ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 6

January 22, 2026

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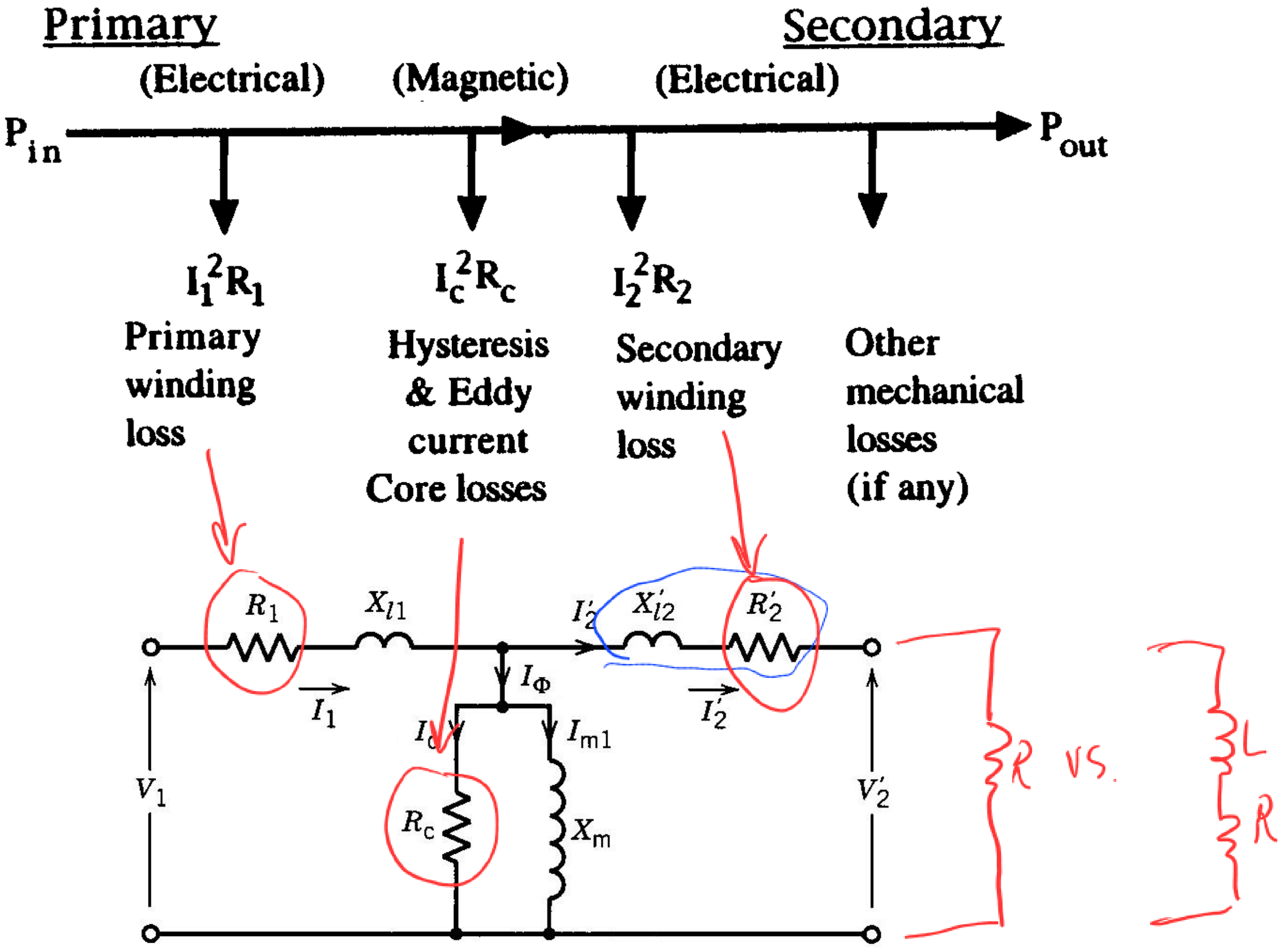
Course Logistics

- Lab 0 and Lab 1 materials posted
  - There will be a safety quiz during Lab 0
  - Pre-Labs: Bring them to actual lab so TA can check you off, but also include them in your lab report submission
  - Reminder: Lab 0 starts next week – go to your enrolled sections

- Assignment 2 posted
  - Covers magnetic circuits (Module 1, Part 2)

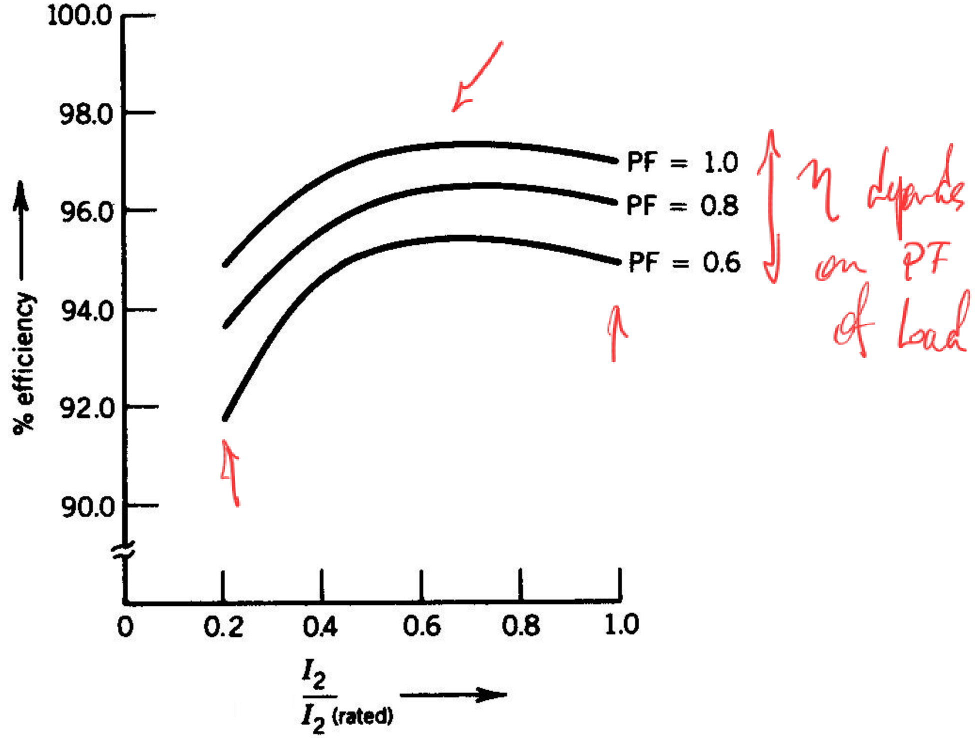
- Quiz 1
  - Coverage: up to and including end of Module 1
  - Date: January 30 (Friday) at the beginning of the tutorial
  - Closed book and notes
  - Calculators allowed

Efficiency of a transformer



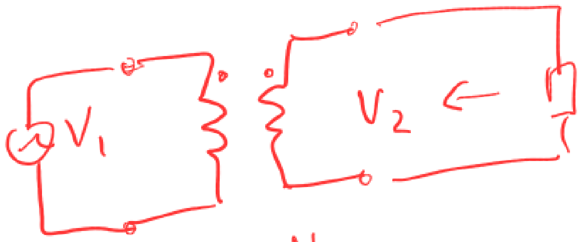
$$\eta = \frac{P_{out}}{P_{in}} =$$

$$= \frac{P_{out}}{P_{out} + P_{core} + P_{resistive}}$$



Most power transformers are designed for max efficiency at 60-80% of the rated power (current)

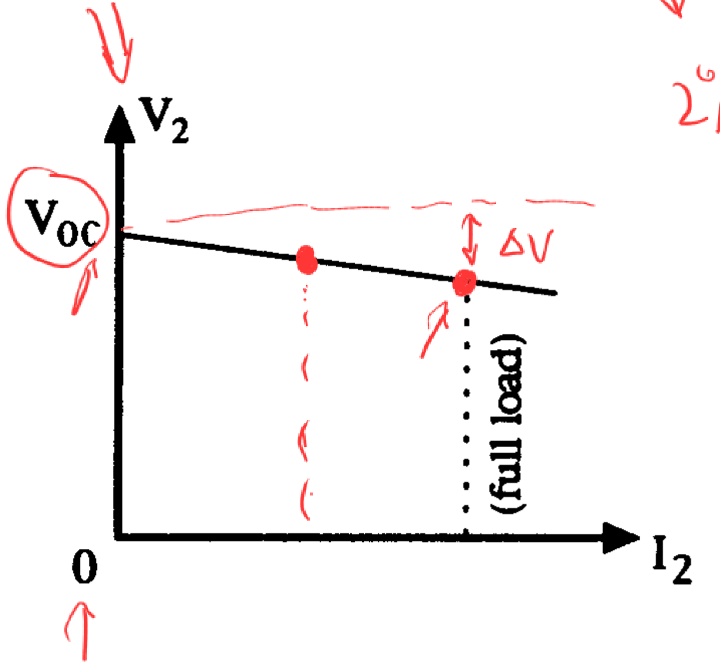
Voltage regulation limits of transformer



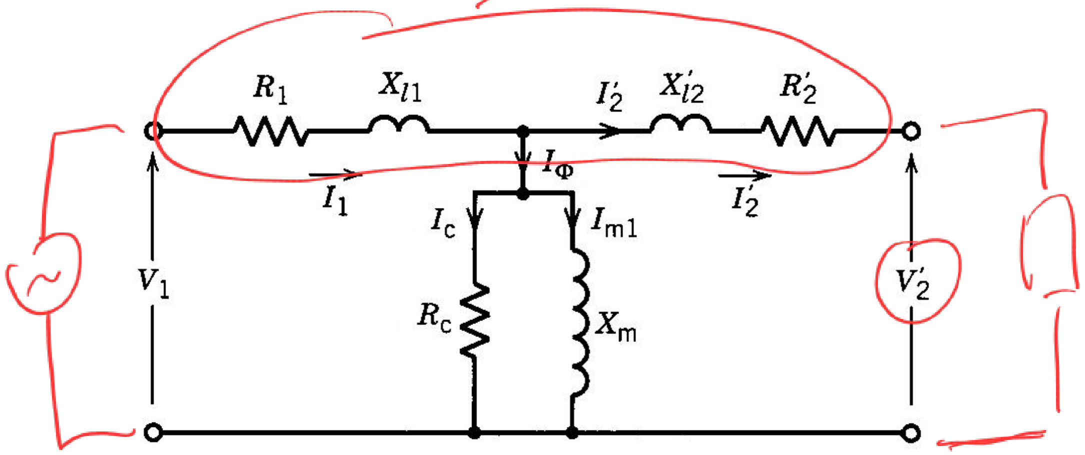
Transformer output (secondary) voltage changes with load:

$$VR = \frac{V_{2,oc} - V_{2,load}}{V_{2,load}} = \frac{V_{2,oc} - V_{2,rated}}{V_{2,rated}} 100\%$$

2% - 7%



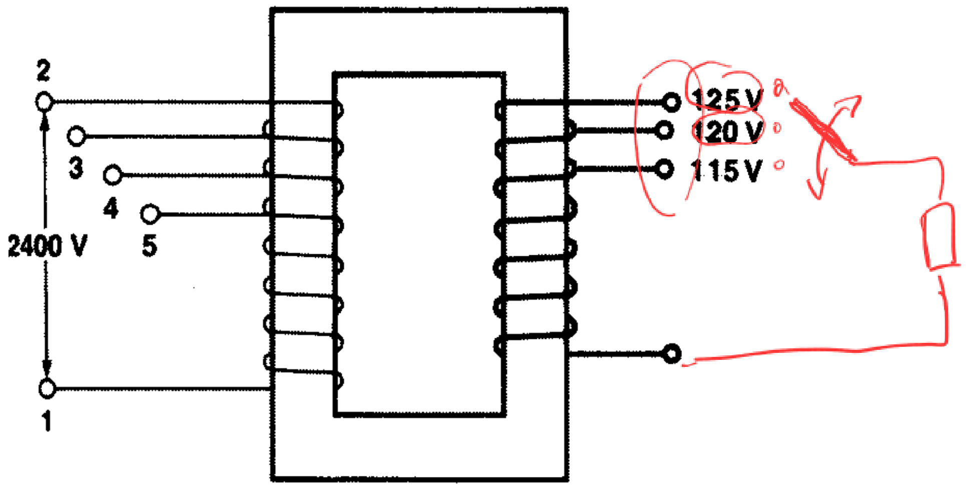
introduces voltage drop when current flows



Want VR as small as possible

Tap-Changing Capability and Multiple Winding Structures

Additional voltage control / regulation



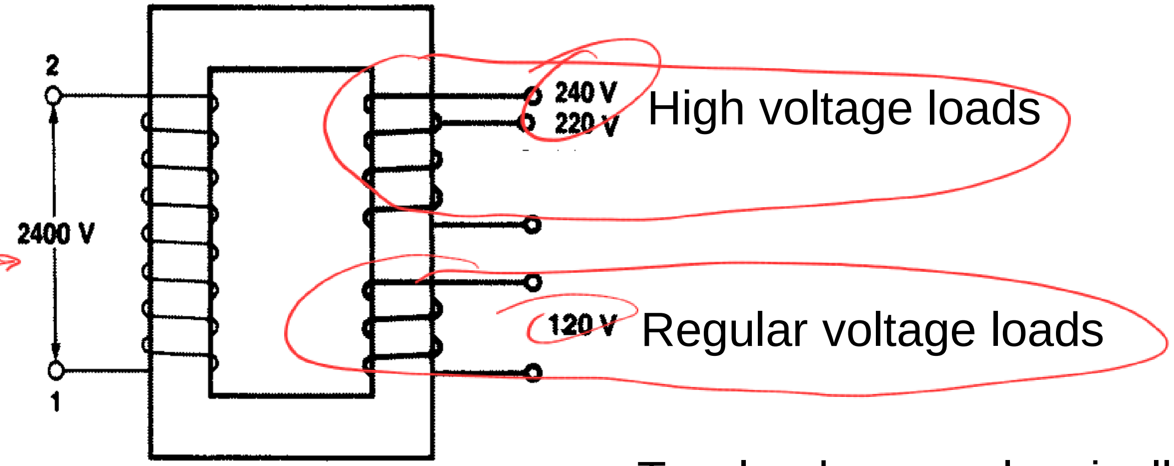
high-voltage terminals	percent tap	primary voltage	secondary voltage
1 - 2	0	2400 V	120 V
1 - 3	$4\frac{1}{2}$	2292 V	120 V
1 - 4	9	2184 V	120 V
1 - 5	$13\frac{1}{2}$	2076 V	120 V

Changing N_1 changes the overall turns-ration

Many substation & distribution transformers have taps
Taps often expressed in percent, such as -2%, +2%, +5%, etc.

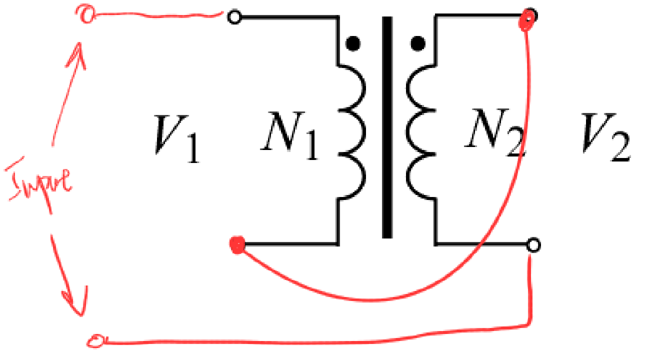
Additional windings provide different output voltages

Distribution side



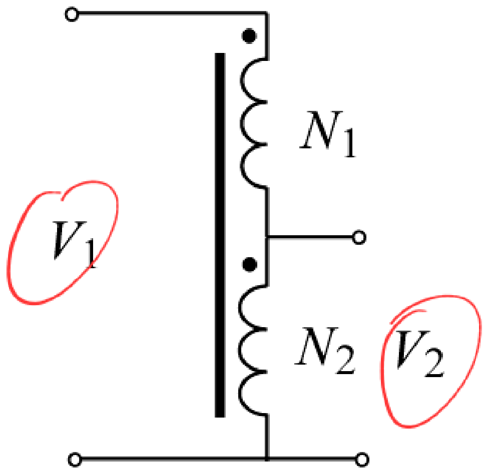
Two loads are galvanically decoupled

Autotransformers



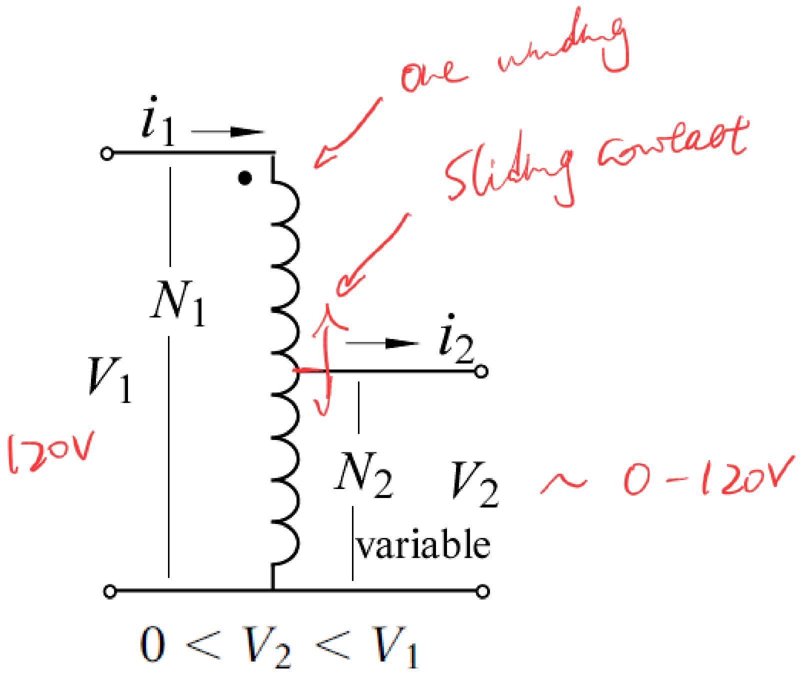
Regular transformer

$P_1 \simeq P_2$



Autotransformer

connect windings in series and use as voltage divider



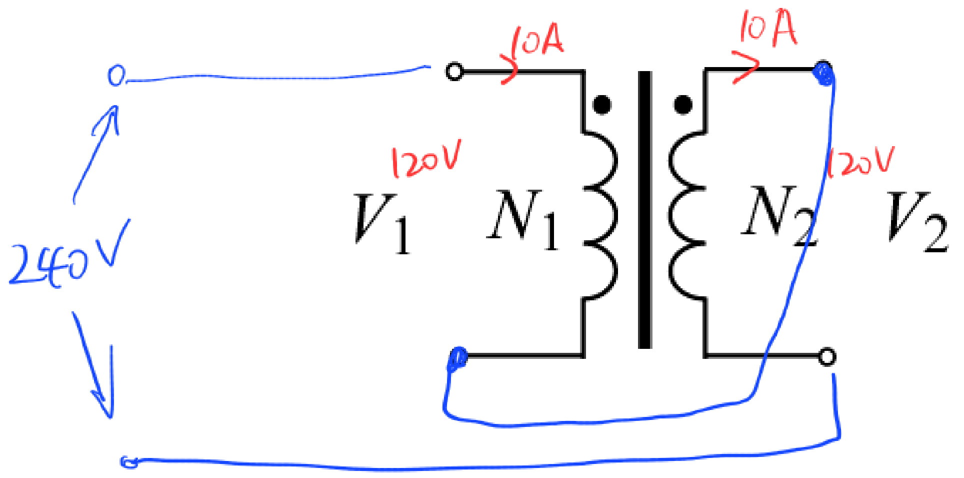
$0 < V_2 < V_1$

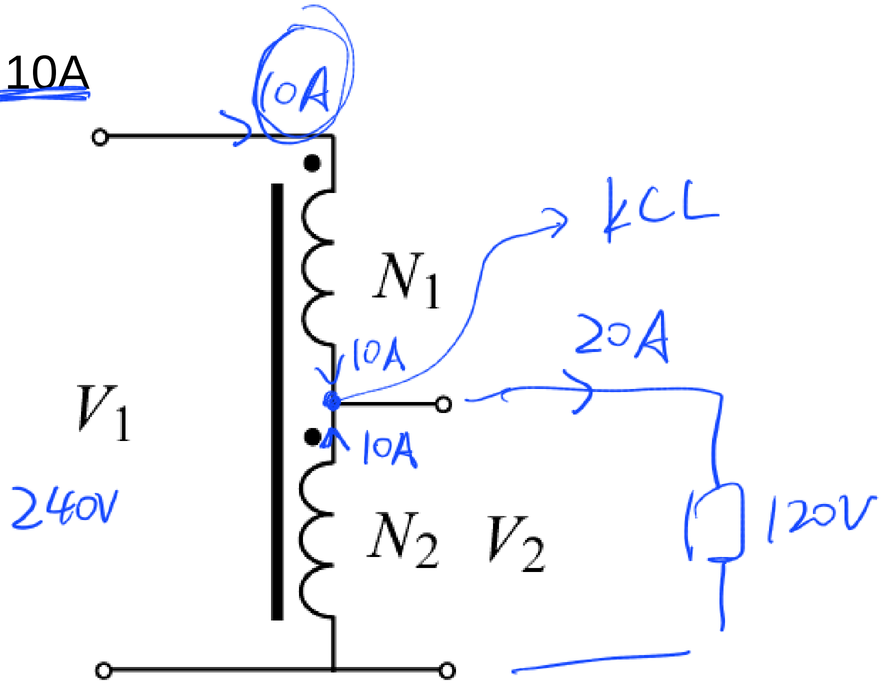
- No electric isolation between primary & secondary
- Safety is a concern
  120/110 or 240/120 is OK
  12kV/240 or 2.4kV/120 is not safe
- Smaller than regular transformers
- Economical (material, losses, etc.)

✓ Often used in Labs as a source of variable AC voltage

Autotransformer Example 1

Consider a 120/120 V, 1.2kVA isolation transformer with windings rated for 10A





Connect it as a 240/120 step-down autotransformer and determine its power rating

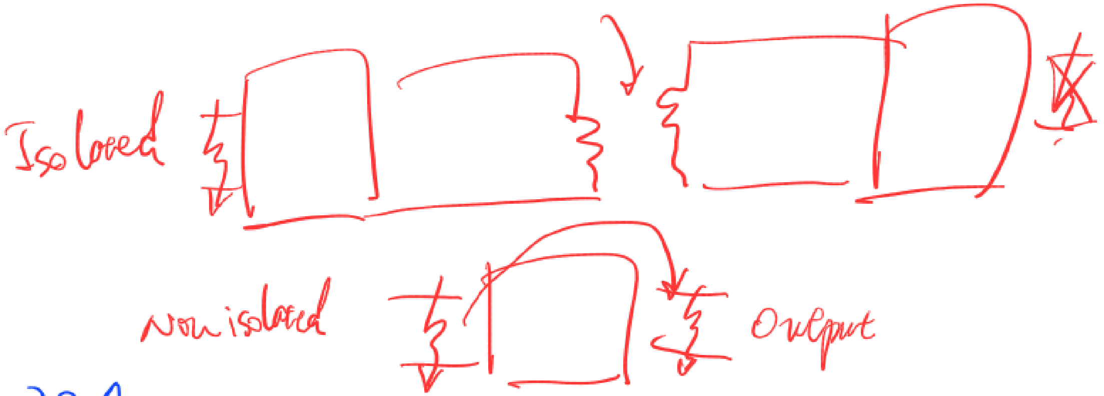
$I_{rated} = 10A$

$V_1 = 240V \qquad V_2 = 120V$

$S_1 = 240V \times 10A = 2400\ VA$

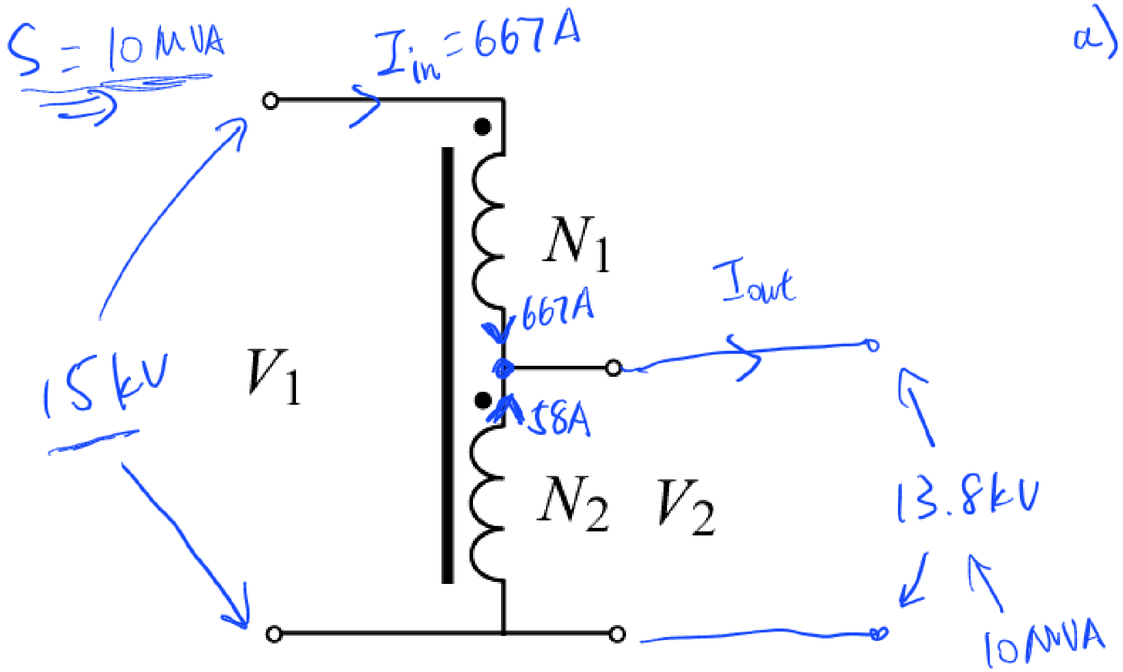
$S_2 = 2400\ VA \Rightarrow I_2 = \frac{2400\ VA}{120\ V} = 20A$

Now TF rating is 2.4 kVA (twice the orignal capacity)



Autotransformer Example 2

Distribution systems sometimes use range of voltages that are close, e.g. 12.47kV, 13.2kV, 13.8kV, 15kV.



a) $I_{in} = \frac{P}{V_1} = \frac{10\,MVA}{15\,kV} = 667\,A$

$I_{N_1} = 667\,A$

$I_{out} = \frac{P}{V_2} = \frac{10\,MVA}{13.8\,kV} = 725\,A$

$I_{N_2} = 725\,A - 667\,A = 58\,A$

b) $N_2 \rightarrow 13.8\,kV$

$N_2 = 13.8\,kV \times \frac{10\ \text{turns}}{kV} = 138\ \text{turns}$

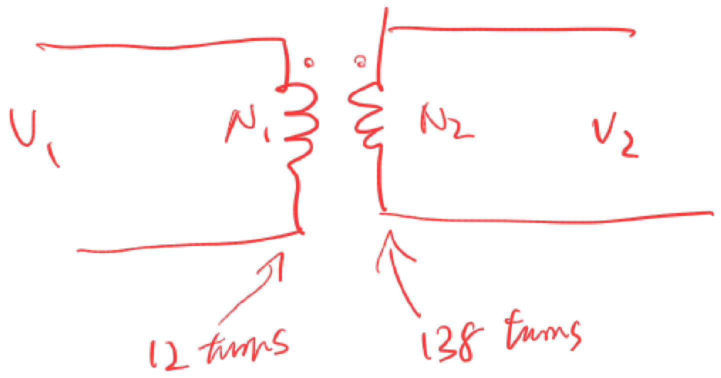
$N_1 + N_2 \rightarrow 15\,kV$

$N_1 + N_2 = 150\ \text{turns} \Rightarrow N_1 = 150 - 138 = 12\ \text{turns}$

Connect it as a 15kV/13.8kV step-down autotransformer for 10 MVA

a) Calculate currents in each winding?

b) Assuming 10 turns per kV, what are the turns in each section?



$V_2 = 13.8\,kV$

$V_1 = 1.2\,kV$

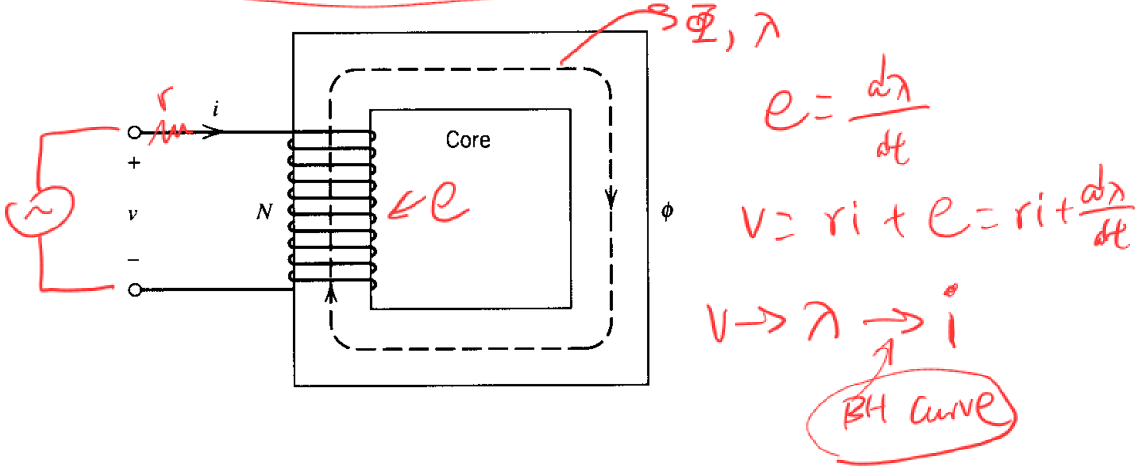
$S = I_2 V_2$

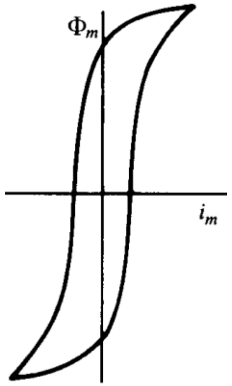
$= 58\,A \times 13.8\,kV$

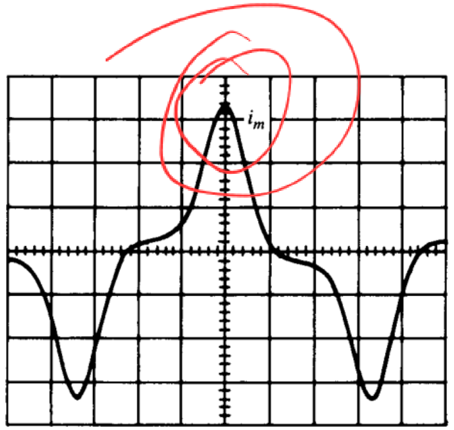
$= 800\,kVA$

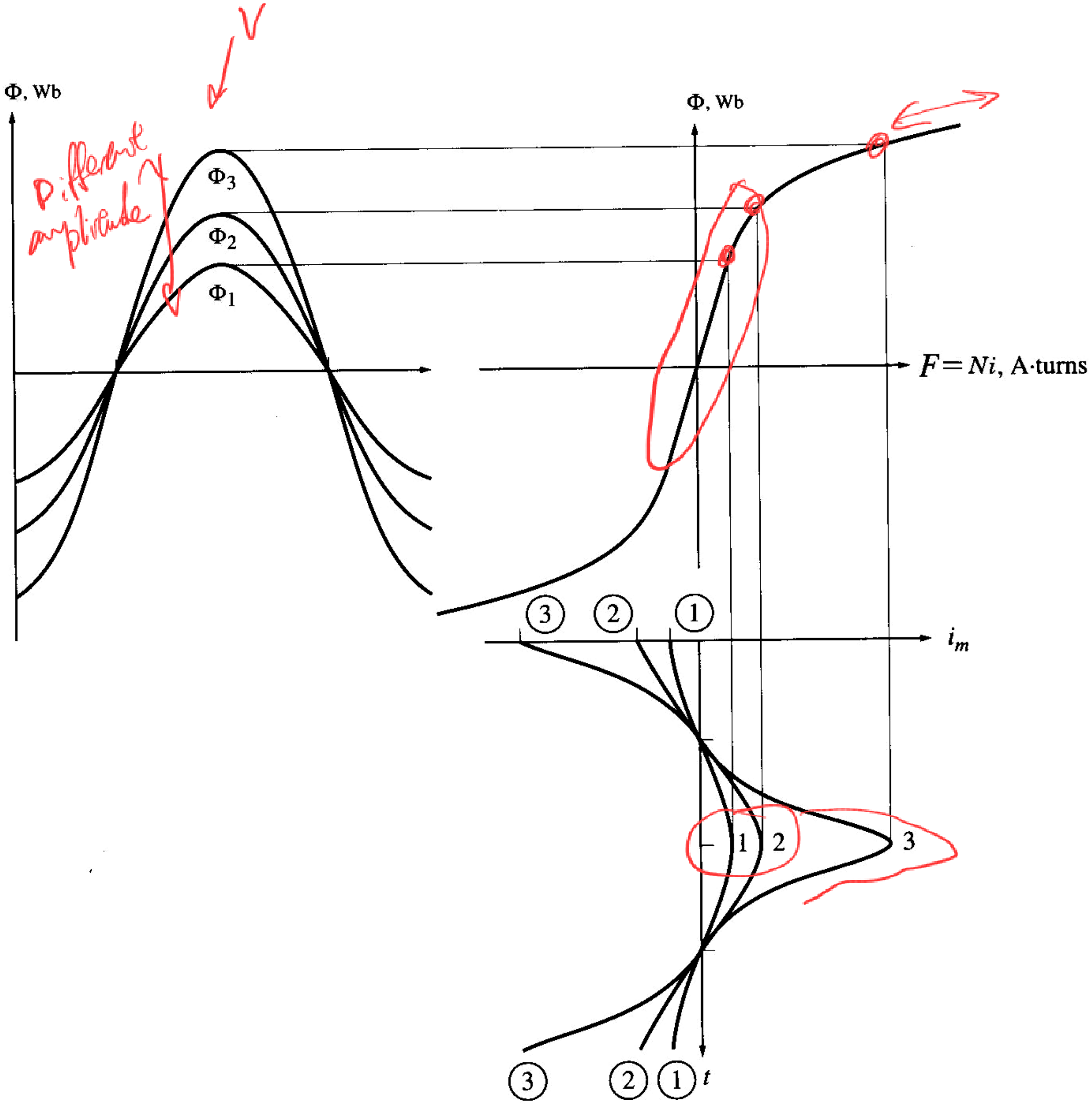
smaller than 10 MVA

Harmonics in Transformers





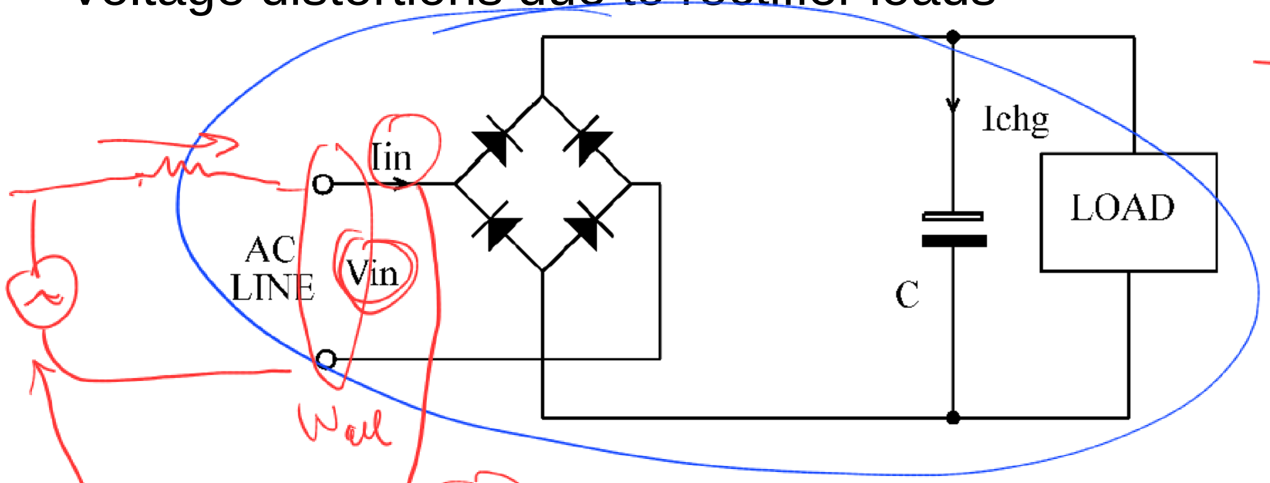


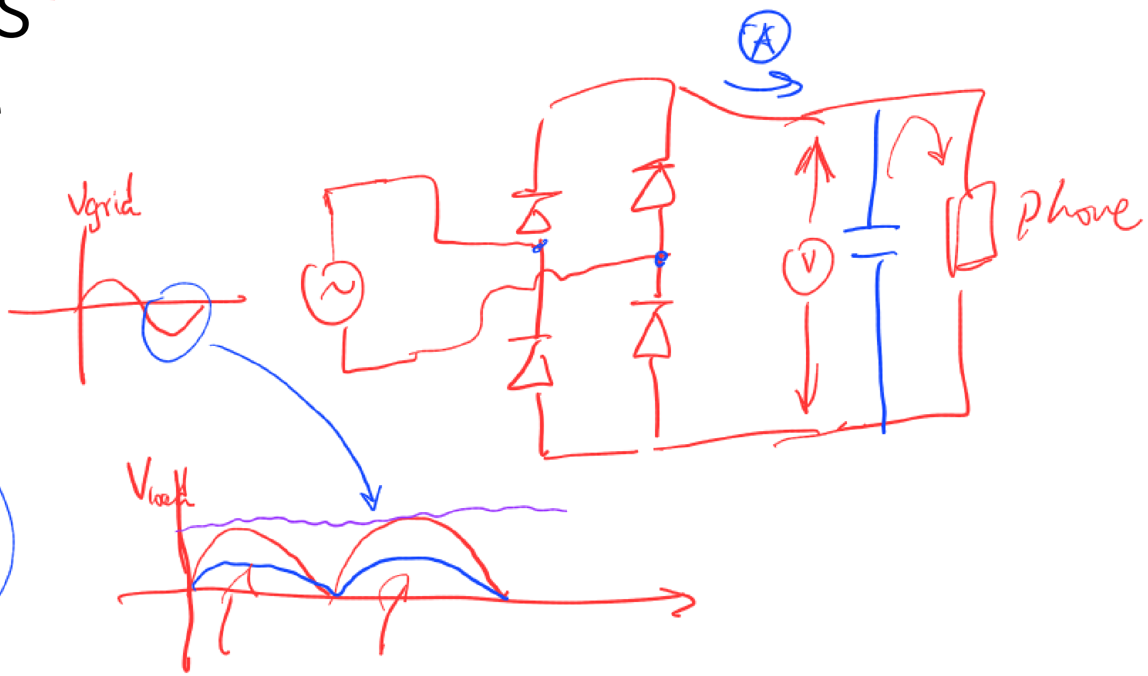


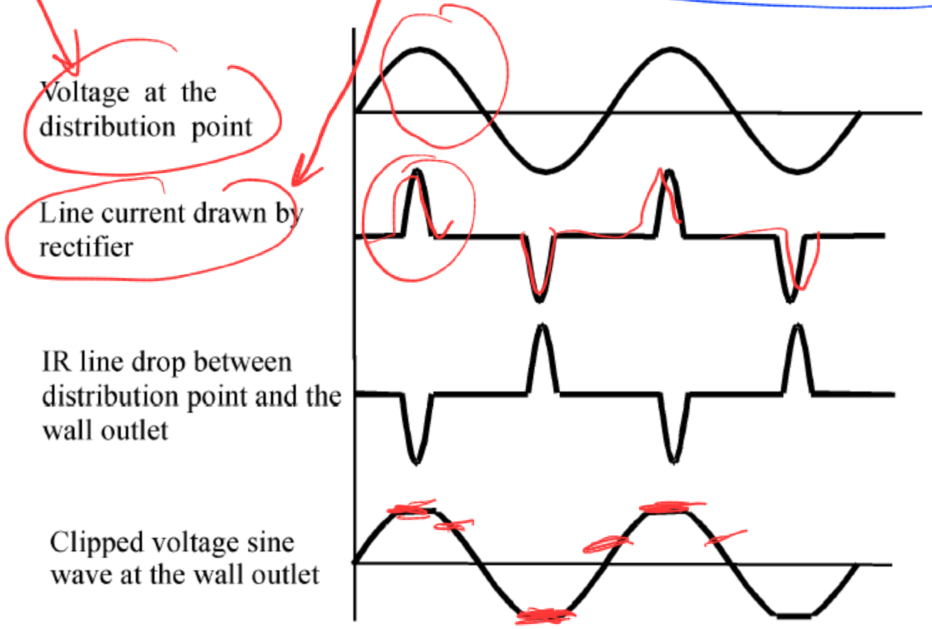
Magnetization Current may have significant distortions, i.e. presence of 3rd harmonic

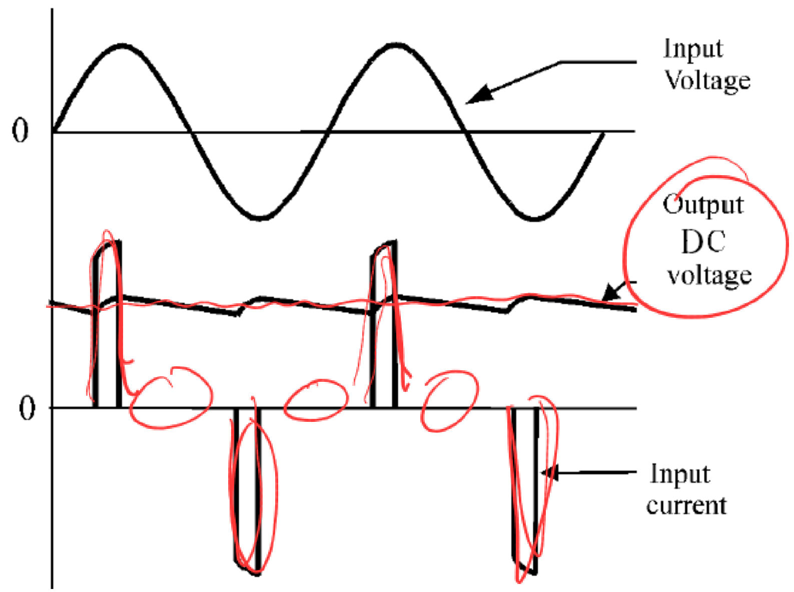
Other Harmonics in AC Mains

Many office/household loads use DC power inside
Require rectification of AC to DC
Voltage distortions due to rectifier loads









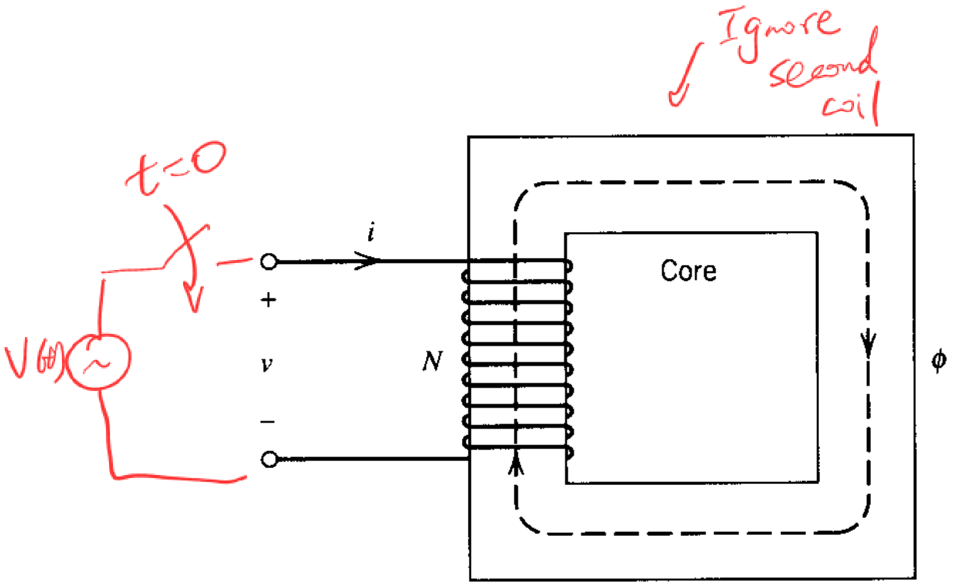
Inrush Current in Transformers

Time of energizing transformer is important

Case 1) Transformer is turned on at v(t) = V_peak

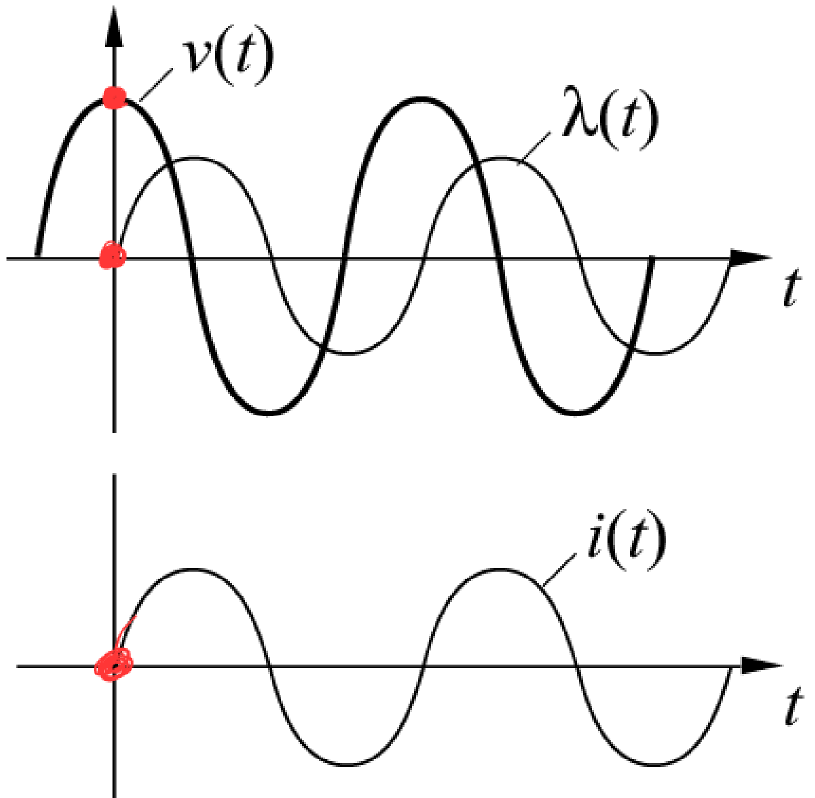
$$v(t)=\sqrt{2}V_{rms}\cos(\omega t)=\frac{d\lambda}{dt}$$

(ignore series impedance)

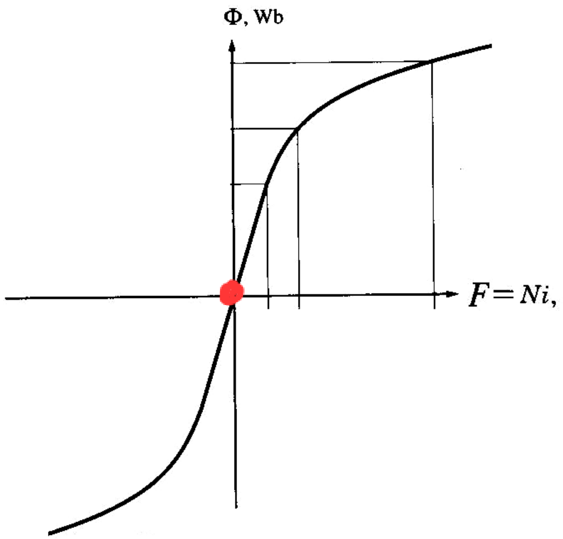


$$\lambda=\int v(t)dt$$

$$\lambda=\int_0^t v(t)dt=\frac{\sqrt{2}\cdot V_{rms}}{\omega}\sin(\omega t)$$



Recall B-H curve



Inrush Current in Transformers

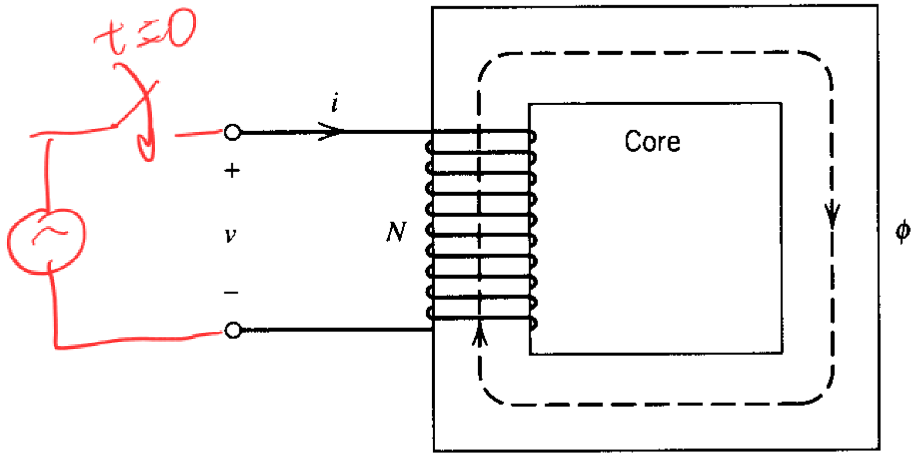
Time of energizing transformer is important

Case 2) Transformer is turned on at $v(t) = 0$

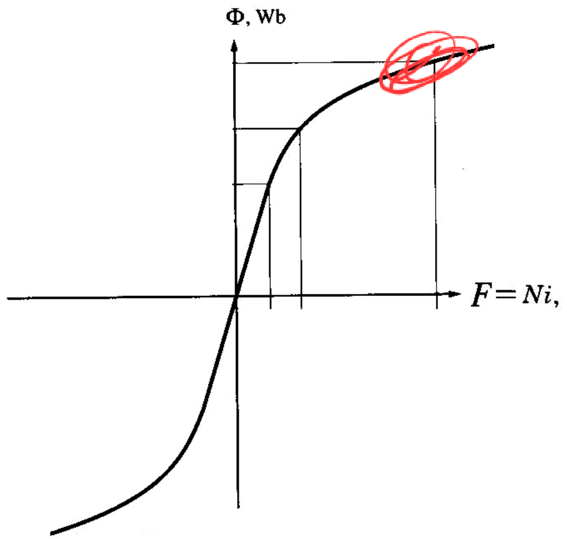
$$v(t) = \sqrt{2}V_{rms}\sin(\omega t) = \frac{d\lambda}{dt}$$

$$\lambda = \int v(t)dt$$

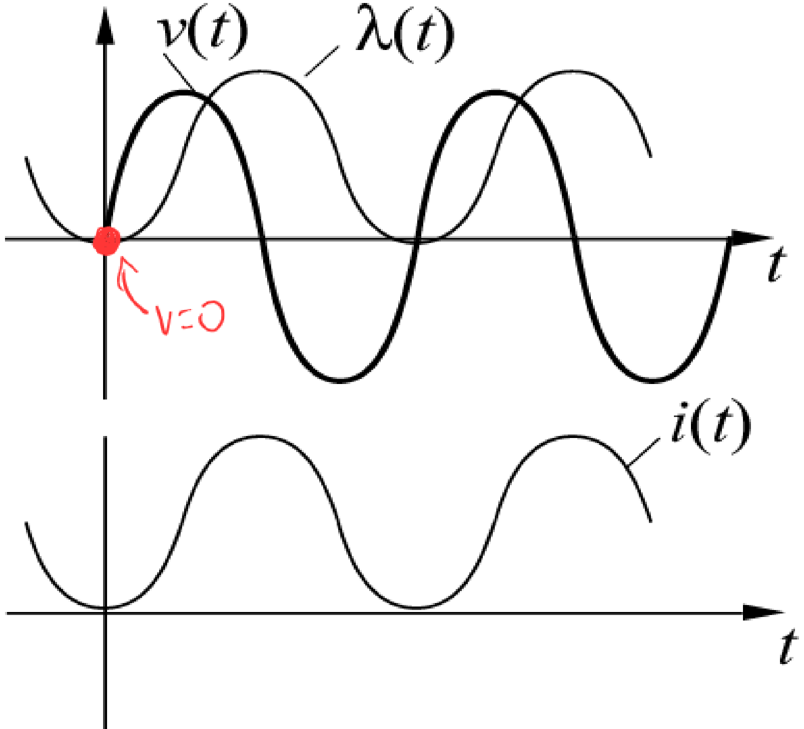
$$\lambda = \int_0^t v(t)dt = \frac{\sqrt{2}\cdot V_{rms}}{\omega}(1-\cos(\omega t))$$



Recall B-H curve



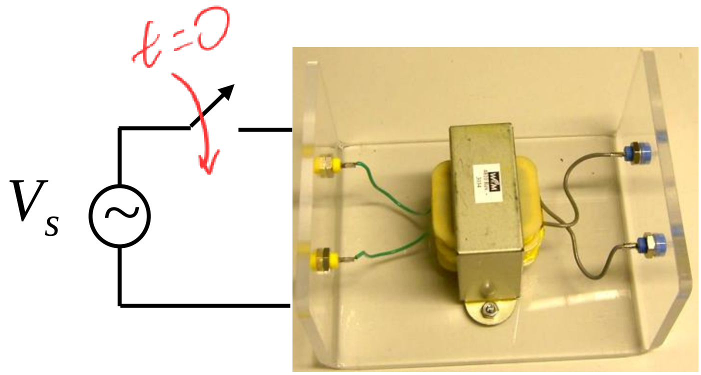
Current during inrush transient can peak up to 20 times of the nominal value: See p. 485 in the book.



Inrush Current in Transformers → Lab 2

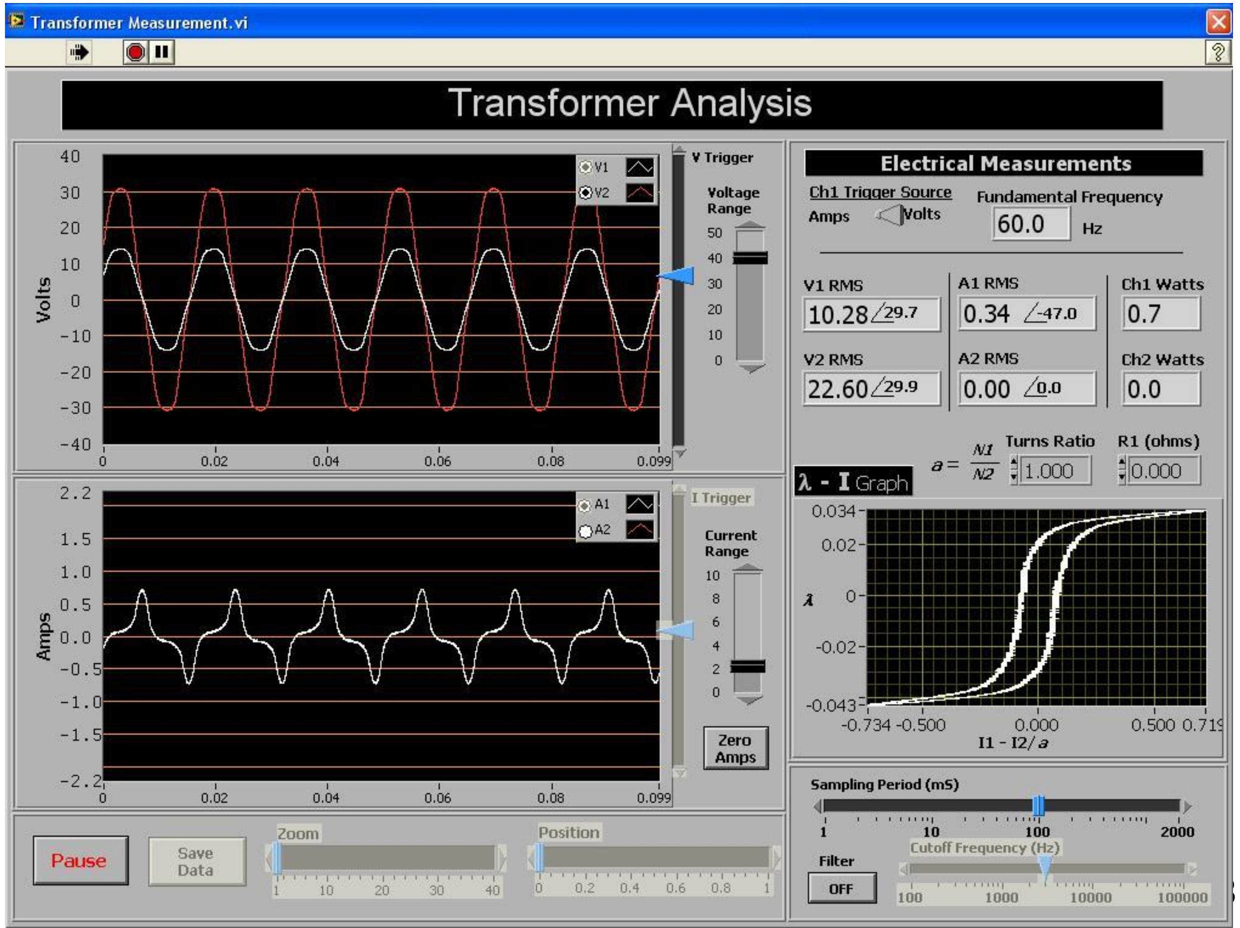
Time of energizing transformer is important

Consider small experiment in the lab



Good waveforms →

0.7 A

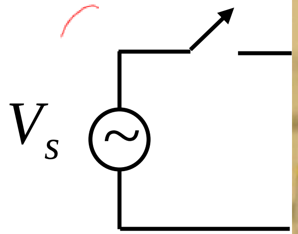


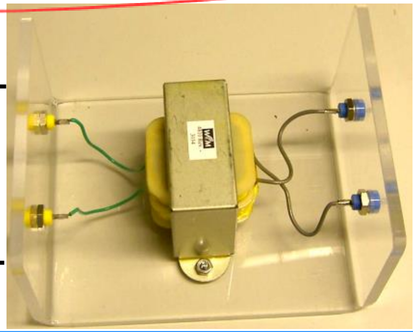
Inrush Current in Transformers

Do not torture TF

Time of energizing transformer is important

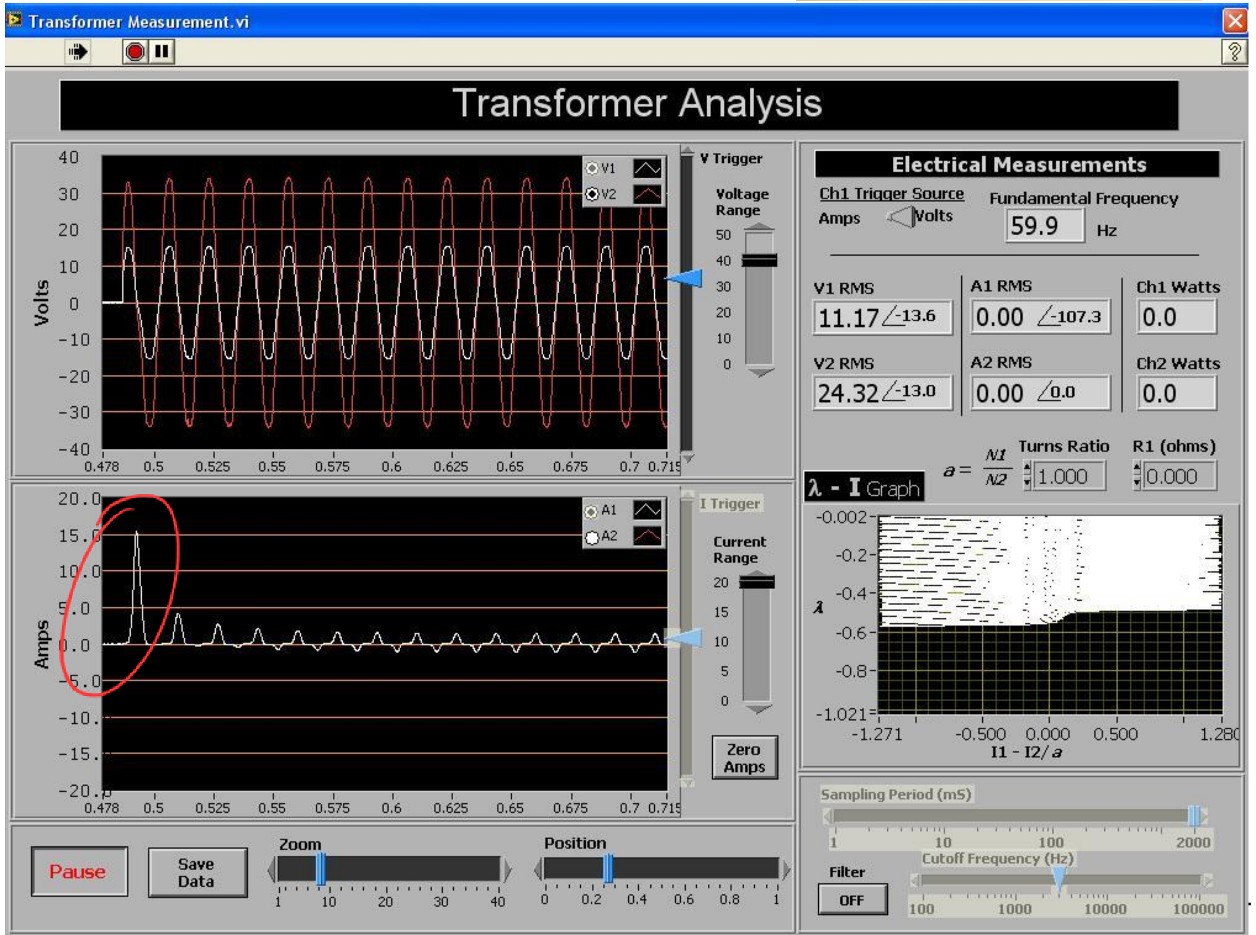
Consider small experiment in the lab





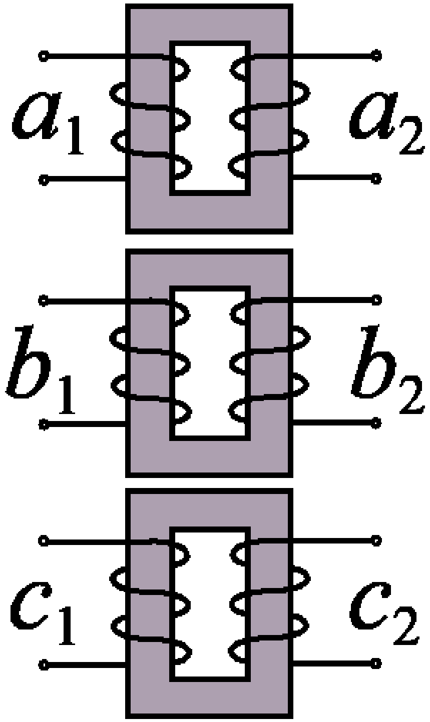
Conclusion: Do not close switch at zero crossing point of v(t)

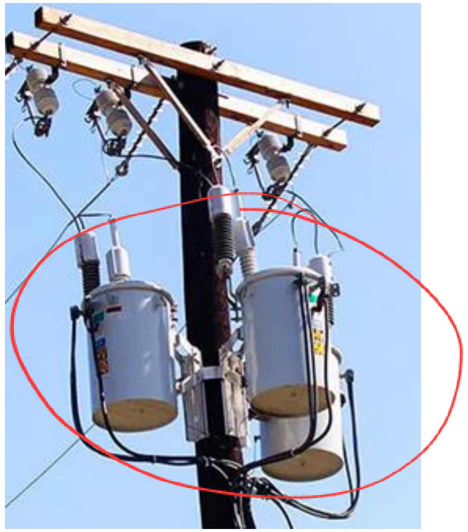
Do it at max (or min) of v(t)



Construction of Three Phase Transformers

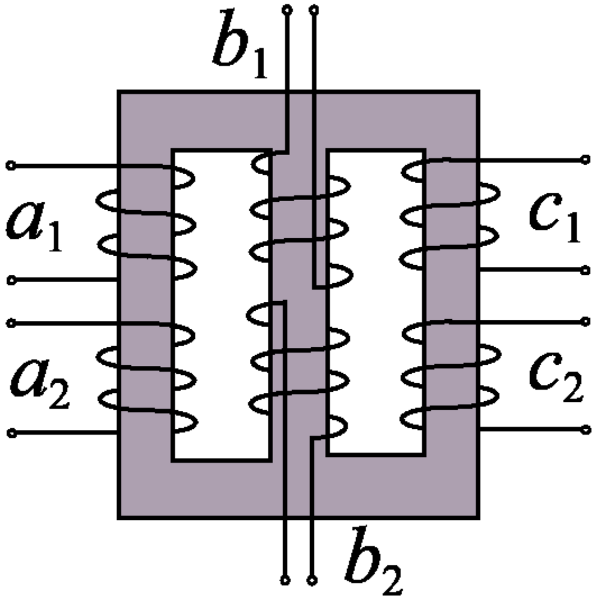
- Use 3 identical transformers
  - no magnetic coupling between phases

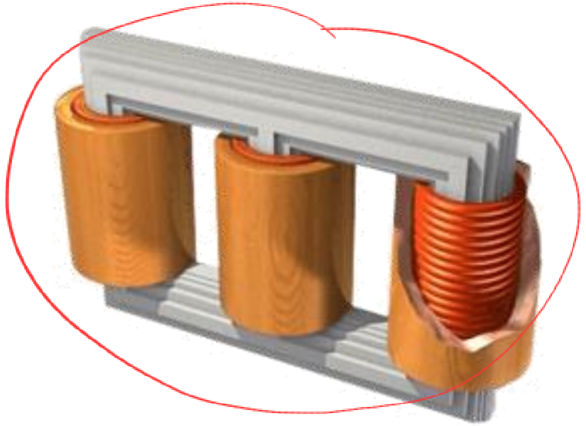




connect in Y or Δ

- Use 3-phase magnetic core
  - efficient use of materials

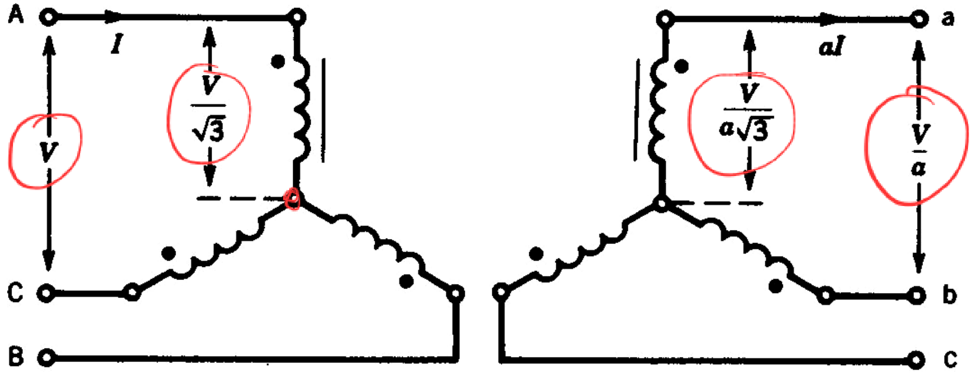




Winding Connections

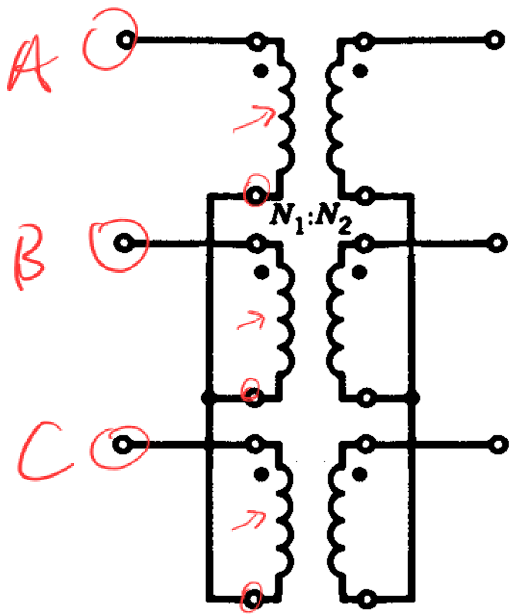
turns ratio = a

Y-Y connection

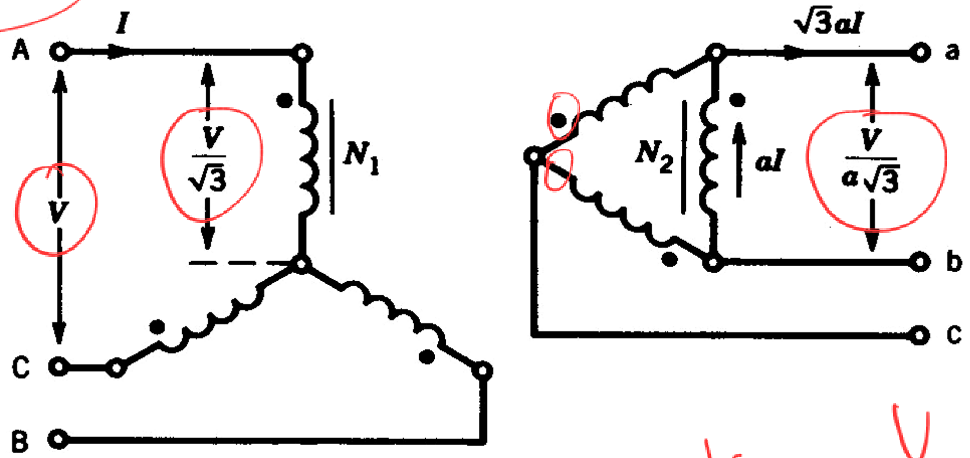


- Neutrals may or may not be grounded

$V : \frac{V}{a}$

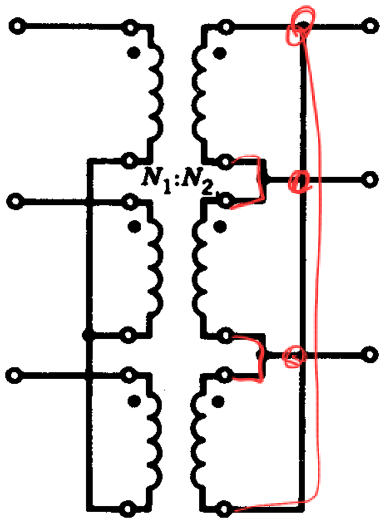


Y-Δ connection



- Δ connection has no neutral

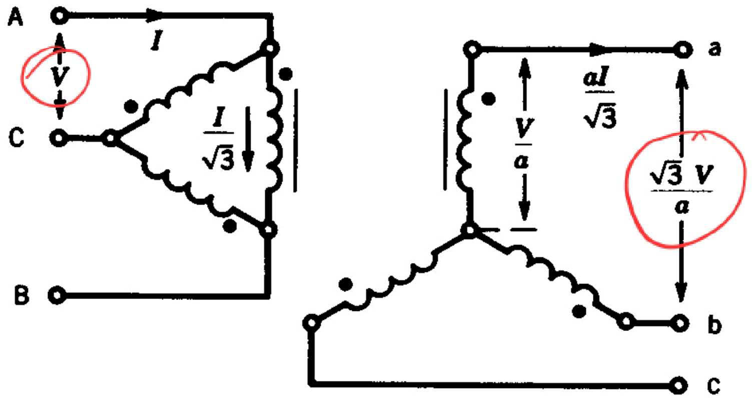
$V : \frac{V}{a\sqrt{3}}$



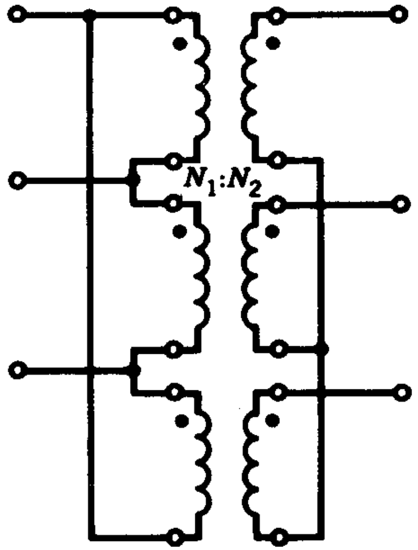
Winding Connections

Make sure to know how to draw connection

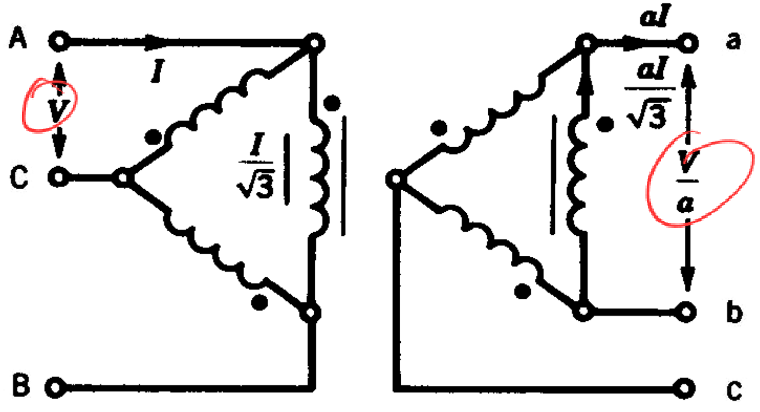
Δ-Y connection



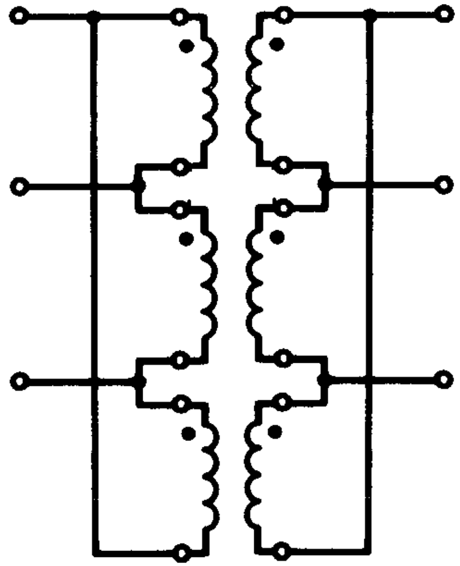
$V : \frac{\sqrt{3}V}{a}$



Δ-Δ connection



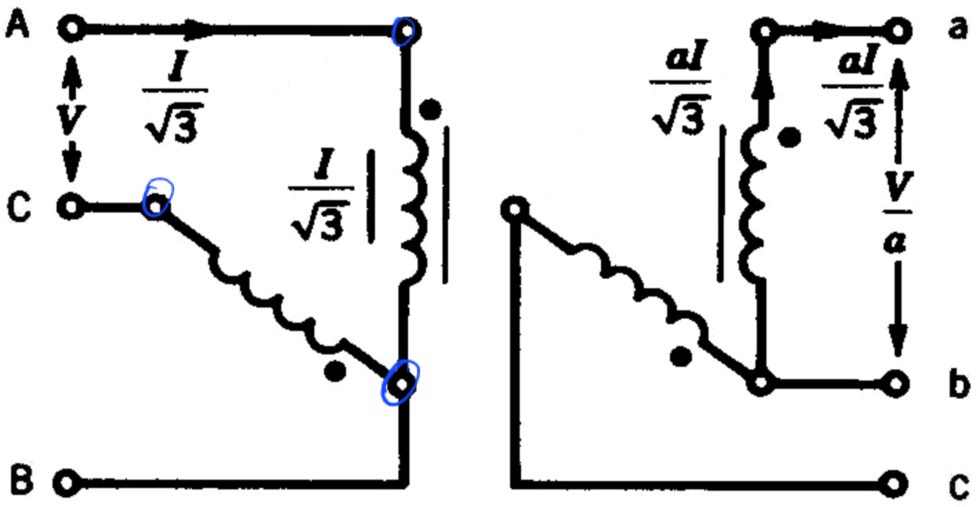
$V : \frac{V}{a}$

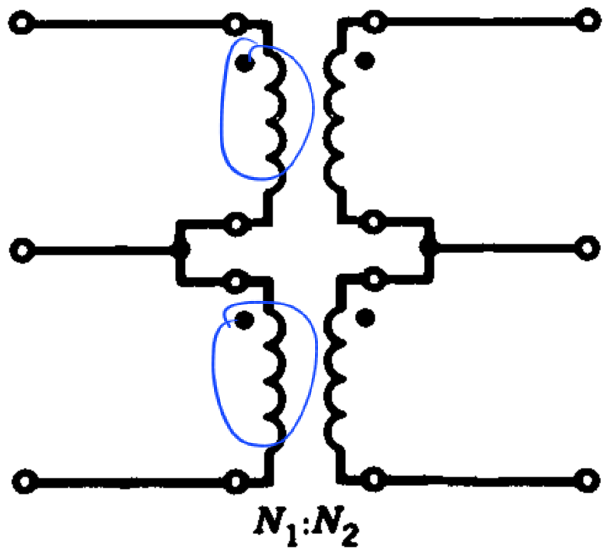


Winding Connections

2 sets of windings

Open - Δ connection





- Voltages are the same as in Δ-Δ
- But less current capacity
- More prone to unbalance conditions

Per-Unit (PU) System

- Parameters (calculations) in power engineering are often given in so-called per-unit (pu) form
- Can be used with quantities: $V, I, P, Q, S, R, X,$ etc.
- Think of it as **normalization** used in power engineering

$$\text{Quantity }(pu) = \frac{\text{Actual Value }(Volt, Amps, \ldots)}{\text{Base Value }(Volts, Amps, \ldots)}$$

Advantages:

- Narrow numerical range, fractions – allows comparisons
- Easy to check
- Simplifies calculations, analysis of systems with many electrical components (e.g., transformers, power converters, electrical machines) with different ratings

Per-Unit (PU) System

you choose

Analysis Steps:

- Select base values: base power S_{base}, base voltage V_{base}
- Convert actual parameters to pu:

$$\text{Quantity }(pu) = \frac{\text{Actual Value }(Volt, Amps, \ldots)}{\text{Base Value }(Volts, Amps, \ldots)}$$

- Do calculations in per-unit world

I = 0.9 p.u.

- Convert the results back to original units (**if needed**):

$$\text{Actual Value } = \text{Quantity }(pu) \cdot \text{Base Value}$$

Per-Unit Calculations for Single Phase Systems

Consider a transformer

- Select **base power S_{base}**
  normally take transformer rated capacity in [VA] or [kVA]

- Select **base voltage V_{base}**
  normally take rated voltage on each side in [V] or [kV]

$V_{base\,1}$ $V_{base\,2}$

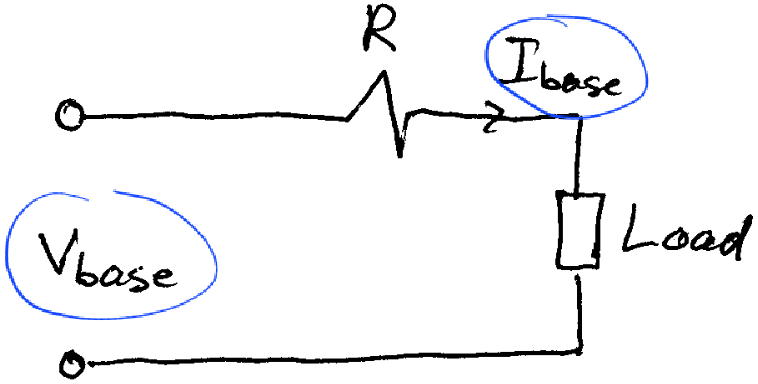
- Calculate base current

$$I_{base} = \frac{S_{base}}{V_{base}}, \quad [A]$$

- Calculate base impedance

$$Z_{base} = \frac{V_{base}}{I_{base}}, \quad [\Omega]$$

Per-Unit System for Single Phase



$$Z_{base} = \frac{V_{base}}{I_{base}}, \quad [\Omega]$$

Find the per-unit value of R

$$R_{pu} = \frac{R}{Z_{base}} = R\frac{I_{base}^2}{S_{base}}$$

$$R_{pu}S_{base} = RI_{base}^2$$

% of rated power → % power loss

Actual copper losses at rated current

$$R_{pu} = \frac{R}{Z_{base}} = R\frac{I_{base}}{V_{base}}$$

$\frac{1}{Z_{base}}$

$$R_{pu}V_{base} = RI_{base}$$

% of rated voltage

Voltage drop at rated current across R

$R_{pu} = 0.05$ p.u.

→ 5% voltage drop at rated current

Transformation from One base to Another

BC Hydro uses V_{base1}, S_{base1}, Siemens use V_{base2}, S_{base2}

$$\text{Quantity (base 2)} = \text{Quantity (base 1)}\frac{\text{Value Base 1}}{\text{Value Base 2}}$$

p.u. p.u.

$$V_{pu(base2)} = V_{pu(base1)}\frac{V_{base1}}{V_{base2}}$$

$$I_{pu(base2)} = I_{pu(base1)}\frac{I_{base1}}{I_{base2}}$$

$$Z_{pu(base2)} = Z_{pu(base1)}\frac{Z_{base1}}{Z_{base2}} = Z_{pu(base1)}\frac{S_{base2}}{S_{base1}}\left(\frac{V_{base1}}{V_{base2}}\right)^2$$

Per-Unit System for Three Phase

Consider a Y-connected system

- Base power

$$S_{base,3\phi} = 3S_{base,per-phase}$$

$$S_{base,3\phi} = \sqrt{3} I_{base,3\phi} V_{base,3\phi}$$

- Base voltage

$$V_{base,3\phi} = V_{line-to-line}$$

- Base current

$$I_{base,3\phi} = I_{base,line} = I_{base,phase}$$

- Base impedance

$$Z_{base,3\phi} = Z_{base,1\phi} = \frac{V_{base,phase}}{I_{base,phase}} = \frac{V_{base,3\phi}}{\sqrt{3} I_{base,3\phi}}$$

→ LL

Looking ahead…

- Finish **Module 2**
- Start **Module 3**