ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 11

February 10, 2026

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Course Logistics

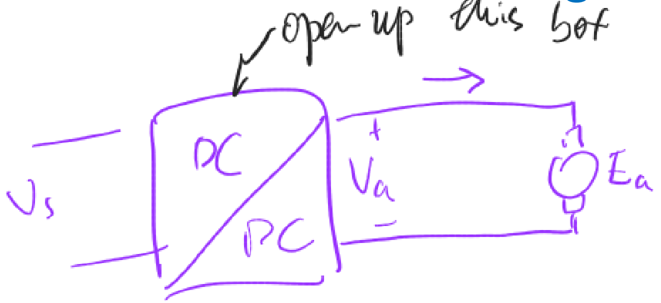
- **Labs:**
  - **Please be on time and come prepared**
  - **Each person needs to complete prelab and submit as a part of the final lab report**
  - **You need to submit all five (5) lab reports in order to write the final exam**

- Quiz 2 will take place on Feb. 13
  - Detailed information already announced on Canvas
    - Beginning of tutorial and you will be given 20 minutes
    - Closed book and notes, calculators allowed
    - Coverage: Modules 2 and 3 (Hint: review transformers and energy/co-energy/areas)

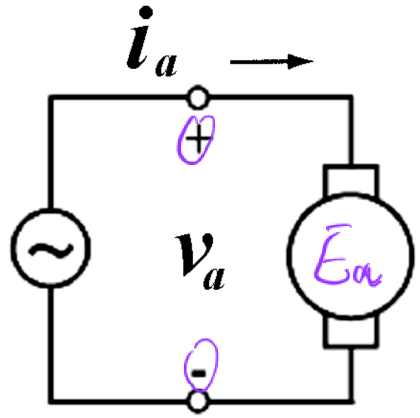
- Midterm will take place on Feb. 26 → no tutorial during that week

DC-DC Converters Quadrants

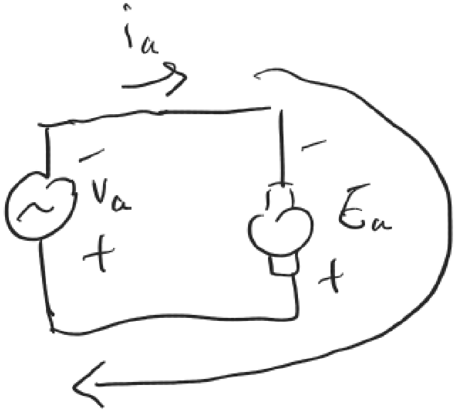
Assume a DC voltage source wherein the averaged output voltage can be controlled

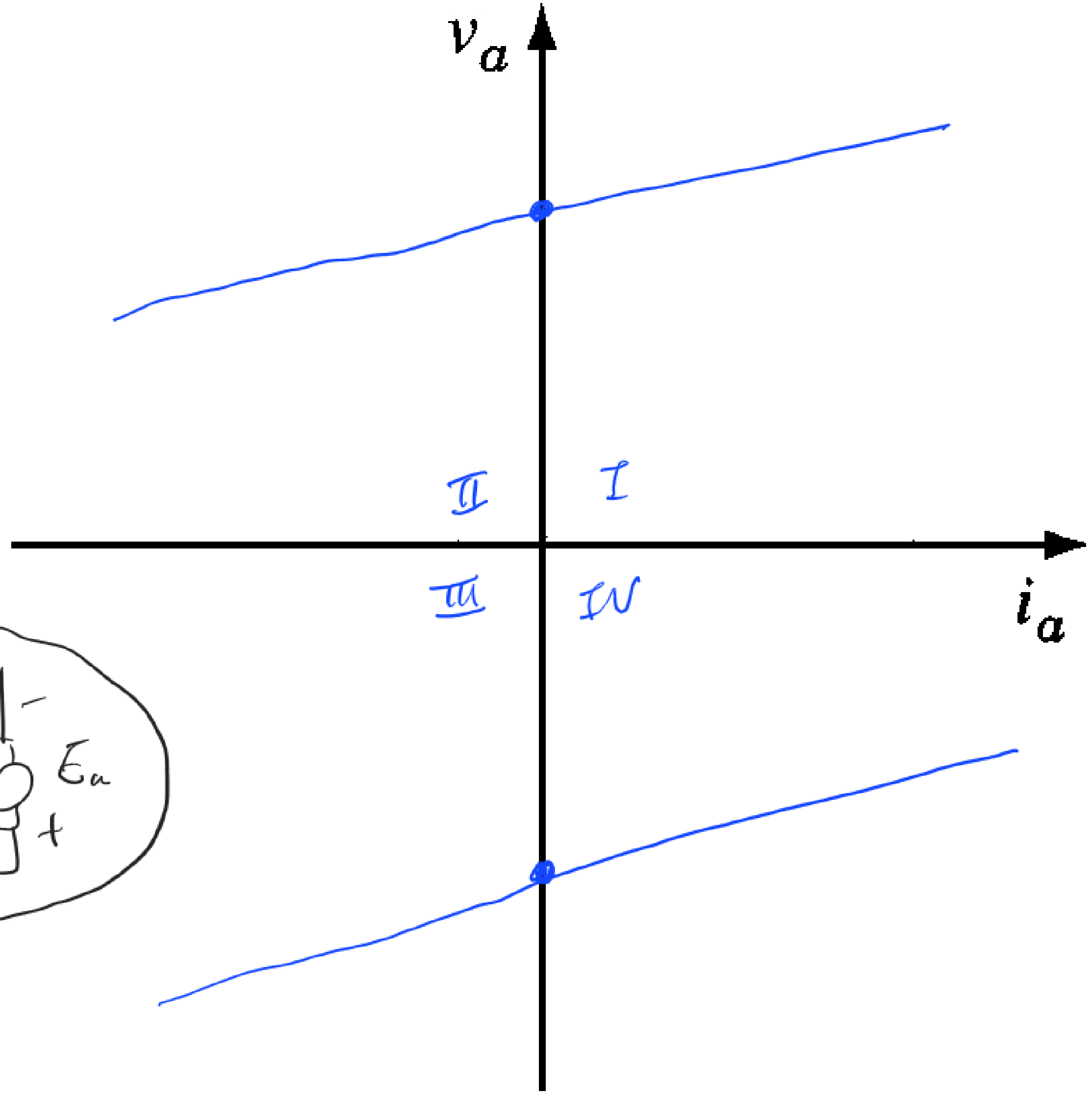


Positive direction of current: from source to machine



$$\bar{V}_a = \frac{1}{T}\int_0^T v_a(t)\,dt$$





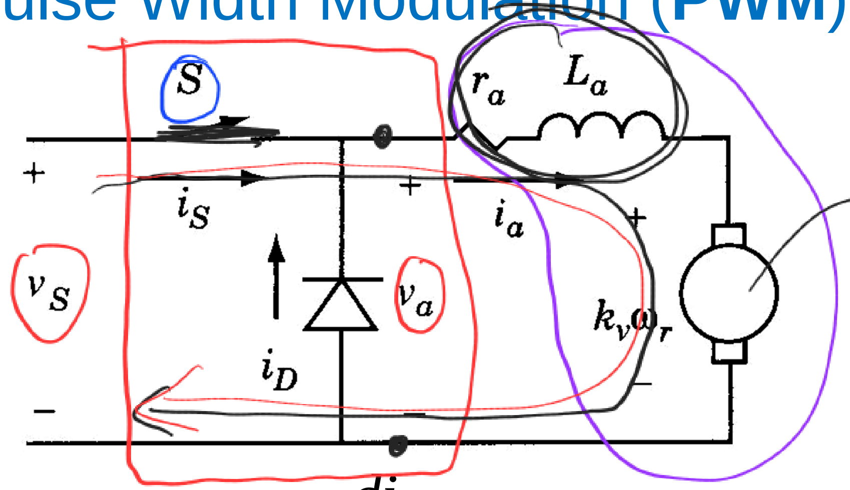
DC-DC Converters Quadrants - Summary

	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
Name	Forwarding motoring	Forward generating	Reverse motoring	Reverse generating
$\boldsymbol{E_a, Va}$	$V_a > E_a > 0$	$E_a > V_a > 0$	$V_a < E_a < 0$	$E_a < V_a < 0$
$\boldsymbol{I_a}$	$I_a > 0$	$I_a < 0$	$I_a < 0$	$I_a > 0$
$\boldsymbol{T_e}$	Positive (CCW)	Negative (CW)	Negative (CW)	Positive (CCW)
$\boldsymbol{T_m}$	Negative (CW)	Positive (CCW)	Positive (CCW)	Negative (CW)
$\boldsymbol{\omega_r}$	Positive (CCW)	Positive (CCW)	Negative (CW)	Negative (CW)
Example	Car in forward (drive) mode going uphill	Car in forward (drive) mode going downhill	Car in reverse mode going uphill	Car in reverse mode going downhill

One-Quadrant DC-DC Converter (Chopper)

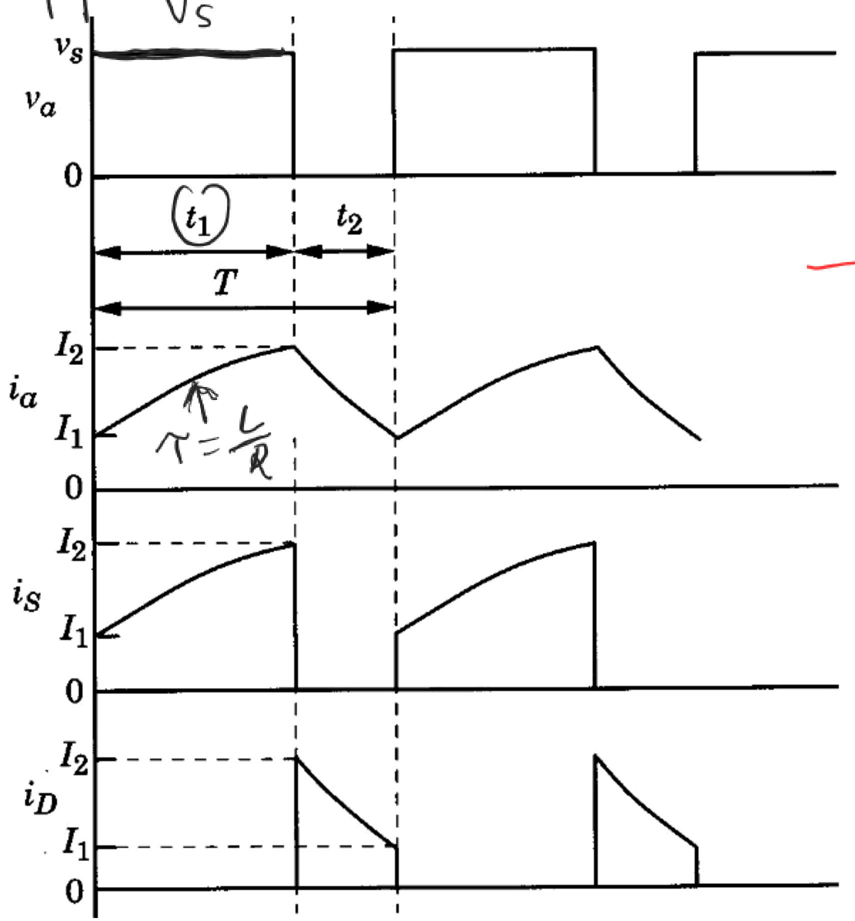
Pulse Width Modulation (**PWM**)

S - ON



→ $E_a \sim$ constant

$E_a < V_s$



$V = L\frac{di}{dt}$

$\frac{di}{dt} = \frac{V}{L}$

$$v_a = r_a i_a + L_a \frac{di_a}{dt} + e_a$$

Switching period : $T \rightarrow f = \frac{1}{T}$

t_1 : S conducting (closed)

$T - t_1 = t_2$: S non-conducting (open)

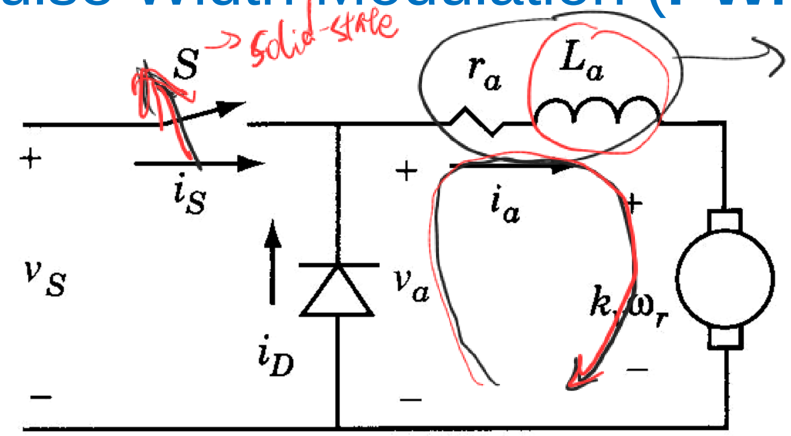
Duty cycle

Average voltage

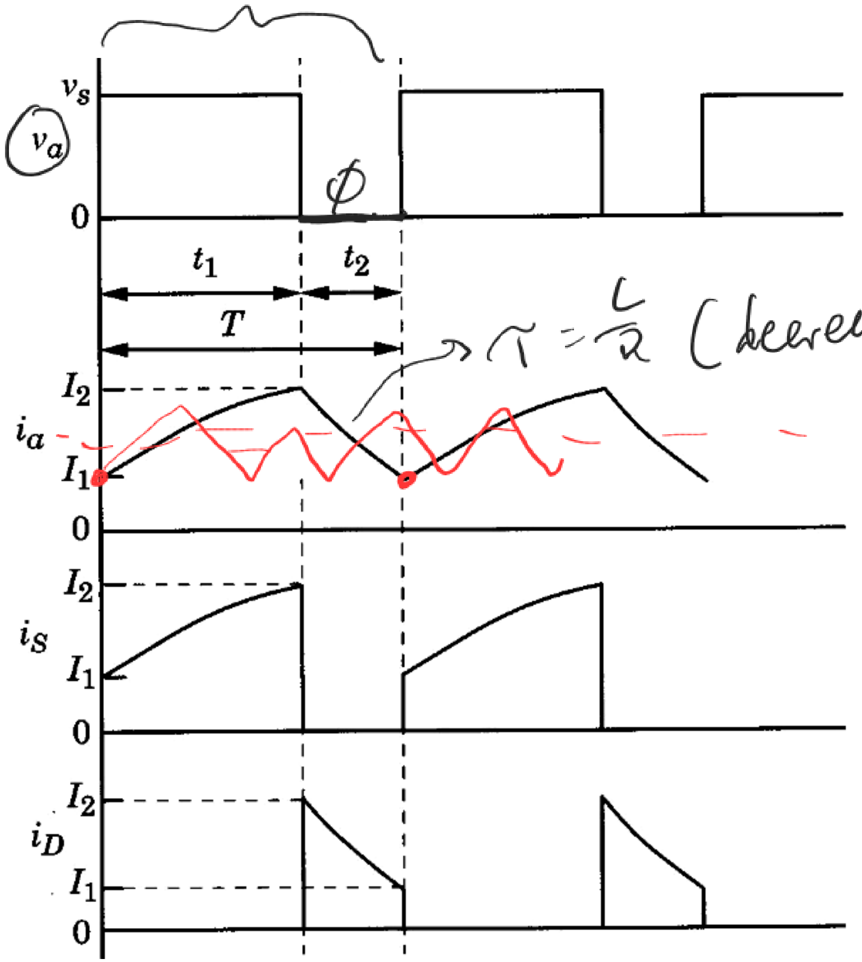
$$\bar{v}_a = \frac{1}{T}\int_0^T v_a(t)dt = \frac{t_1}{t_1 + t_2} V_s$$

One-Quadrant DC-DC Converter

Pulse Width Modulation (**PWM**)



→ negative voltage across



→ $\tau = \frac{L}{R}$ (decreasing)

$\bar{i}_a$

$$v_a = r_a i_a + L_a \frac{di_a}{dt} + e_a$$

t_1 = S on

t_2 = S off

① i_a can only be positive

② $V_a \le V_s$

↳ $0 \le D \le 1$

③ Buck converter

④ Ripple: $i_a \propto T_e$

↳ To reduce ripple

↳ Inc f

↳ Inc L

Duty cycle

$$D = \frac{t_1}{T} \Rightarrow \boxed{\bar{V}_a = D V_s}$$

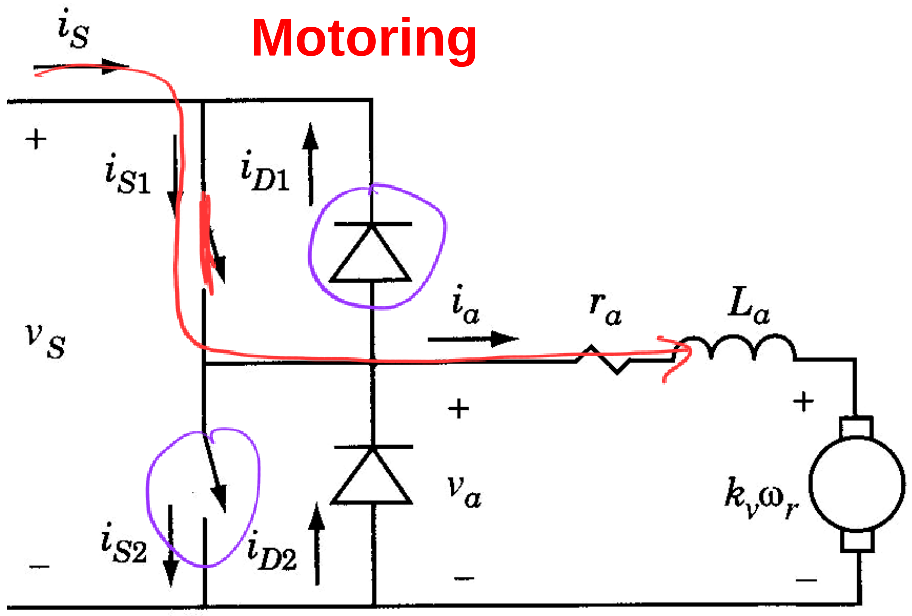
Average voltage

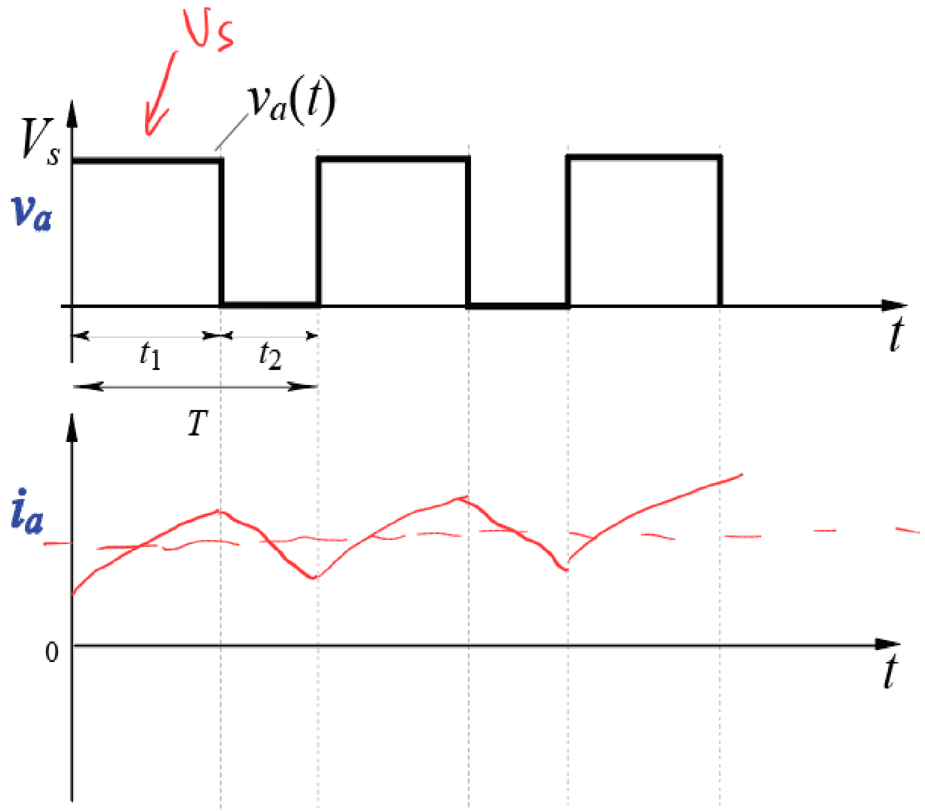
$$\bar{v}_a = \frac{1}{T}\int_0^T v_a(t)\,dt = \frac{t_1}{t_1 + t_2} V_s = \frac{t_1}{T} V_s$$

$T \bar{V}_a = t_1 V_s + t_2 \times \phi$

$\bar{V}_a = \frac{t_1}{T} V_s$

Two-Quadrant DC-DC Converter





Pulse Width Modulation (**PWM**)

Assume complementary switching

$S_1 = \text{NOT}(S_2)$ ↳ Can't have both S_1 and S_2 on at same time

$S_2 = \text{NOT}(S_1)$

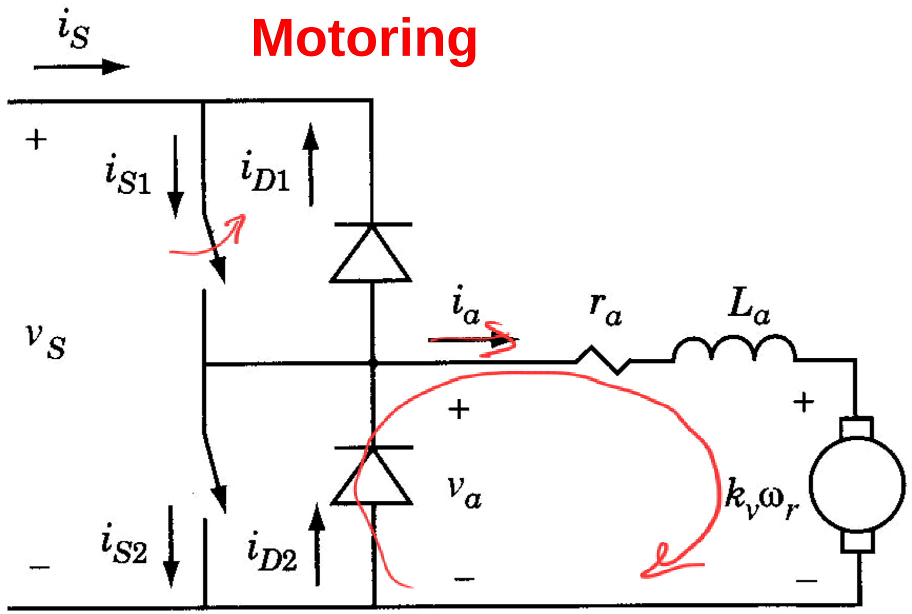
Duty cycle

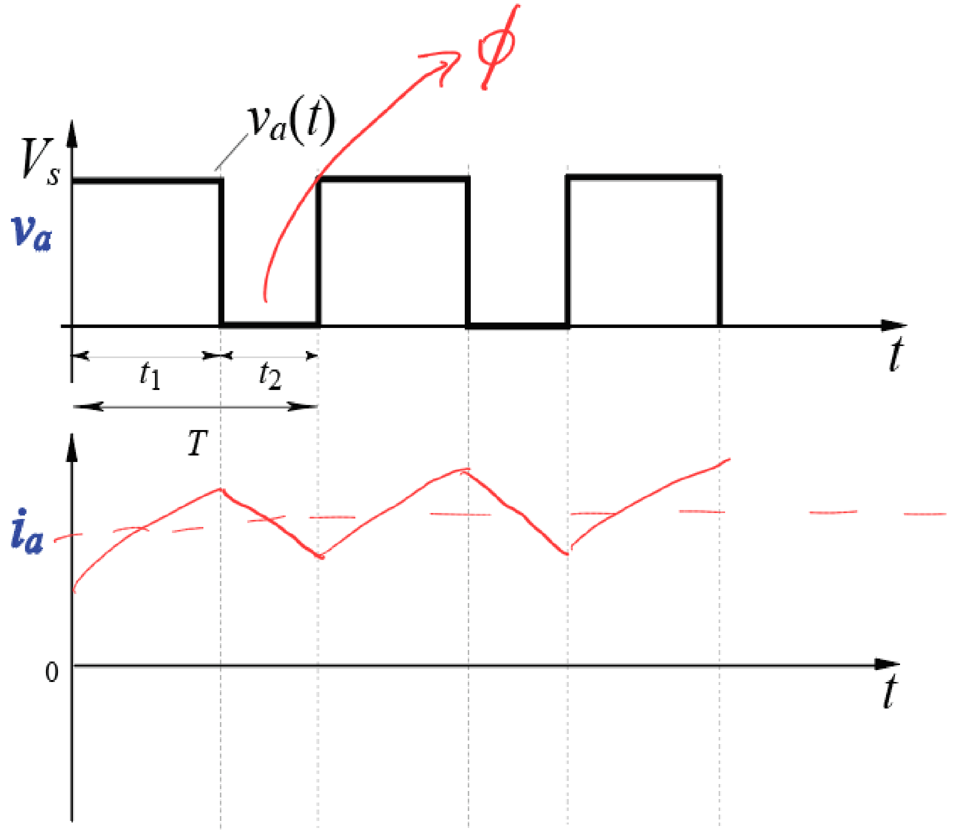
$$D = \frac{t_1}{T} = \frac{V_a}{V_s}$$

Ex. $V_s = 20V$, $V_a = 15V$

$D = 0.75$

Two-Quadrant DC-DC Converter



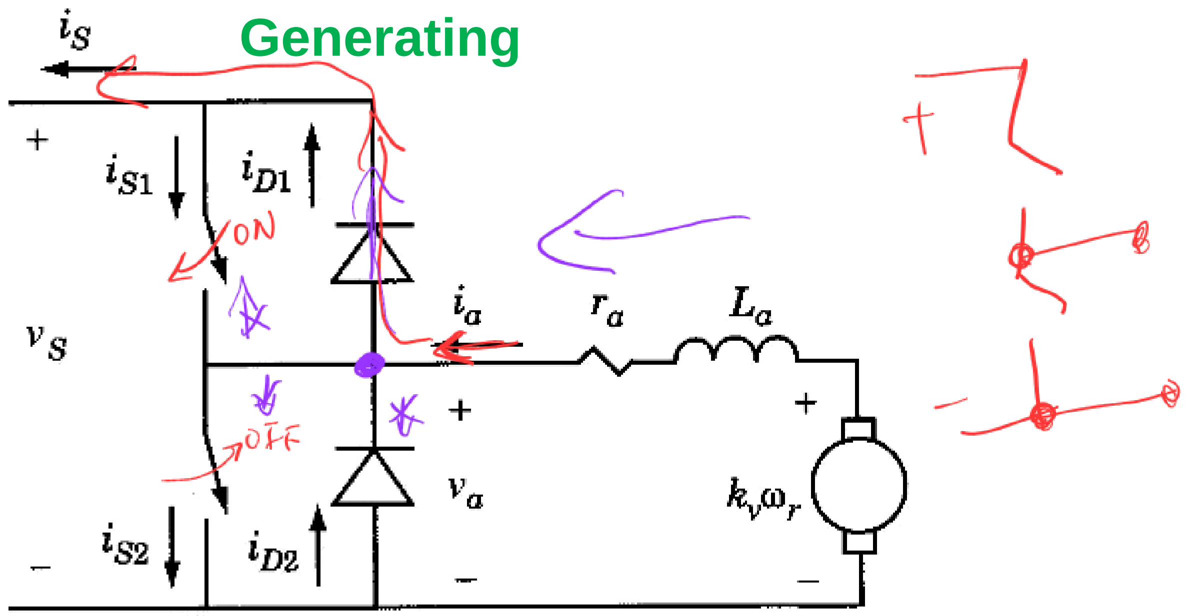


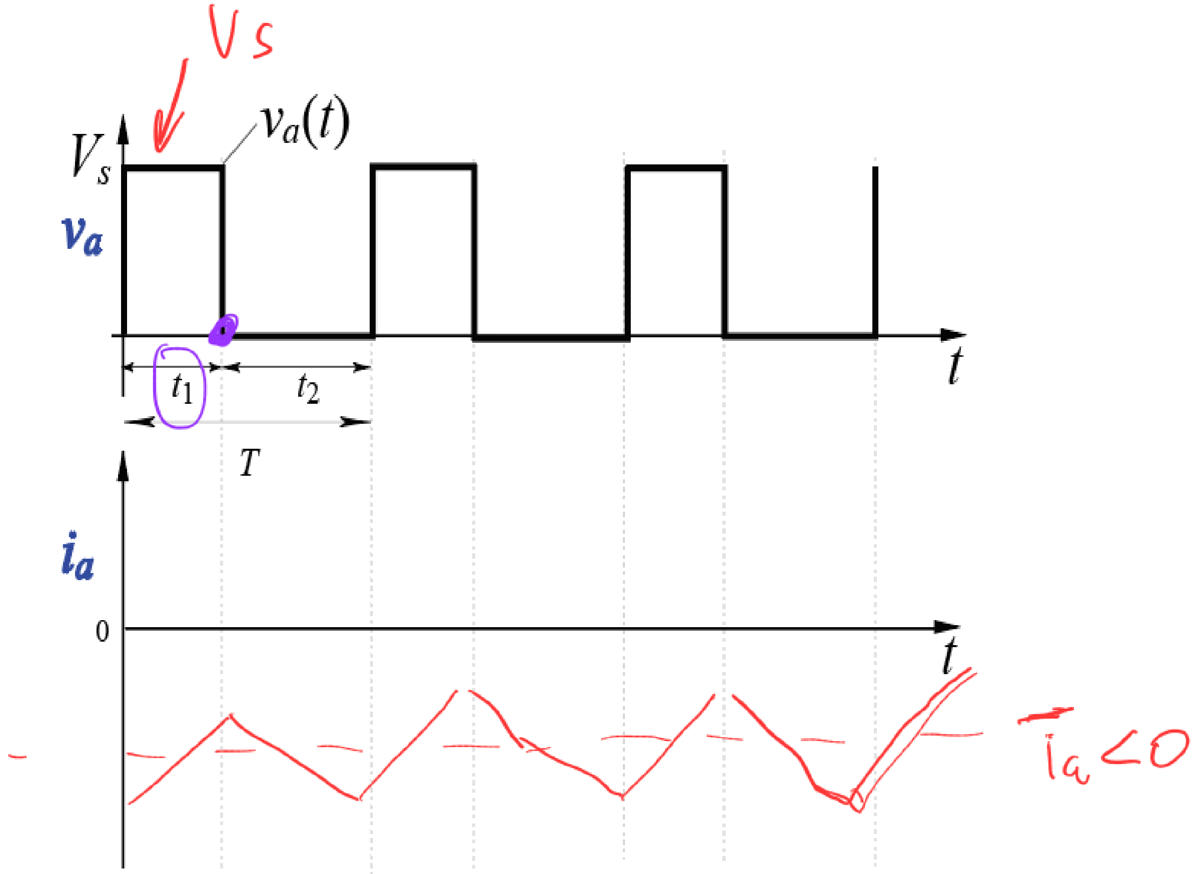
Pulse Width Modulation (**PWM**)

Assume complementary switching

Duty cycle

Two-Quadrant DC-DC Converter





Pulse Width Modulation (**PWM**)

Assume complementary switching

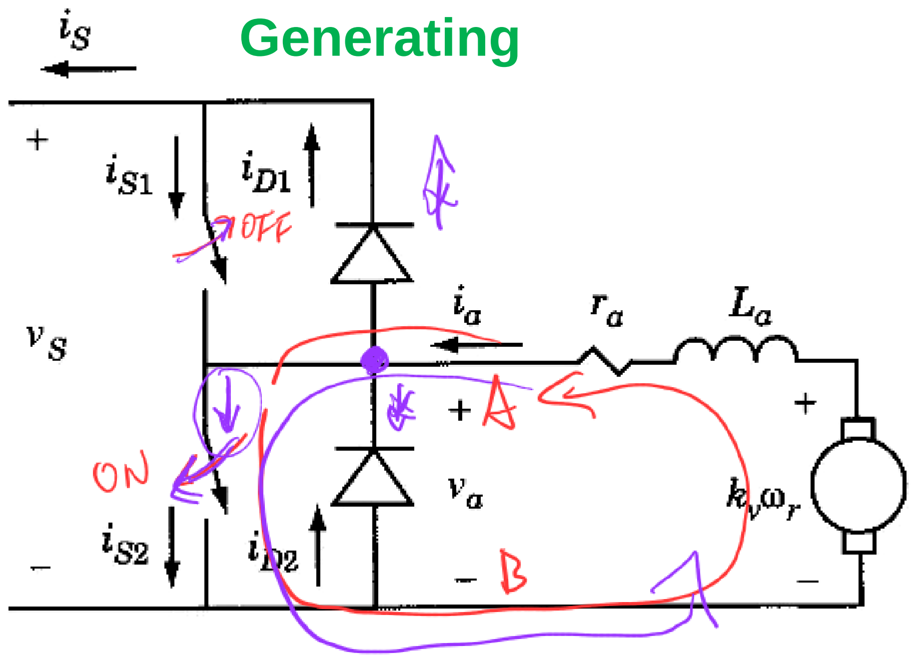
Duty cycle

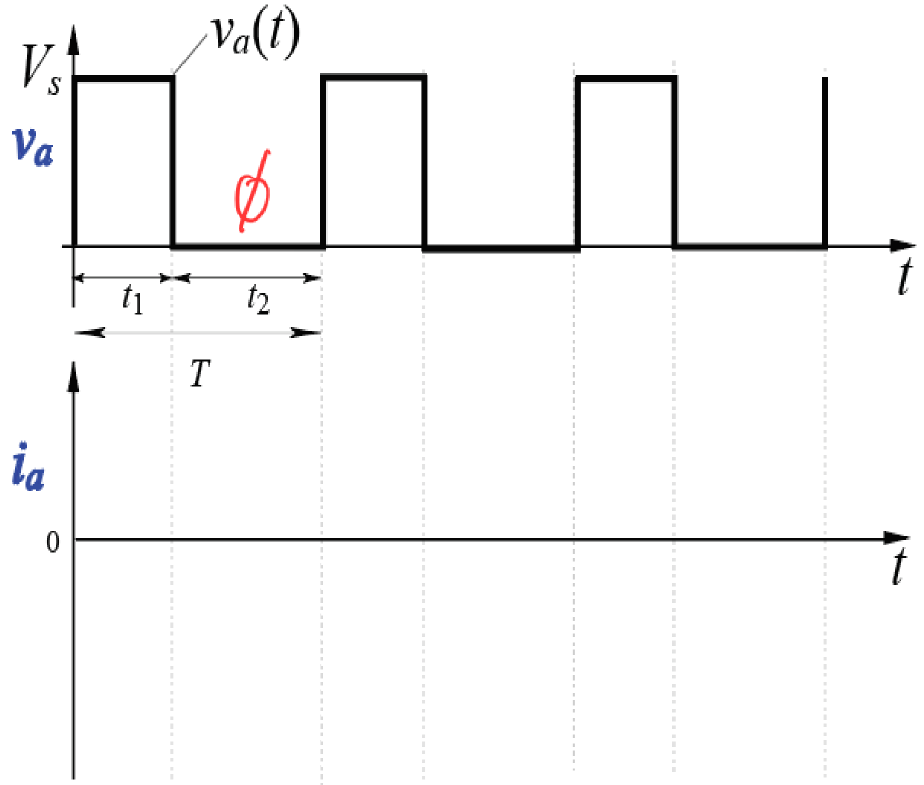
$V_a = DV_s$

① Synchronous buck converter
↳ Bidirectional current flow

② Half bridge → Only quadrants I and II

Two-Quadrant DC-DC Converter





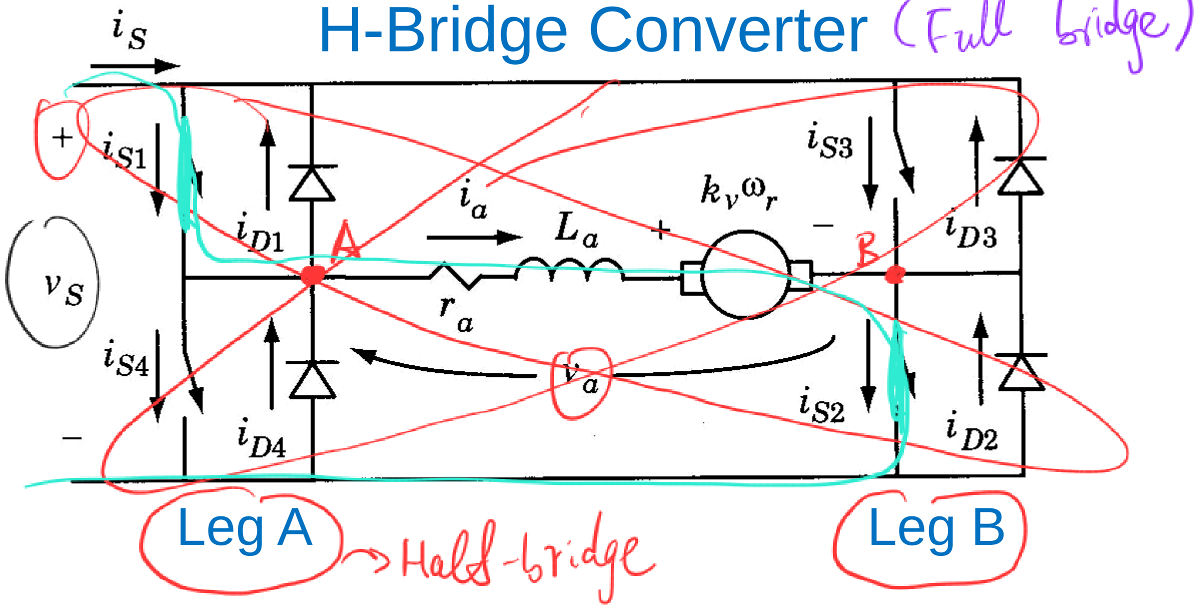
Pulse Width Modulation (**PWM**)

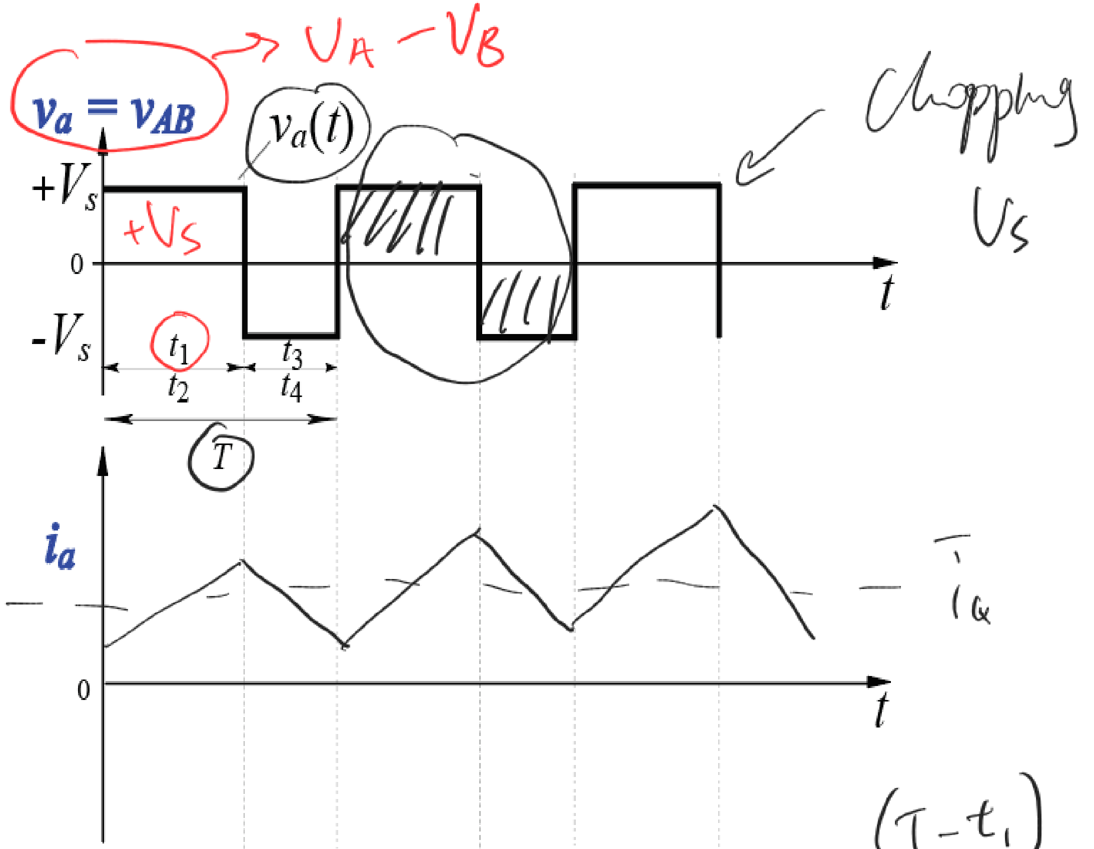
Assume complementary switching

Duty cycle

Four-Quadrant DC-DC Converter

H-Bridge Converter (Full bridge)





Complementary PWM

S_1 and S_2 on → t_1

S_3 and S_4 on → t_2

$+$ V_s $-$ $+$ V_a $-$

Duty cycle

$$D = \frac{t_1}{T}, \quad 0 \le D \le 1$$

Average voltage

$$T\bar{V}_a = t_1 V_s + t_2(-V_s) \qquad (T - t_1)$$

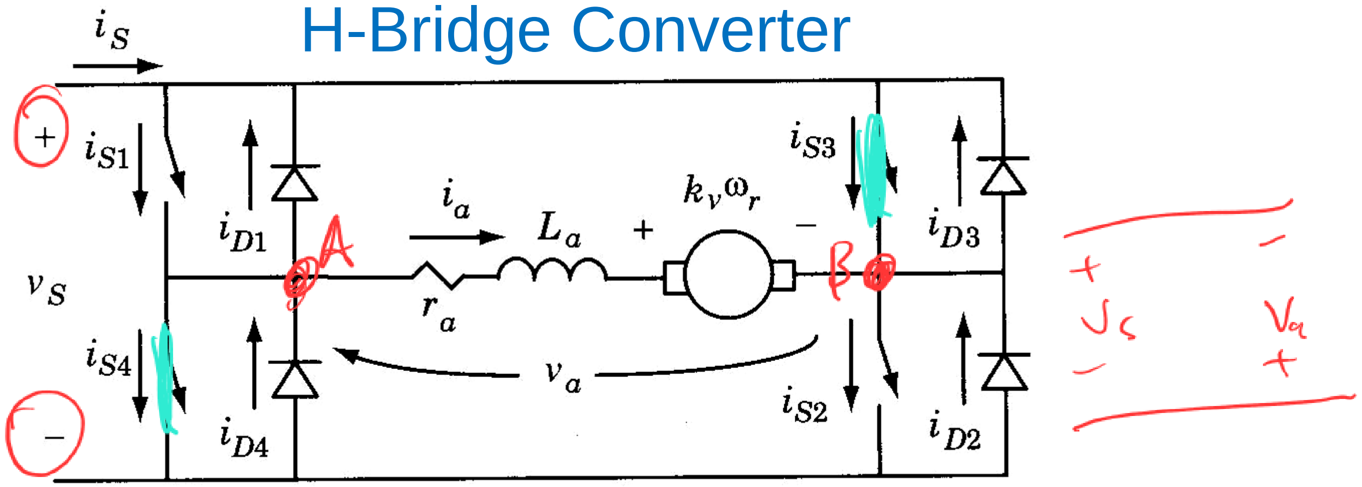
$$T\bar{V}_a = t_1 V_s - V_s(T - t_1)$$

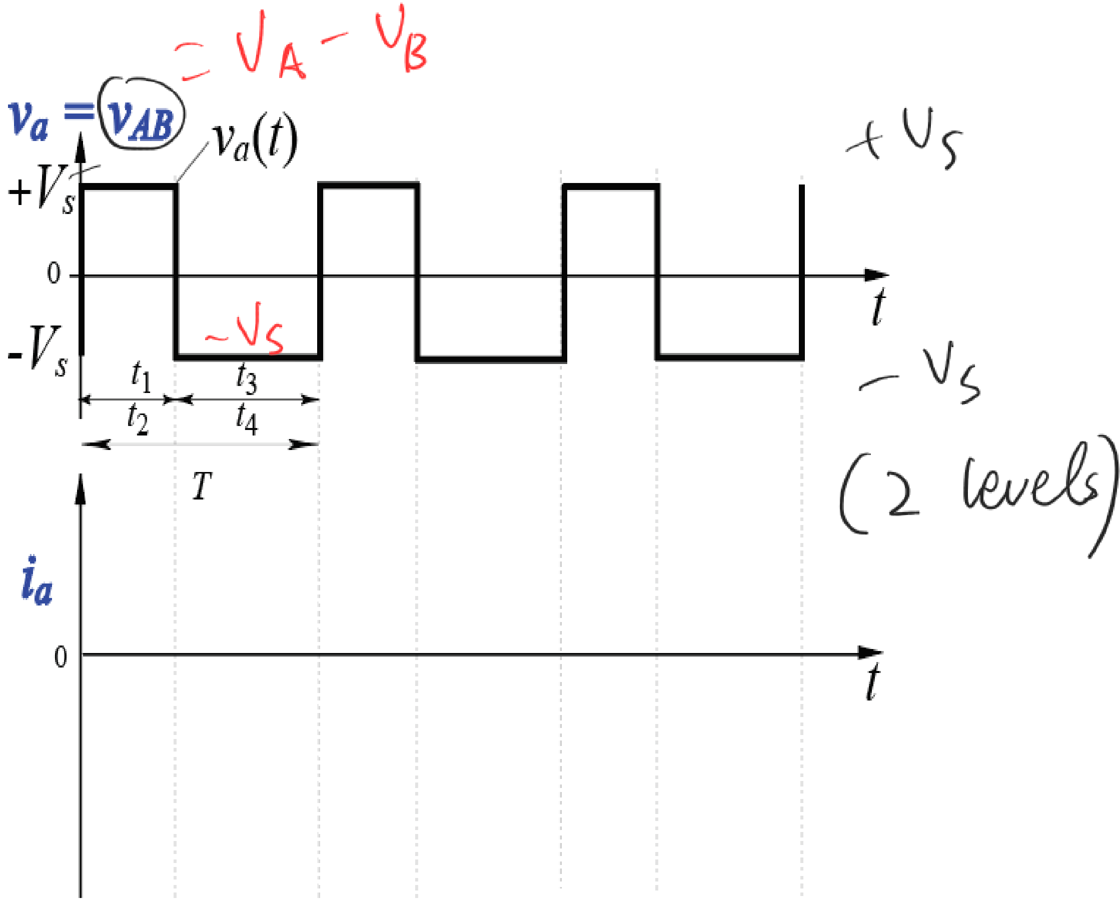
$$\bar{v}_a = \frac{1}{T}\int_0^T v_a(t)\,dt = (2D-1)\,V_s$$

$$-V_s \le \bar{V}_a \le V_s$$

Four-Quadrant DC-DC Converter

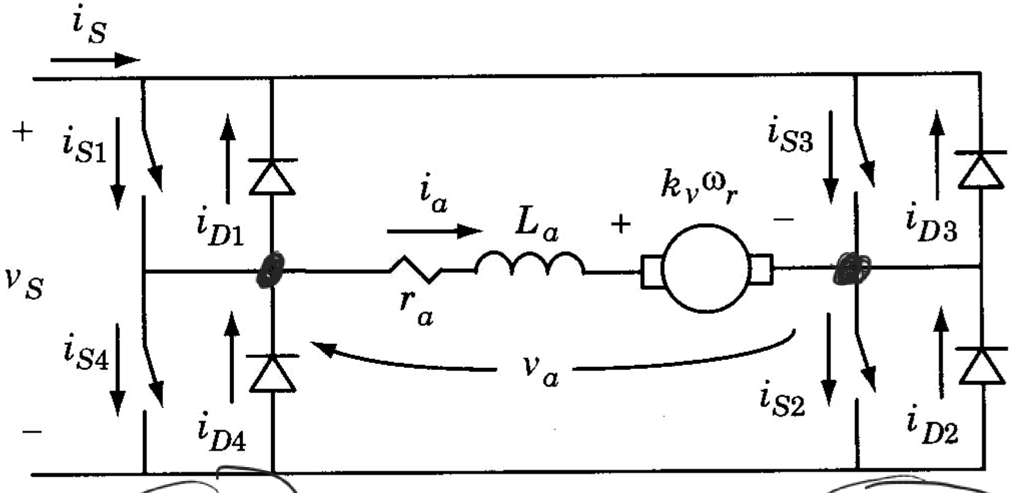
H-Bridge Converter





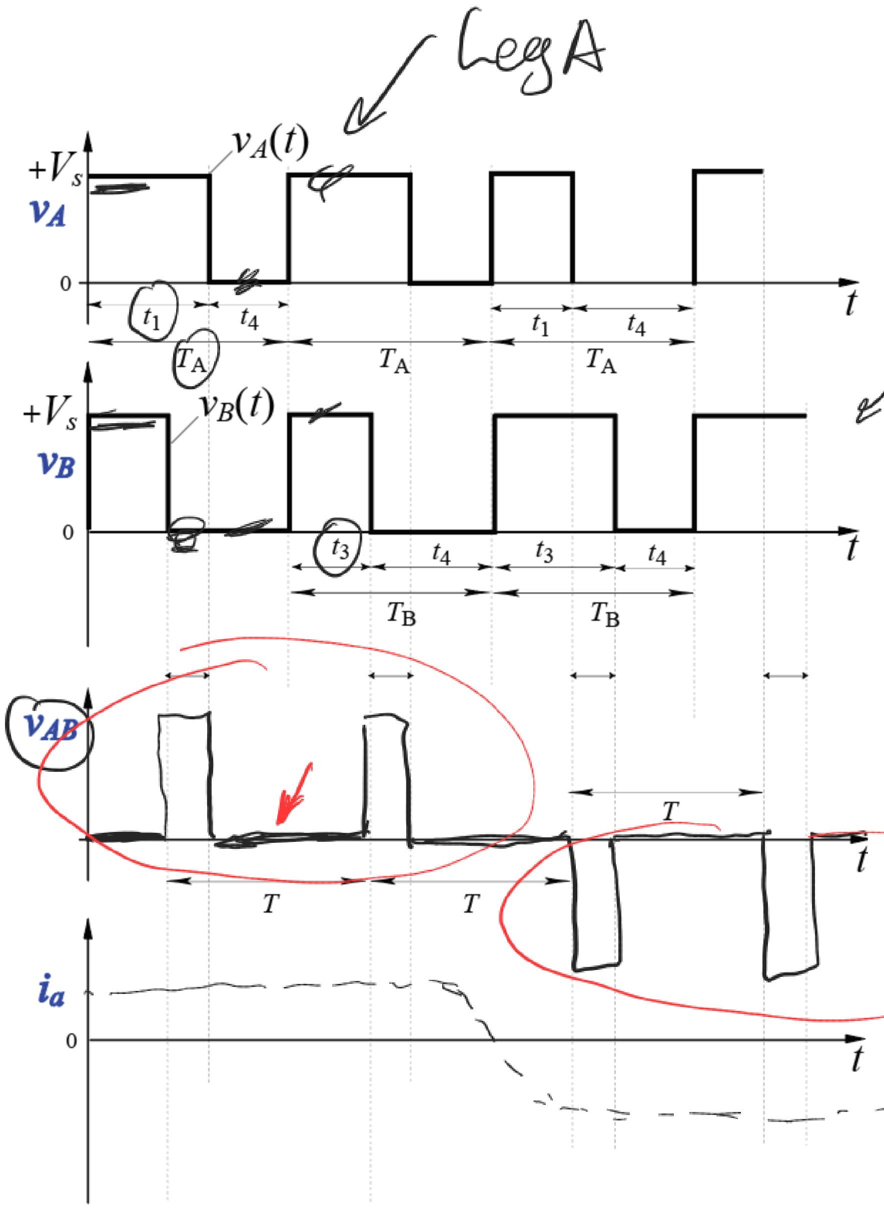
Complementary **PWM**

Four-Quadrant DC-DC Converter



Leg A — D_1

Leg B — D_2



$\frac{t_1}{T_A} = D_1$

$\frac{t_3}{T_B} = D_2$

$+V_s$

ϕ

$-V_s$

Differential **PWM**

(Non-complementary)

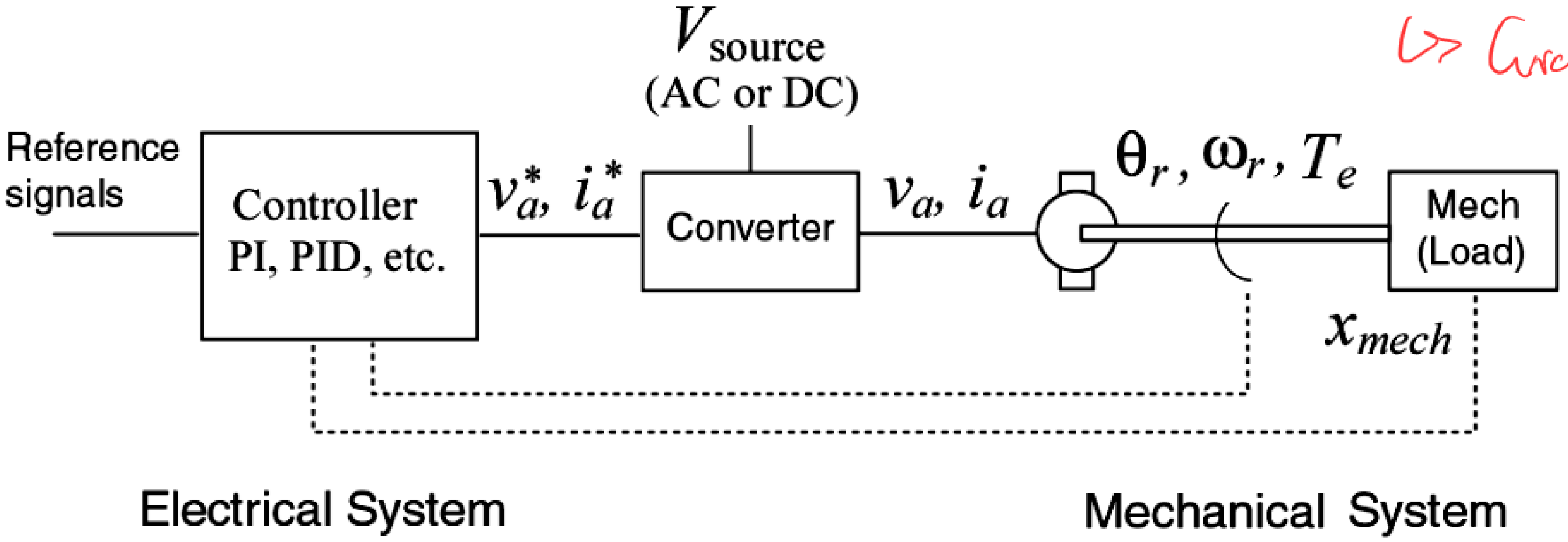
$S_1 \rightarrow D_1 \rightarrow S_4 = \text{NOT}(S_1)$

$S_3 \rightarrow D_3 \rightarrow S_2 = \text{NOT}(S_3)$

$T_A = T_B$

Closed-Loop Electric Drive Systems

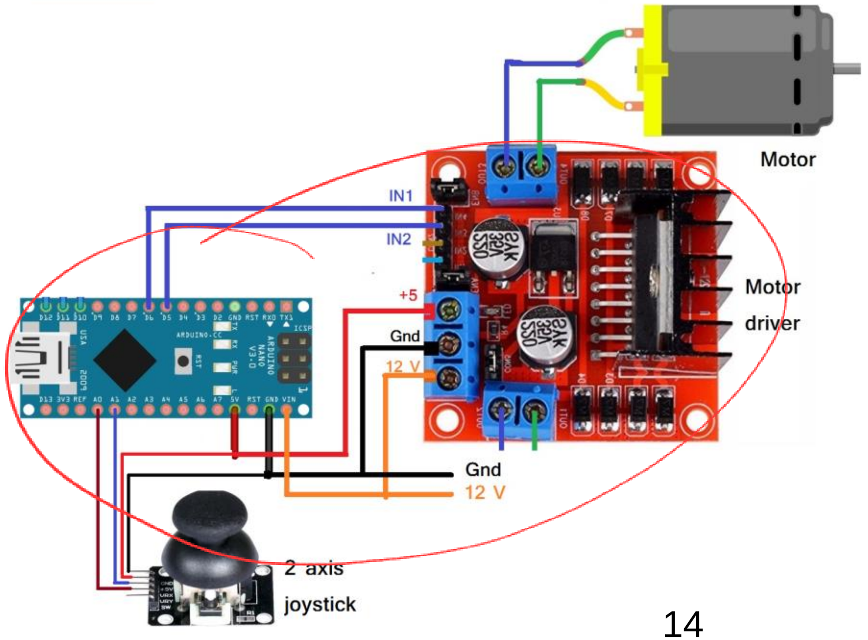
Dr Jatskevich
↳ Great course on machines + machine drives



Systems are more complicated

- Models are needed for analysis, design, tuning of controllers, and investigating interactions among components
- Simple steady state models may not be sufficient anymore

- Dynamic models (differential equations)
- Model → controller → simulation → Hardware

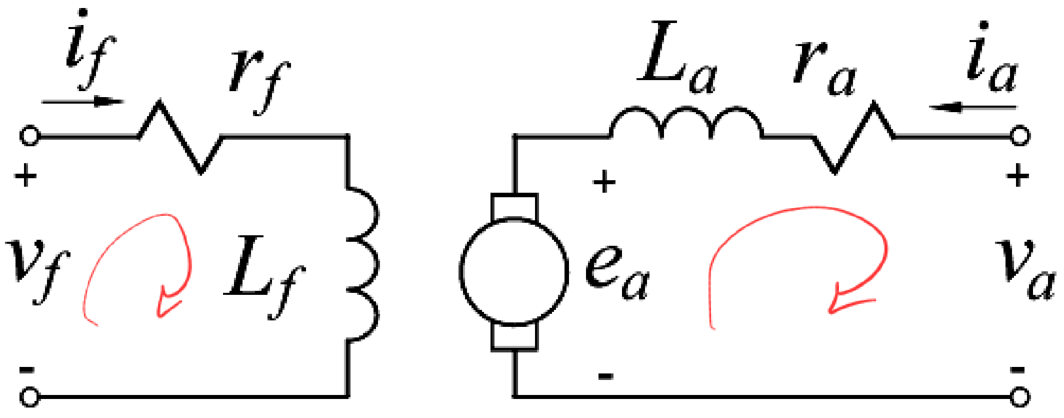


Dynamic Modeling of DC Machine with Field Winding

Circuit is valid for transient analysis:

↳ Circuit has all equations

↳ no need for formula sheet



Voltage Equations

$$v_a = r_a i_a + e_a + L_a \frac{di_a}{dt}$$ → armature KVL

$$v_f = r_f i_f + \frac{d\lambda_f}{dt} = r_f i_f + L_f \frac{di_f}{dt}$$ → field KVL

Speed Equation

friction

$$T_e = J\frac{d\omega_r}{dt} + T_m + D_m\omega_r$$

Coupling Terms

$$e_a = \omega_r L_{af} i_f$$

$$T_e = L_{af} i_f i_a$$

State Variables → i_f, i_a, ω_r

- Define the energy state of the system
- Cannot change instantaneously

Input variables: voltage, driving torque

Re-Write the Equations to Express Derivatives

State Equations -> ODEs

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{di_f}{dt} = -\frac{r_f}{L_f} i_f + \frac{1}{L_f} v_f$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J} \omega_r + \frac{1}{J}(T_e - T_m)$$

Coupling Terms

$$e_a = \omega_r L_{af} i_f$$

$$T_e = L_{af} i_f i_a$$

Are these equations coupled or decoupled ? Coupled

Are these equations linear or non-linear ? Non-linear -> variables multiplied together

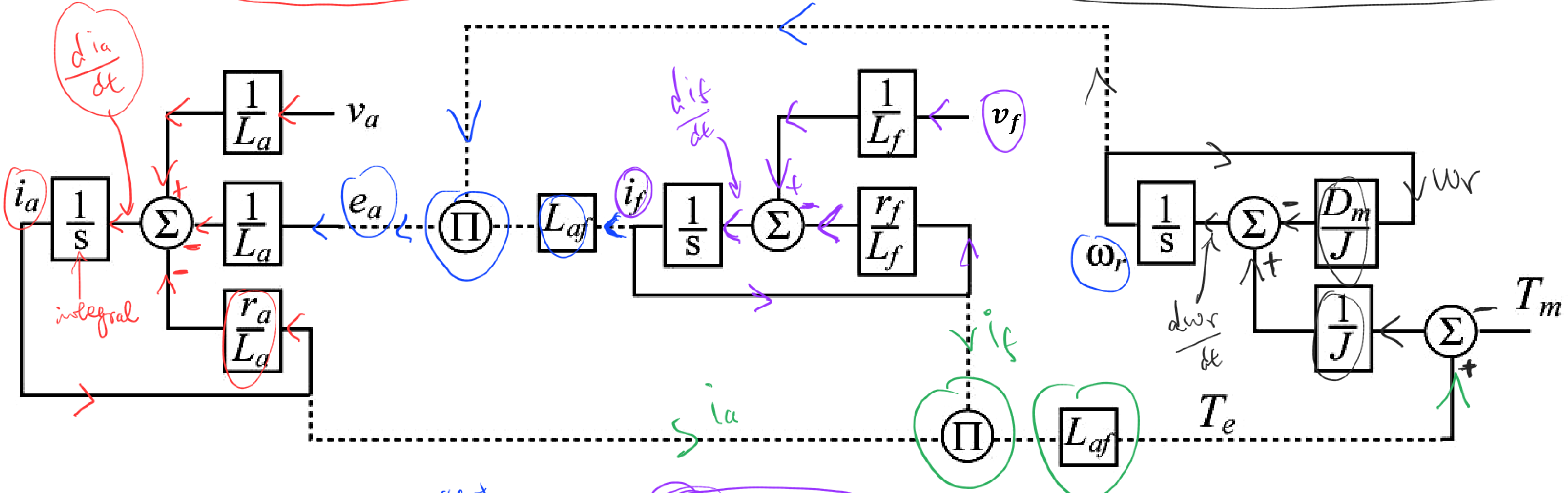
How do we solve these equations ? Analytical -> challenging

Numerically → numerical integration → software (Simulink)

Implementation of the State Equations → Signal diagram

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}(T_e - T_m)$$



constant

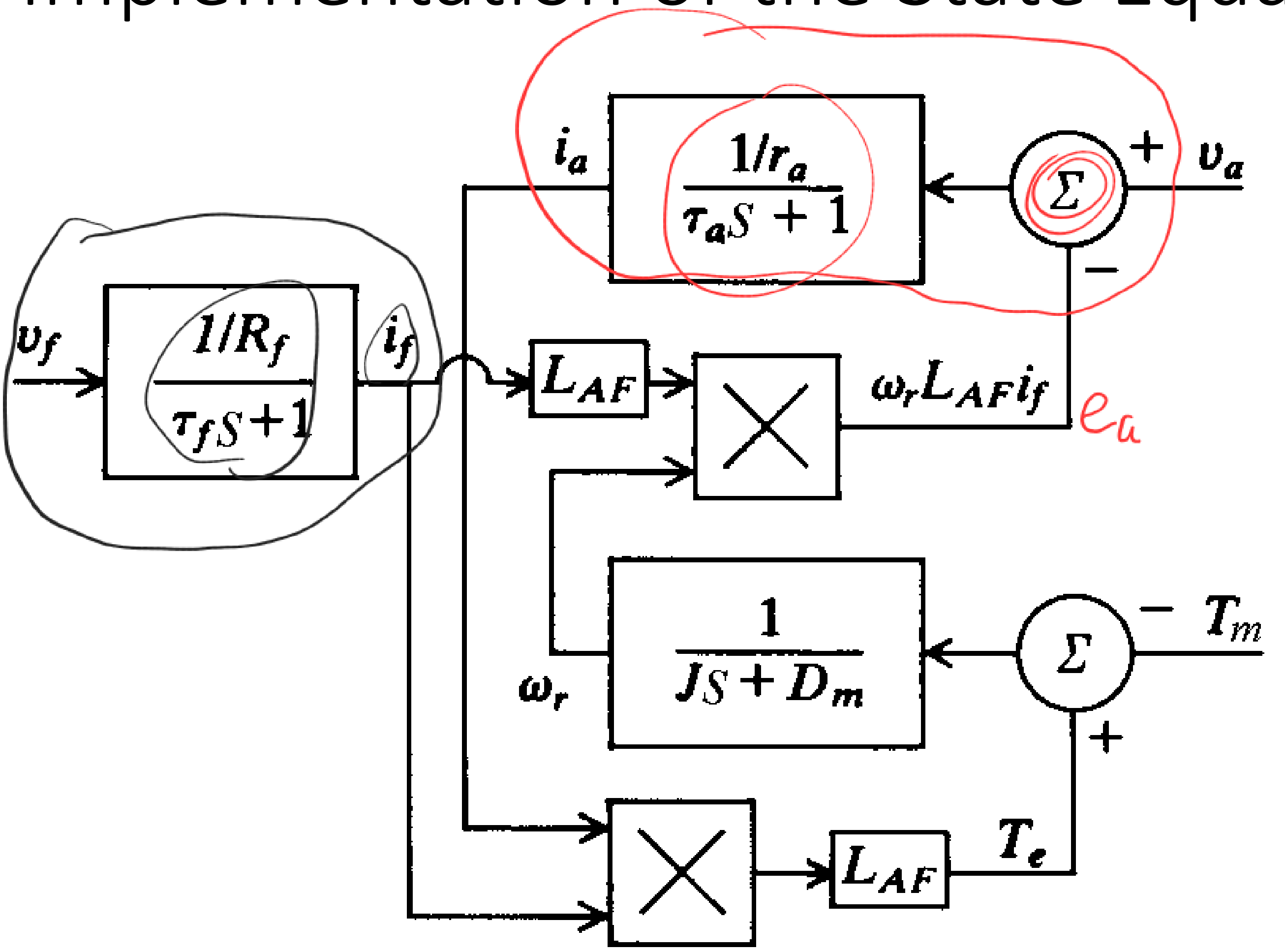
$$e_a = \omega_r L_{af} i_f$$

$$\frac{di_f}{dt} = -\frac{r_f}{L_f} i_f + \frac{1}{L_f} v_f$$

$$T_e = L_{af} i_f i_a$$

Is this a unique implementation ? No

Implementation of the State Equations



Re-write the equations in *s*-domain

$$v_f = r_f(1+\tau_f s)i_f \Rightarrow i_f = \frac{1/r_f}{1+\tau_f s} v_f$$

$$v_a = r_a(1+\tau_a s)i_a + e_a$$

$$\Rightarrow (v_a + e_a)\frac{1/r_a}{\tau_a s+1} = i_a$$

$$e_a = \omega_r L_{af} i_f$$

$$T_e - T_m = (D_m + Js)\omega_r$$

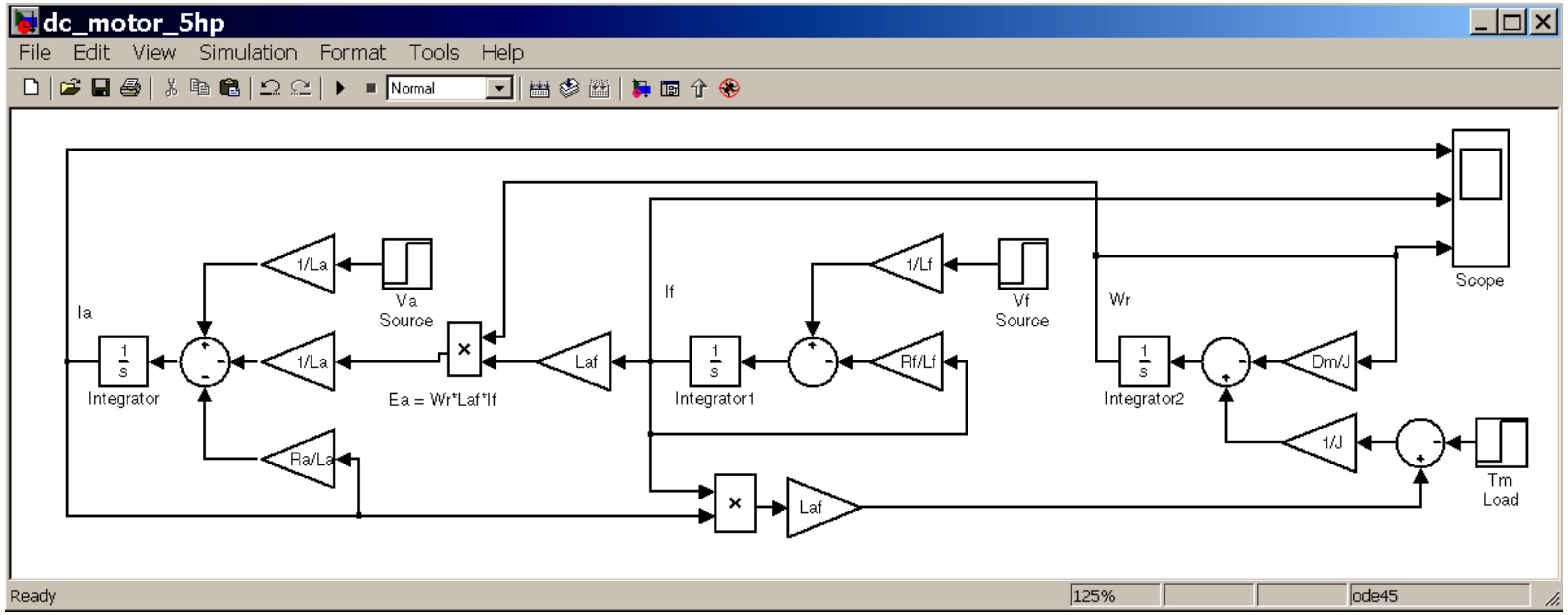
$$T_e = L_{af} i_f i_a$$

Define time constants

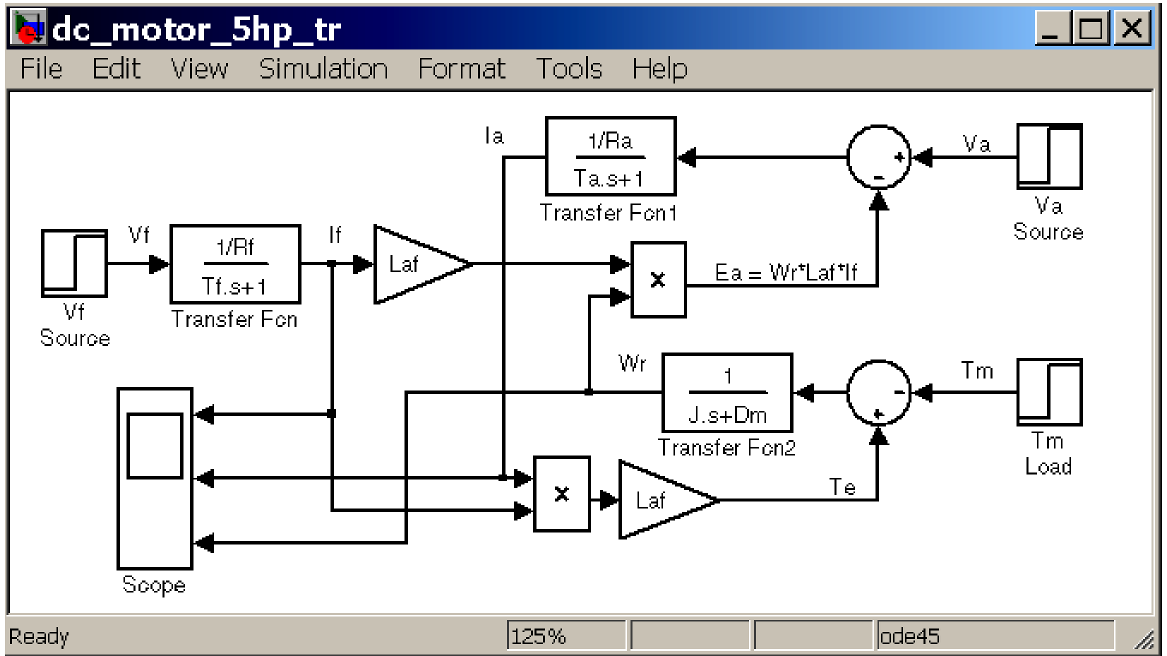
$$\tau_a = \frac{L_a}{r_a} \qquad \tau_f = \frac{L_f}{r_f}$$

Same system as previous slide

5-HP DC Motor (With Field Winding) Simulink Model



Slide 17



Slide 18

Try to do it yourself

- Both models are absolutely equivalent
- Models behave like “virtual” DC machine

Looking ahead…

- Finish **Module 4**