ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 5

January 20, 2026

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Course Logistics

- Lab 0 and Lab 1 materials posted
  - There will be a safety quiz during Lab 0
  - Pre-Labs: Bring them to actual lab so TA can check you off, but also include them in your lab report submission
  - Reminder: Lab 0 starts next week – go to your enrolled sections

- Assignment 2 will be posted this Thursday
  - Covers magnetic circuits (Module 1, Part 2)

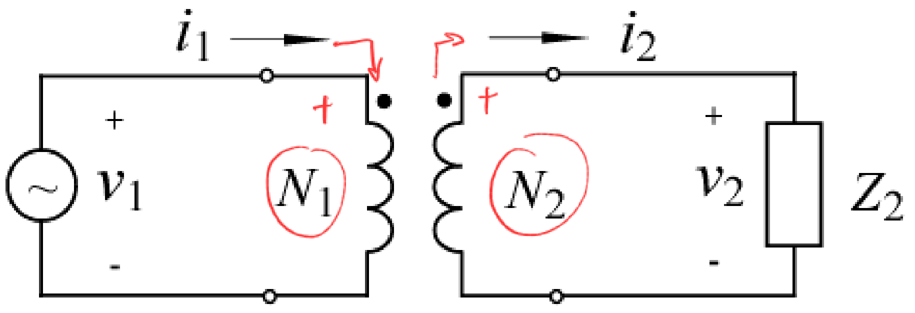
- Quiz 1
  - Coverage: up to and including end of Module 1
  - Date: January 30 (Friday) at the beginning of the tutorial
  - Closed book and notes
  - Calculators allowed

Module 2: Transformers

TX or TF

Ideal Transformer Impedance Transfer

$a = \frac{N_1}{N_2}$



$$V_1 = \frac{N_1}{N_2} V_2 \qquad I_1 = \frac{N_2}{N_1} I_2$$

$$Z_1 = Z_2 a^2 = Z_2'$$

Example 1: Consider ideal transformer loaded with impedance $Z_2 = 1 + j4\ \Omega$. Assume the turns ration a = 5, and source voltage $V_1 = 120V(rms)$ → $120\angle 0°$

1) Determine secondary current; 2) Determine primary current

1) $I_2 = \frac{V_2}{Z_2} = \frac{V_1 \rightarrow V_2}{a \cdot Z_2} = \frac{120}{5(1+j\cdot 4)} = 1.412 - j\cdot 5.647\ A$ (rms)

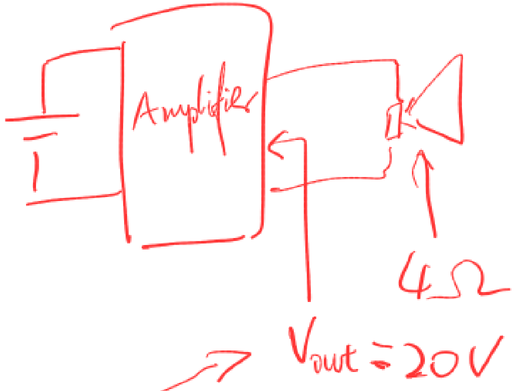
2) $I_1 = \frac{I_2}{a} = 0.28 - j\cdot 1.13\ A$ (rms)

$I_1 = \frac{V_1}{\underbrace{a^2 \cdot Z}_{Z_1'}} = \frac{120}{25 + j\cdot 100} = 0.28 - j\cdot 1.13\ A$ (rms)

Ideal Transformer Example 2

Consider a 100W audio (voltage) amplifier designed to work with 4Ω speaker.
Assume you have only 8Ω speaker

1) Determine the output power
2) Determine the turns-ratio for appropriate matching impedance transformer

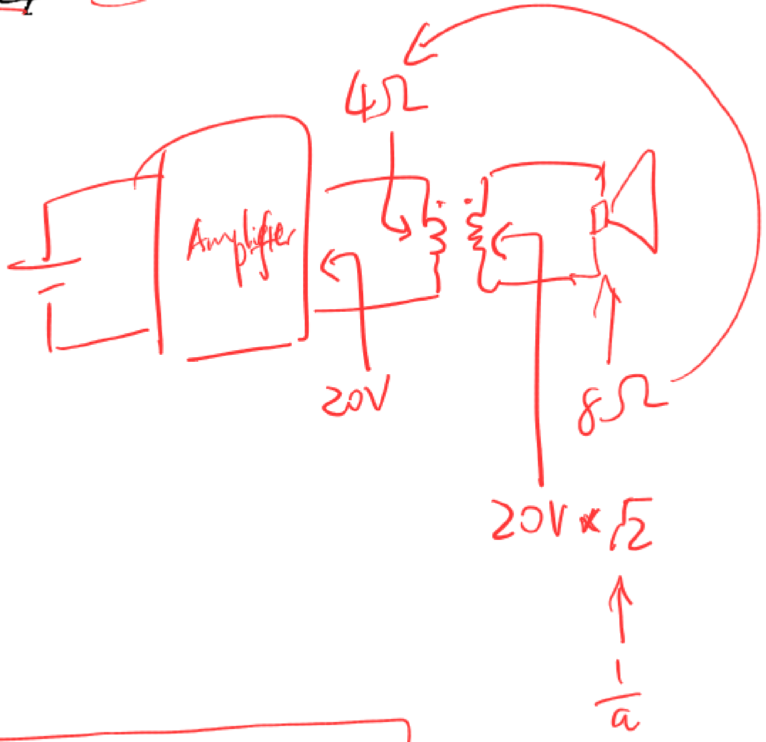


Assume the amplifier is voltage-source output

$P = i \cdot V = \frac{V^2}{R}$;

$V = \sqrt{P \cdot R} = \sqrt{400} = 20V$

$$P = \frac{\overset{\to 20V}{V^2}}{\underset{\to 8\Omega}{R}} = \frac{400}{8} = 50W$$



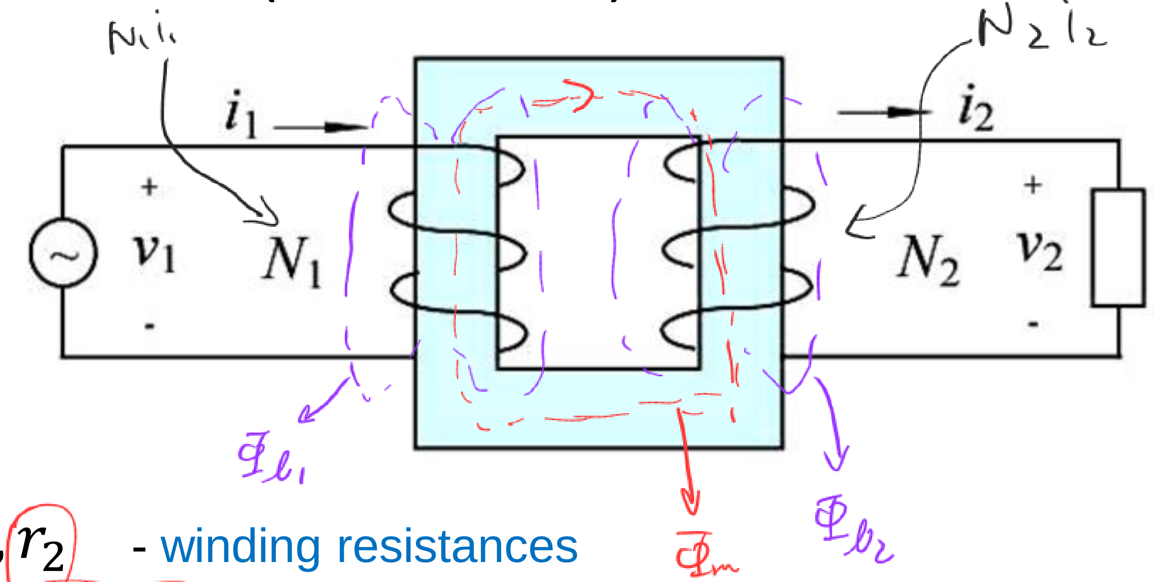
$R_2' = R_2 \cdot a^2 = 4\Omega$

$a^2 = \frac{4}{8} = \frac{1}{2}$; $a = \frac{1}{\sqrt{2}}$

$\boxed{N_1 = 100 \text{ turns} \qquad N_2 = 141 \text{ turns}}$

Non-Ideal (Practical) Transformer

$-\mathcal{R}_m \neq \phi$



r_1, r_2 - winding resistances

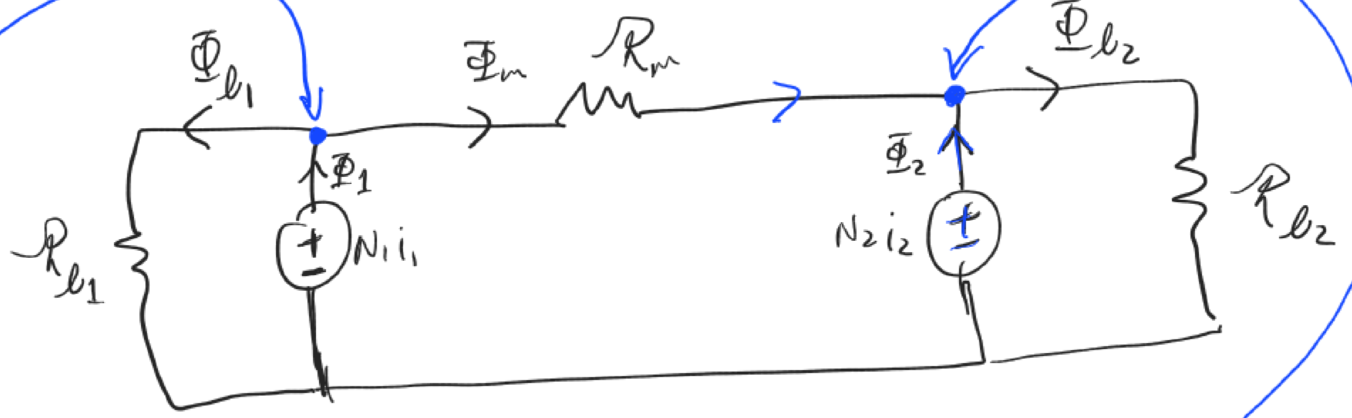
Φ_m - mutual or magnetizing flux (goes inside the core and links both windings)

Φ_{l1}, Φ_{l2} - flux leakage (goes in the air and links only one winding)

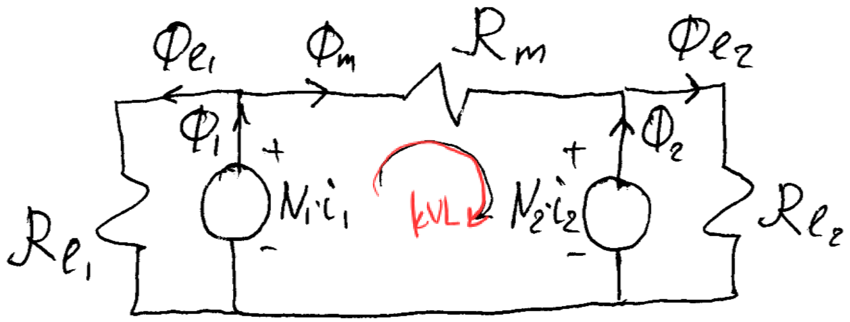
$$\boxed{\Phi_1 = \Phi_m + \Phi_{l_1}}$$

$$\boxed{\Phi_2 = -\Phi_m + \Phi_{l_2}}$$

Equivalent Magnetic Circuit:



Non-Ideal (Practical) Transformer



Consider Fluxes

KVL

$$\Phi_1 = \Phi_{l1} + \Phi_m = \frac{N_1 i_1}{\Re_{l1}} + \frac{N_1 i_1 - N_2 i_2}{\Re_m}$$

$$\Phi_2 = \Phi_{l2} - \Phi_m = \frac{N_2 i_2}{\Re_{l2}} - \frac{N_1 i_1 - N_2 i_2}{\Re_m}$$

Consider Flux Linkages

$$\lambda_1 = N_1\Phi_1 = \frac{N_1^2}{\Re_{l1}} i_1 + \frac{N_1^2}{\Re_m} i_1 - \frac{N_1 N_2}{\Re_m} i_2$$

$$= L_{l1} i_1 + L_{m1} i_1 - L_{m12} i_2$$

contribution to λ_1 by i_2

$$\lambda_2 = N_2\Phi_2 = \frac{N_2^2}{\Re_{l2}} i_2 + \frac{N_2^2}{\Re_m} i_2 - \frac{N_2 N_1}{\Re_m} i_1$$

$$= L_{l2} i_2 + L_{m2} i_2 - L_{m21} i_1$$

contribution to λ_2 by i_1

Recall inductance

$$L = \frac{N^2}{\Re}$$

$L_{\ell_1} = \frac{N_1^2}{\mathcal{R}_{\ell_1}}$ $\quad L_{\ell_2} = \frac{N_2^2}{\mathcal{R}_{\ell_2}}$ (leakage)

$L_{m_1} = \frac{N_1^2}{\mathcal{R}_m}$ $\quad L_{m_2} = \frac{N_2^2}{\mathcal{R}_m}$ (core inductance)

$L_{m_{12}} = L_{m_{21}} = \frac{N_1 N_2}{\mathcal{R}_m}$

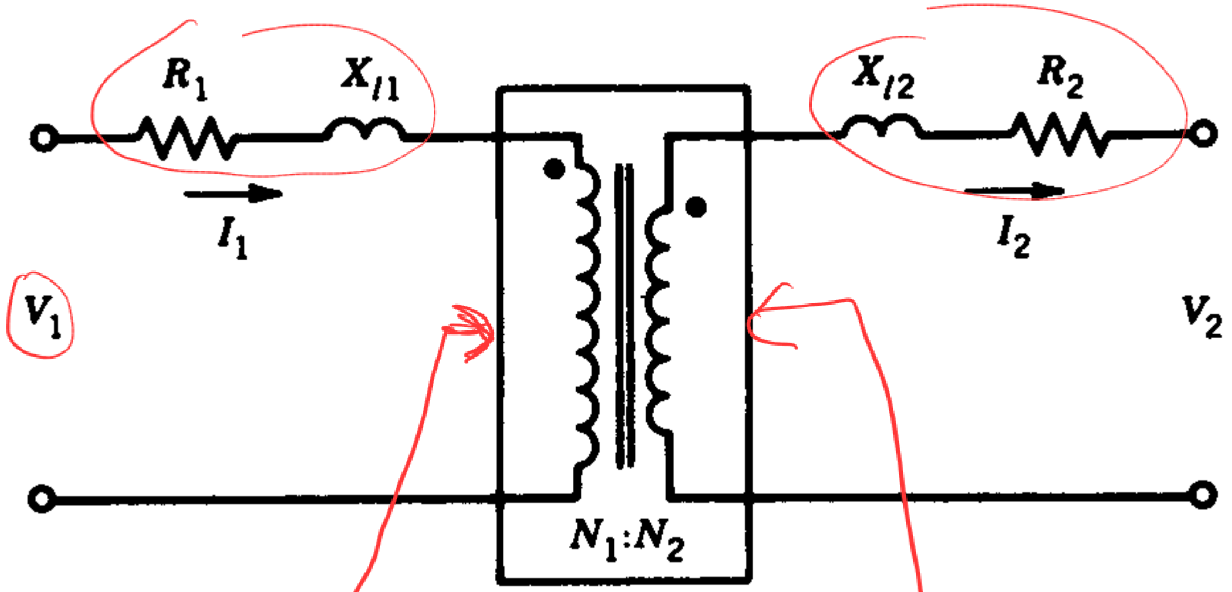
mutual inductance

T – Equivalent Circuit for Practical Transformer

Define reactance $X = L\omega, \left[H \dfrac{rad}{\sec} = \Omega \right]$

$X_{l1} = L_{l1}\omega$
$X_{l2} = L_{l2}\omega$ } leakage inductance

R_1 and R_2 → winding resistance

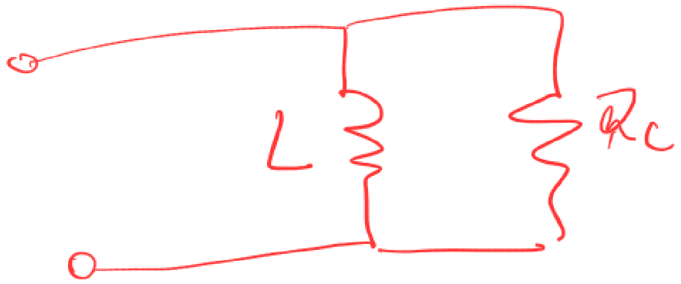


Tightly coupled transformer
(ideal TF)

($V_1 f$) e_1

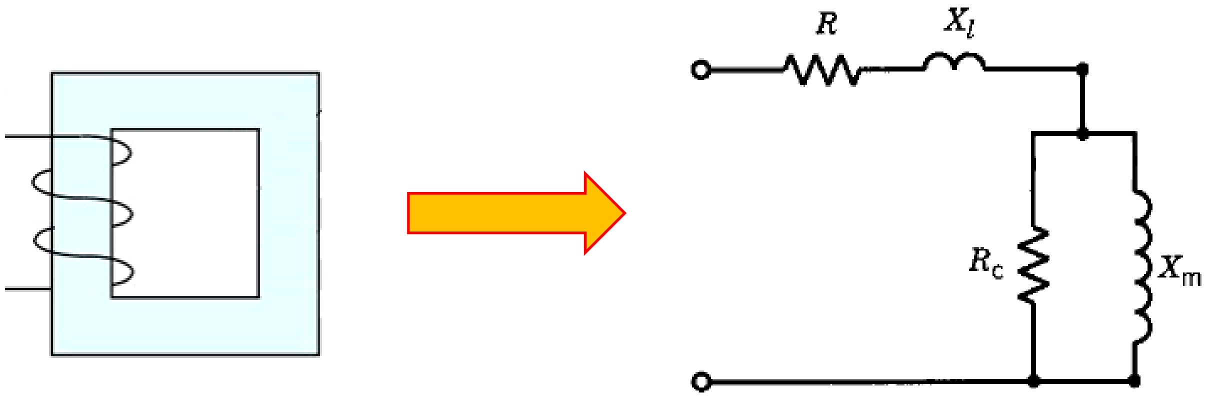
e_2

How to model core non-idealities?



T – Equivalent Circuit for Practical Transformer

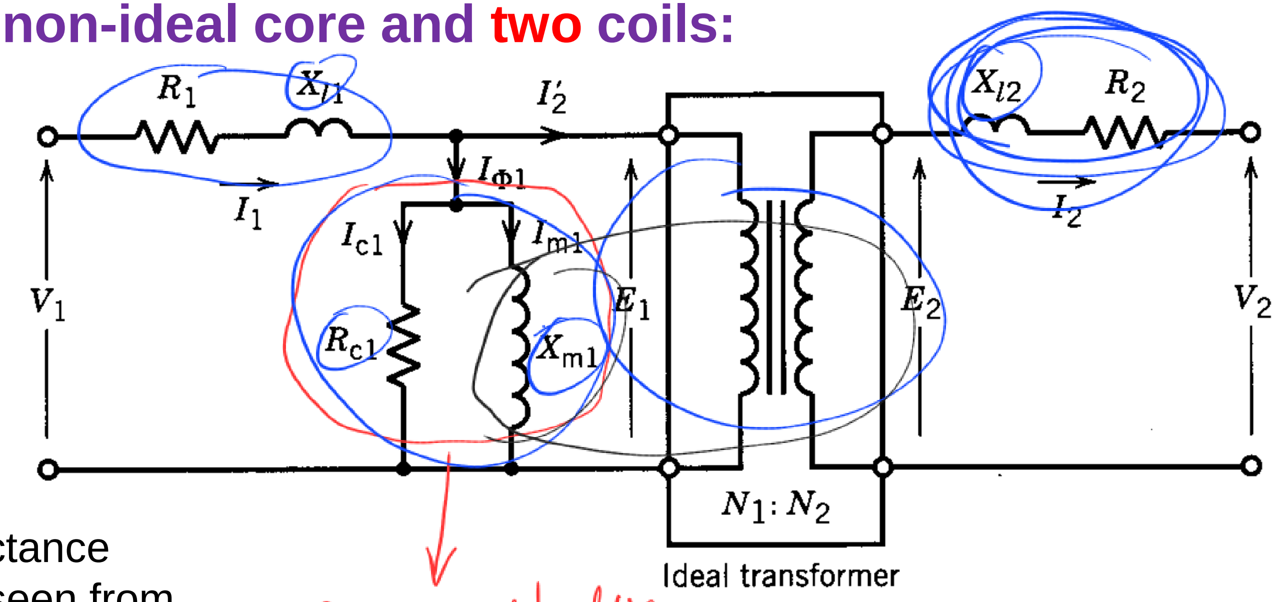
➢ **Recall the equivalent circuit (model) of a non-ideal core and one coil:**



Use reactances instead or inductances (to obtain impedances in Ohms)

$$\boxed{X = L\omega}$$

➢ **Model of a non-ideal core and two coils:**



Core nonidealties

where

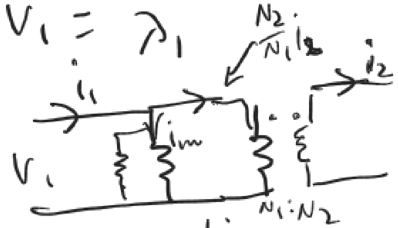
$$X_{m1} = L_{m1}\omega$$

magnetizing reactance (inductance) as seen from the primary side

Ignore leakage:

$\lambda_1 = L_{m_1} i - L_{m_{12}} i_2$

$v_1 = \dot{\lambda}_1$



$v_1 = L_m \frac{di_m}{dt}$ KCL

$= L_m \frac{d}{dt}\left(i_1 + \frac{N_2}{N_1} i_2\right)$

$= L_m \frac{d}{dt} i_1 + L_m \frac{N_2}{N_1} \frac{d}{dt} i_2$

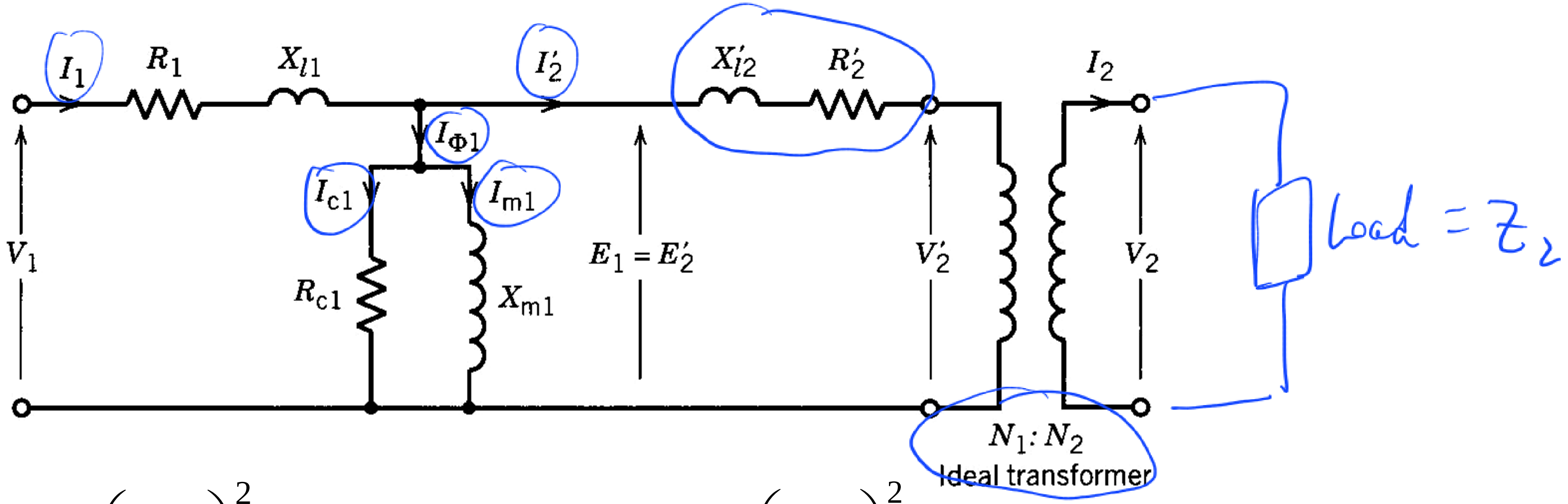
$= \frac{N_1^2}{\mathcal{R}_m} \frac{d}{dt} i_1 + \frac{N_1 N_2}{\mathcal{R}_m} \frac{d}{dt} i_2$

L_{m_1} $\qquad$ $L_{m_{12}}$

(sign not strictly followed)

T – Equivalent Circuit for Practical Transformer

Move the Secondary Side parameters to the Primary Side



where $R_2' = R_2\left(\dfrac{N_1}{N_2}\right)^2 = R_2 a^2$ and $X_{l2}' = X_{l2}\left(\dfrac{N_1}{N_2}\right)^2 = X_{l2} a^2$

R_{c1} - represents core losses (Eddy current + Hysteresis)

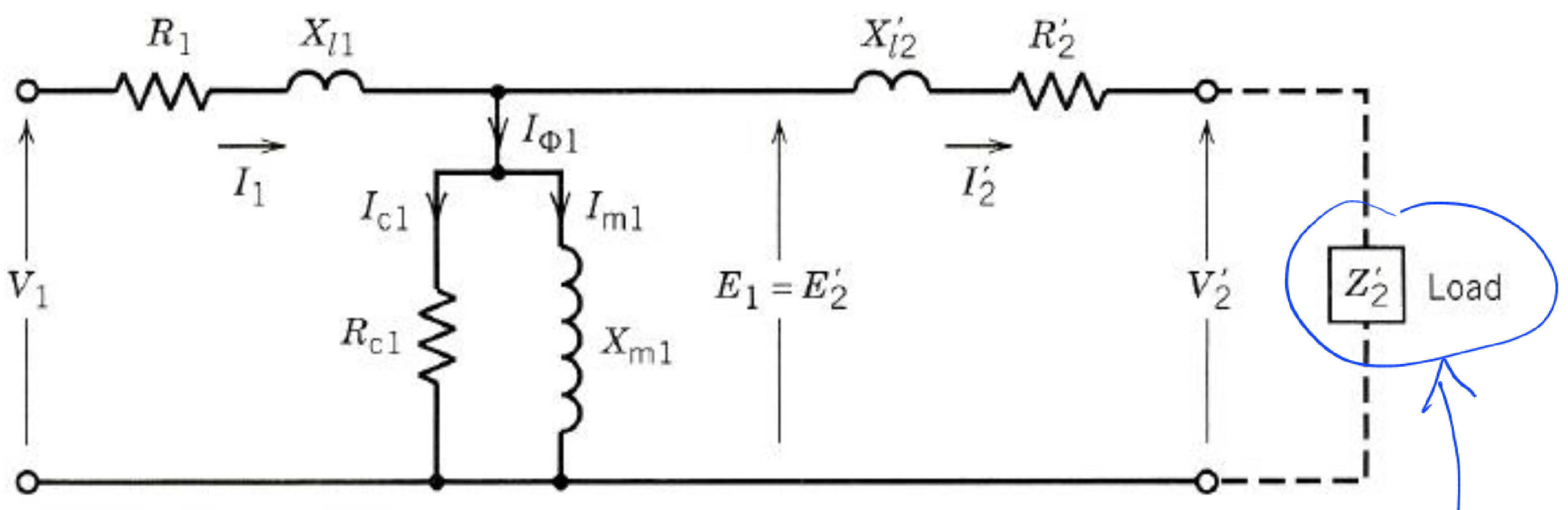
$I_{\Phi 1}$ - exciting current

I_{m1} - magnetizing current

T – Equivalent Circuit for Practical Transformer

❑ **Equivalent circuit (model) referred to the primary side:**

➢ **Transfer (re-scale) the secondary variables/parameters to the primary:**



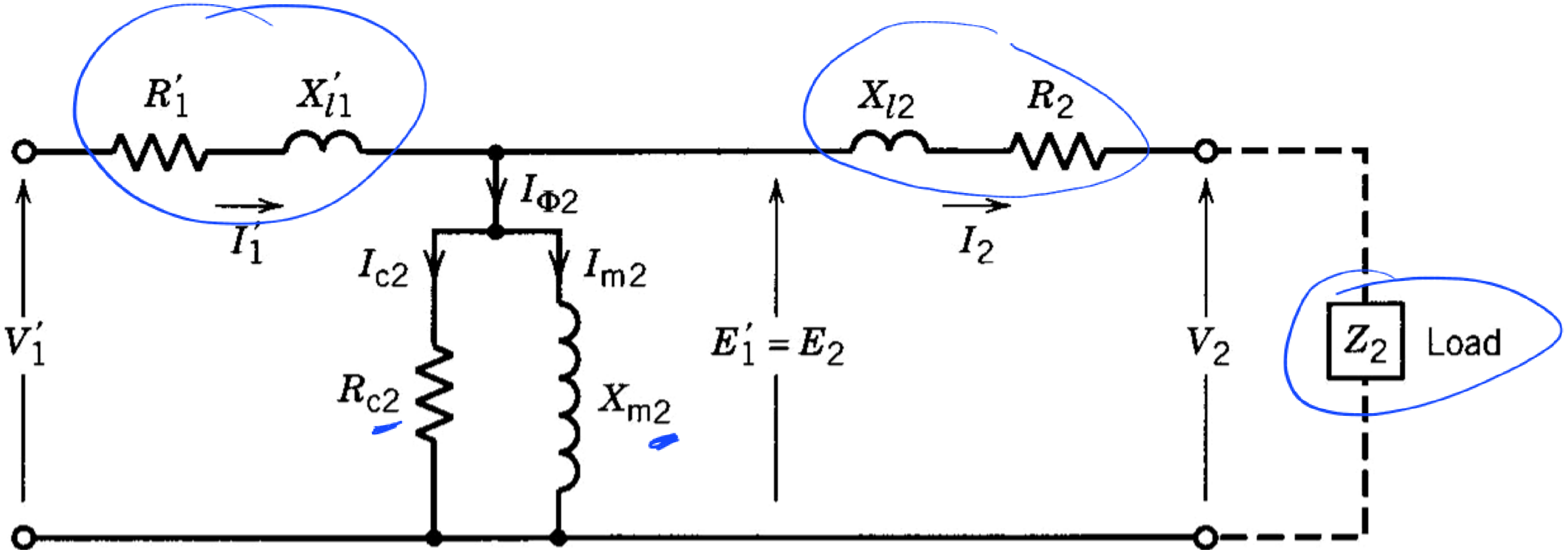
$$\begin{cases} E_2' = aE_2 = E_1 \\ V_2' = aV_2 \\ I_2' = I_2 / a \end{cases}$$

$$\begin{cases} X_{l2}' = a^2 X_{l2} \\ R_2' = a^2 R_2 \\ Z_2' = a^2 Z_2 \end{cases}$$

$$\boxed{X = L\omega}$$

T – Equivalent Circuit for Practical Transformer

- **Equivalent circuit (model) referred to the secondary side:**
- **Transfer (re-scale) the primary variables/parameters to the secondary:**



$$\begin{cases} E_1' = E_1 / a = E_2 \\ V_1' = V_1 / a \\ I_1' = aI_1 \end{cases}$$

$$\begin{cases} X_{l1}' = X_{l1} / a^2 \\ R_1' = R_1 / a^2 \\ X_{m2} = X_{m1} / a^2 \\ R_{c2} = R_{c1} / a^2 \end{cases}$$

$$\boxed{X = L\omega}$$

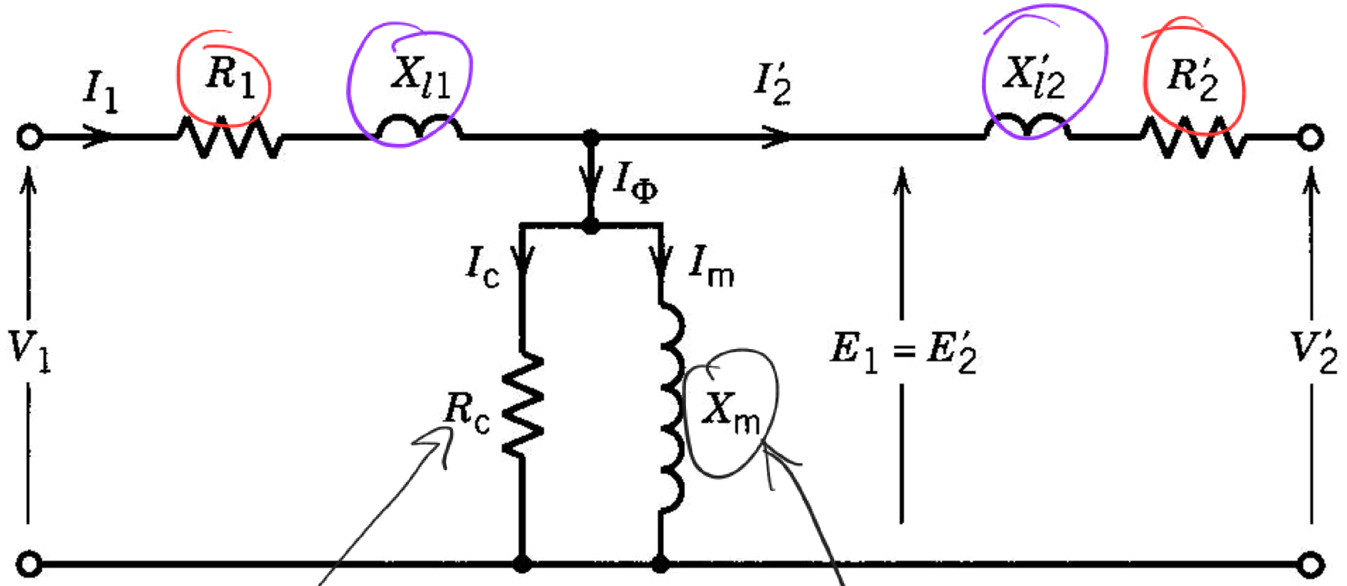
T – Equivalent Circuit for Practical Transformer

Most commonly considered **T- equivalent circuit** (Referred to Primary)

for consistency

Resistances of the primary and secondary windings

Leakage reactances of the primary and secondary windings



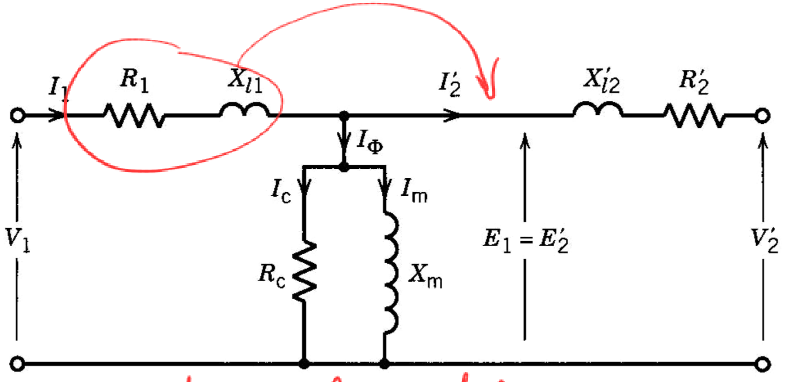
Core losses

Magnetizing reactance

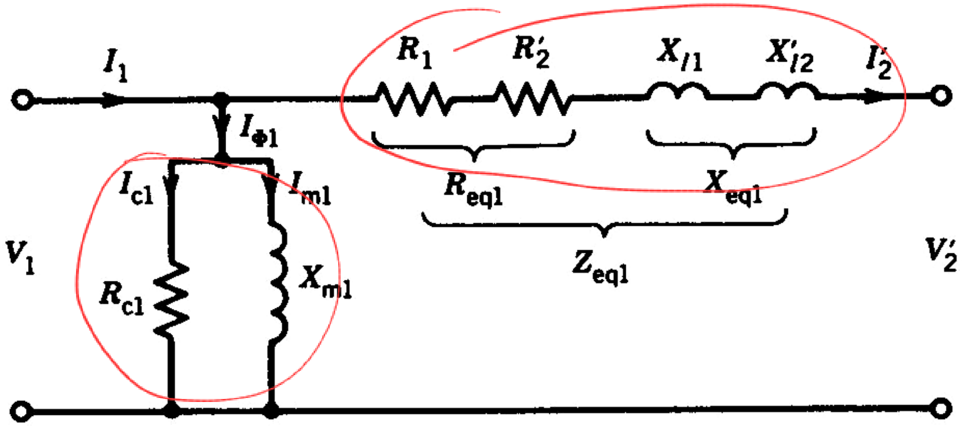
Approximate Equivalent Circuits

Not 100% accurate

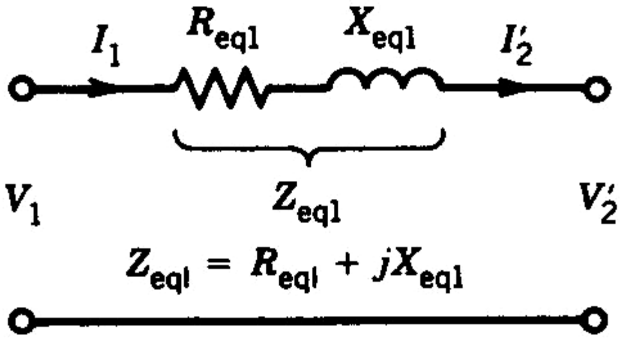
Sometimes used to simplify (approximate) calculations



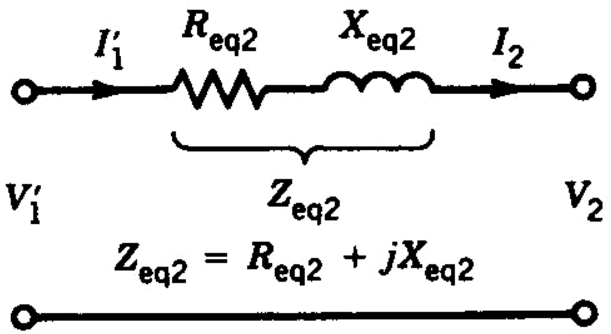
Lumped model



$V_2' = aV_2, \quad I_2' = I_2/a$



ignored shunt branch

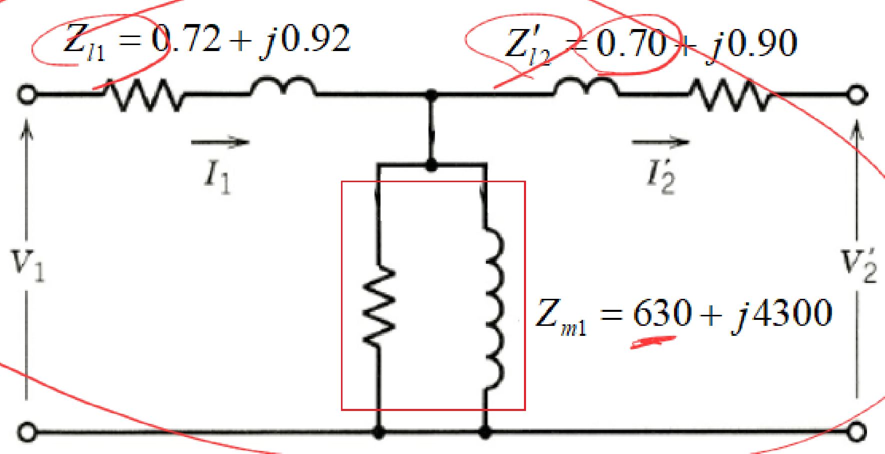


Example: T – Equivalent Circuit for Practical Transformer

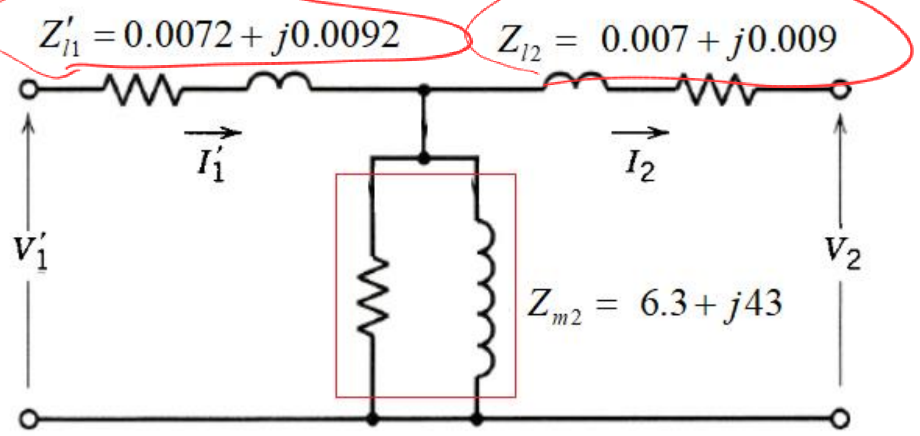
- **Example 4:** Consider a 50 kVA, 2400/240 V transformer with leakage impedances Z_{l1}=0.72+j0.92 Ω, Z_{l2}= 0.007+j0.009 Ω, and shunt impedance combined as Z_{m2}= 6.3+j43 Ω.
- Draw the equivalent circuits referred to the HV and LV sides and label parameters.
- **Solution:**

Compute turns ratio: $$a = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{2400}{240} = 10$$

Model Referred to HV Side



Model Referred to LV Side



Tests for Determination of Parameters

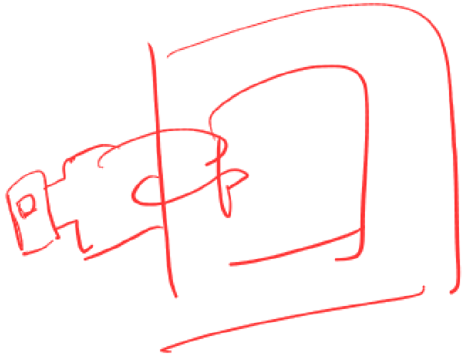
How to find TF parameters without a datasheet?

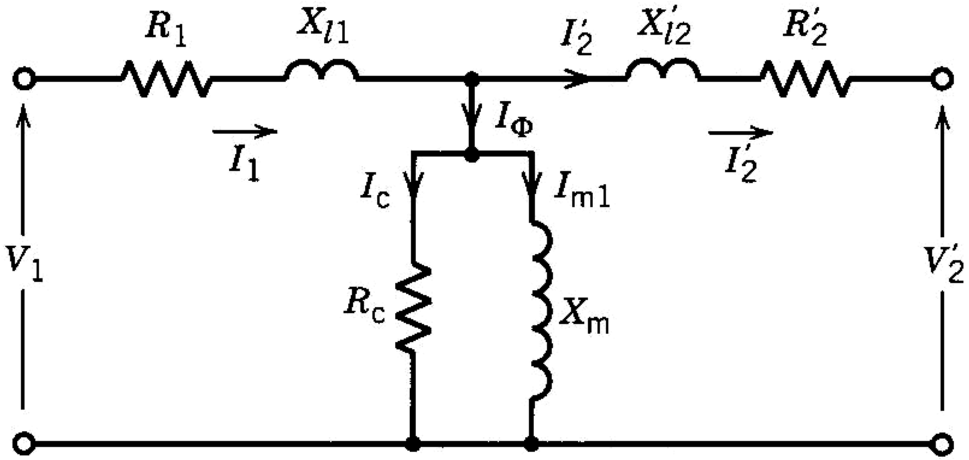
→ Rc? Not impacted
→ AC flux

1) Use DC Measurements to get R1 and R2

2) Open-Circuit (OC) No Load Test to get Rc and Xm

3) Short-Circuit (SC) Test to get Req and Xeq





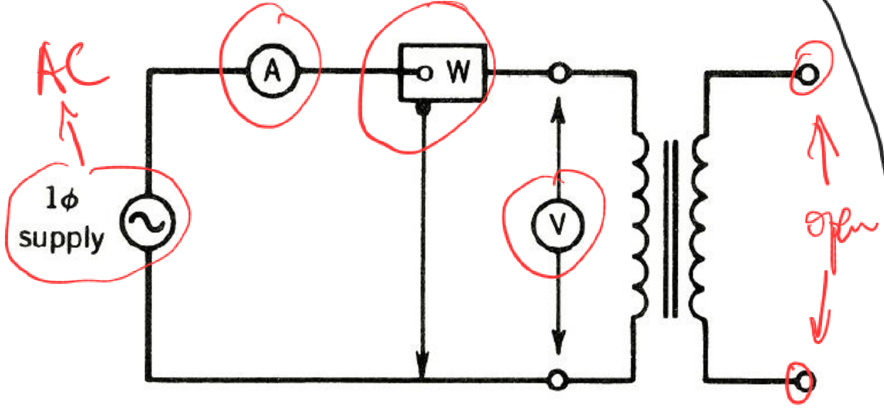
Tests for Determination of Parameters

How to Empirically find nominal voltage?

$V \to \lambda \to i$

When saturated: small V increase induces large current increase

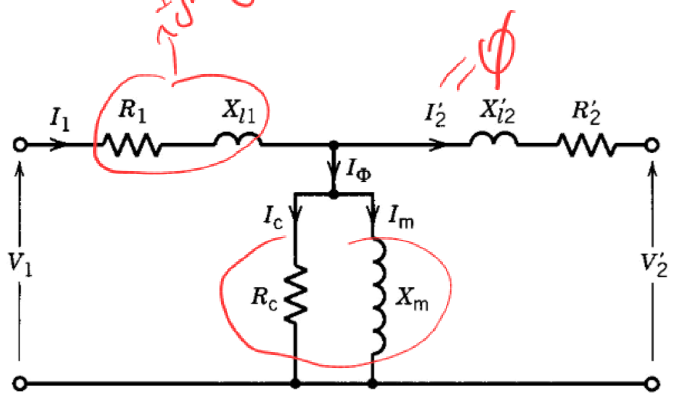
❑ Open-Circuit (OC) Test:

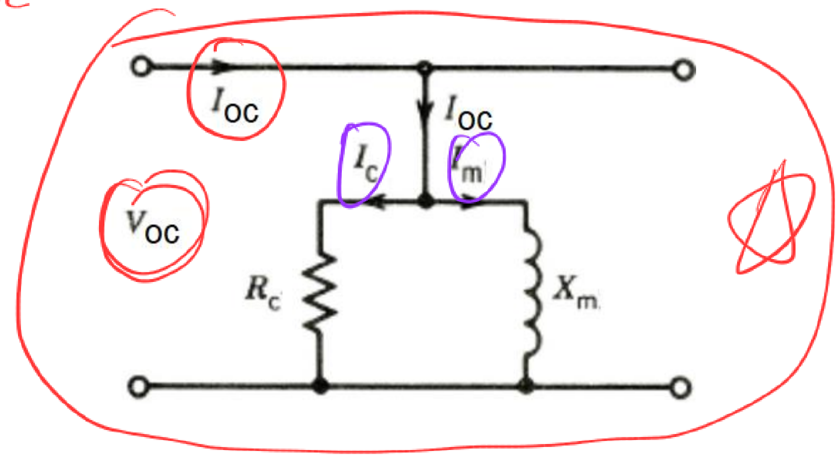


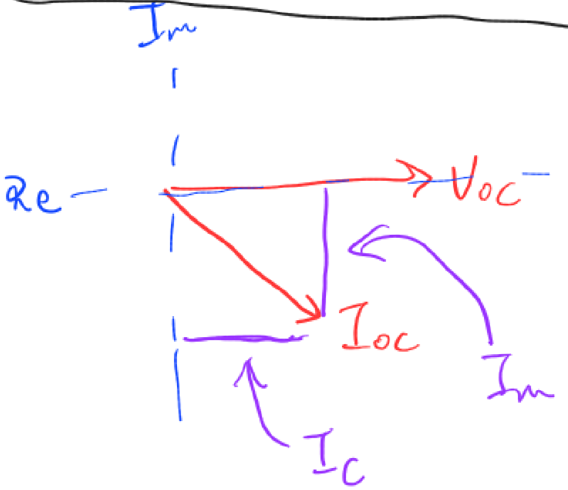
- Leave one side open circuit
- Apply nominal voltage to the other side
- Read voltmeter, ammeter, wattmeter
- Calculate parameters Rc and Xm

measure V_{oc}, I_{oc}, P_{oc}

Ignore







$$P_{oc} = \frac{V_{oc}^2}{R_c} \Rightarrow R_c = \frac{V_{oc}^2}{P_{oc}}$$

$$I_c = \frac{V_{oc}}{R_c} \Rightarrow I_m = \sqrt{I_{oc}^2 - I_c^2}$$

$$X_m = \frac{V_{oc}}{I_m}$$

$\to \omega L$

Assume shunt Z >> series Z

Tests for Determination of Parameters

❑ Short-Circuit (SC) Test:

Apply nominal current

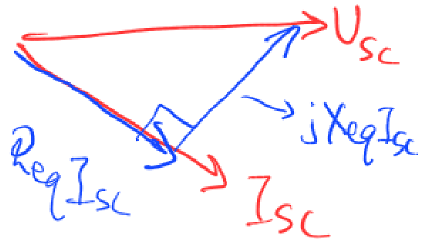
Short circuit one winding

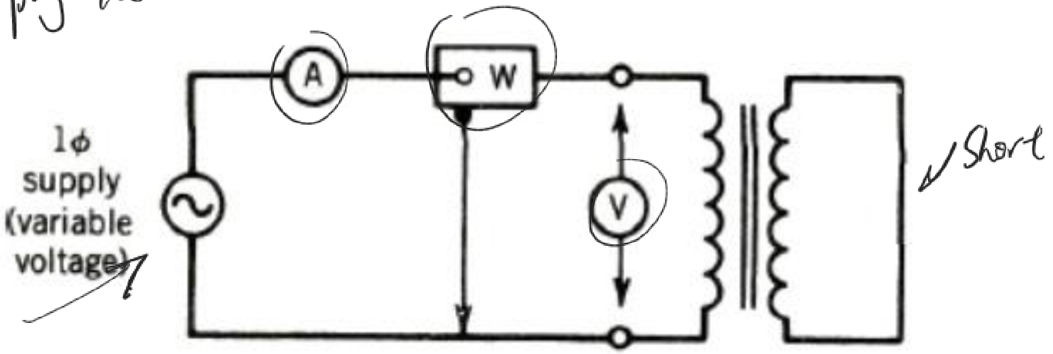
Apply nominal current to the other side

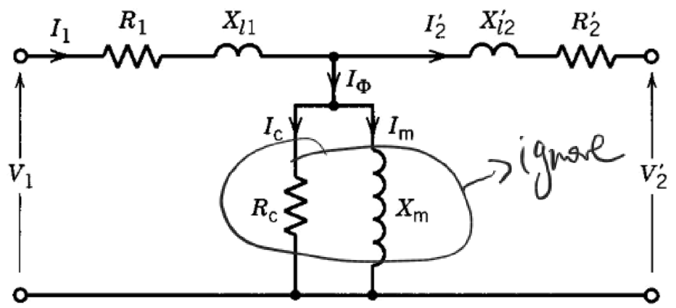
Read voltmeter, ammeter, wattmeter

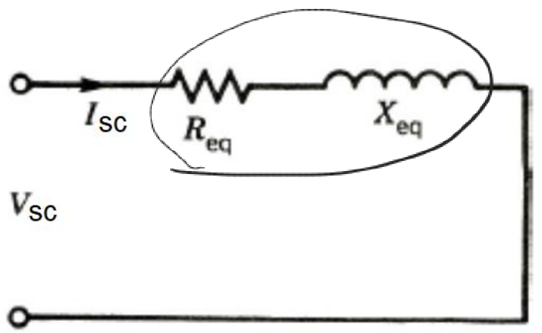
measure I_{sc}, V_{sc}, P_{sc}

Calculate parameters *Req* and *Xeq*









$$\begin{cases} X_{eq} = X_{l1} + X'_{l2} \\ R_{eq} = R_1 + R'_2 \end{cases}$$

$Z_{eq} = R_{eq} + jX_{eq}$

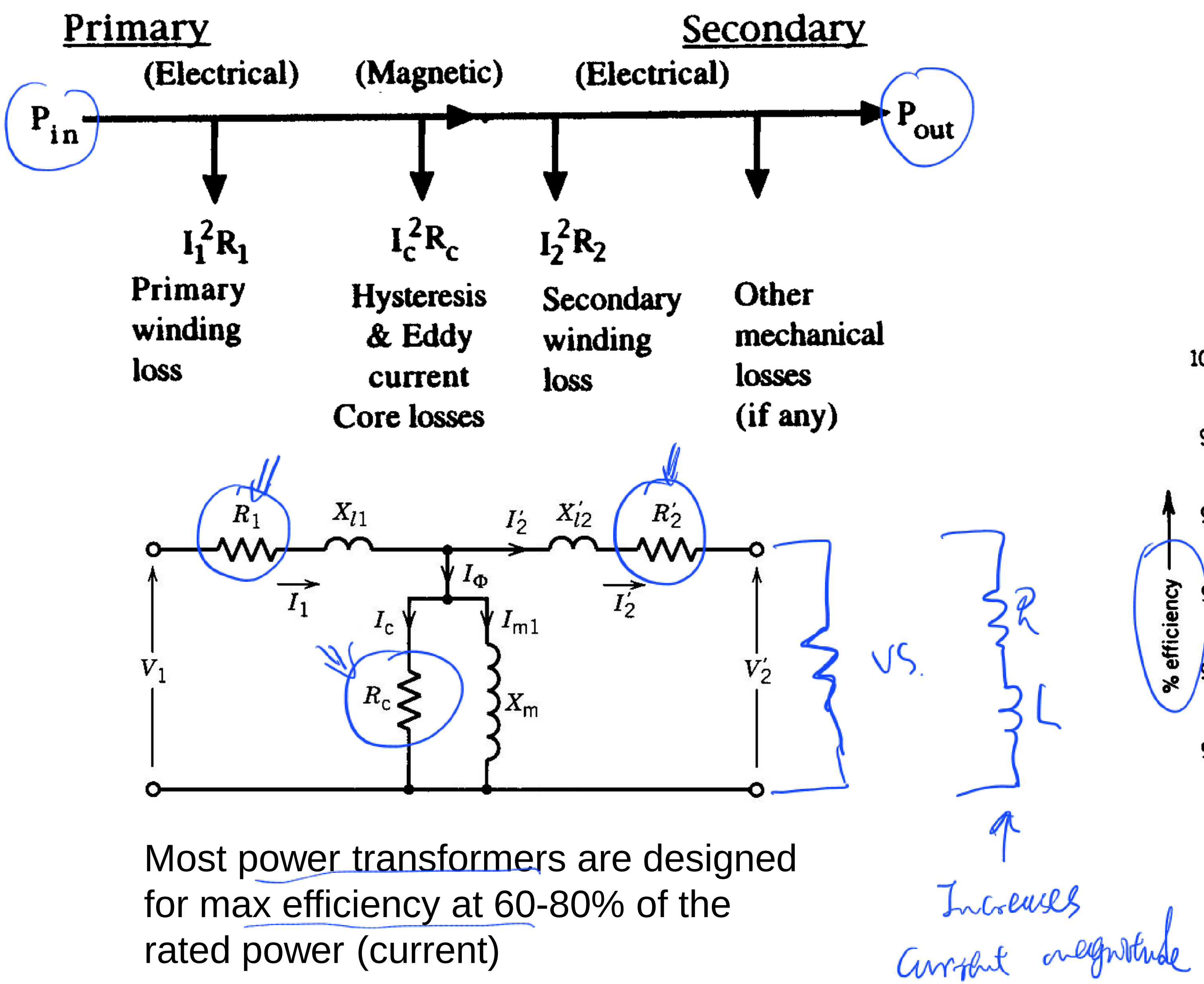
$$P_{sc} = I_{sc}^2 R_{eq} \quad \Rightarrow \quad R_{eq} = \frac{P_{sc}}{I_{sc}^2} \quad \Rightarrow \quad Z_{eq} = \frac{V_{sc}}{I_{sc}} \quad \Rightarrow \quad X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

We may assume

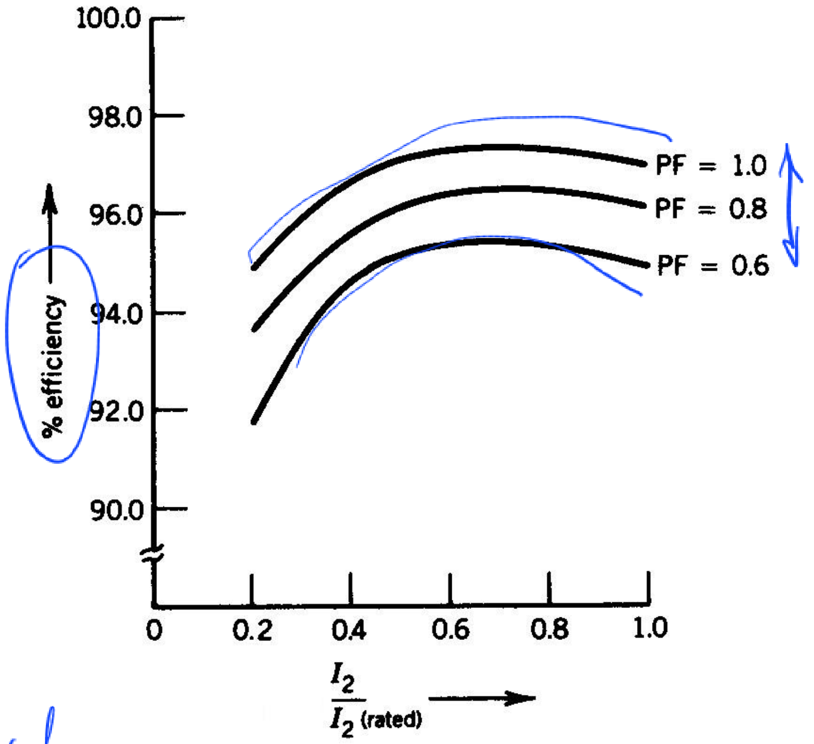
$$\begin{cases} X_{l1} = X'_{l2} = 0.5X_{eq} \\ R_1 = R'_2 = 0.5R_{eq} \end{cases}$$

Assume equal when referred to one side

Efficiency of a transformer



$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{core} + P_{resistive}}$$



Most power transformers are designed for max efficiency at 60-80% of the rated power (current)

Looking ahead…

- Finish **Module 2**