ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 9

February 3, 2026

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Course Logistics

- Assignment 3 due on Feb. 11
- Quiz 2 will take place on Feb. 13 ⇒ Modules 2 + 3
- Midterm will take place either Feb. 24 or Feb. 26 (there is a poll on Piazza)

Fundamentals: Force & Torque

Consider a conductor frame (single-coil)

Lorentz Force $F = lBi_a$

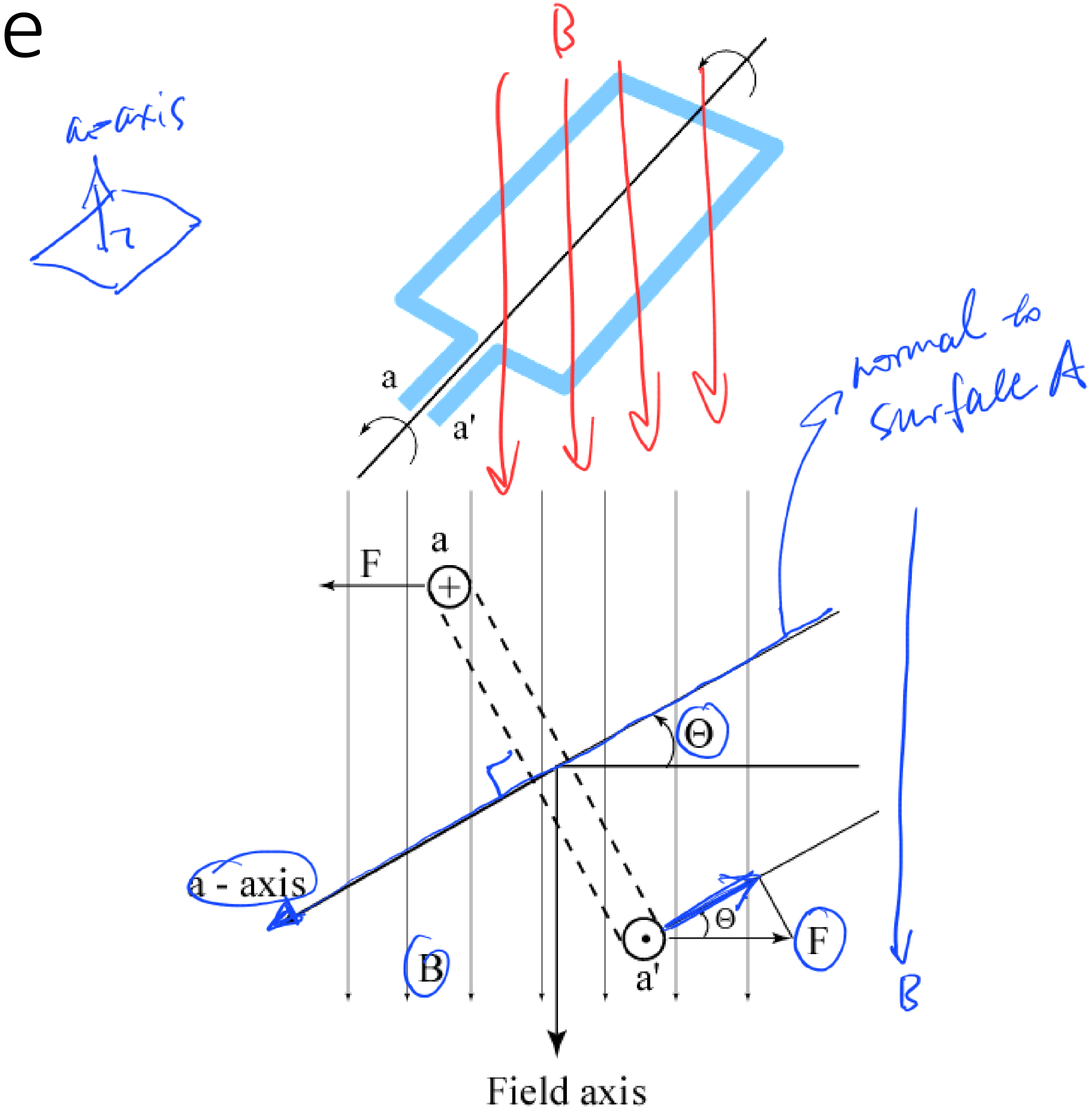
Torque $T_e = 2RF\cos(\theta)$

$$T_e = 2RlBi_a\cos(\theta)$$

$$T_e = ABi_a\cos(\theta) = \Phi_p i_a\cos(\theta)$$

NOTE: Maximum torque is when $a-axis \perp B$

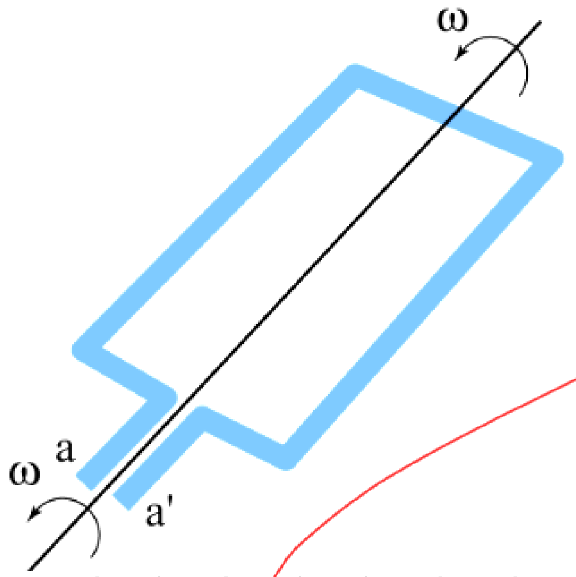
and ⇨ $T_e \sim \Phi_p i_a$

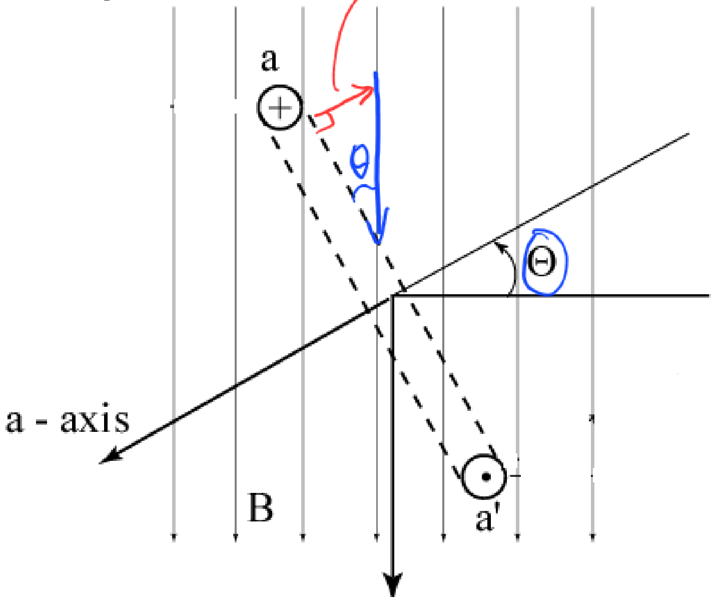


Fundamentals: Induced Voltage

$\Phi = \int B \cdot dA \rightsquigarrow B \perp A$

Consider a conductor frame (single-coil)





$$\Phi = BA\sin(\theta) = B2Rl\sin(\theta)$$

Let us rotate the frame with speed ω

$$\theta = \omega t + \theta_o$$

$$\Phi = BA\sin(\theta) = \Phi_p \sin(\theta)$$

Faraday's Law $\quad e = {d\Phi}/{dt}$

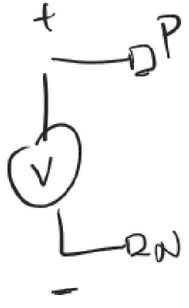
For coil with one turn

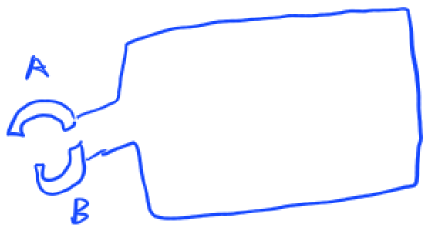
$$e = \frac{d\Phi}{dt} = \omega\Phi_p \cos(\theta) \Rightarrow e \sim \omega\Phi_p$$

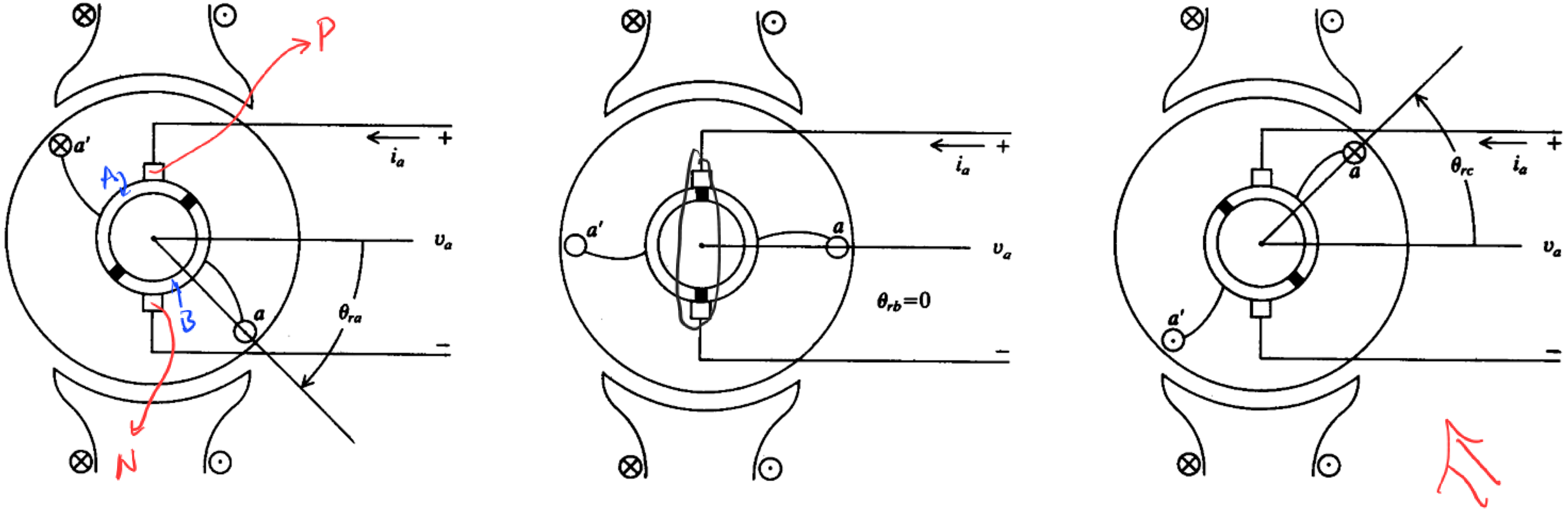
For coil with N turns

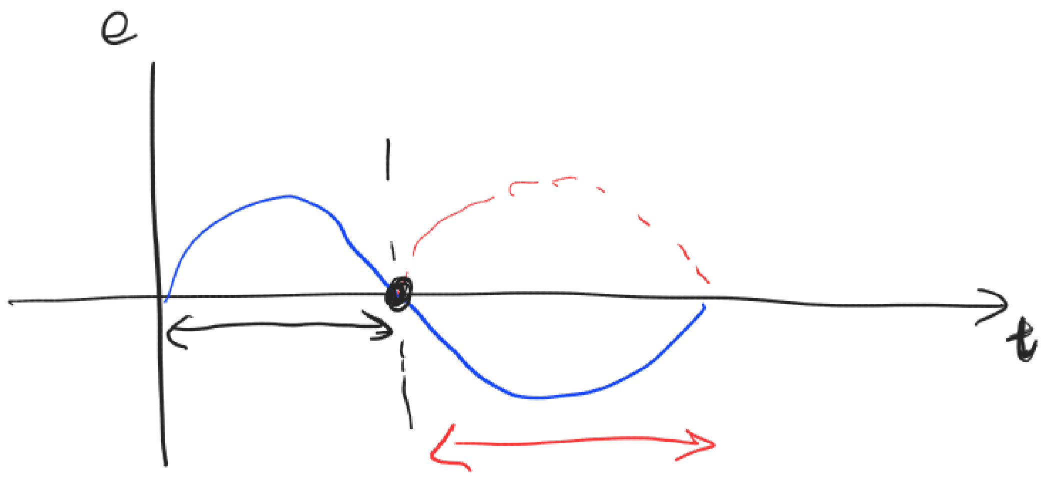
$$e = N\frac{d\Phi}{dt} = \omega\lambda \cos(\theta)$$

Commutation of Elementary DC Machine





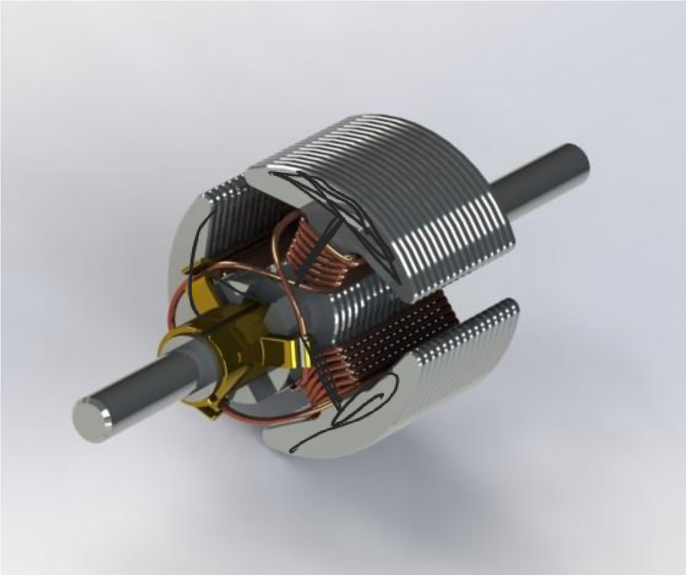


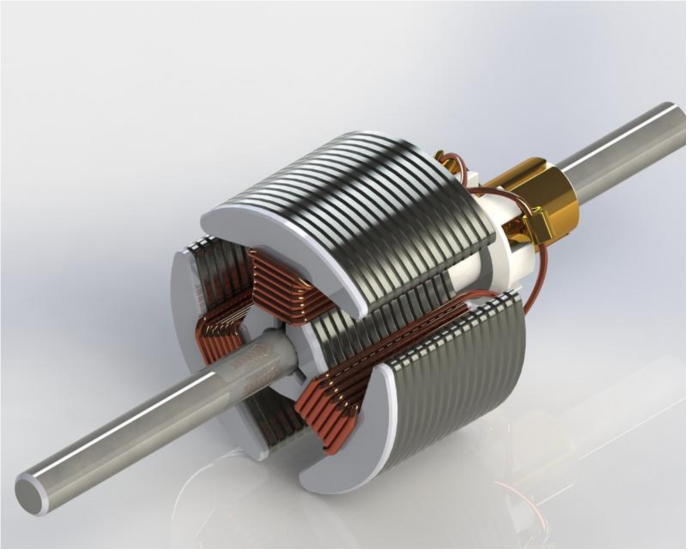


Add more frames to reduce ripple

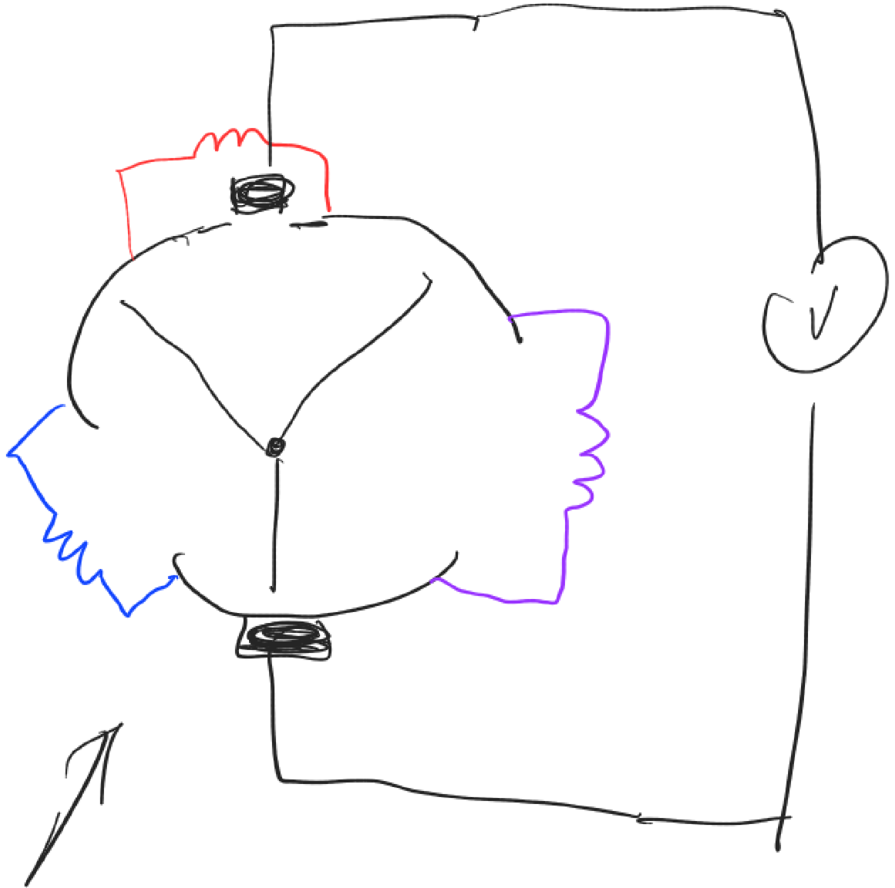
More Realistic DC Machine

3 windings on rotor (also called armature)



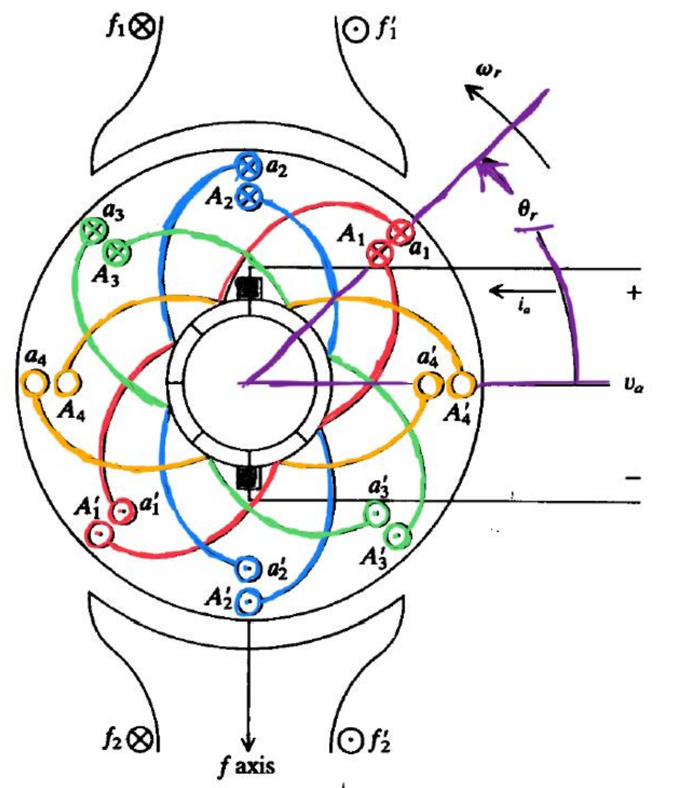


Winding Connection

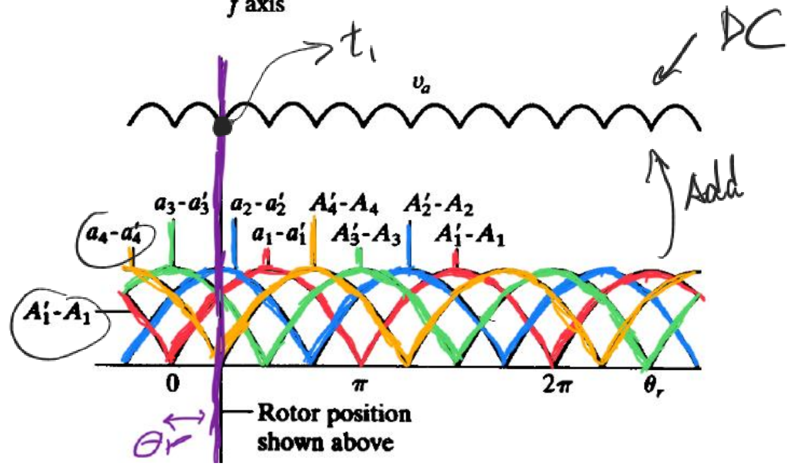


No short at any given moment

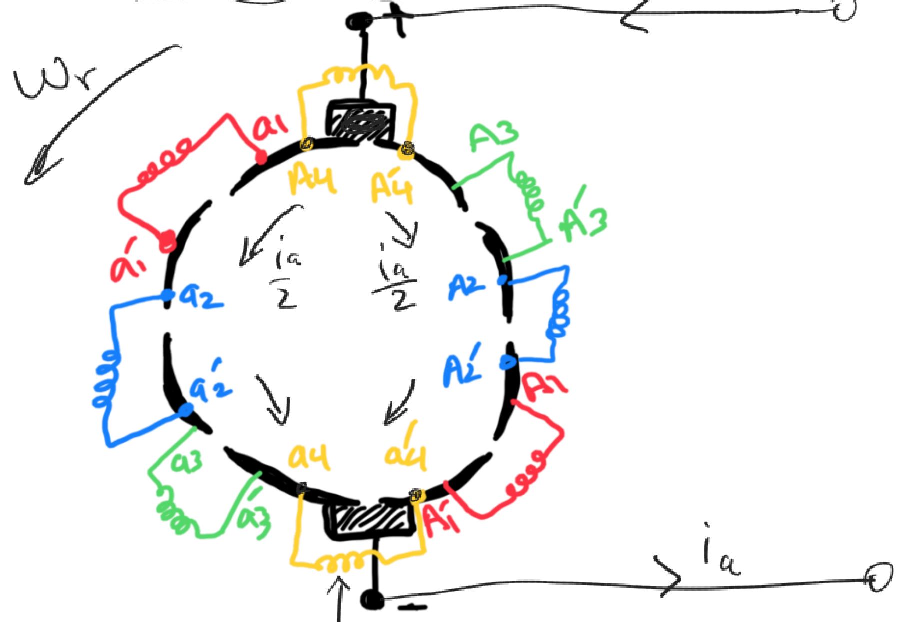
More Realistic DC Machine



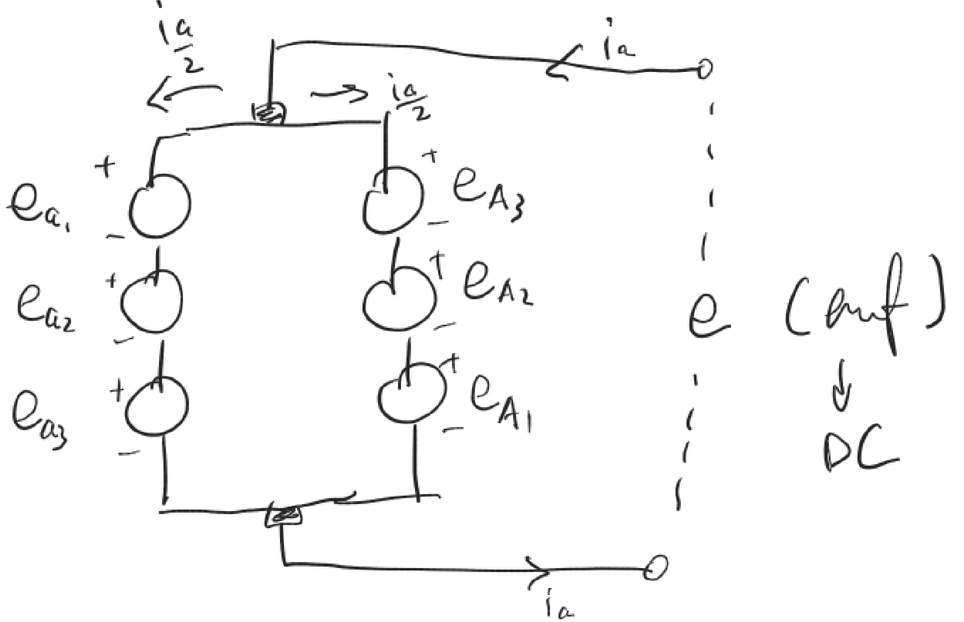
f windings:
- $A_1 - A_1'$
- $A_2 - A_2'$
- $A_3 - A_3'$
- $A_4 - A_4'$
- $a_1 - a_1'$
- $a_2 - a_2'$
- $a_3 - a_3'$
- $a_4 - a_4'$



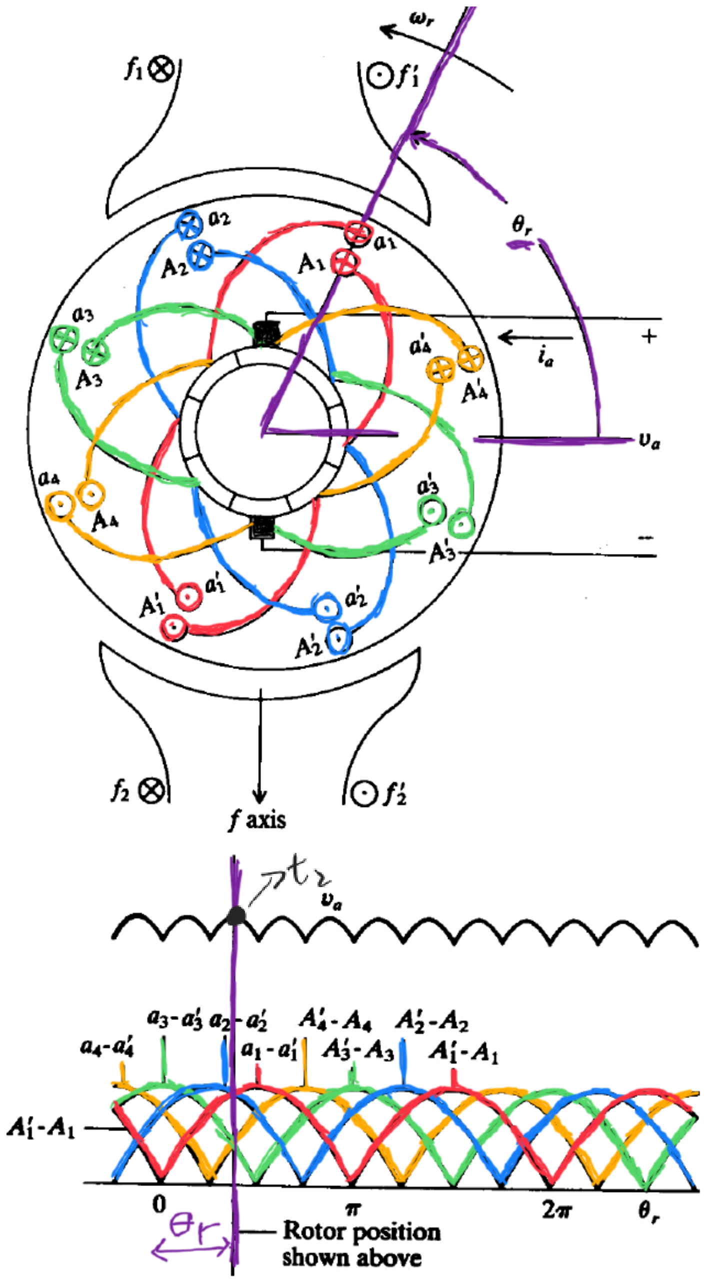
t_1: Winding Connection



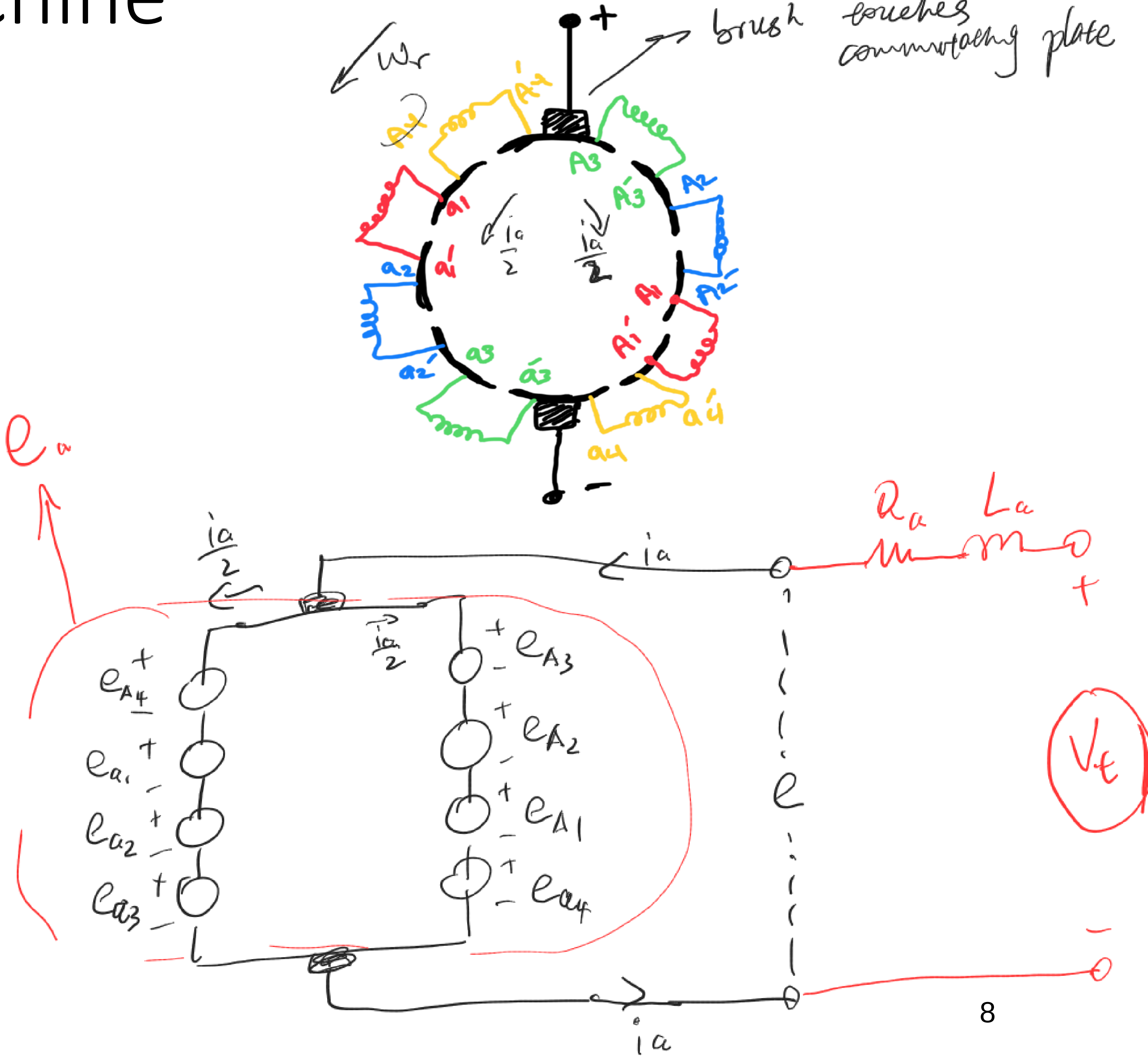
yellow coils are shorted are



More Realistic DC Machine

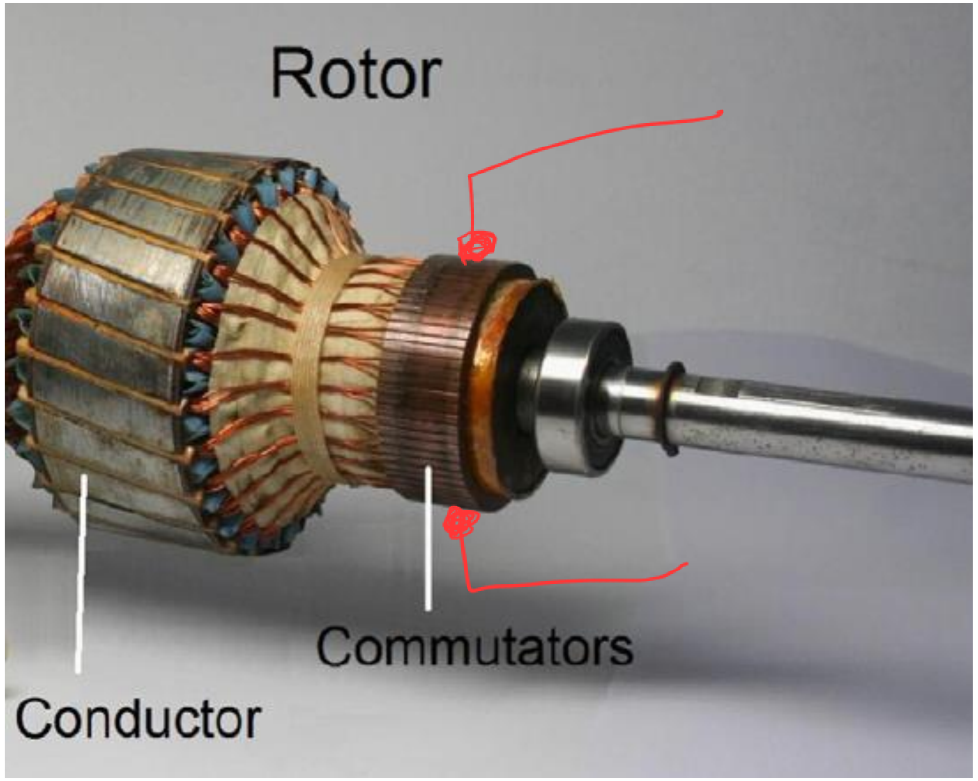


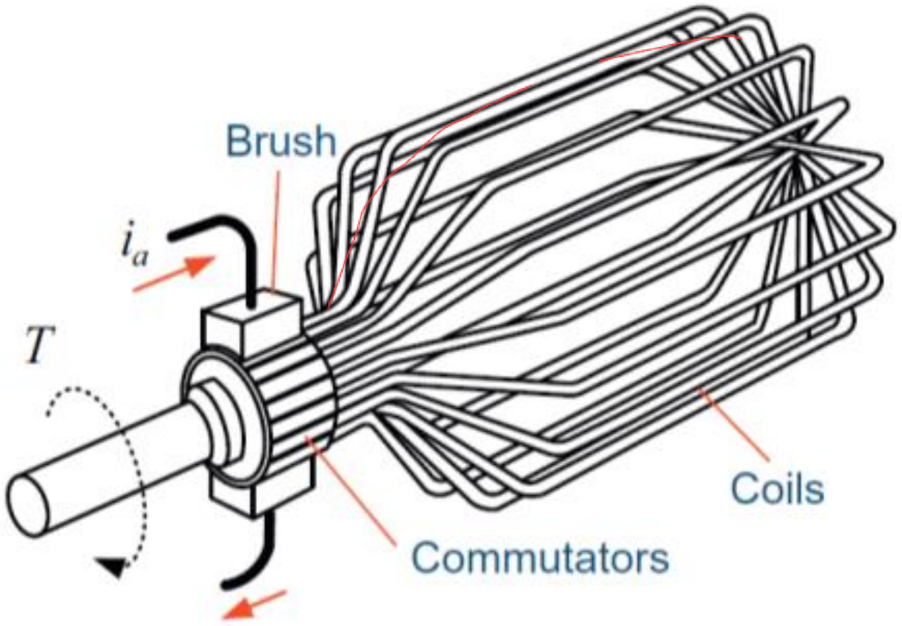
t_2: Winding Connection

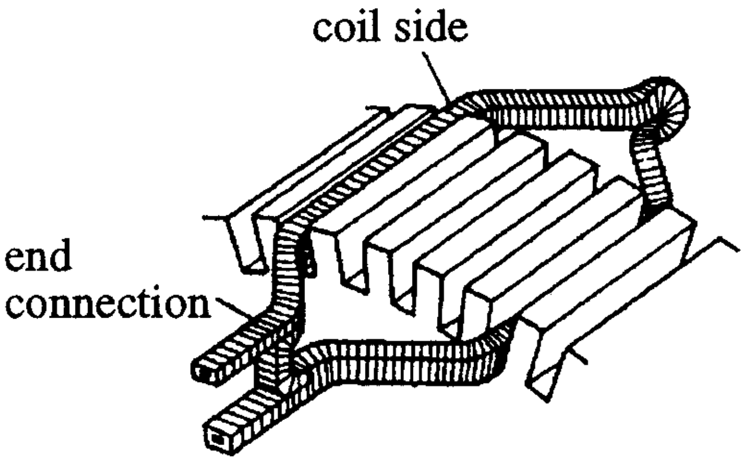


More Realistic DC Machine (rotor)

Conductor frame with several coils:

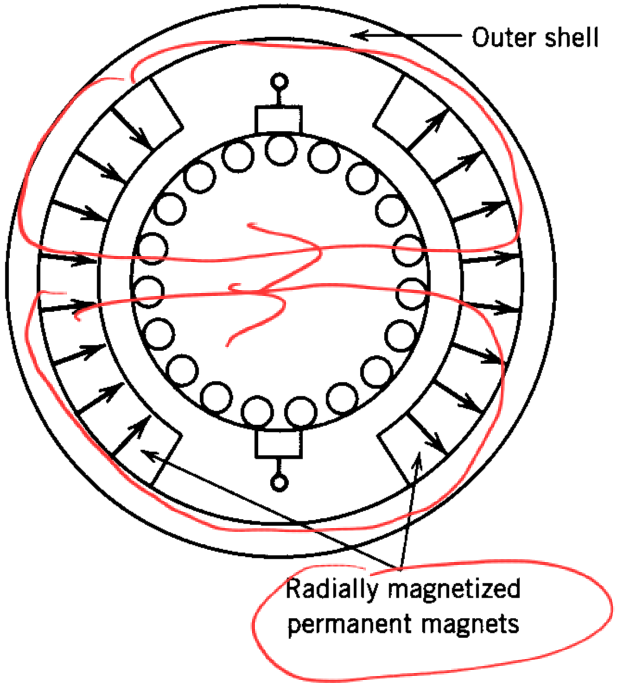






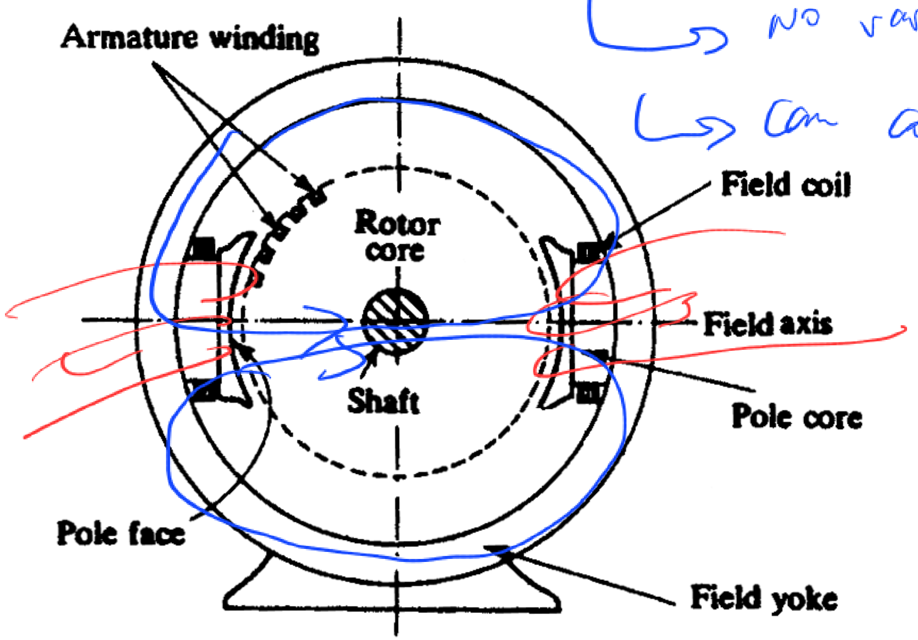
DC Machine Construction

Permanent Magnet (PM) DC machine Stator

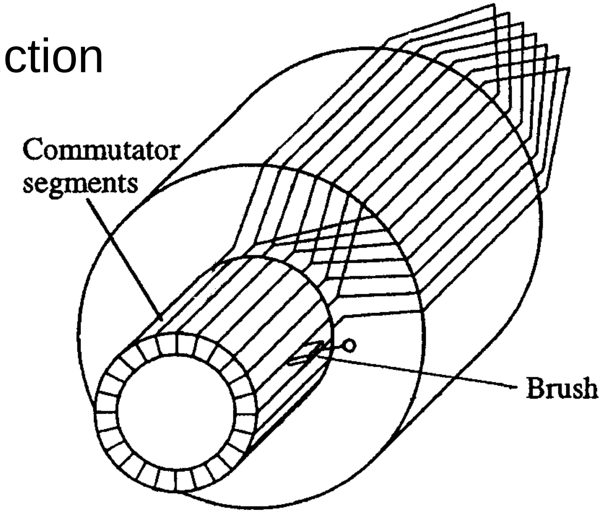


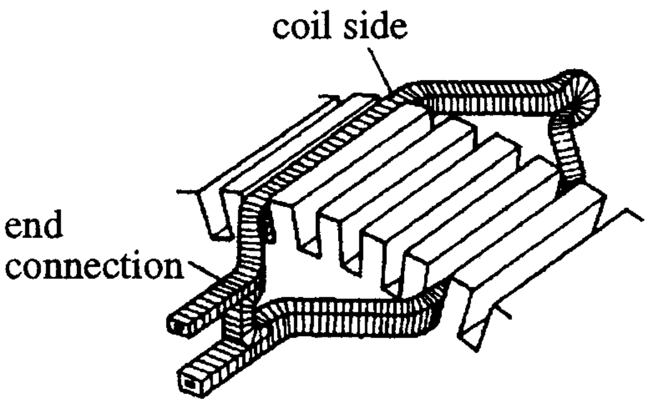
DC machine with field winding Stator

→ no rare earth materials
→ can control B

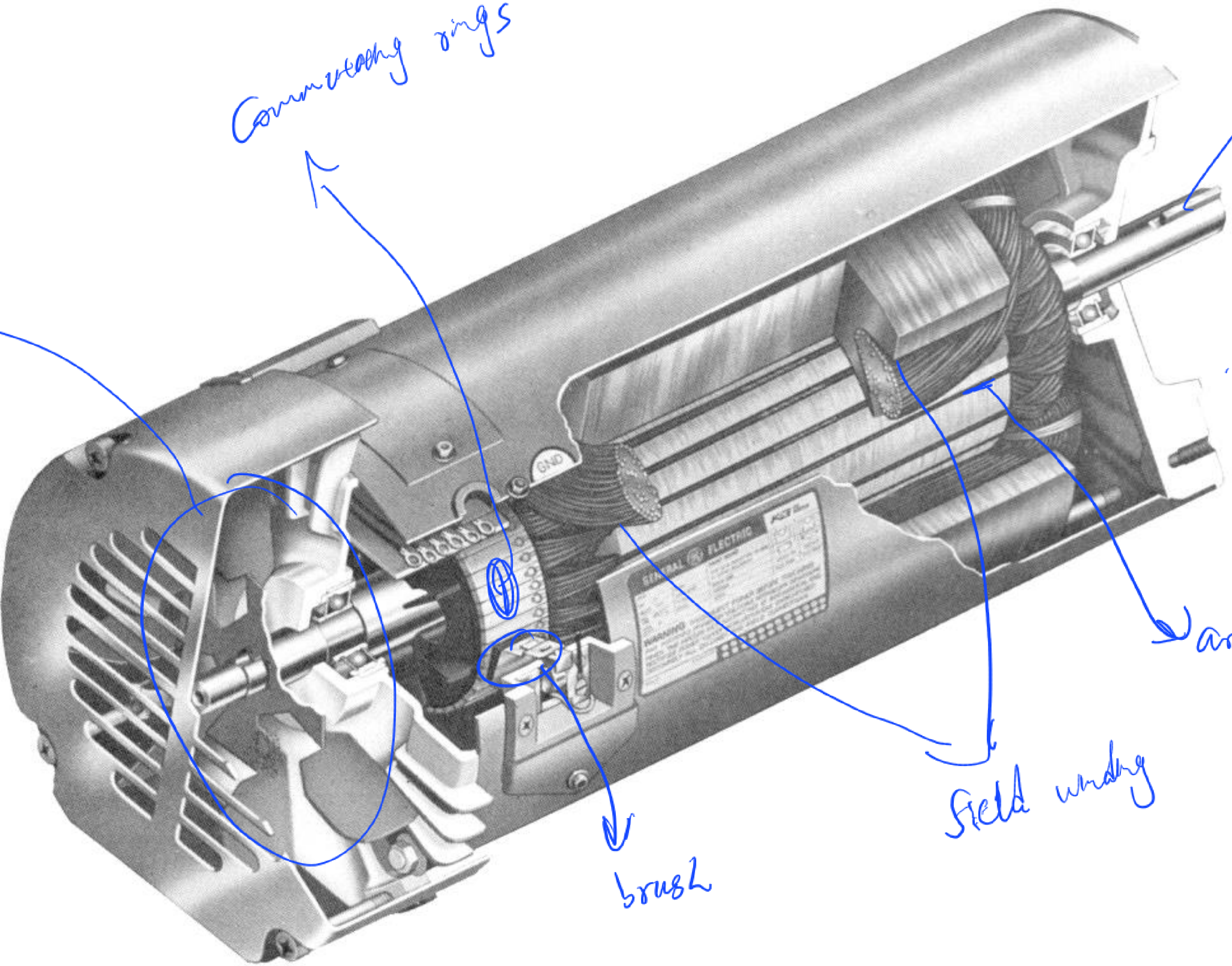


Rotor (Armature) construction



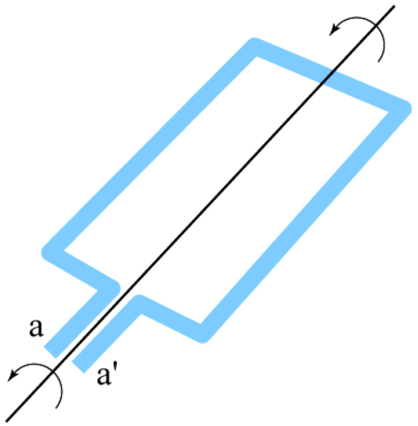


Cutaway View of a Two-Pole DC Machine



Induced Voltage & Torque

Recall a simple coil-frame – one turn



For armature winding we need to consider:

- Type of winding, rectification/averaging by the commutator action, etc.

$$e_{turn} \sim \omega \Phi_p \quad \Phi_p \to BA \quad \Rightarrow \quad \boxed{E_a = K_a \omega_r \Phi_p}$$

$$T_{turn} \sim \Phi_p i_a \quad \Rightarrow \quad T_e = K_? \Phi_p I_a$$

K_a - is the machine armature constant – determined by construction / design → mechanical design

E_a - is the generated voltage or back emf

Power Balance $E_a I_a = P_e$ (electrical power) $\omega_r T_e = P_e$ (electrical torque)

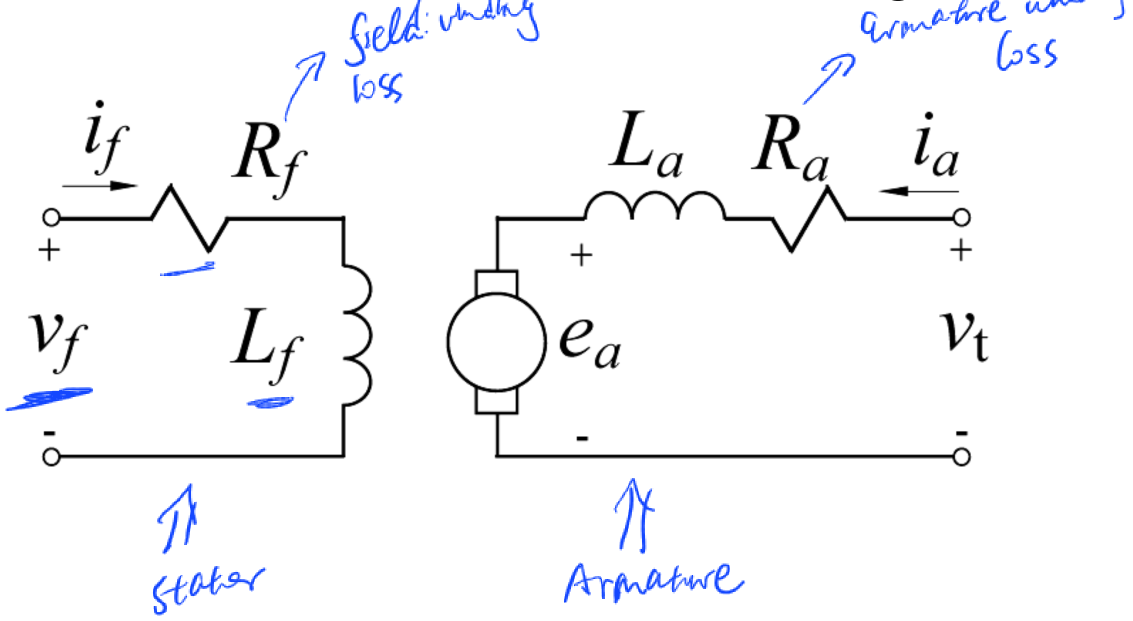
$$P_e = E_a I_a = P_e = \omega_r T_e \quad \text{(lossless)}$$

$$\underbrace{K_a \omega_r \Phi_p}_{E_a} I_a = \omega_r \underbrace{K_? \Phi_p I_a}_{T_e} \quad \Rightarrow \quad T_e = K_a \Phi_p I_a$$

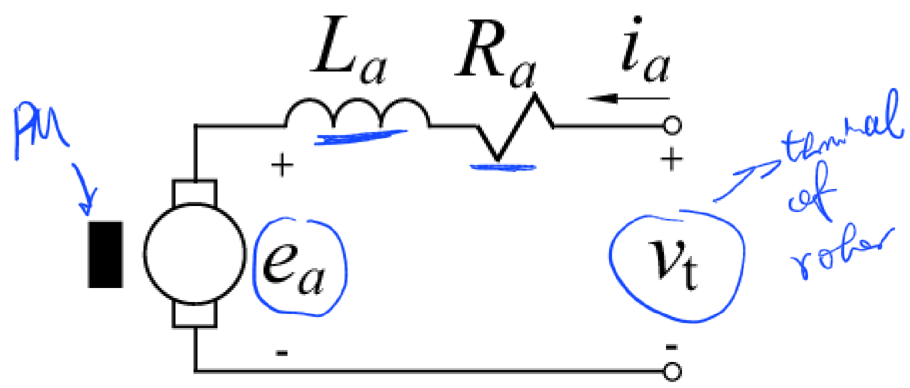
Commonly-Used Equivalent Circuit

Quiz questions

DC Machine with Field Winding



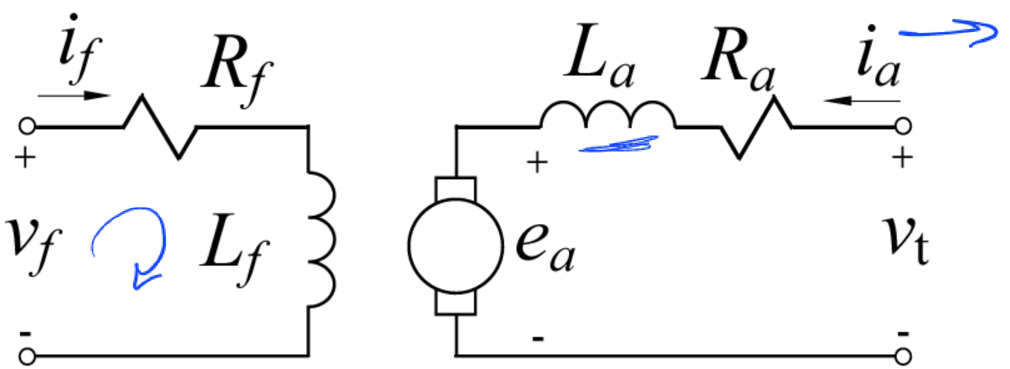
PM DC Machine



- v_f - is the field voltage
- i_f - is the field current
- R_f - field winding resistance
- L_f - field winding inductance

- v_t - applied terminal voltage
- i_a - is the armature current
- R_a - armature winding + brush resistance
- L_a - armature winding inductance
- e_a - Induced back emf (voltage)

Governing Dynamics State Equations



Depending on motoring or generating, polarity is different

Motoring: $v_t > e_a$

Generating: $v_t < e_a$

Field Winding Equations

$$v_f = R_f i_f + \frac{d\lambda_f}{dt}$$

$$\lambda_f = N_f \Phi_p = L_f i_f$$

Armature Equations

$$v_t = \pm R_a i_a + L_a \frac{di_a}{dt} + e_a$$

$$e_a = K_a \Phi_p \omega_r$$

$$T_e = K_a \Phi_p i_a$$

Mechanical Motion

$$J \frac{d\omega_r}{dt} = T_e - T_m - T_{fric}$$

$J = J_{dc\_machine} + J_{mech\_load}$, $[kg \cdot m^2]$ - is the combined moment of inertia of the motor and load

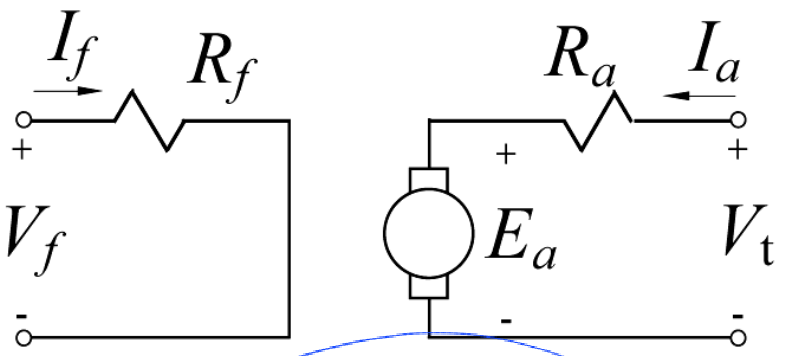
Steady-State Equivalent Circuit $\Rightarrow$ no L, $\frac{dw}{dt} = \emptyset$

Field-Wound Machine

Field Winding Equations

$$V_f = R_f I_f$$

$$\lambda_f = N_f \Phi_p = L_f I_f$$

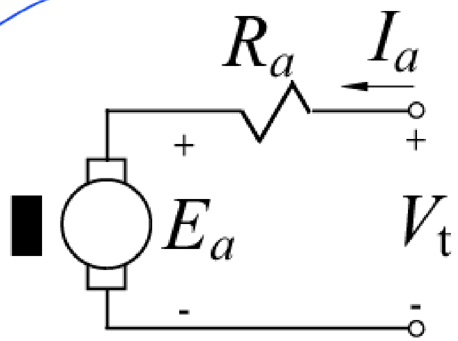


Armature Equations

$$V_t = \pm R_a I_a + E_a$$

$$E_a = K_a \Phi_p \omega_r$$

PM Machine



Torque / Mechanical Side $T_e = K_a \Phi_p I_a$

$$T_e = T_m + T_{mech\_loss} = T_m + K_{fric}\omega_r$$

Friction loss
(for large machines
often can be neglected)

Torque & Voltage Constant for PM Machines

BA

Recall field winding flux: $\Phi_p = \dfrac{\lambda_f}{N_f} = \dfrac{L_f}{N_f} i_f$

Back emf: $e_a = K_a \Phi_p \omega_r = K_a \dfrac{L_f}{N_f} i_f \omega_r$

Electromagnetic torque: $T_e = K_a \Phi_p i_a = K_a \dfrac{L_f}{N_f} i_f i_a$

Define $L_{af} = K_a \dfrac{L_f}{N_f}$ - is the mutual inductance between field and rotating armature coils

↳ field ⟺ armature

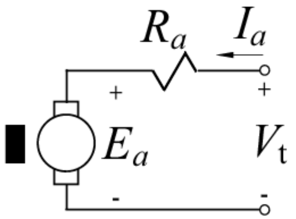
We can use alternative expression for back EMF and Torque for **Field-Wound Machine**

$$e_a = L_{af} I_f \omega_r = \lambda_{af} W_r$$

$$T_e = L_{af} I_f I_a = \underbrace{\lambda_{af}}_{\text{constant in PM}} I_a$$

$LI = \lambda$

For PM Machine



$$e_a = K_v \omega_r$$

$$T_e = K_t i_a$$

Torque / voltage machine constant $K_t = K_v$

Often you find these equal
↳ Because different units
it's not in datasheet

DC Machine: Example 1

Consider a small PM DC Motor with the following parameters:

$V_{t\_rated} = 6$ V, $I_{no\_load} = 0.15$ A, $R_a = 7\ \Omega$, and torque constant $K_t = 0.014$ [Vs/rad].

Find no-load speed n_{nl} [rpm], back emf E_a, and friction torque T_{fric} at $V_t = 6$ V

$$V_t = R_a \cdot I_a + E_a = R_a I_a + K_t \cdot \omega$$

$$\omega_{nl} = \frac{V_t - R_a I_a}{K_t} = \frac{6 - 7 \cdot 0.15}{0.014} = 353.57 \text{ rad/sec}$$

$$n_{nl} = \frac{30}{\pi} \omega_{nl} = 3{,}376.4 \text{ rpm}$$

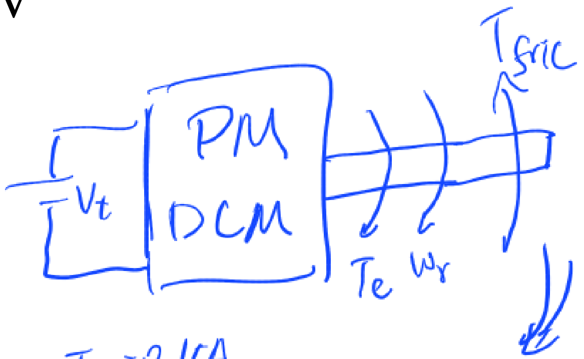
Back emf $E_a = K_t \cdot \omega = 0.014 \cdot 353.57 = 4.95$ V

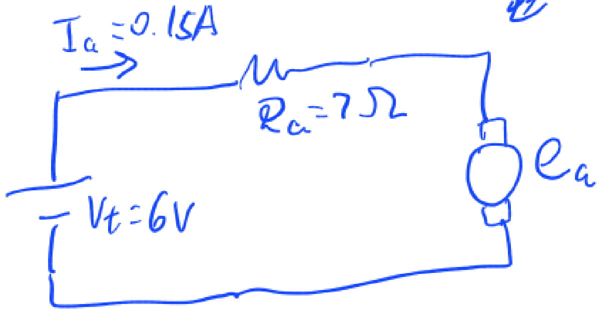
Electromagnetic torque

$$T_e = K_t \cdot I_a = 0.014 \cdot 0.15 = 0.0021 \text{ N·m}$$

$$T_e = \frac{P_e}{\omega} = \frac{I_a \cdot E_a}{\omega} = \frac{0.15 \cdot 4.95}{353.57} = 0.0021 \text{ N·m}$$

$$T_{fric} = T_e = 0.0021 \text{ N·m}$$

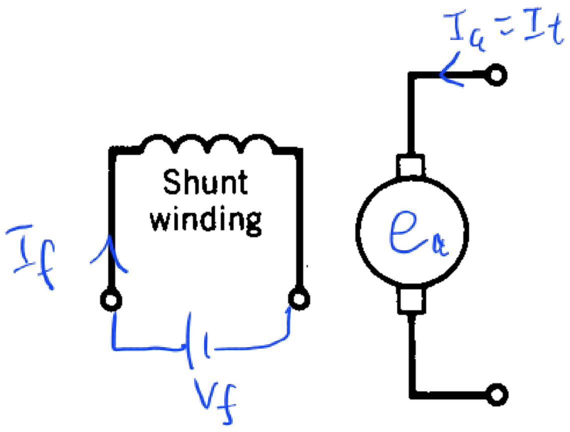




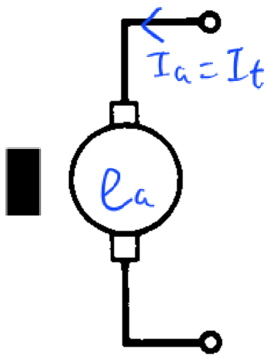
Need to be good at this on tests

Basic Types of DC Machines → 4 main types testable in this class

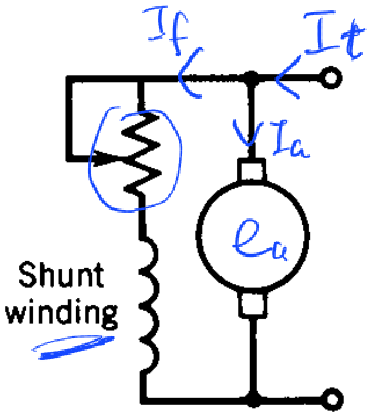
➢ Shunt or Series field windings are possible



Separately Excited
DC Machine

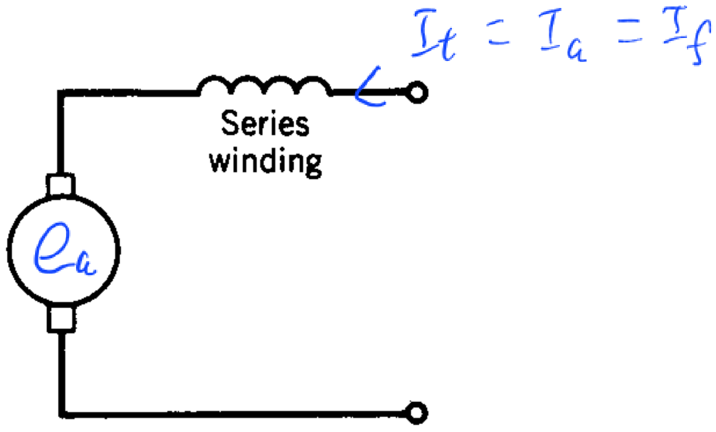


PM
DC Machine



Shunt
DC Machine

- Field winding is designed for up to rated armature voltage
- Field winding has large number of turns
- Field current is small compared to armature current



Series
DC Machine

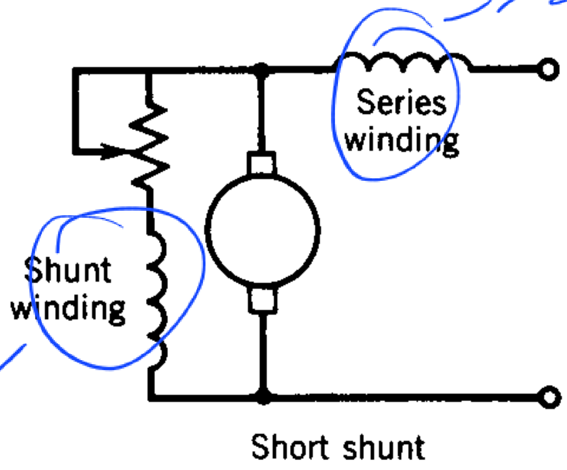
- Field winding is designed for up to rated armature current
- Field winding has small number of turns
- Field current is the same as the armature current

Basic Types of DC Machines → Not on test

Compound DC Machines

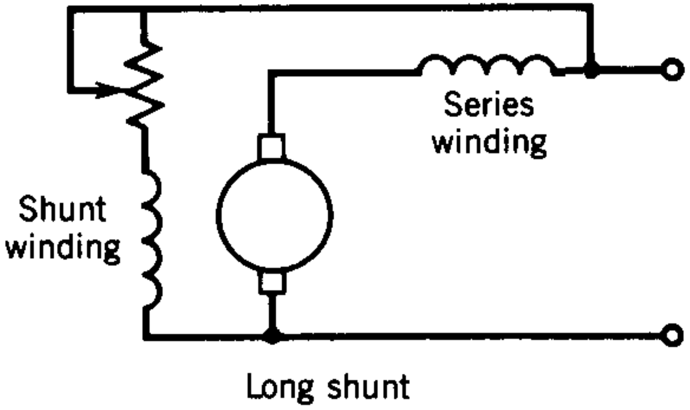
- Both Shunt and Series field windings are present
- Various connections are possible
- Field windings are designed/connected to achieve a desirable torque/speed characteristics

$\rightarrow \Phi_{f,ser}$



$\Phi_{f,sh}$

Short-Shunt Compound DC Machine



Long-Shunt Compound DC Machine

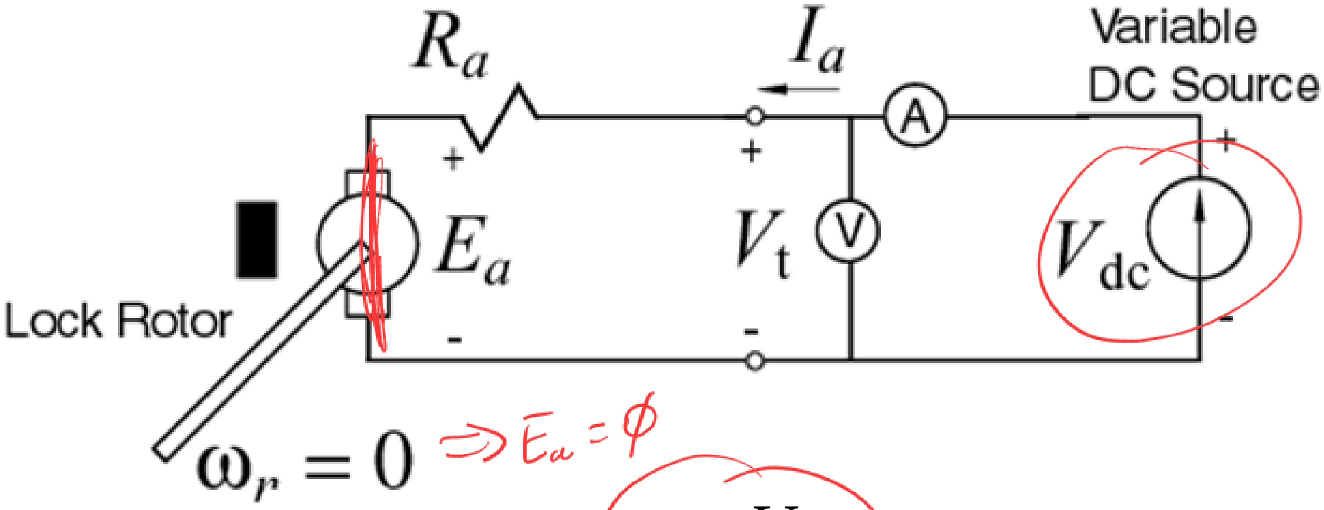
$$\Phi_f = \pm \Phi_{f,ser} \pm \Phi_{f,sh}$$

Parameters of PM DC Machine (Lab 3)

means below nominal voltage

- **Blocked-Rotor Test** (DC Measurements)

  Slowly ramp up DC voltage and measure DC current up to nominal value
  Determine resistance of armature winding:

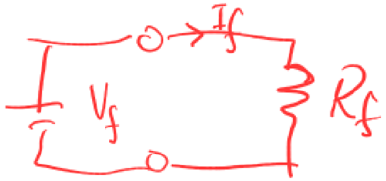


$\Rightarrow E_a = 0$

$$R_a = \frac{V_t}{I_a}$$

(armature + brush combined resistance at no speed!)

Use similar measurement for field winding to estimate its resistance for field-wound DC machines



$$R_f = \frac{V_f}{I_f}$$

Parameters of PM DC Machine

- **No Load Test** (Friction vs. Speed) ⇒ No load, machine rotates up to nominal speed

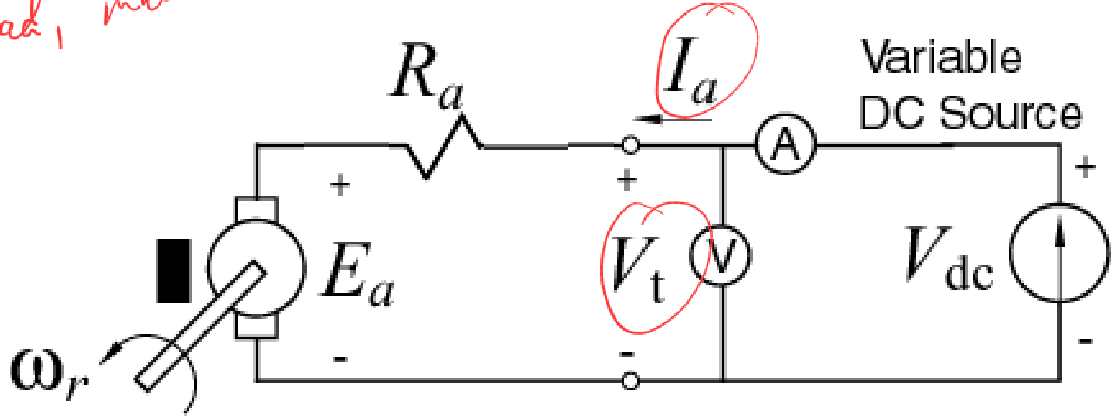
Assume you know R_a ⇒ from blocked rotor test

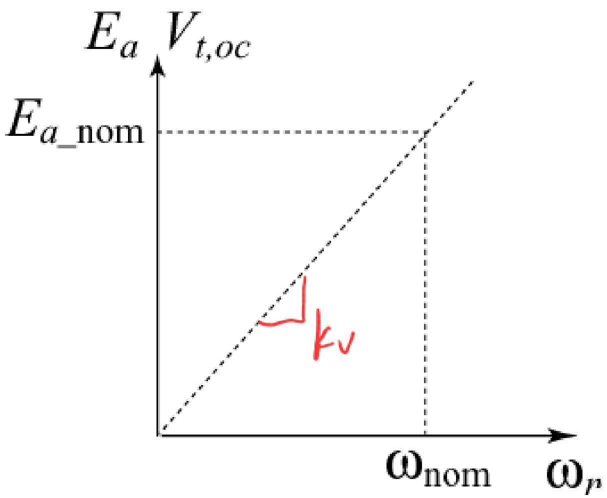
Measure $I_a, V_t, \omega_r \neq \emptyset$

$$E_a = k_v \omega_r$$

$$V_t = R_a I_a + E_a$$

Torque / voltage machine constant $k_t = k_v = \dfrac{V_t - R_a I_a}{\omega_r}$

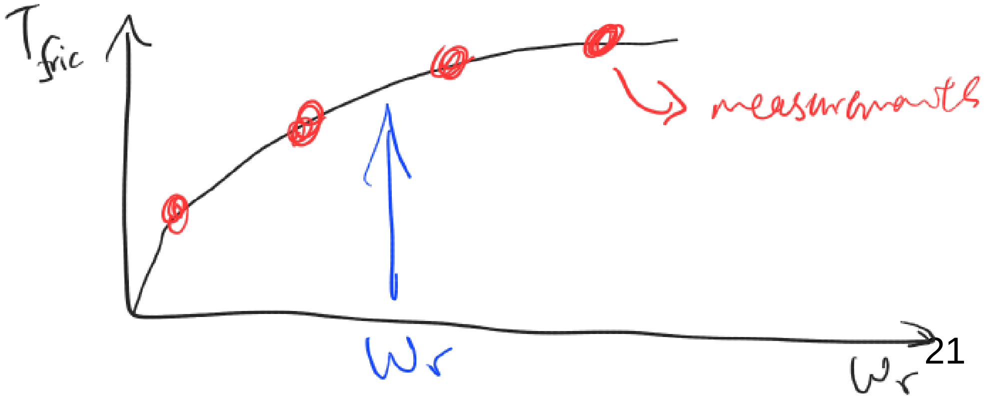




Obtain Friction Torque-Speed Characteristic

$$T_e = k_t I_a$$

$$T_e = T_{fric}(\omega_r) = \omega_r K_{fric}$$

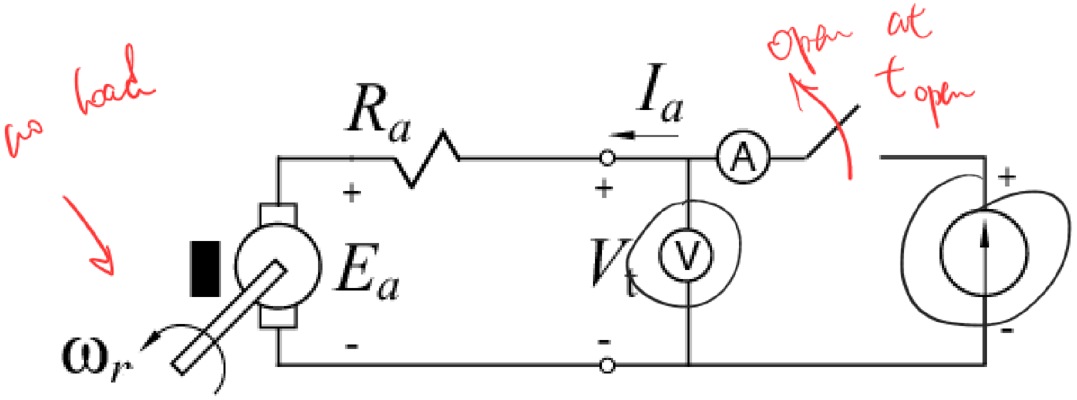


Parameters of DC Machine

- **Stopping Transient** (for Determining Inertia)

Assume you know $T_{fric}(\omega_r)$

Measure & Record I_a, V_t, ω_r



First, run motor to a fixed speed, then open switch

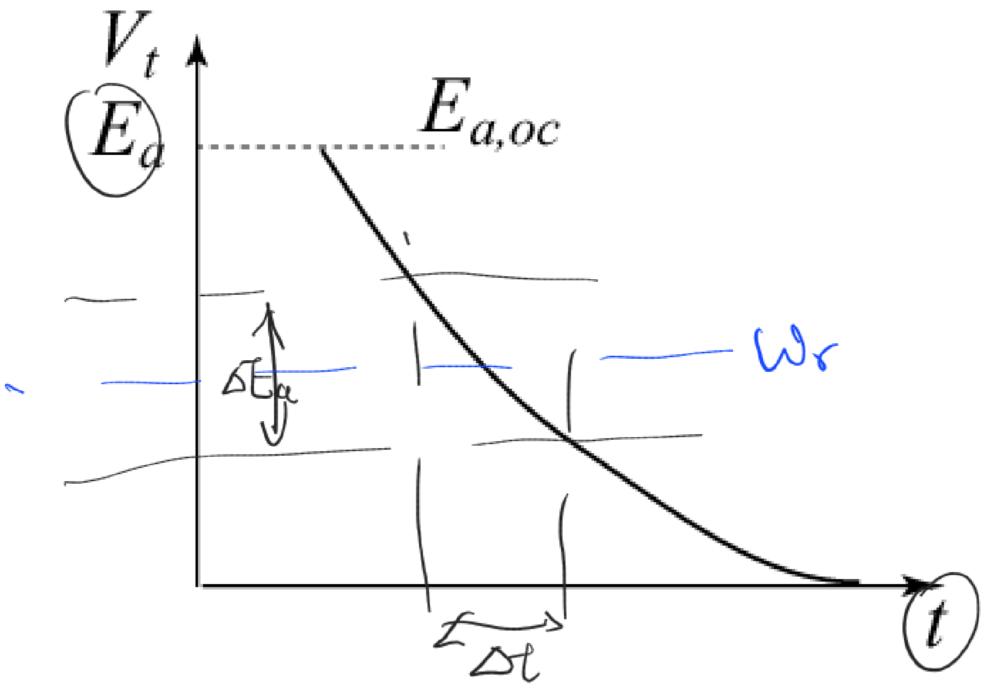
$$\overset{\phi}{\cancel{T_e}} = \overset{\phi}{\cancel{T_m}} + T_{fric}(\omega_r) + J\frac{d\omega_r}{dt}$$

$$E_a = K_a \Phi_p \omega_r = K_v \omega_r$$

$$T_{fric}(\omega_r) = -J\frac{\Delta\omega_r}{\Delta t} = -J\frac{\Delta E_a}{K_v \Delta t}$$

$$J = \frac{T_{fric}(\omega_r)\cdot K_v \cdot \Delta t}{\Delta E_a}$$

ΔE_a → measured



Looking ahead…

- Continue **Module 4**