ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 8

January 29, 2026

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Course Logistics

- Office hour:
  - Same time, but location changed from KAIS 3047 to KAIS 3049

Module 3: Electromechanical Energy Conversion

Electromechanical Energy Conversion

$$W_e \longrightarrow \boxed{\text{Coupling Field}} \longrightarrow W_m$$

Energy Balance $W_e = W_f + W_m = \int eidt = W_f + \int f_m dx$

How do you convert the energy ?

$$\boxed{\Delta W_e = \Delta W_f + \Delta W_m} \rightarrow \text{change in } W_e \text{ will change } W_f \text{ and } W_m$$

First, lets consider fixed position, and assume $dx = 0$ $\Rightarrow \Delta W_m = 0$

$$W_f = \int eidt = \int \frac{d\lambda}{\cancel{dt}} i\cancel{dt} = \int_0^{\lambda_{final}} i d\lambda$$

Energy & Co-Energy Coupling in Field

Consider a state of the system

$$i = i_a \qquad \lambda = \lambda_a \qquad \lambda = L_m(x)i$$

Energy going in coupling field

$$W_f = \int i d\lambda$$

Co-Energy associated with this state

$$W_c = \int \lambda di$$ (algebraic quantity)

Energy and Co-Energy Balance

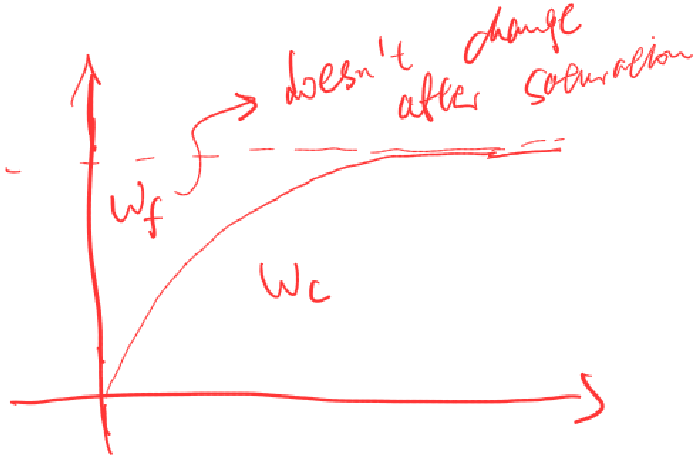
$$\lambda i = W_f + W_c$$

For magnetically linear systems

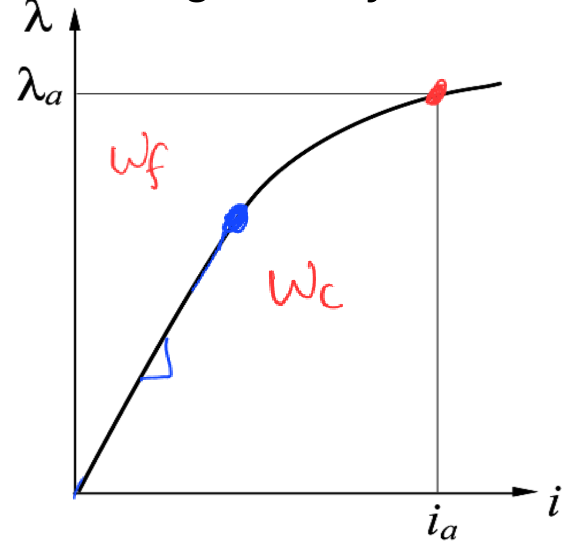
$$W_f = W_c = \frac{1}{2}\lambda i = \frac{1}{2}Li^2$$

energy in magnetic field

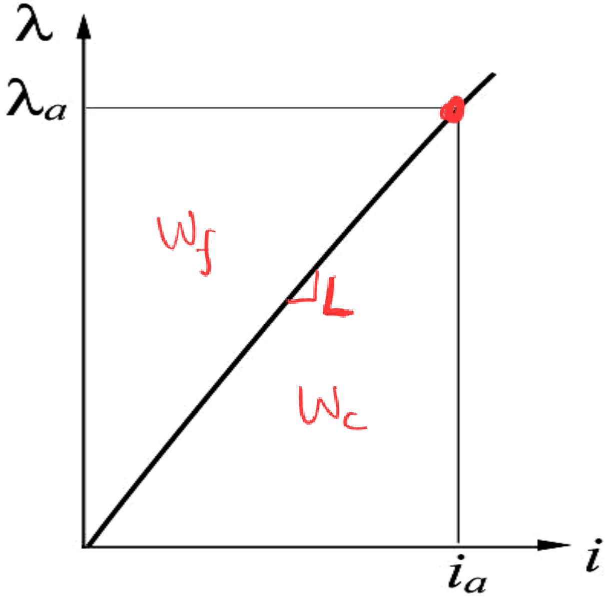
<u>Coupling Field is Conservative</u> – The stored energy does not depend on the history of electromechanical variables, it depends only on their final state/values



Non-linear magnetic system



Magnetically linear system

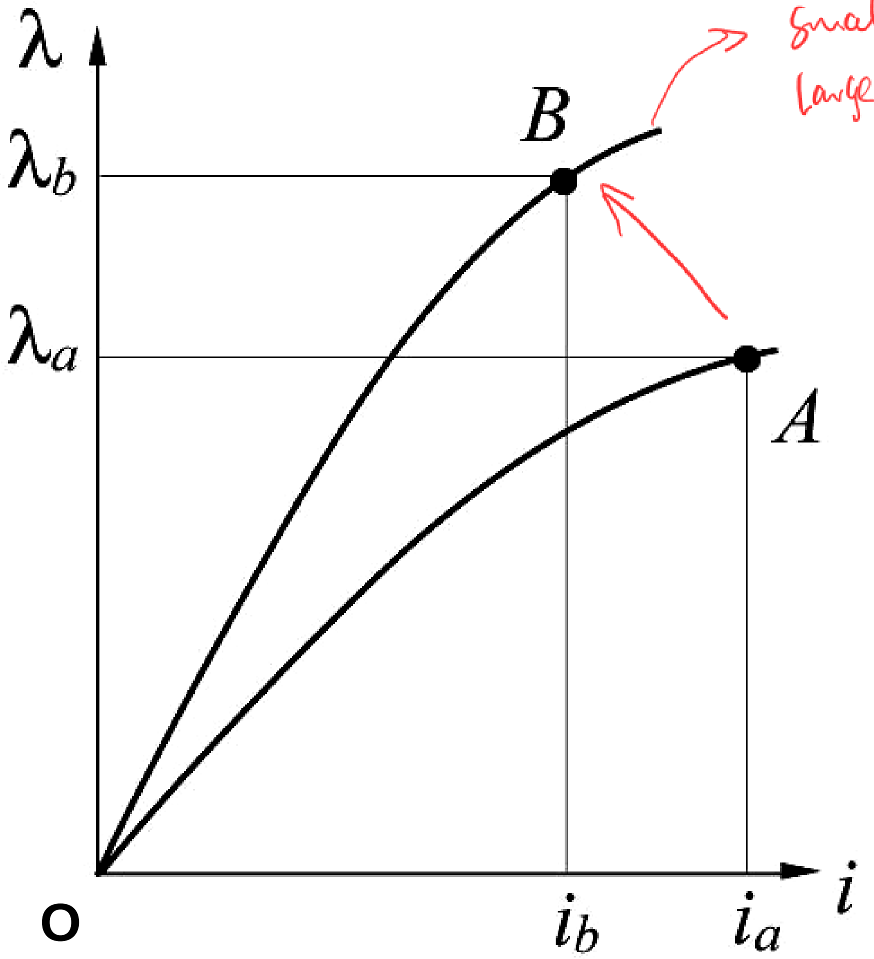


Electromechanical Energy Conversion

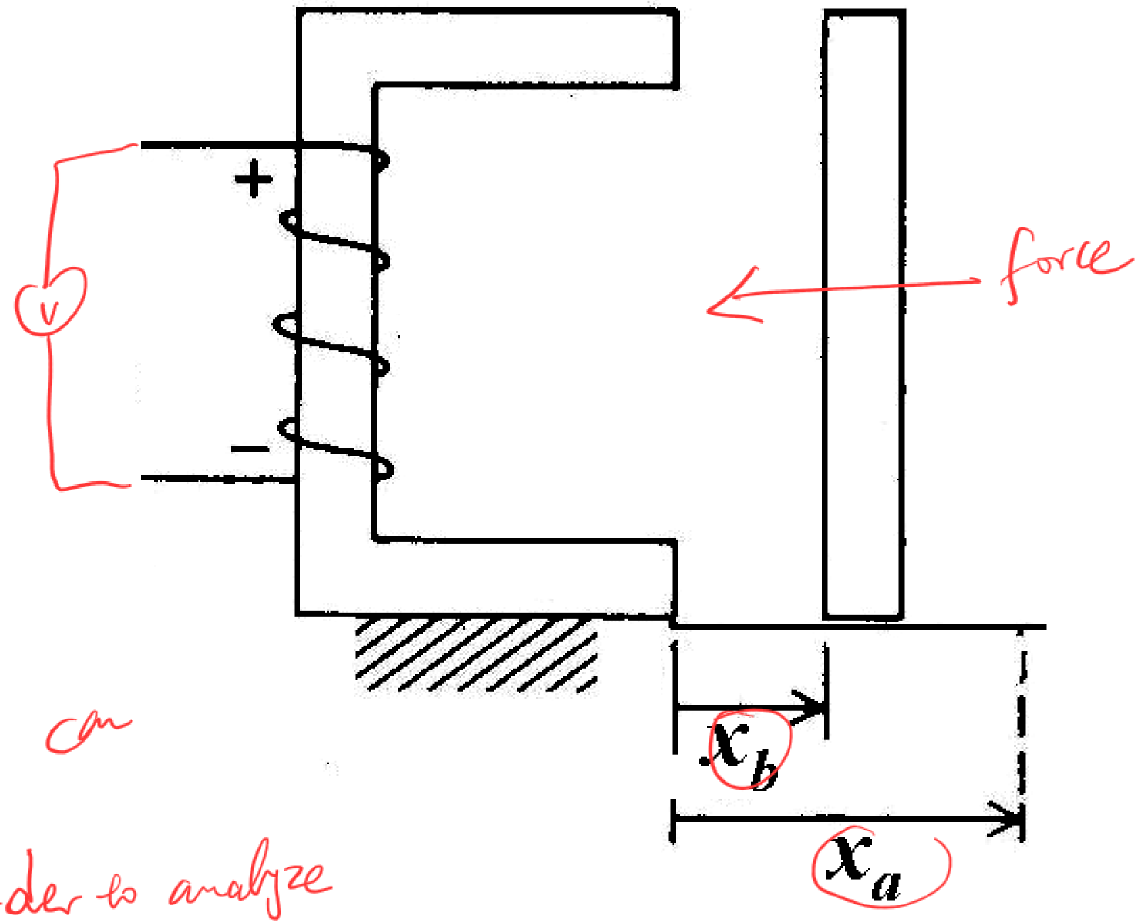
Graphical interpretation

Assume move from x_a to x_b

Consider $\lambda - i$ relationship



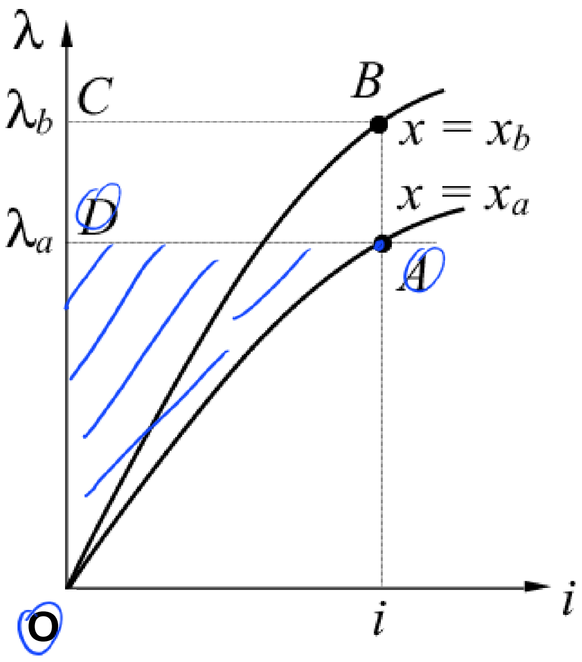
smaller $\mathcal{R}$
larger L, λ



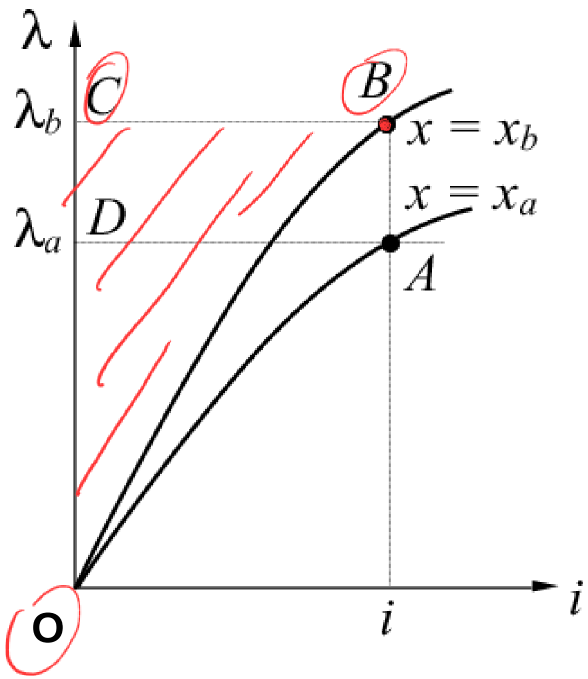
Both λ, i can change
↳ Harder to analyze
↳ Examine λ, i changes separately

Change in Energy at Constant Current

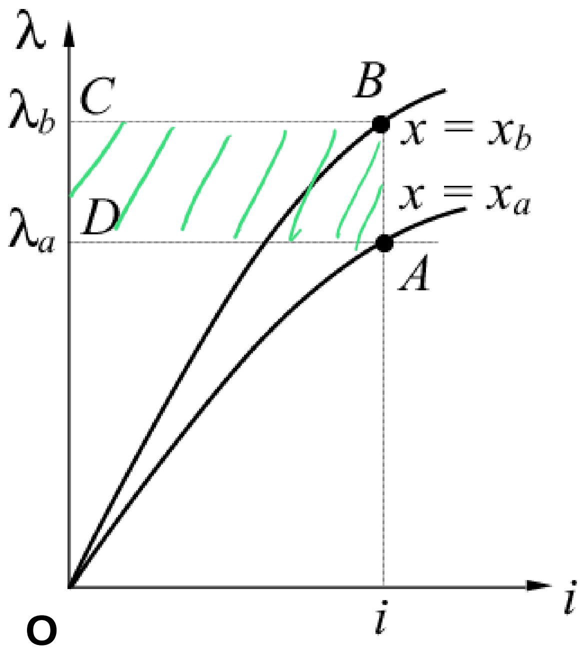
State x_a



State x_b



Change in W_e



Coupling Field Energy $\quad \Delta W_f = OBC - OAD$

Change in electrical input $\quad \Delta W_e = \int iedt = \int_{\lambda_a}^{\lambda_b} id\lambda = ABCD$

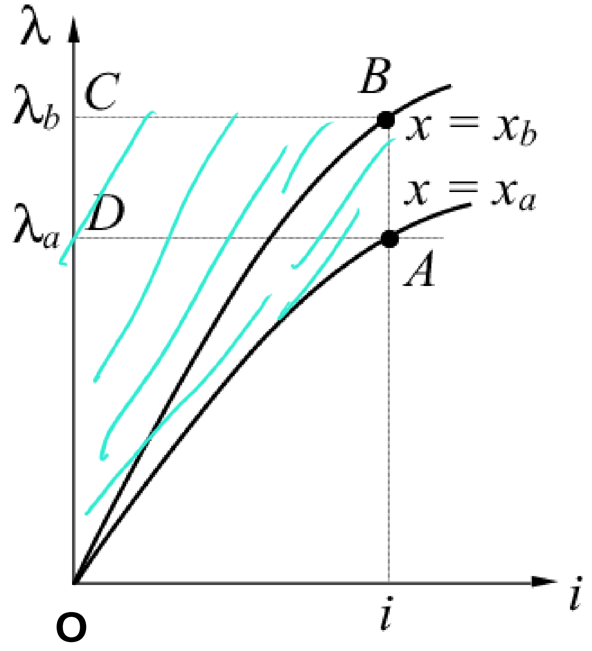
$\int \left(\frac{d\lambda}{dt}\right) i\, dt \rightarrow$ constant

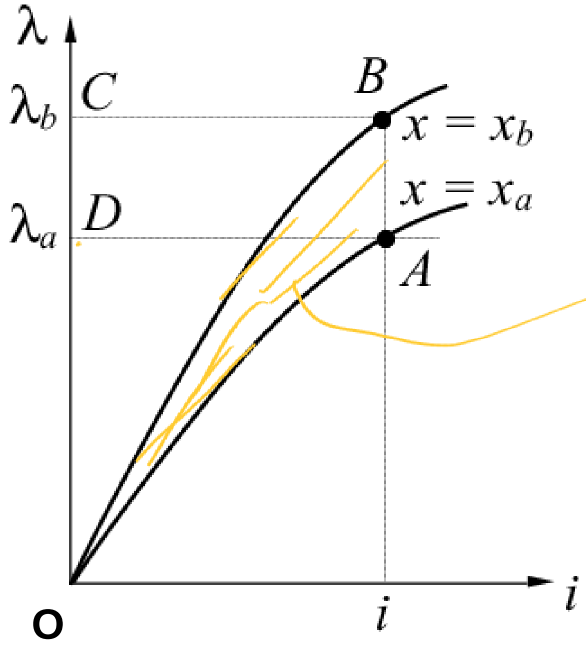
$\Delta W_m = ?$

$= \Delta W_e - \Delta W_f$

Change in Energy at Constant Current

Change in Mechanical Energy $\Delta W_m = \Delta W_e - \Delta W_f = ABCD - (OBC - OAD)$





Balance of areas: $OAD + ABCD = ABO + OBC$
$= ABCO$

$$\boxed{\Delta W_m = ABO}$$

Remember Co-Energy:

$$W_c = \int \lambda di = W_{c_{BOi}} - W_{c_{AOi}}$$

$$\boxed{\Delta W_m = \Delta W_c}$$

Change in mechanical work under constant current $\Delta W_m = \Delta W_c = f_m \Delta x$

Developed electromagnetic Force $f_m(i,x) = \left.\dfrac{\partial W_c(i,x)}{\partial x}\right|_{i=const}$

assumption

So change in co-energy is related to change in mechanical energy → Test question

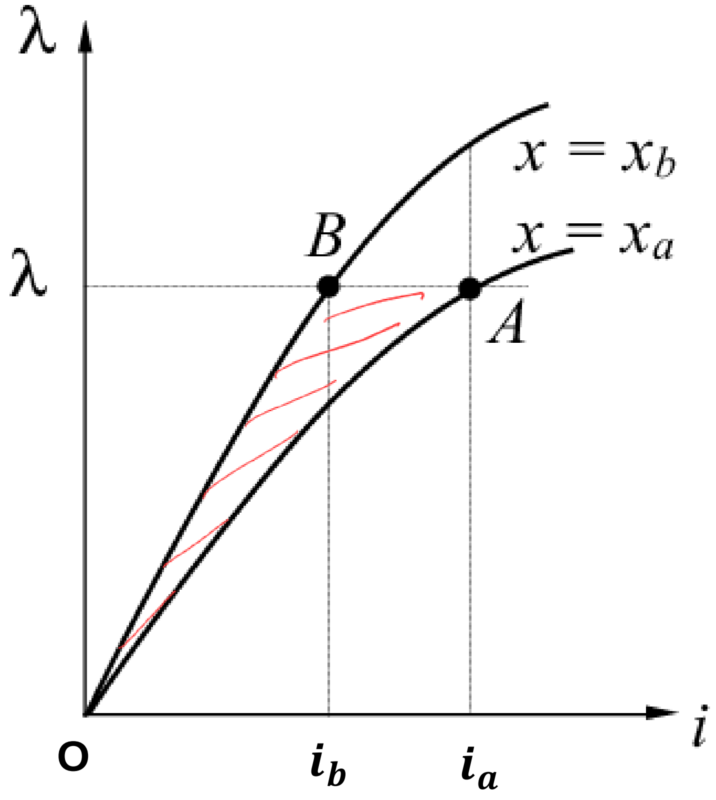
Change in Energy at Constant Flux

$\phi \Rightarrow \int i \, d\lambda$

Change in Mechanical Energy at constant flux $\Delta W_m = \Delta W_e - \Delta W_f = OAB$

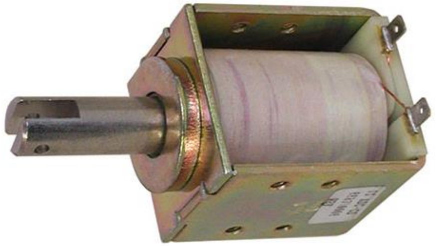
Change in mechanical work under constant flux $\Delta W_m = f_m \Delta x = -\Delta W_f$

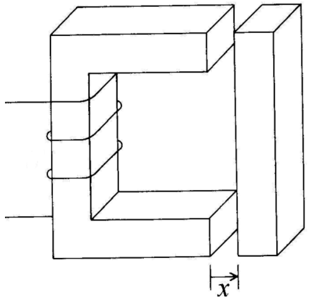
Developed electromagnetic Force $f_m(i,\lambda) = -\left.\dfrac{\partial W_f(i,\lambda)}{\partial x}\right|_{\lambda = const}$ → assumption



Linear vs. Rotating Devices

Linear Devices





Mechanical Energy/Work

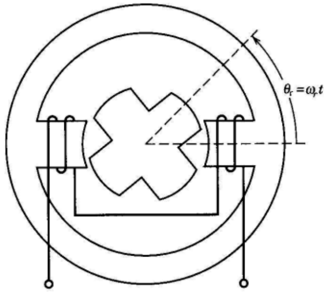
$$W_m = \int f_m dx$$

Electromagnetic Force f_m

$$dW_m = f_m dx$$

$$f_e(i, x) = \frac{\partial W_c}{\partial x} \quad \text{or} \quad f_e(\lambda, x) = -\frac{\partial W_f}{\partial x}$$

Rotating Devices



Mechanical Energy/Work

$$W_m = \int T_m d\theta$$

Electromagnetic Torque T_m

$$dW_m = T_m d\theta$$

[N.m]

[rad]

$$T_e(i, \theta) = \frac{\partial W_c}{\partial \theta} \quad \text{or} \quad T_e(\lambda, \theta) = -\frac{\partial W_f}{\partial \theta}$$

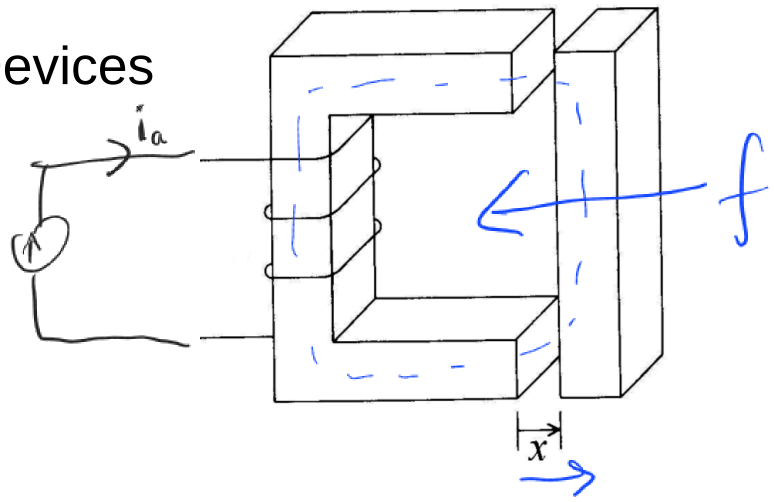
For magnetically linear systems:
Energy and Co-Energy are the same

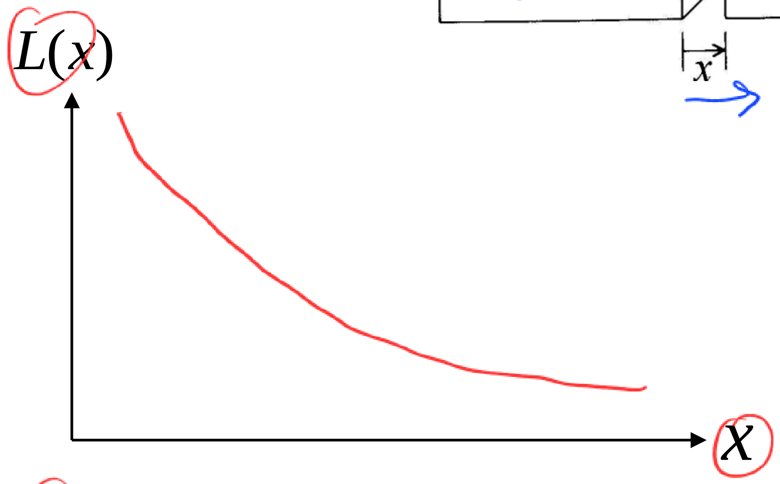
$$W_f = W_c = \frac{1}{2}\lambda i = \frac{1}{2}L(x)i^2$$

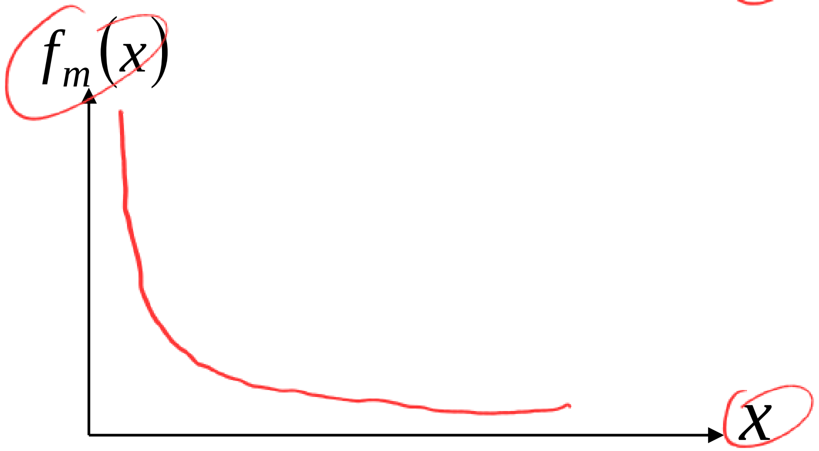
Example

Neglect saturation and leakage

Linear Devices







Given that

constant   neglect $\frac{k_1}{k_2+x}$ approximate

$$\lambda(i,x) = Li = \left[L_l + L_m(x)\right]i = \left(\cancel{L_l} + \frac{k}{x}\right)i$$

Calculate $f_m(i,x)$ at given current $i = i_a$

$$W_f(i,x) = W_c(i,x) = \frac{1}{2}\left(\frac{k}{x}\right)i_a^2$$

$$f_m(i,x) = \frac{\partial W_c}{\partial x} = \frac{1}{2}i_a^2\frac{\partial L}{\partial x} = \boxed{-\frac{1}{2}i_a^2\frac{k}{x^2}}$$

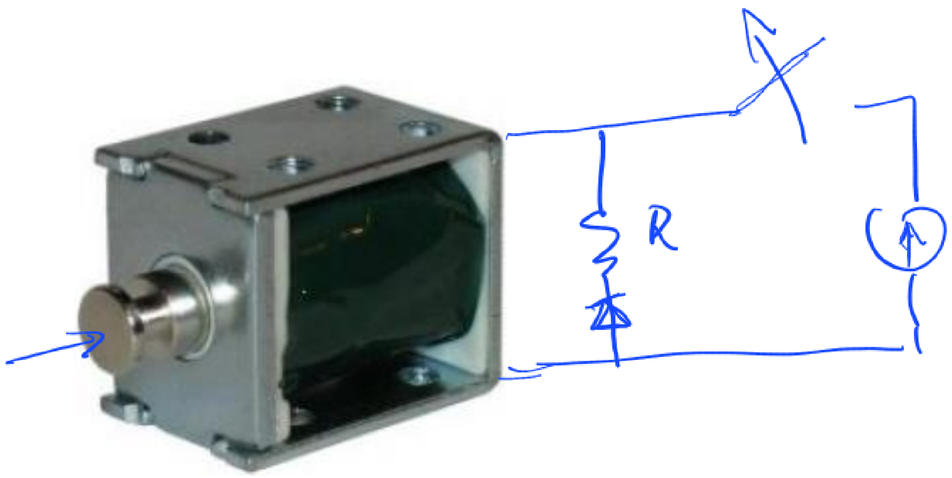
What does the negative sign mean?

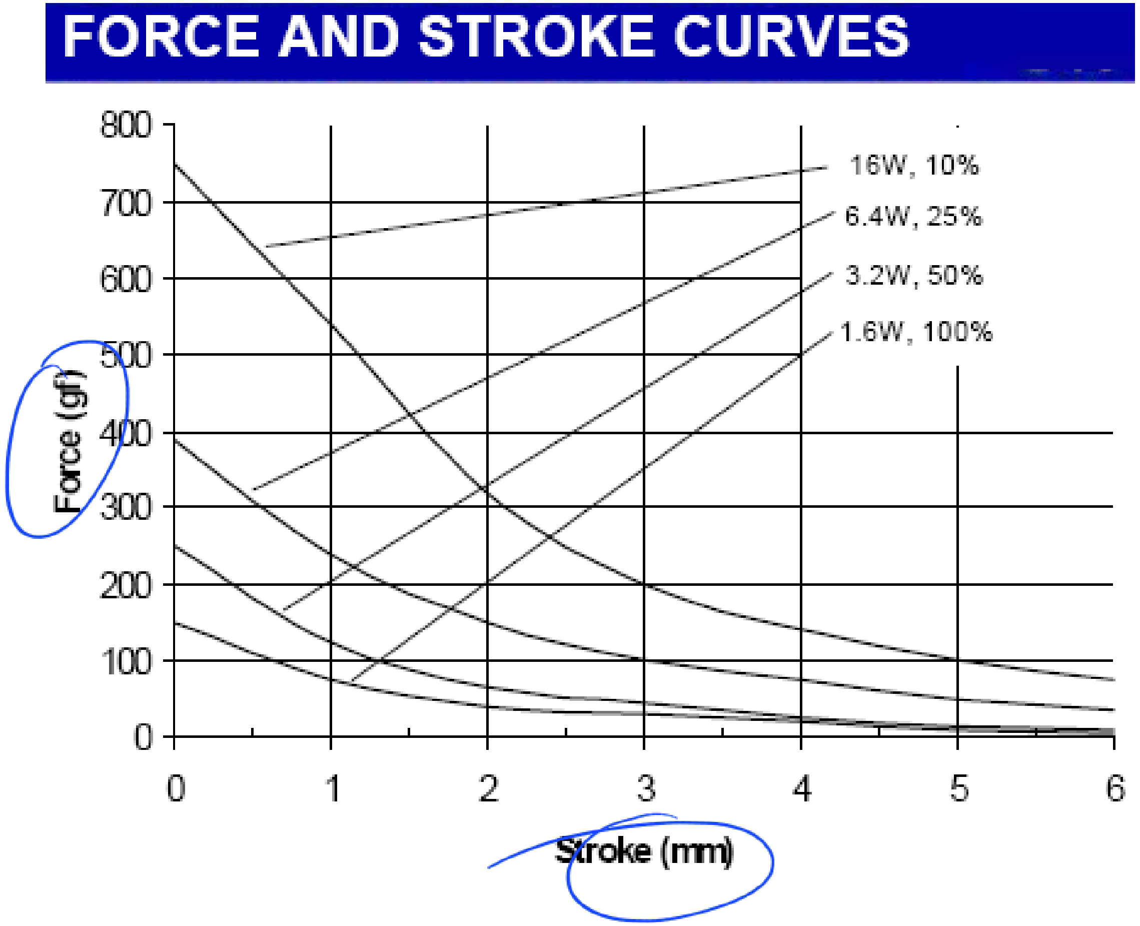
Can the direction of the force be changed?

→ Electromagnet attracts

→ For passive system, can't change direction

Typical Industrial Push-Pull Solenoids



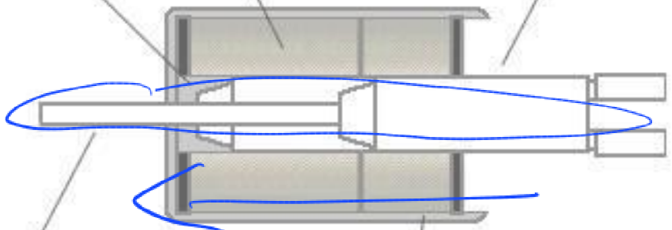


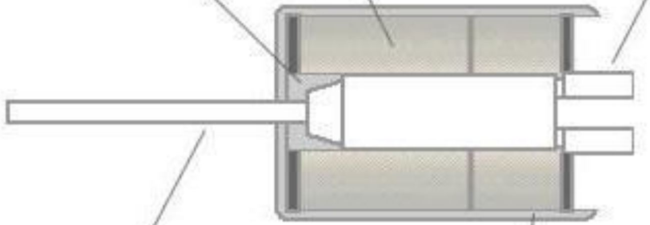
Typical Push-Pull Solenoids

Naturally, the solenoid coil pulls-in the plunger
To get a push action, a thrust pin is added

pull in

push out



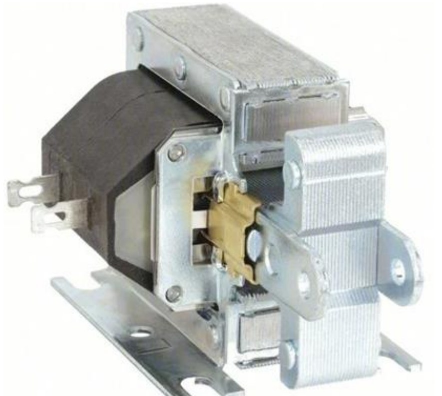


DC vs AC Solenoids:

$\frac{d\lambda}{dt}$

- DC solenoids may use solid core
- AC solenoids may use laminated core

$\frac{d\lambda}{dt} \rightarrow i$



Module 4: DC Machines

↳ Lab 3

Objectives & important concepts

- Basic construction & types of DC machines
- Principle of induced voltage and electromagnetic torque
- Speed, torque, motoring / generating principle
- Equivalent circuits, tests & determination of parameters
- Classification of dc machines & torque-speed characteristics
- Starting issues
- Basic types of DC motor drives, DC to DC choppers, four quadrants of operations
- AC to DC controlled rectifiers
- Dynamic modeling of DC motors

- Must read **Chapter 4**

DC Machines

Range of Sizes

- Miniature dc motors – mW range
- Fractional HP (1HP = 746W)
- Medium Size – 1 … 500 HP
- Large DC Machines – 500 HP and up

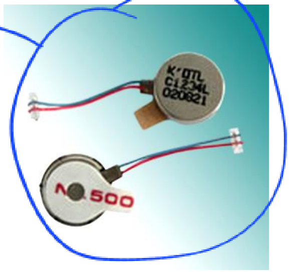
Applications

- Very popular and easy to use in various applications
- Tools, robotics, toys, medical, automotive & other industries

General Properties

- Very easy to control !
- Good torque-speed performance
- Brushes is a weak point of design
    - Limits the application
    - Limits the lifetime
    - Maintenance

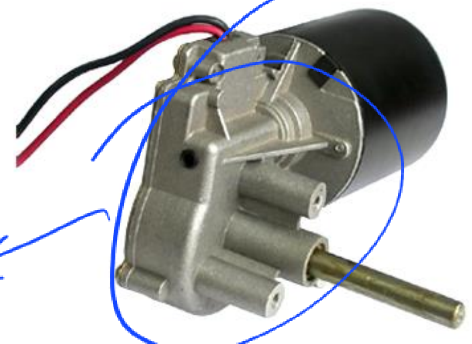
Phone vibration



Toys

Small Motors

Car window



Gear-head Motors

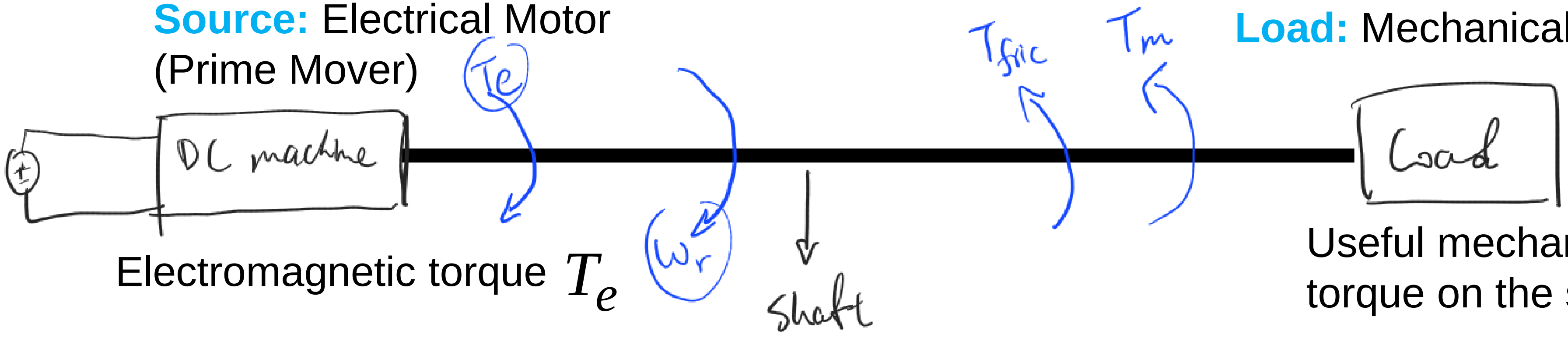
Motor Speed and Torque

$P_m = T_m \omega_r$

Transmitting power through the mechanical shaft

Source: Electrical Motor (Prime Mover)

Load: Mechanical system



Electromagnetic torque T_e

Useful mechanical torque on the shaft T_m

Commonly used units of speed

ω - $[\text{rad}/\text{sec}]$

$$m = \frac{\omega}{2\pi} \quad [\text{rev}/\text{sec}]$$

$$n = 60\frac{\omega}{2\pi} = \frac{30}{\pi}\omega \quad [\text{rev}/\text{min}]$$

RPM => Most commonly used

Mechanical Motion

- Use motor sign convention

$$J\frac{d\omega_r}{dt} = T_e - T_m - T_{fric}$$

always opposite to rotation

$$J = J_{dc\_machine} + J_{mech\_load}, \quad [kg \cdot m^2]$$

- combined moment of inertia of the motor and load

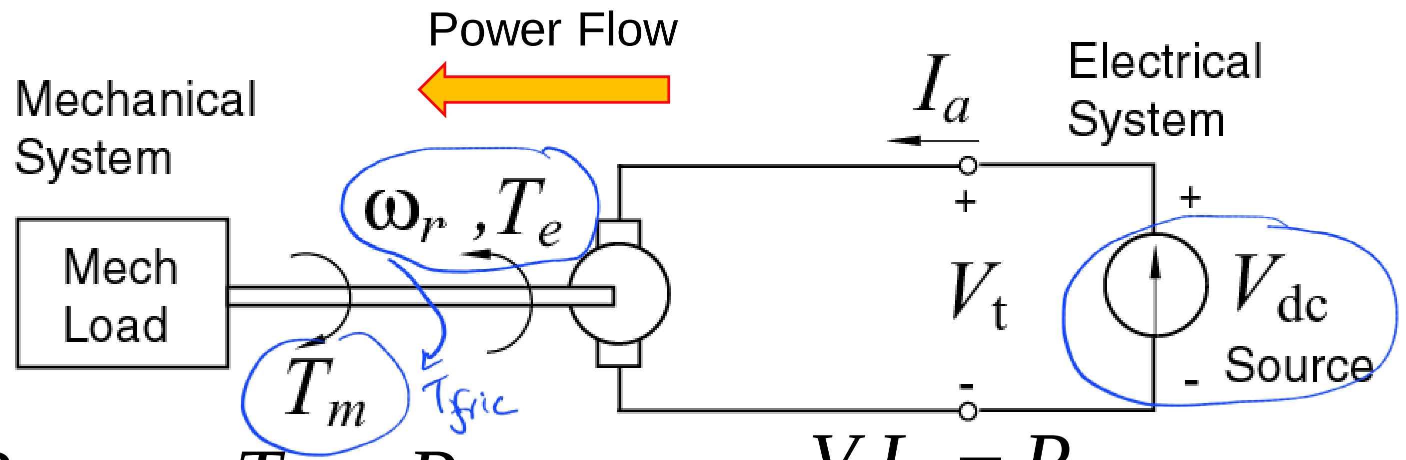
$\frac{d\omega_r}{dt} = 0$ (Steady state)

$\hookrightarrow T_e = T_m + T_{fric}$

DC Machine Motoring and Generating

Motoring

Mechanical load torque is applied in the direction opposite to rotation

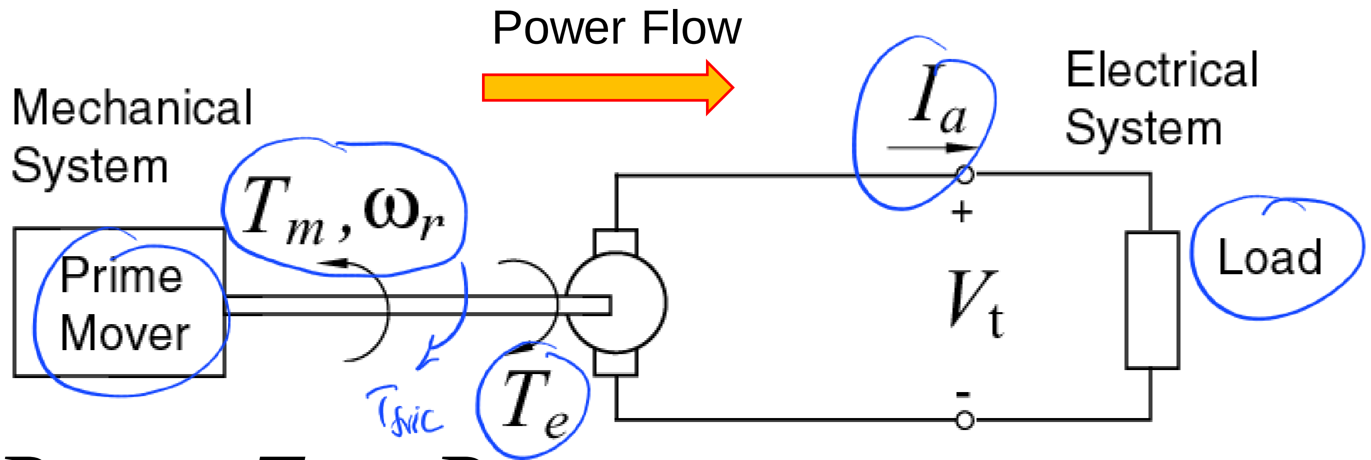


$$P_{out} = \omega_r T_m = P_m$$

$$V_t I_a = P_{in}$$

Generating

Prime mover torque is applied in the direction of rotation

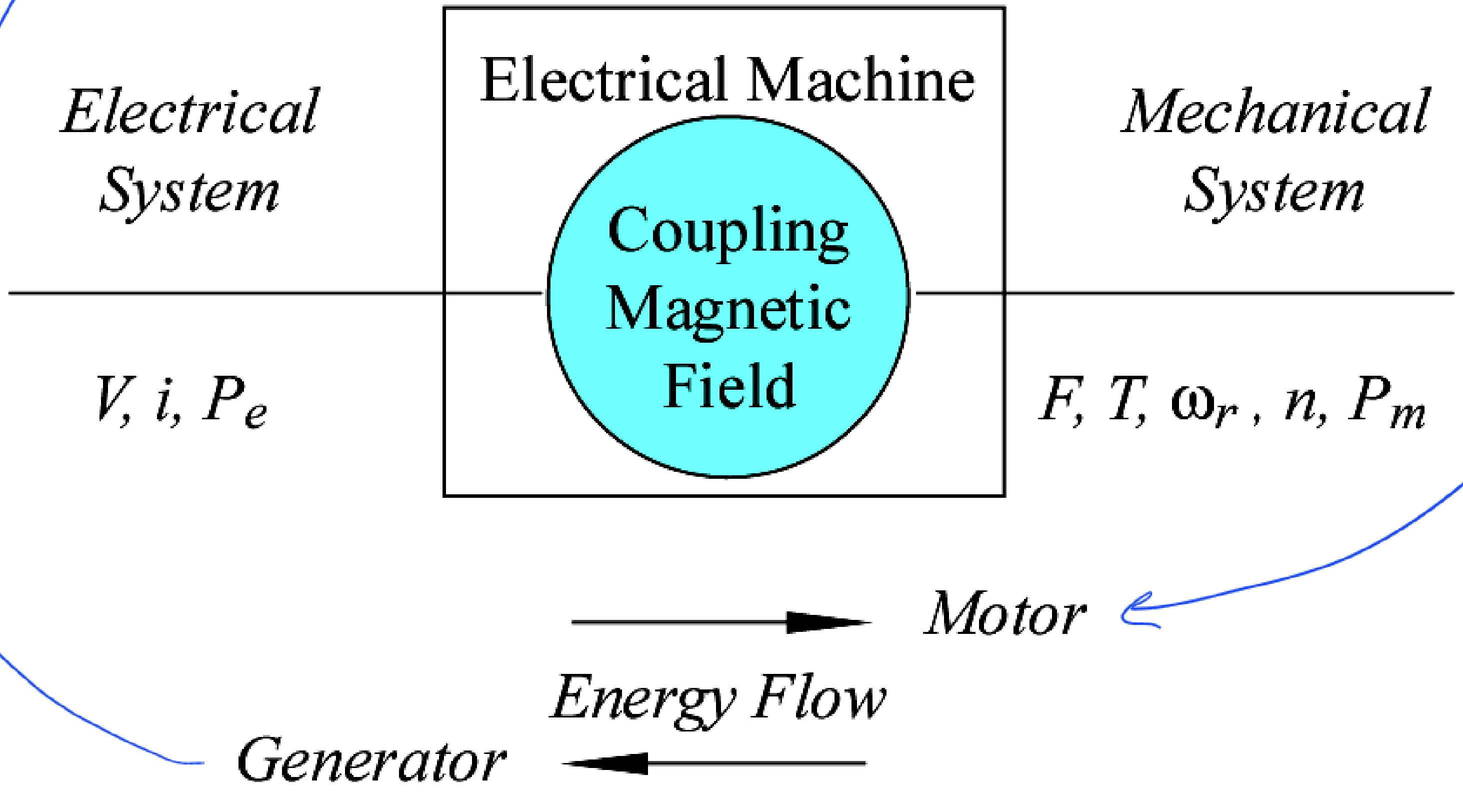


$$P_{in} = \omega_r T_m = P_m$$

$$V_t I_a = P_{out}$$

Electromechanical Fundamentals

- Current-carrying conductor in magnetic field
  => mechanical force => Lorentz force
- Conductor moves in magnetic field
  => voltage induced, emf => Faraday's Law

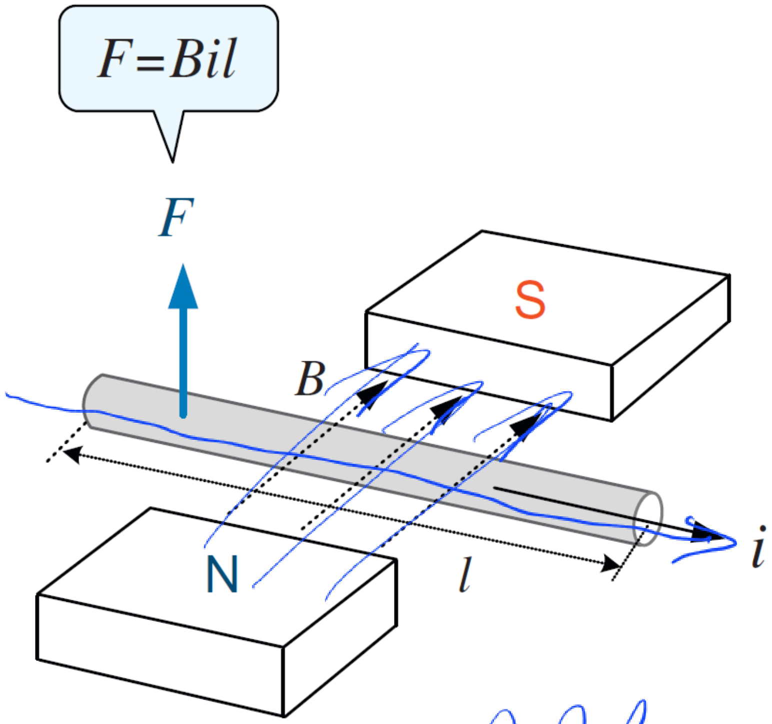


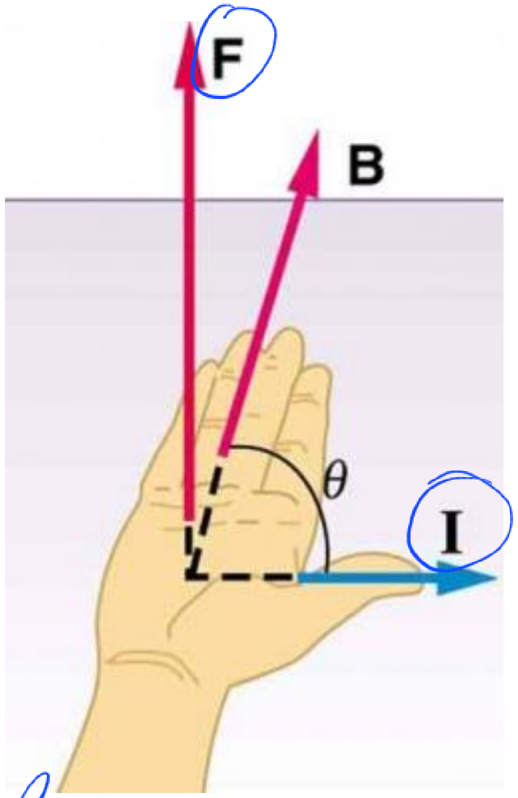
Fundamentals: Force

$\underline{F} = \underline{q}(\cancel{E} + \underline{v} \times \underline{B})$

❖ A force is exerted on a current-carrying conductor that is placed in a magnetic field:

Lorentz Force: $F = Bil$





Fundamentals: Force & Torque

Consider a conductor frame (single-coil)

Lorentz Force $\quad F = lBi_a$

Torque
$$T_e = 2RF\cos(\theta)$$
$$T_e = 2RlBi_a\cos(\theta)$$
$$T_e = ABi_a\cos(\theta) = \Phi_p i_a \cos(\theta)$$

→ Area of frame

→ ωt

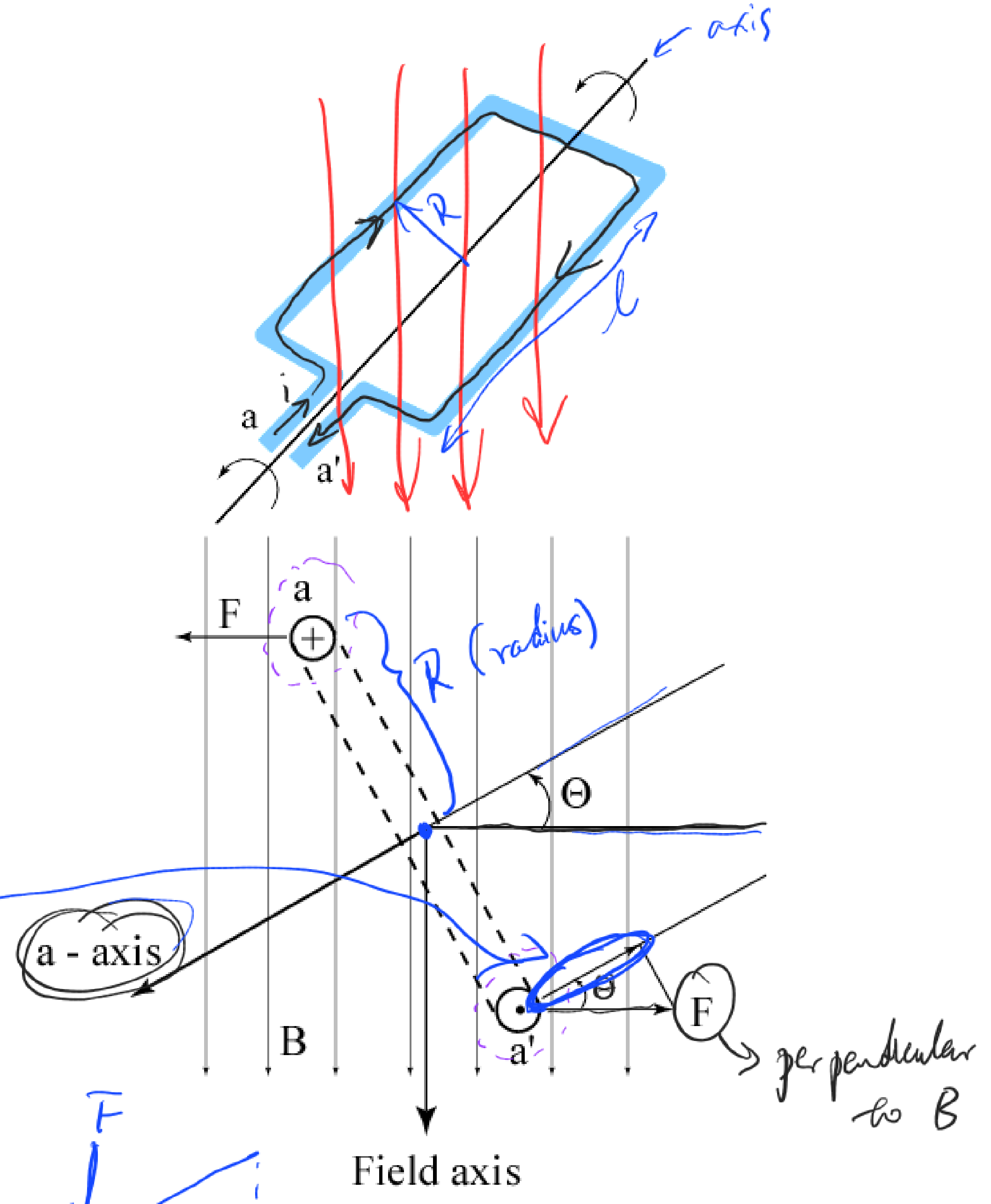
NOTE: Maximum torque is when $a - axis \perp B$

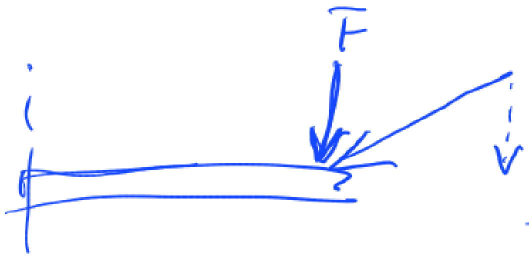
and ⇨ $T_e \sim \Phi_p i_a$ → one frame

$i_a \uparrow \Rightarrow T_e \uparrow$

$\Phi_p \uparrow \Rightarrow T_e \uparrow$

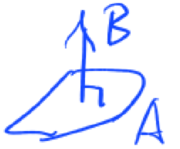
↳ BA



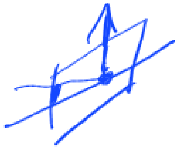


$l F_\perp = T$

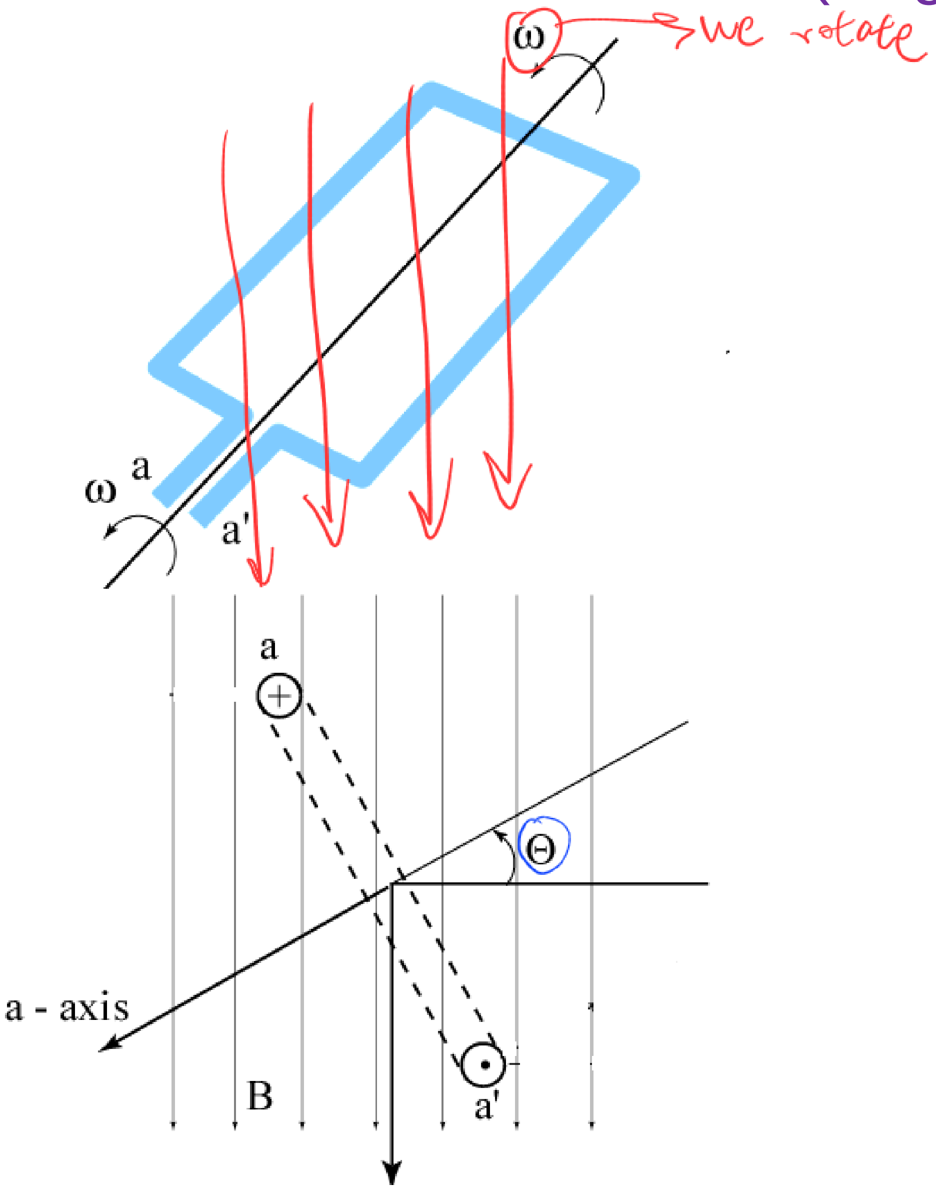
Fundamentals: Induced Voltage



$\Phi = AB$



Consider a conductor frame (single-coil)



$$\Phi = BA\sin(\theta) = B2Rl\sin(\theta)$$

Let us rotate the frame with speed ω

$$\Rightarrow \theta = \omega t + \theta_o$$

$$\Rightarrow \Phi = BA\sin(\theta) = \Phi_p \sin(\theta)$$

$\omega t + \theta_o$

Faraday's Law $e = {d\Phi}/{dt}$

For coil with one turn

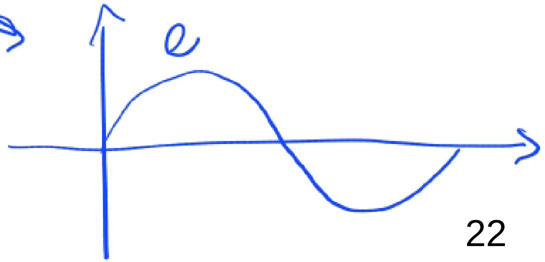
emf

$$e = \frac{d\Phi}{dt} = \omega\Phi_p \cos(\theta) \quad \Rightarrow \quad e \sim \omega\Phi_p$$

comes from differentiation

For coil with *N* turns

$$e = N\frac{d\Phi}{dt} = \omega\lambda \cos(\theta)$$

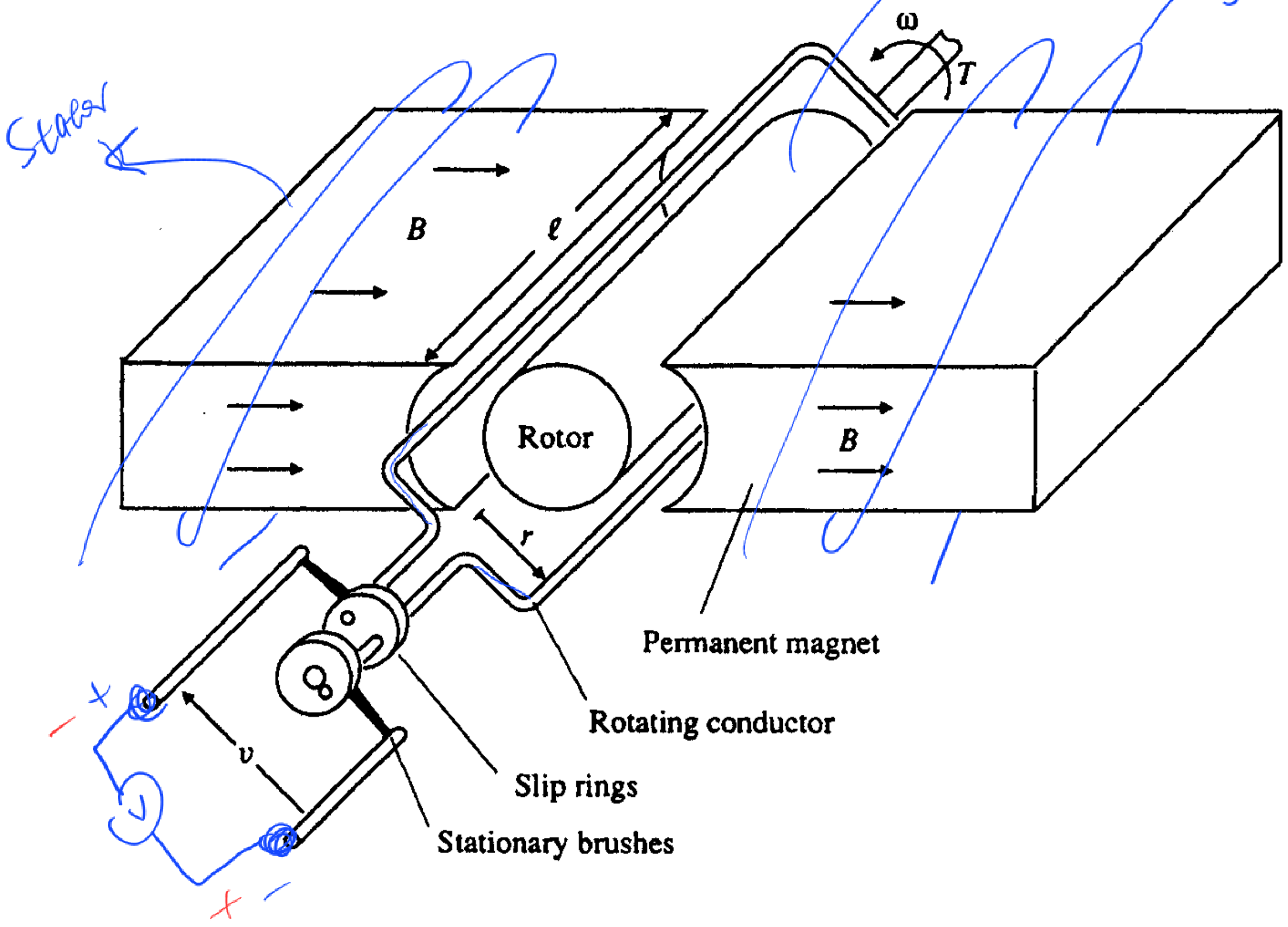


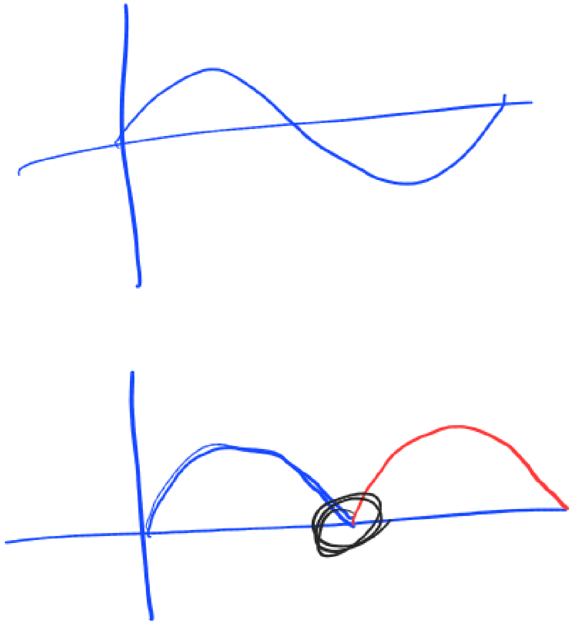
Conductor Frame (1-turn coil)

Want to fill the air gap to minimize $\mathcal{R}$

❖ Consider a conductor frame (single-coil) placed between magnetic poles:

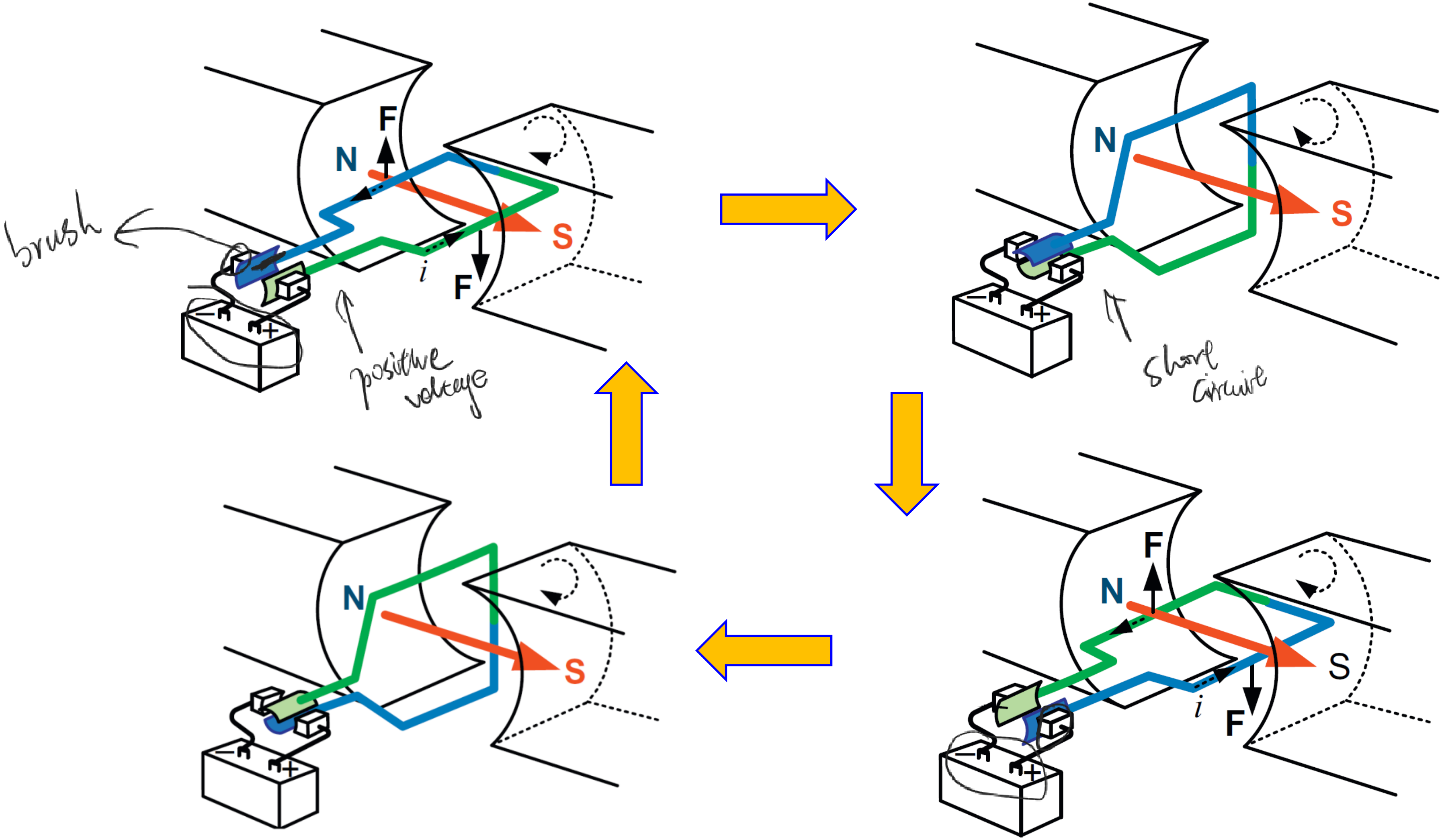
field winding





Conductor Frame (1-turn coil)

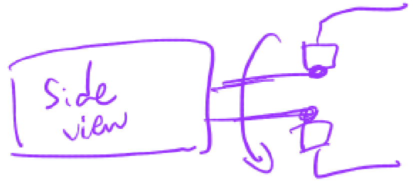
- Consider a conductor frame (single-coil) placed between magnetic poles:



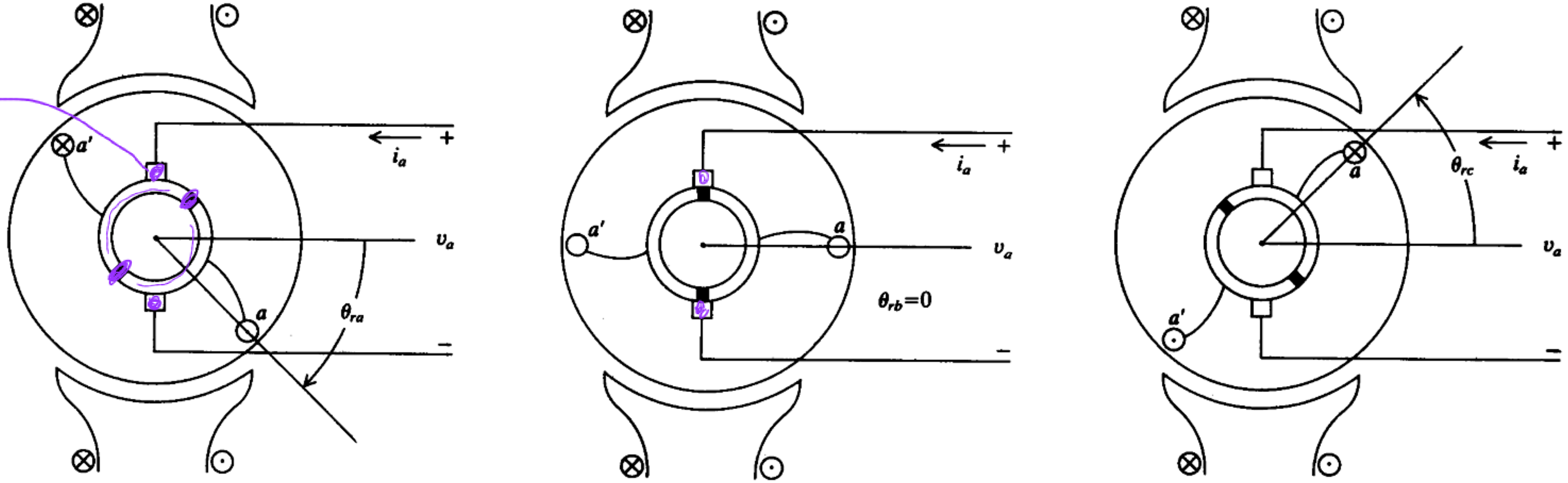
Preserves polarity mechanically
↓
in our frame of reference, brush is stationary
↓
in conductor's frame of reference, brush is rotating

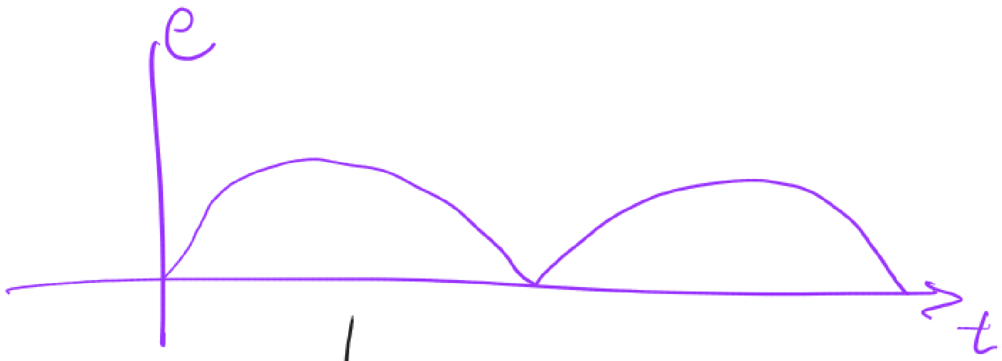
Is continuous torque produced? No

Commutation of Elementary DC Machine



brushes typically made of carbon
↓
won't melt or weld together with conductor





Mechanical rectification of voltage

↳ Add more frames to reduce discontinuity

Looking ahead…

- Continue **Module 4**