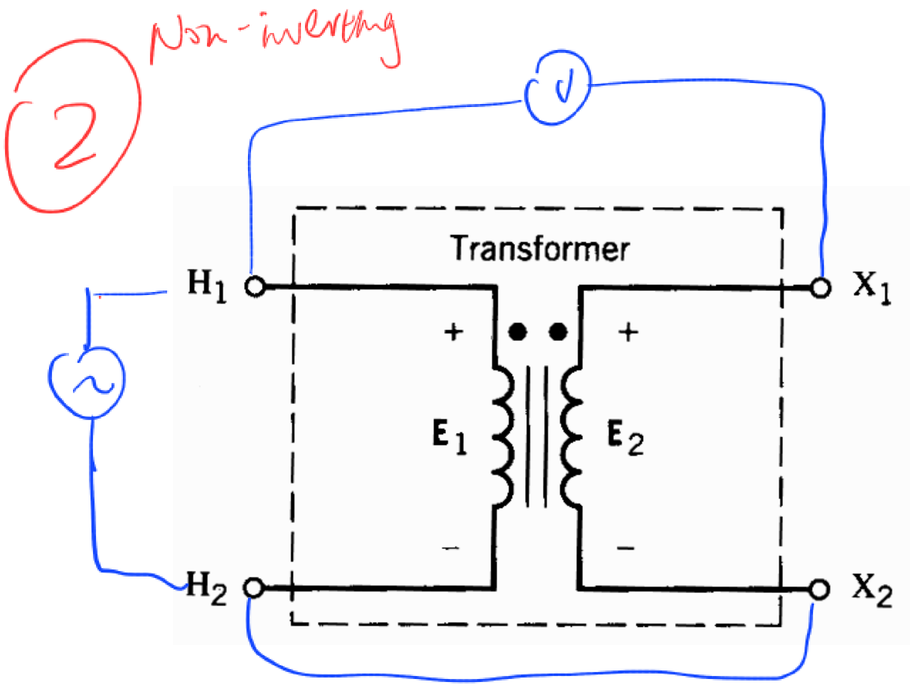


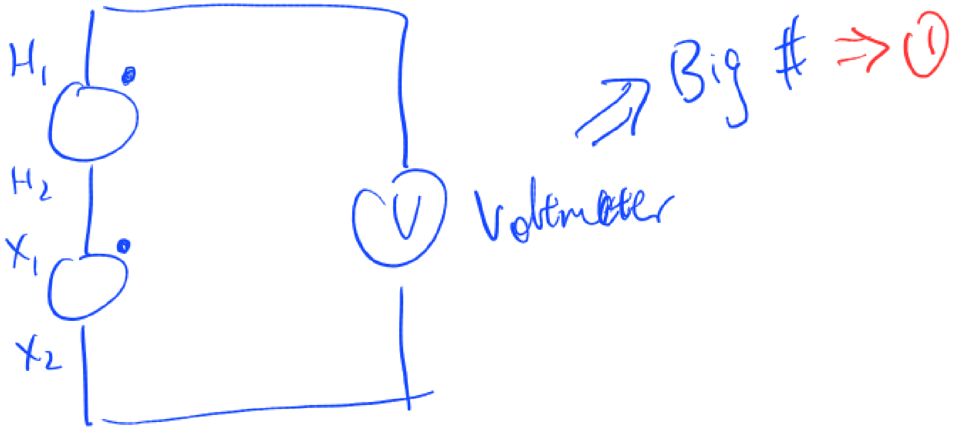


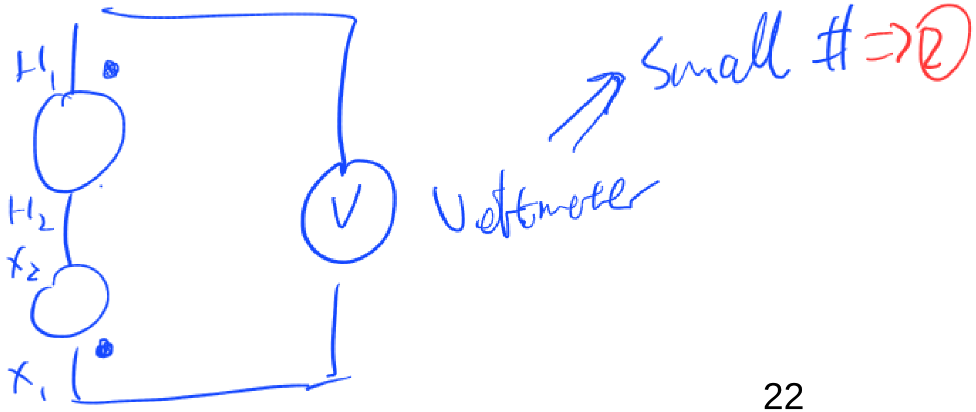
Transformer Polarity

Polarity Test (assuming only a basic RMS voltmeter and no oscilloscope)







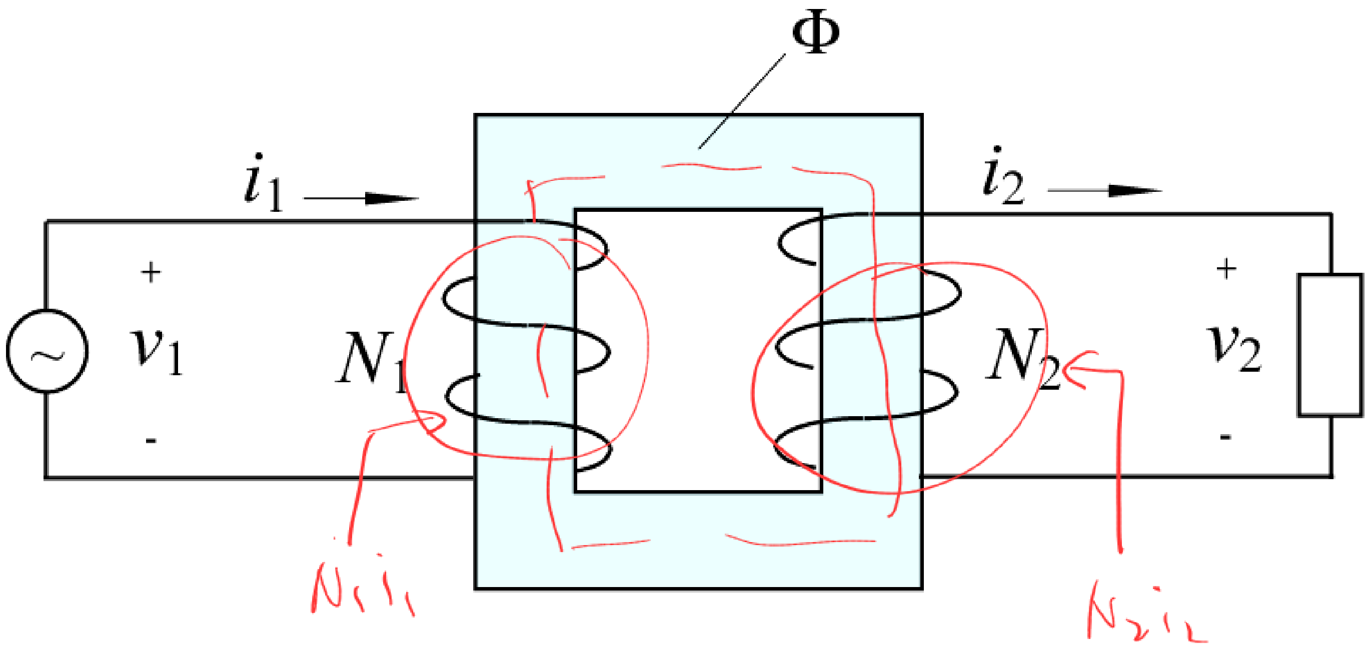


Ideal Transformer

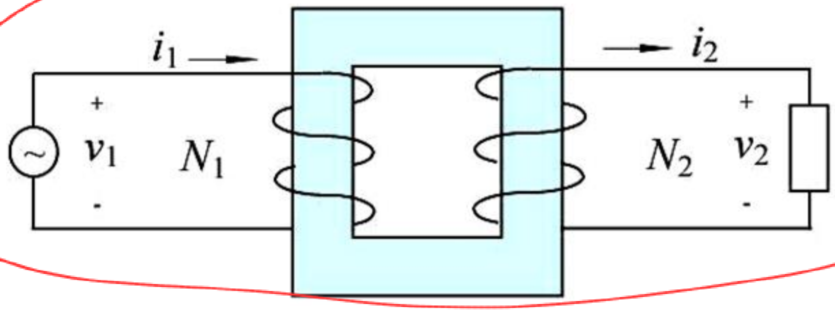
Assumptions

- Winding resistance is negligible $\quad r_1, r_2 \to 0$
- Infinite core permeability $\quad \mu_{core} \to \infty$
- All flux is confined inside the core
- No core losses

$\mathcal{R}_m = \frac{l}{\mu A} = \phi$



Properties of Ideal Transformer



Equivalent Magnetic Circuit

MMF $\quad F = N_1 i_1 - N_2 i_2 = \Phi \Re_{total} = 0$

$$\Rightarrow \quad N_1 i_1 = N_2 i_2 \quad \Rightarrow \quad \boxed{\frac{i_2}{i_1} = \frac{N_1}{N_2} = a}$$

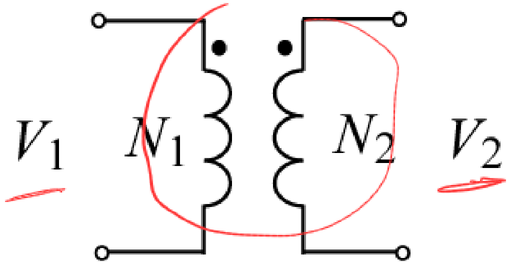
→ turns ratio

→ Current

Power $\quad P_{in} = i_1 v_1 = i_2 v_2 = P_{out}$

Assuming no losses

Equivalent Electric Circuit

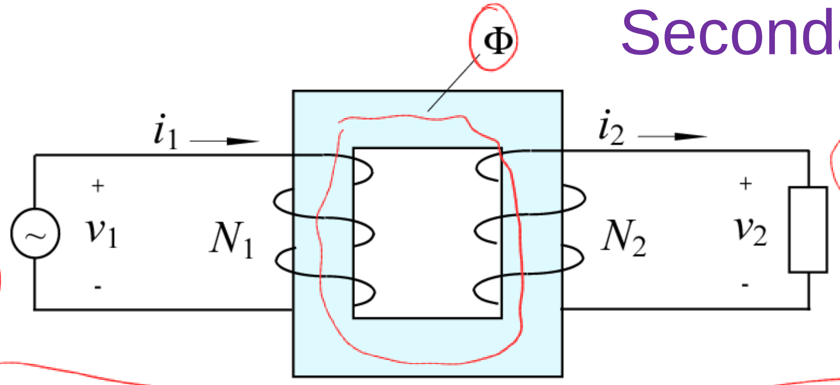


Properties of Ideal Transformer

Primary Side

$$v_1 = \sqrt{2}V_{1,rms}\cos(\omega t)$$

$$v_1 = e_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\Phi}{dt}$$



Secondary Side

$$v_2 = e_2 = \frac{d\lambda_2}{dt} = N_2\frac{d\Phi}{dt}$$

$$\Phi = \frac{\sqrt{2}V_{1,rms}}{N_1\omega}\sin(\omega t) = \Phi_{peak}\sin(\omega t)$$

$$\Phi_{peak} = \frac{\sqrt{2}V_{1,rms}}{N_1\omega} = \frac{V_{1,rms}}{\sqrt{2}N_1\pi \cdot f}$$

$$\frac{d\Phi}{dt} = \frac{v_1}{N_1} = \frac{v_2}{N_2}$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a$$ Turns ratio

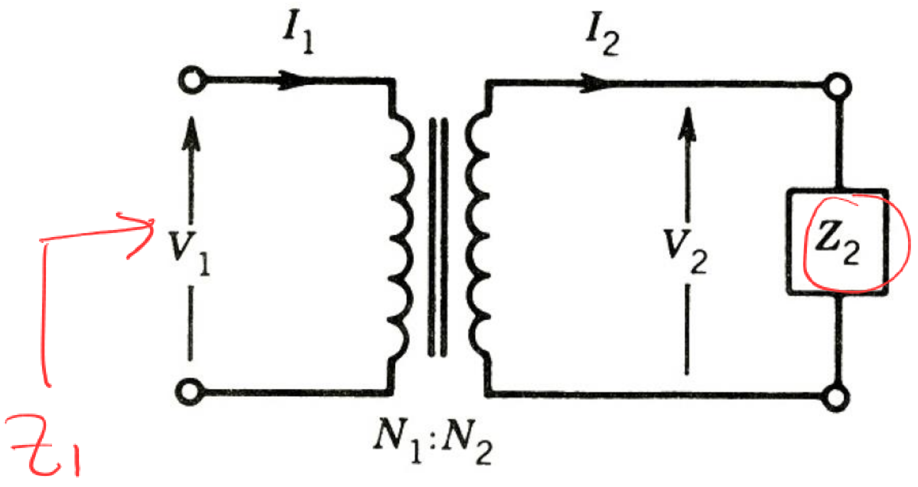
Maximum flux density for common magnetic materials

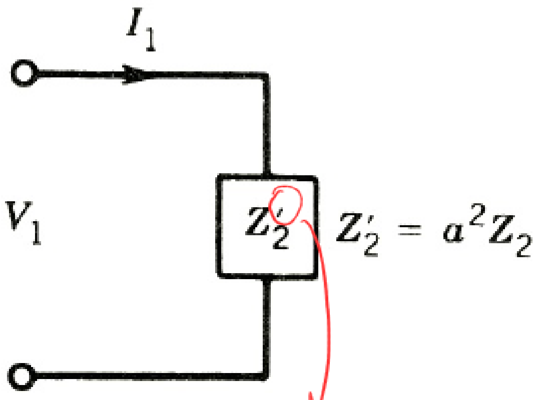
$$B_{peak} = \frac{\Phi_{peak}}{A_c} \approx 1\ldots1.5\,\mathrm{T}$$

Properties of Ideal Transformer

- Scale voltages and currents
- Impedance transfer:

$$Z_2 = \frac{V_2}{I_2}$$





referred to primary side

$$Z_1 = \frac{V_1}{I_1} = $$

$$= \frac{N_1^2}{N_2^2}\frac{V_2}{I_2}$$

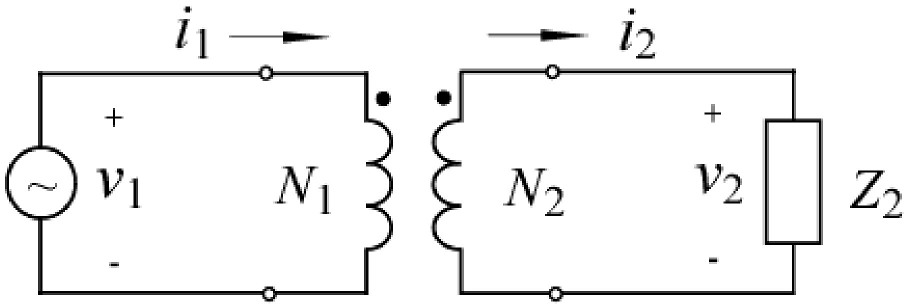
$$= a^2 Z_2$$

$$V_1 = \frac{N_1}{N_2}V_2$$

$$I_1 = \frac{N_2}{N_1}I_2$$

$$Z_1 = Z_2'$$

Ideal Transformer Impedance Transfer



$$V_1 = \frac{N_1}{N_2} V_2 \quad \text{and} \quad I_1 = \frac{N_2}{N_1} I_2$$

$\frac{N_1}{N_2} \to a$, $\frac{N_2}{N_1} \to \frac{1}{a}$

$$\frac{V_1}{I_1} = \frac{V_2}{I_2}\left(\frac{N_1}{N_2}\right)^2 = \frac{V_2}{I_2} a^2$$

$$Z_1 = Z_2 a^2 = Z_2'$$

Example 1: Consider ideal transformer loaded with impedance Z_2 = 1 + j4 Ω. Assume the turns ration a = 5, and source voltage V_1 = 120V(rms)

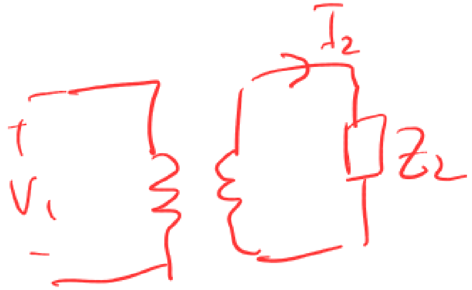
$\Rightarrow V_1 = 120\angle 0^\circ$ → assume angle = ϕ

1) Determine secondary current; 2) Determine primary current

1) $I_2 = \frac{V_2}{Z_2} = \frac{V_1}{a \cdot Z_2} = \frac{120}{5(1 + j\cdot 4)} = 1.412 - j\cdot 5.647\ A$ (rms)

2) $I_1 = \frac{I_2}{a} = 0.28 - j\cdot 1.13\ A$ (rms) (option 1)

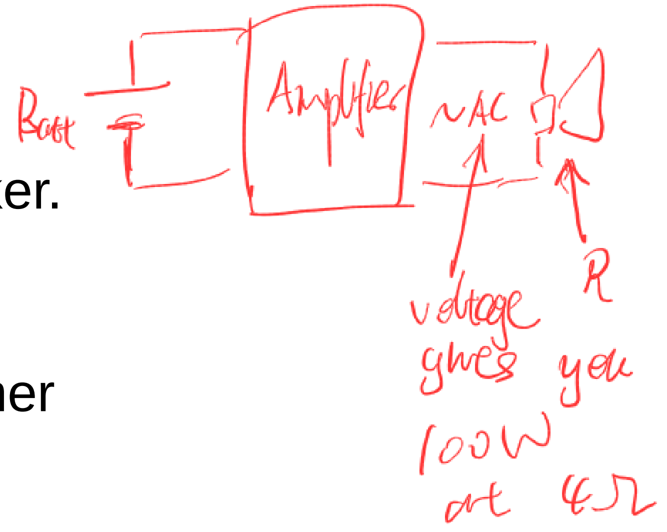
$I_1 = \frac{V_1}{a^2 \cdot Z} = \frac{120}{25 + j\cdot 100} = 0.28 - j\cdot 1.13\ A$ (rms) (option 2)



Ideal Transformer Example 2

Consider a 100W audio (voltage) amplifier designed to work with 4Ω speaker.
Assume you have only 8Ω speaker

1) Determine the output power
2) Determine the turns-ratio for appropriate matching impedance transformer

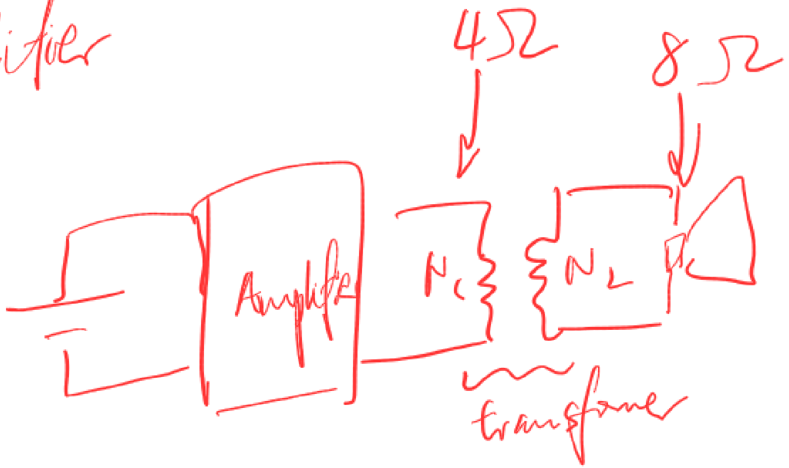


Assume the amplifier is voltage-source output

$$P = i \cdot V = \frac{V^2}{R}$$

$$V = \sqrt{P \cdot R} = \sqrt{400} = 20V$$ → output of the amplifier

$$P = \frac{V^2}{R} = \frac{400}{8} = 50W$$



$$R_2' = R_2 \cdot a^2 = 4\Omega$$

$$a^2 = \frac{4}{8} = \frac{1}{2}, \quad a = \frac{1}{\sqrt{2}}$$

$N_1 = 100$ turns, $N_2 = \sim 141$ turns

Looking ahead…

- Continue **Module 2**