ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 7

January 27, 2026

Zhi Jin (Justin) Zhang

Course Logistics

- Assignment 2 posted

- Quiz 1
  - Coverage: up to and including end of Module 1
  - Date: January 30 (Friday) at the beginning of the tutorial
  - Closed book and notes
  - Calculators allowed

Per-Unit (PU) System

Analysis Steps:

- Select base values: base power S_{base}, base voltage V_{base}
- Convert actual parameters to pu:

$$\text{Quantity } (pu) = \frac{\text{Actual Value } (Volt, Amps, \ldots)}{\text{Base Value } (Volts, Amps, \ldots)}$$

- Do calculations in per-unit world

- Convert the results back to original units (**if needed**):

$$\text{Actual Value} = \text{Quantity } (pu) \cdot \text{Base Value}$$

Per-Unit Calculations for Single Phase Systems

Consider a transformer

- Select **base power S_{base}**
  normally take transformer rated capacity in [VA] or [kVA]

- Select **base voltage V_{base}**
  normally take rated voltage on each side in [V] or [kV]

- Calculate base current

$$I_{base} = \frac{S_{base}}{V_{base}}, \quad [A]$$

- Calculate base impedance

$$Z_{base} = \frac{V_{base}}{I_{base}}, \quad [\Omega]$$

Per-Unit System for Three Phase

Consider a Y-connected system

- Base power

$$S_{base,3\phi} = 3S_{base,per-phase}$$

$$S_{base,3\phi} = \sqrt{3}I_{base,3\phi}V_{base,3\phi}$$

- Base voltage

$$V_{base,3\phi} = V_{line-to-line}$$

- Base current

$$I_{base,3\phi} = I_{base,line} = I_{base,phase}$$

- Base impedance

$$Z_{base,3\phi} = Z_{base,1\phi} = \frac{V_{base,phase}}{I_{base,phase}} = \frac{V_{base,3\phi}}{\sqrt{3}I_{base,3\phi}}$$

Per-Unit Calculations: Example 1

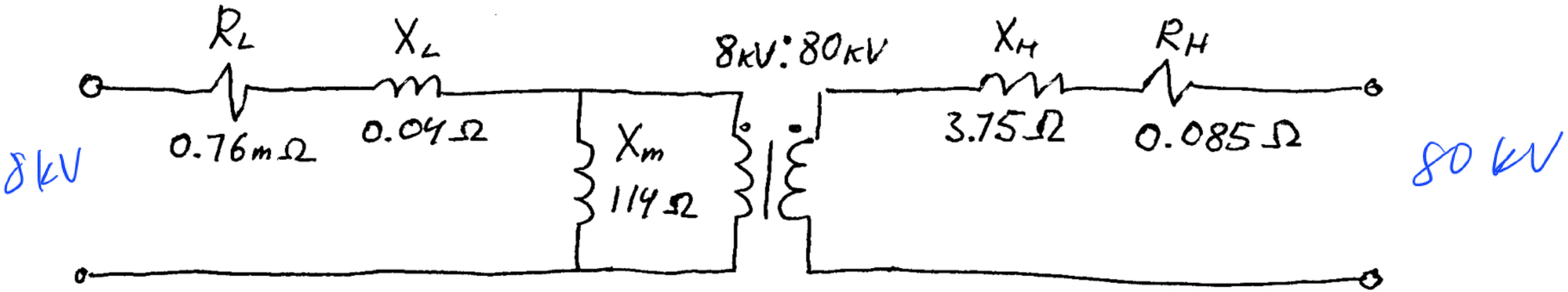
S $\quad a = \frac{1}{10}$

Consider a 100MVA, 8kV/80kV transformer with the following parameters

$$X_L = 0.04\,\Omega \qquad X_H = 3.75\,\Omega$$

$$R_L = 0.76\,\text{m}\Omega \qquad R_H = 0.085\,\Omega$$

$$X_m = 114\,\Omega$$



Strategy: per-unitize each side separately

Per-Unit Calculations: Example 1

$S_{base} = 100\,MVA$; $V_{base,L} = 8\,kV$; $V_{base,H} = 80\,kV$

$$I_{base,L} = \frac{S_{base}}{V_{base,L}} = \frac{100\cdot 10^6}{8\cdot 10^3} = 12.5\,kA$$

$$I_{base,H} = \frac{S_{base}}{V_{base,H}} = \frac{100\cdot 10^6}{80\cdot 10^3} = 1.25\,kA$$

2 current bases

$$Z_{base,L} = \frac{V_{base,L}}{I_{base,L}} = \frac{8\,kV}{12.5\,kA} = 0.64\,\Omega$$

$$Z_{base,H} = \frac{V_{base,H}}{I_{base,H}} = \frac{80\,kV}{1.25\,kA} = 64\,\Omega$$

2 impedance bases

Per-Unit Calculations: Example 1

Per-Unit Parameters

$$R_{L,pu} = \frac{R_L}{Z_{base,L}} = \frac{0.76 \cdot 10^{-3}}{0.64} = 0.0012\,pu$$

$$X_{L,pu} = \frac{X_L}{Z_{base,L}} = \frac{0.04}{0.64} = 0.0625\,pu$$

much larger than R_L, X_L, R_H, X_H, so can be neglected

$$X_{m,pu} = \frac{X_m}{Z_{base,L}} = \frac{114}{0.64} = 178.125\,pu$$

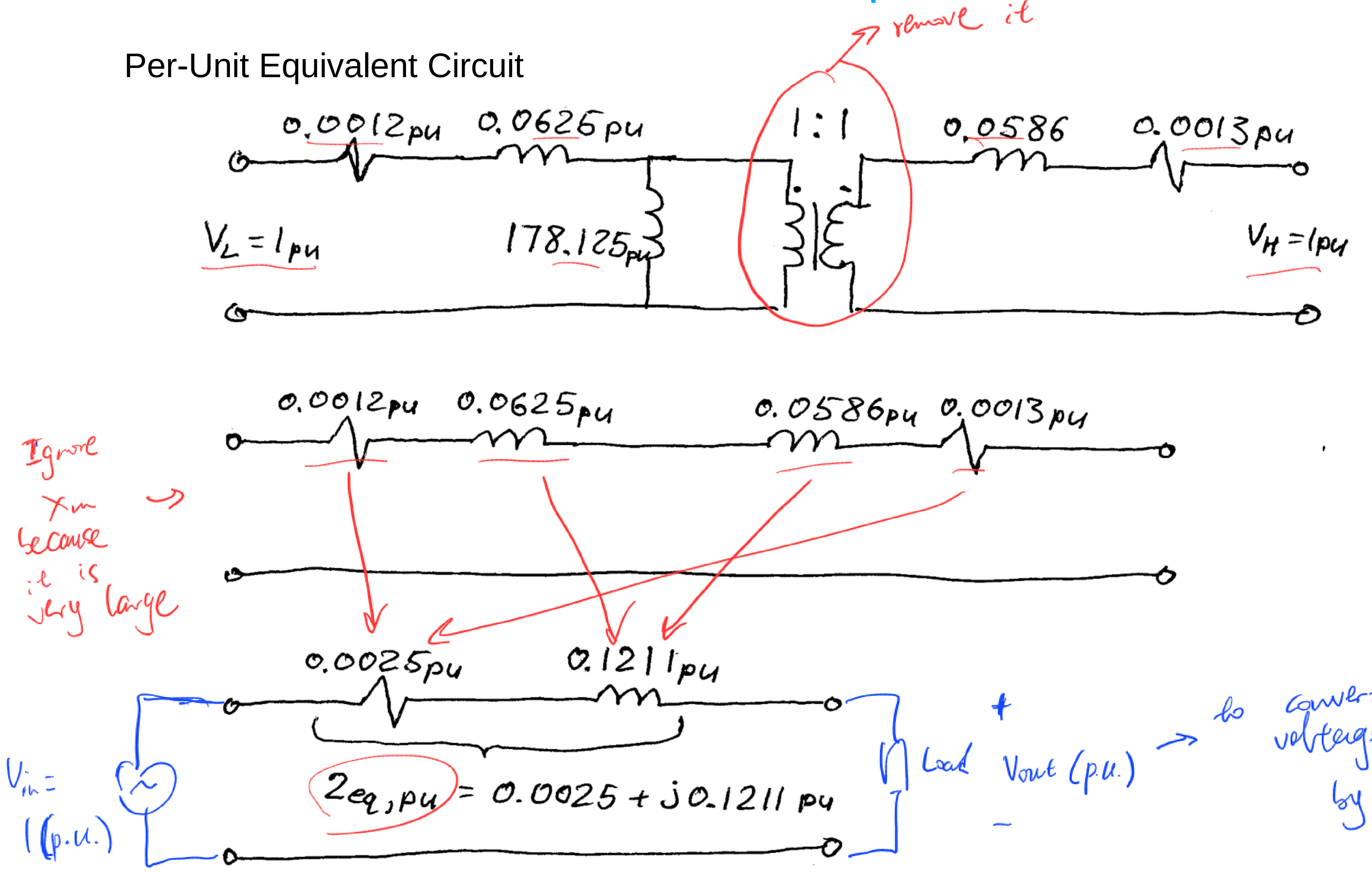
$$R_{H,pu} = \frac{R_H}{Z_{base,H}} = \frac{0.085}{64} = 0.0013\,pu$$

$$X_{H,pu} = \frac{X_H}{Z_{base,H}} = \frac{3.75}{64} = 0.0586\,pu$$

Per-Unit Calculations: Example 1

-Per unit gets rid of transformer turns ratio

Per-Unit Equivalent Circuit



Ignore X_m because it is very large →

→ to convert to actual voltage, multiply by $V_{base,H}$

Per-Unit Calculations: Example 2

Consider two step-down transformers connected in series. Transformers T1 and T2

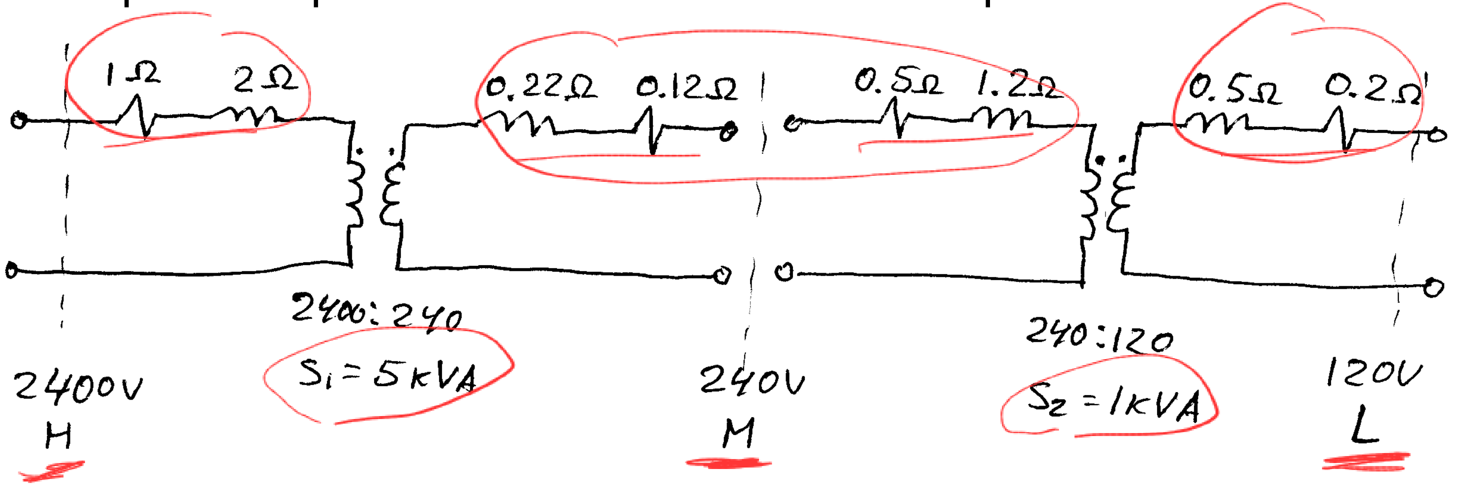
T1
Voltages 2400/240V
Rated power S_1 = 5kVA

$$X_{1H} = 2\,\Omega \quad R_{1H} = 1\,\Omega$$
$$X_{1L} = 0.22\,\Omega \quad R_{1L} = 0.12\,\Omega$$

T2
Voltages 240/120V
Rated power S_2 = 1kVA

$$X_{2H} = 1.2\,\Omega \quad R_{2H} = 0.5\,\Omega$$
$$X_{2L} = 0.5\,\Omega \quad R_{2L} = 0.2\,\Omega$$

Draw the equivalent per-unit circuit and calculate its parameters



→ If use 2 S_{base}, then we cannot connect the 2 per-unit circuits together

Three voltage levels:

3 voltage levels {
- High → 2400V
- Medium → 240V
- Low → 120V

What base power should we use?

Common practice: use the high power as S_{base}

Per-Unit Calculations: Example 2

Determining of base quantities for each voltage level with respect to S_{base},

$$S_{base} = 5\,kVA$$

$$V_{base,H} = 2400\,V;\quad V_{base,M} = 240\,V;\quad V_{base,L} = 120\,V$$

$$I_{base,H} = \frac{S_{base}}{V_{base,H}} = \frac{5\,kVA}{2400\,V} = 2.08\,A$$

$$I_{base,M} = \frac{S_{base}}{V_{base,M}} = \frac{5\,kVA}{240\,V} = 20.83\,A$$

$$I_{base,L} = \frac{S_{base}}{V_{base,L}} = \frac{5\,kVA}{120\,V} = 41.67\,A$$

$$Z_{base,H} = \frac{V_{base,H}}{I_{base,H}} = \frac{2400}{2.08} = 1152\,\Omega$$

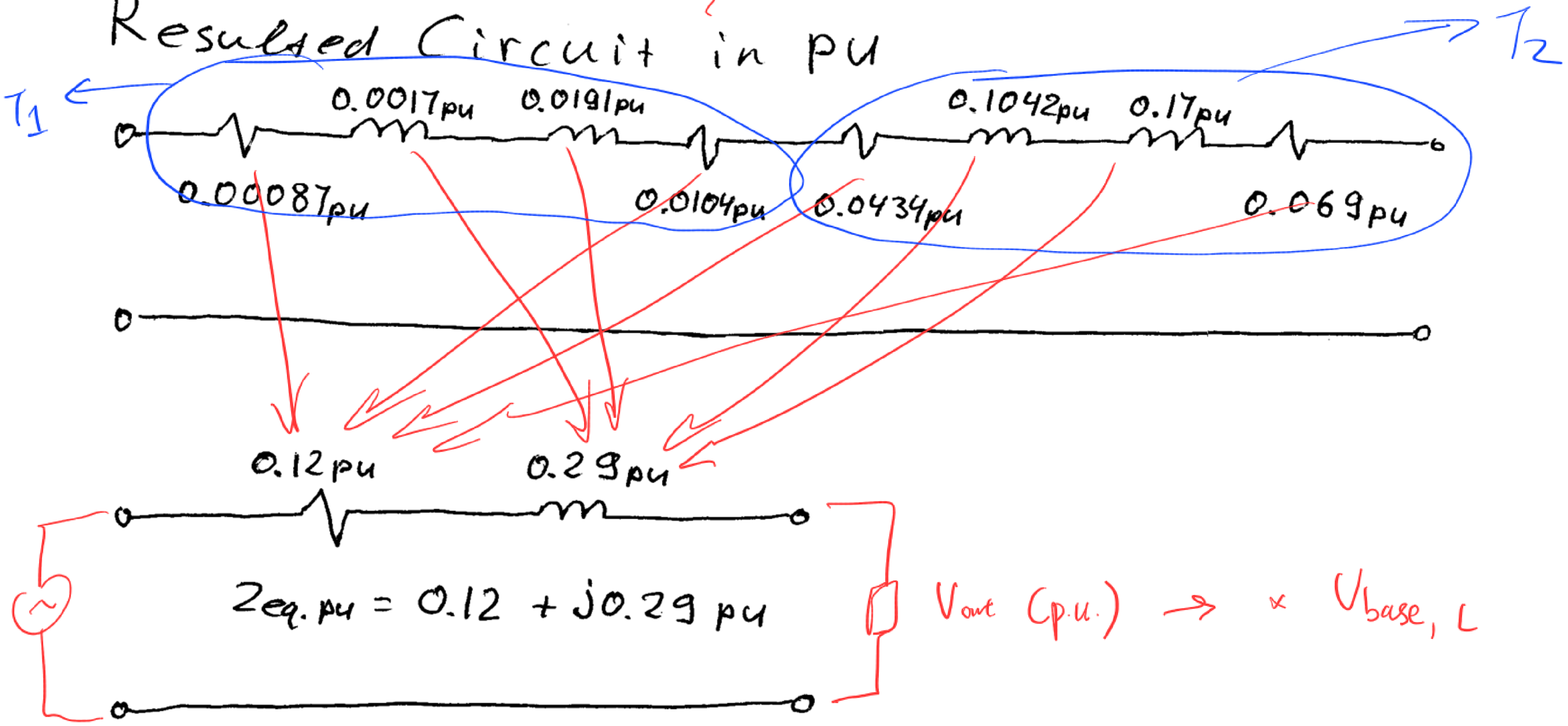
$$Z_{base,M} = \frac{V_{base,M}}{I_{base,M}} = \frac{240}{20.83} = 11.52\,\Omega$$

$$Z_{base,L} = \frac{V_{base,L}}{I_{base,L}} = \frac{120}{41.67} = 2.88\,\Omega$$

Per-Unit Calculations: Example 2

→ no more turns ratios

Resulted Circuit in pu



I_{max} ? ⇒ less or equal to 0.2 p.u.

↳ limited by S of T2

Module 3: Electromechanical Energy Conversion

Objectives & important concepts

- Basic linear devices with position-dependent reluctance & inductance
- Basic rotating devices with position-dependent reluctance & inductance
- Concept of coupling field, stored **energy & co-energy**
- Graphical interpretation of energy conversion
- Electromechanical force and torque → rotation
- Typical practical reluctance solenoids and actuators

linear motion

- Must read Chapter 3

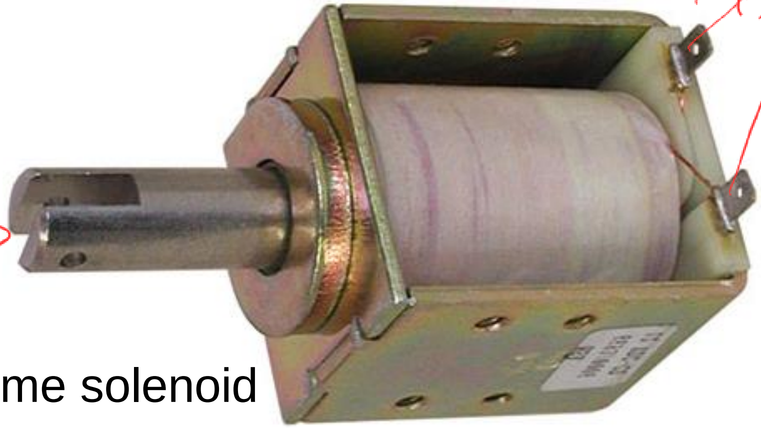
Practical Reluctance Devices

Plunger solenoid

electrical

mechanical

Closed-frame tubular solenoid

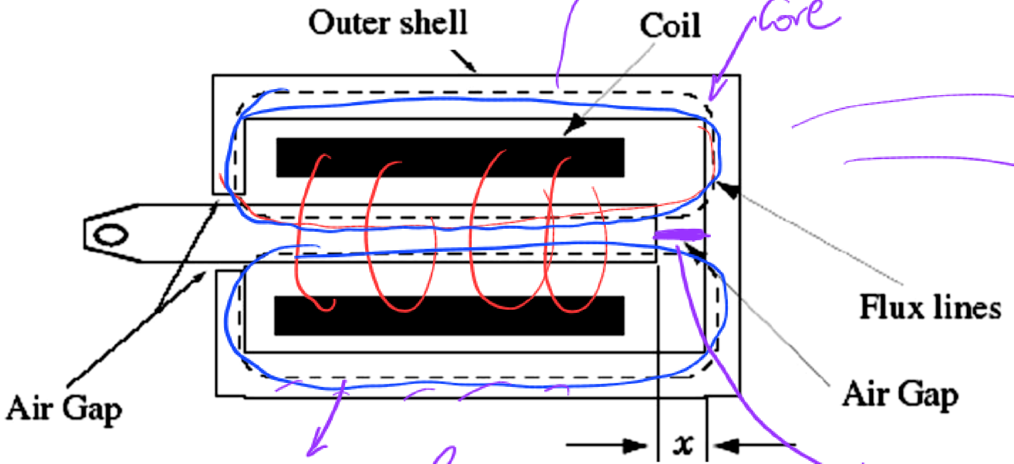


mechanical

Open-frame solenoid

Φ_m

Core



$$\mathcal{R}_m = \frac{l_c}{\mu A_c} \qquad \mathcal{R}_g = \frac{l_g}{\mu_0 A_g}$$

The basic electromagnet is very similar to a plunger solenoid

Core

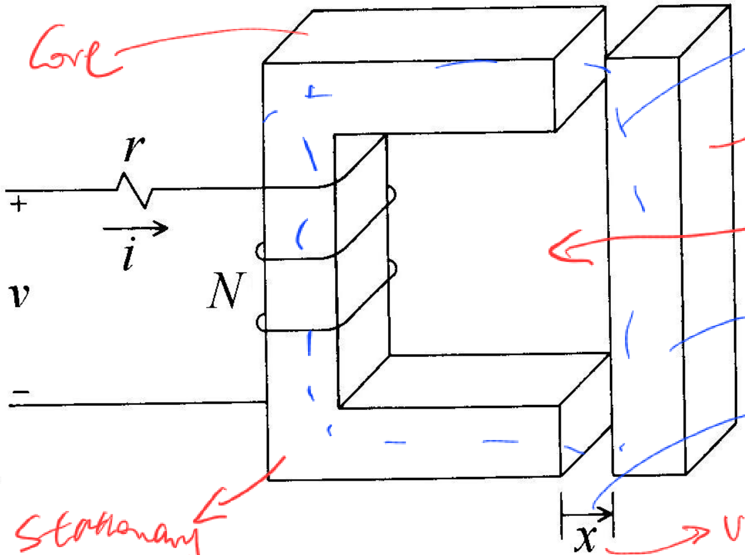
Φ_m

moving

linear motion

$\mathcal{R}_m$

$\mathcal{R}_g$



stationary

variable

Basic Electromagnet

Voltage Equation :
(Faraday's law + KVL)

$$v = ri + \frac{d\lambda}{dt} = ri + e$$

$e \rightarrow$ emf

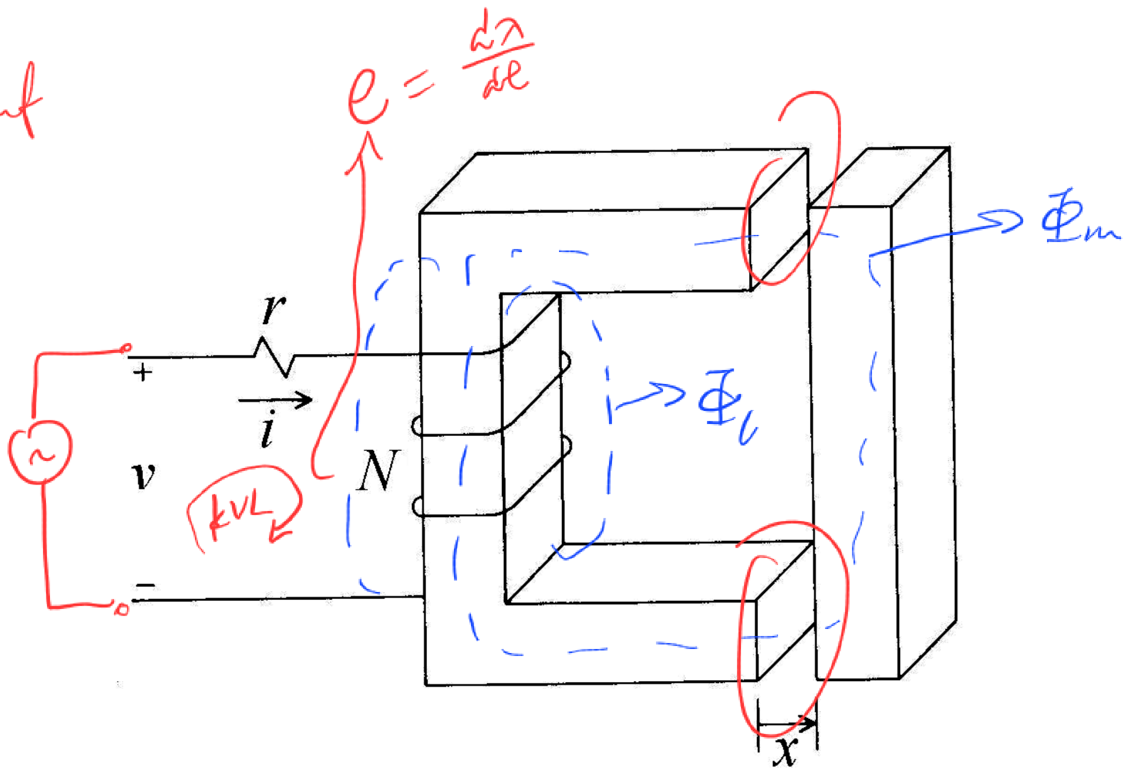
Flux linkage

magnetizing flux   leakage

$$\lambda = N\Phi = N\left(\Phi_m + \Phi_l\right)$$

Magnetizing flux $\Phi_m = Ni/\Re_m$

Flux leakage $\Phi_l = Ni/\Re_l$



Flux linkage & inductances

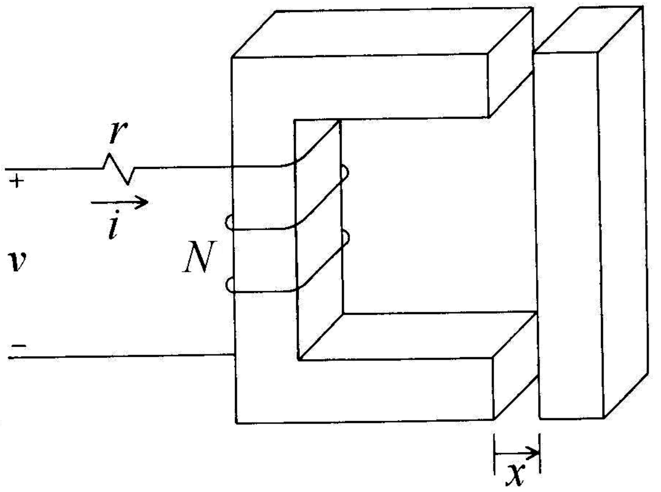
$$\lambda = \left(\frac{N^2}{\Re_l} + \frac{N^2}{\Re_m}\right)i = \left(L_l + L_m\right)i$$

L_l - Leakage inductance (assume constant)

L_m - Magnetizing inductance (depends of position x)

Consider magnetizing path reluctance $\Re_m = \Re_c + 2\Re_g$

Basic Electromagnet



Consider Magnetizing Path $\Re_m = \Re_c + 2\Re_g$

$$\Re_c = \frac{l_c}{\mu_r \mu_0 A_c}$$ - Reluctance of the stationary + movable plunger

$$\Re_g(x) = \frac{x}{\mu_0 A_g}$$ - Reluctance of the air-gap

Assume $A_c = A_g = A$ we get $\Re_m(x) = \dfrac{1}{\mu_0 A}\left(\dfrac{l_c}{\mu_c} + 2x\right)$

Magnetizing inductance $L_m = \dfrac{N^2}{\Re_m} = N^2 \mu_0 A \dfrac{1}{(l_c/\mu_c + 2x)} = \dfrac{k_1}{k_2 + x}$

Total inductance:

where $k_1 = \dfrac{N^2 \mu_0 A}{2}$ and $k_2 = \dfrac{l_c}{2\mu_c}$

$$L = L_m + L_l = \frac{k_1}{k_2 + x} + L_l$$

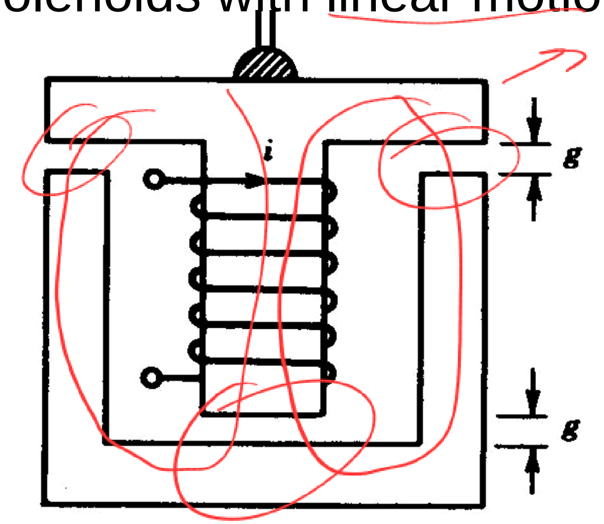
air gap distance (variable) changes inductance

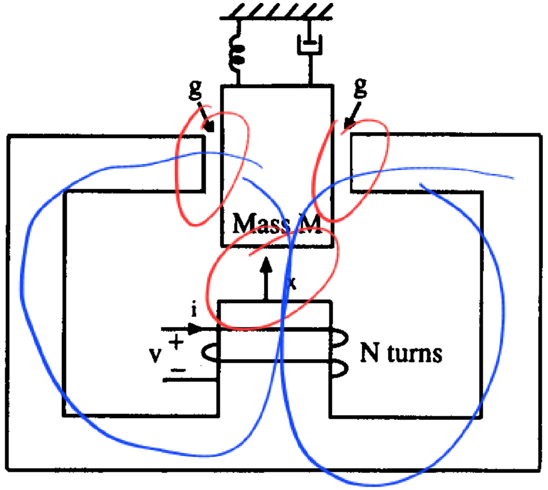
Other Reluctance Devices

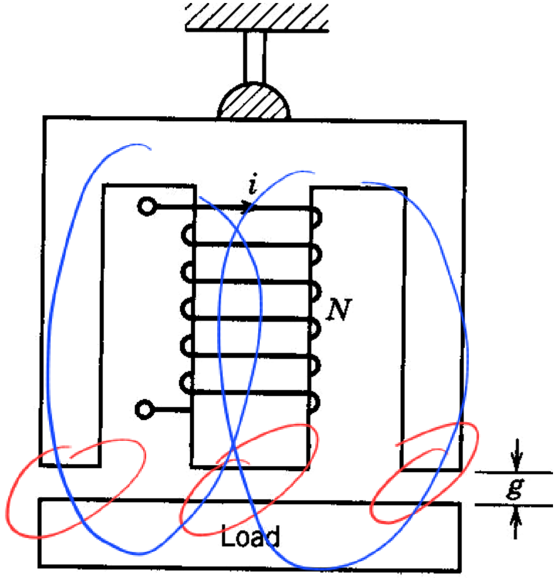
→ l_g → linear distance

Solenoids with linear motion

→ 3 air gaps



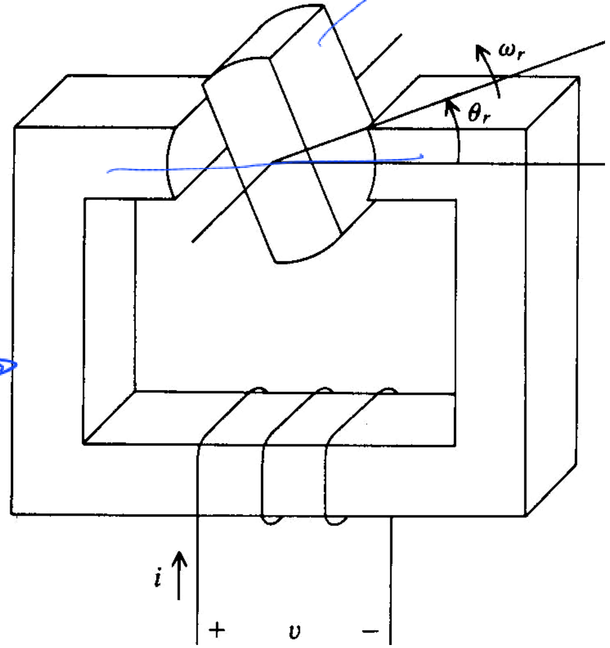




Rotating reluctance devices

→ rotor

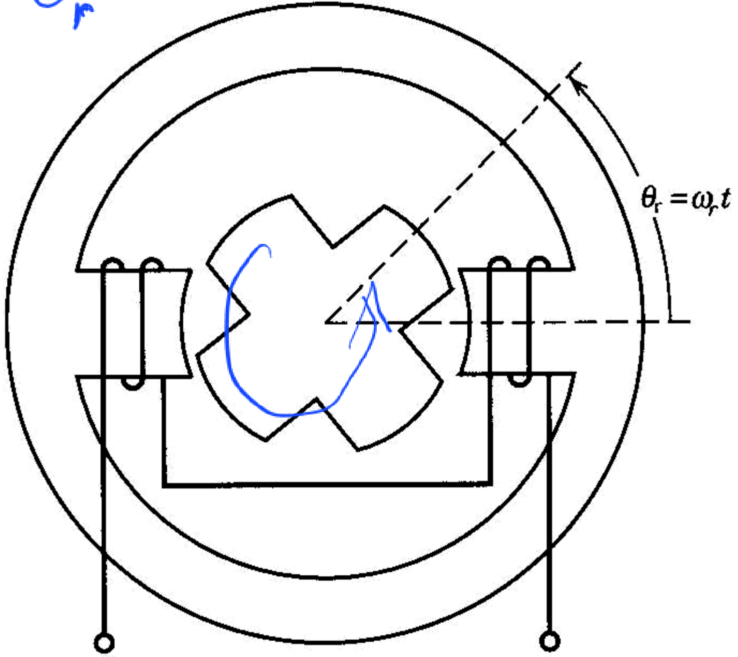
$l_g \to \theta_r$



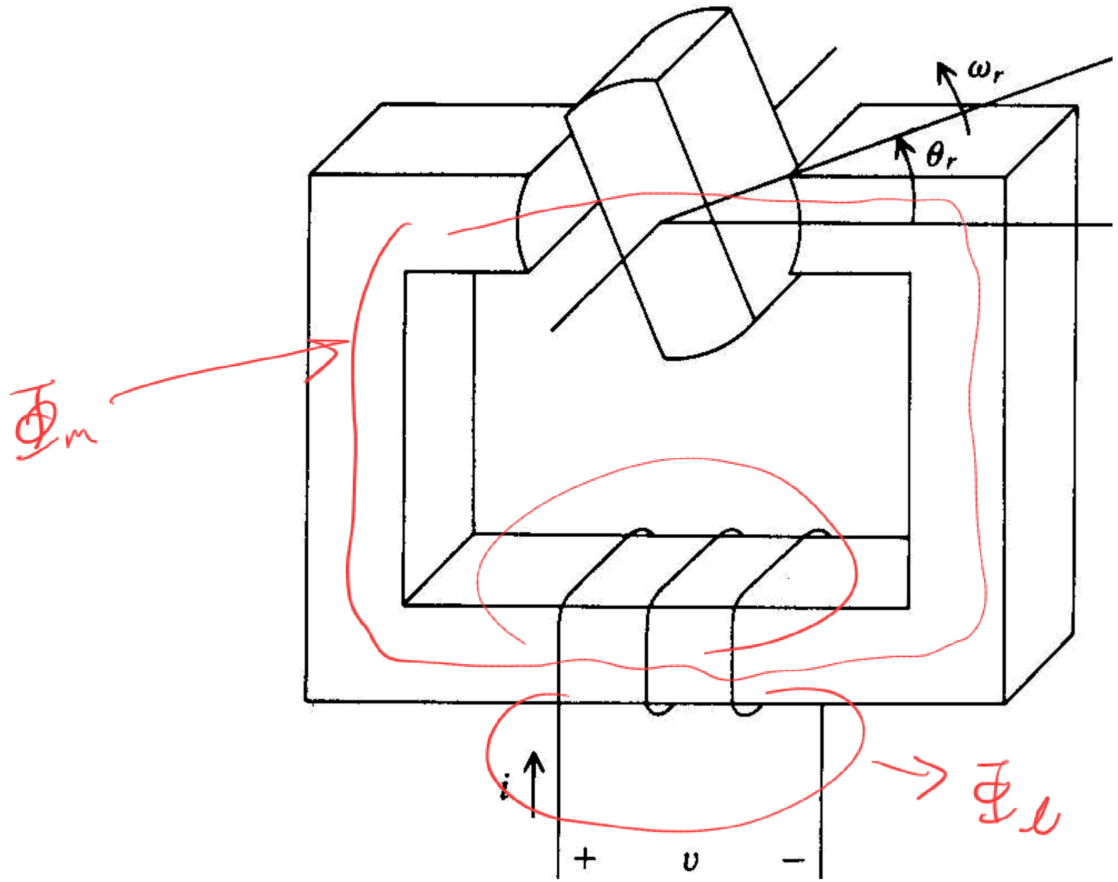
Stator (stationary)

$\theta_r = 0°$
↓
largest air gap

$\theta_r = 90°$
↓
smallest air gap



Rotating Reluctance Devices



Flux linkage & inductances

$$\lambda = \left(\frac{N^2}{\Re_l} + \frac{N^2}{\Re_m} \right) i = (L_l + L_m) i$$

Magnetizing inductance changes with position

$$L_m = L_m(\theta_r) = \frac{N^2}{\Re_m(\theta_r)}$$ → function of angle

$\Re_m(0)$ - Maximum reluctance → $L_m(0)$ ⇒ minimum inductance

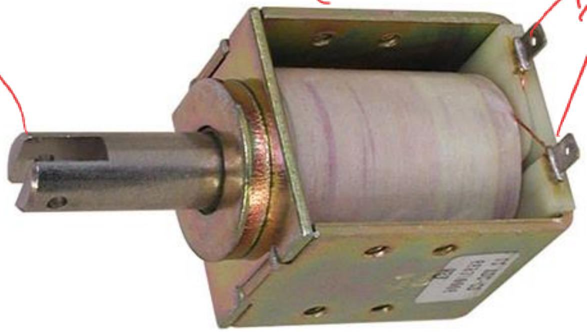
$\Re_m(\pi/2)$ - Minimum reluctance → $L_m(\frac{\pi}{2})$ ⇒ maximum inductance

Electromechanical Energy Conversion

Linear motion

electrical system connection

mechanical system connection



rotating motion

mechanical system connection

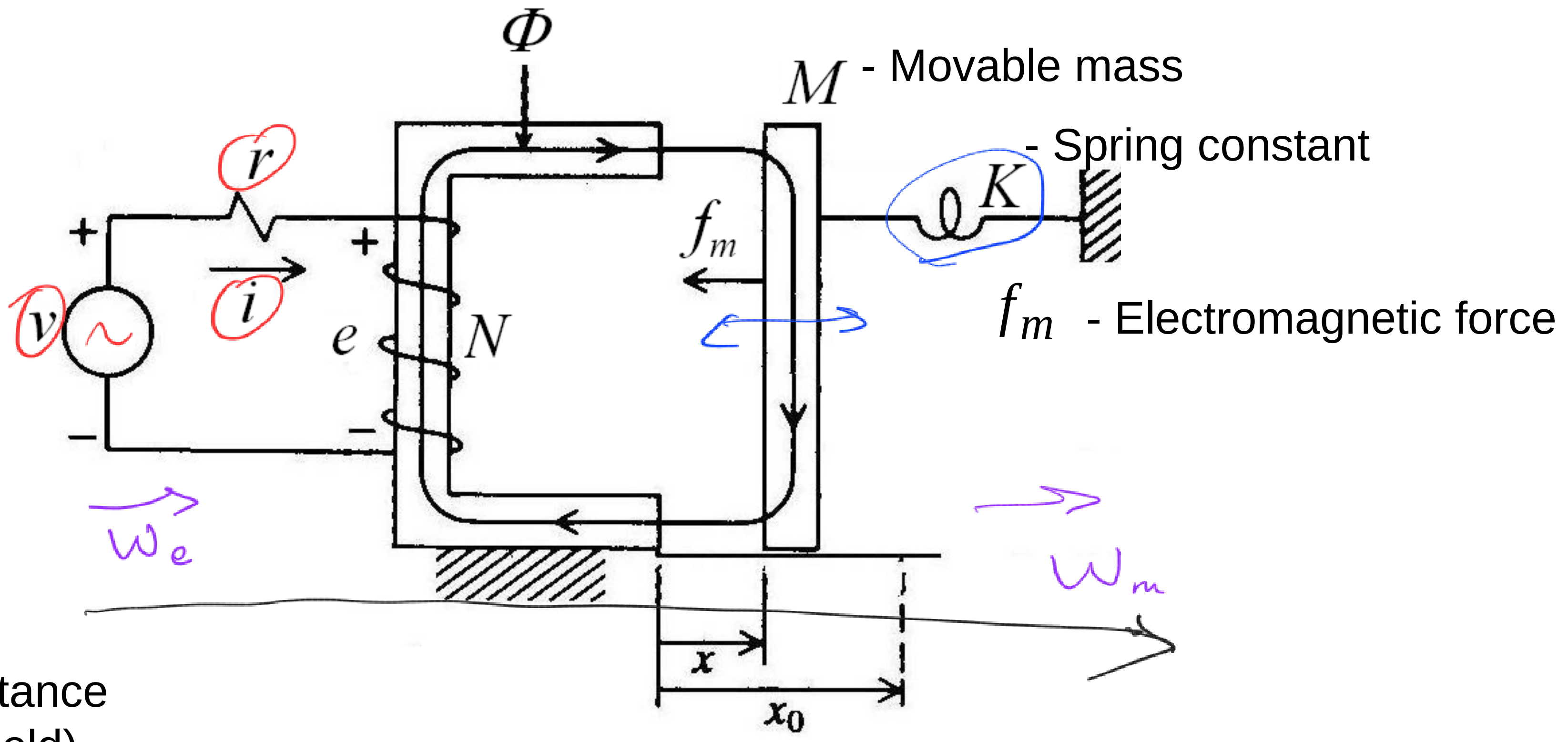
electrical system connection

reversible

Motoring

Generating

Basic Electromechanical System



v - Source voltage

i - Source current

r - Coil resistance

L_m - Magnetizing inductance

e - EMF due to magnetizing inductance (voltage drop due to coupling field)

W_e - Energy supplied from electrical system (going into coupling field)

W_f - Energy in coupling field (magnetic field)

W_m - Energy going into mechanical system (from coupling field)

In following slides, we ignore all losses, just focus on energy conversion.

Electromechanical Energy Conversion

$P = vi$
$= ei$

$W_e \longrightarrow$ Coupling Field $\longrightarrow W_m$

Energy Balance $\quad W_e = W_f + W_m = \int e i\,dt = W_f + \int \underbrace{f_m}_{\text{force}} \underbrace{dx}_{\text{distance}}$

How do you convert the energy ?

$\Delta W_e = \Delta W_f + \Delta W_m$

First, lets consider fixed position, and assume $dx = 0 \Rightarrow W_m = 0$

$$W_e = W_f = \int e i\,dt = \int \underbrace{\frac{d\lambda}{dt}}_{e} i\,\cancel{dt} = \underbrace{\int i\,d\lambda}_{W_f}$$

$\rightarrow$ Energy stored in coupling field at fixed mechanical position

Energy & Co-Energy Coupling in Field

Consider a state of the system

$$i = i_a \qquad \lambda = \lambda_a \qquad \lambda = L_m(x)i$$

→ definition of inductance

Energy going in coupling field

$$W_f = \int i d\lambda$$ ⇒ area above the curve (physical)

Co-Energy associated with this state

$$W_c = \int \lambda di$$ ⇒ dual / companion of W_f (algebraic definition)

Energy and Co-Energy Balance

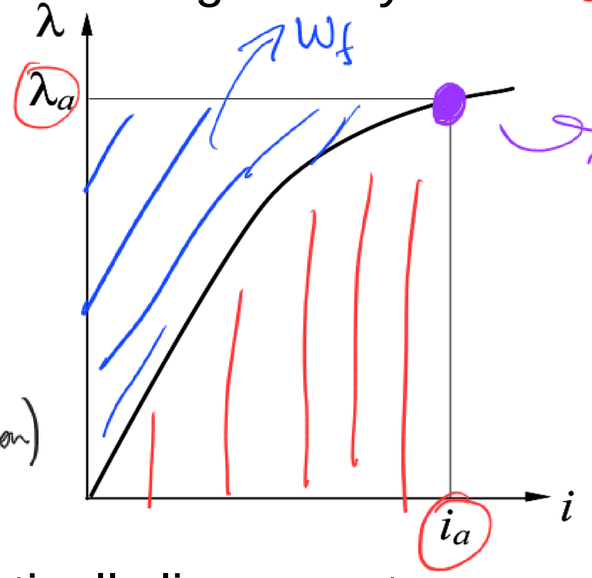
$$\lambda i = W_f + W_c$$

For magnetically linear systems

$$W_f = W_c = \frac{1}{2}\lambda i = \frac{1}{2}Li^2$$

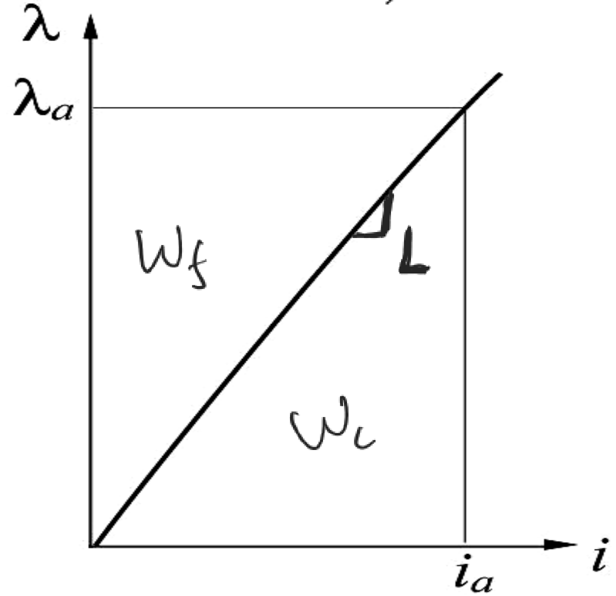
Coupling Field is Conservative – The stored energy does not depend on the history of electromechanical variables, it depends only on their final state/values

Non-linear magnetic system (B-H curve)



→ use this to calculate W_f
↓
How we get to here doesn't matter
↓
no dependence on history

Magnetically linear system



Looking ahead…

- Finish **Module 3**

- Start **Module 4**