ELEC 342 Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 12

February 12, 2026

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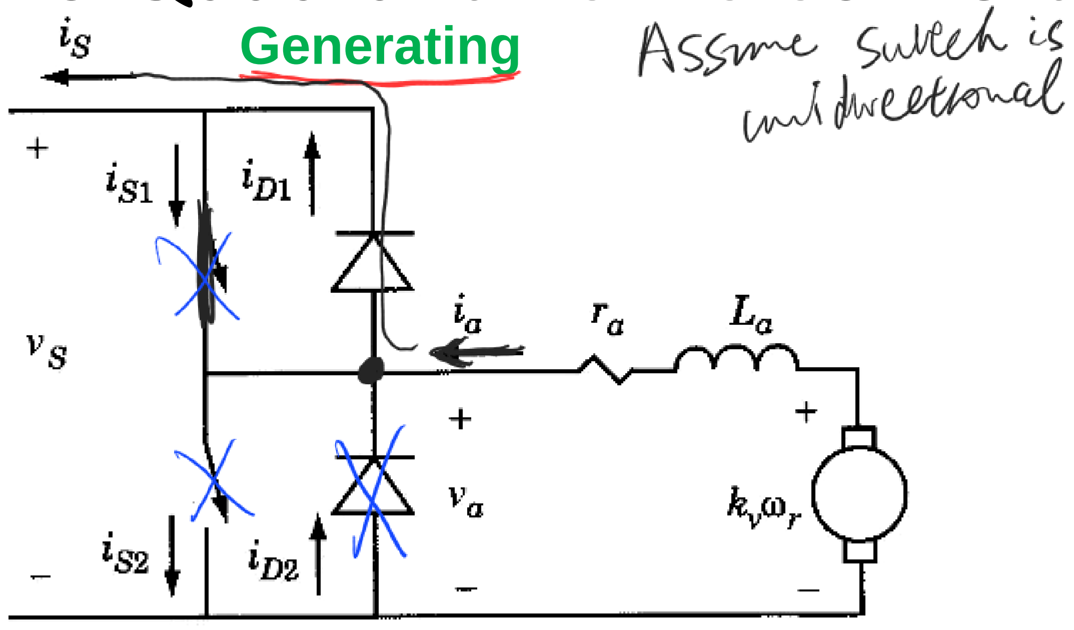
Course Logistics

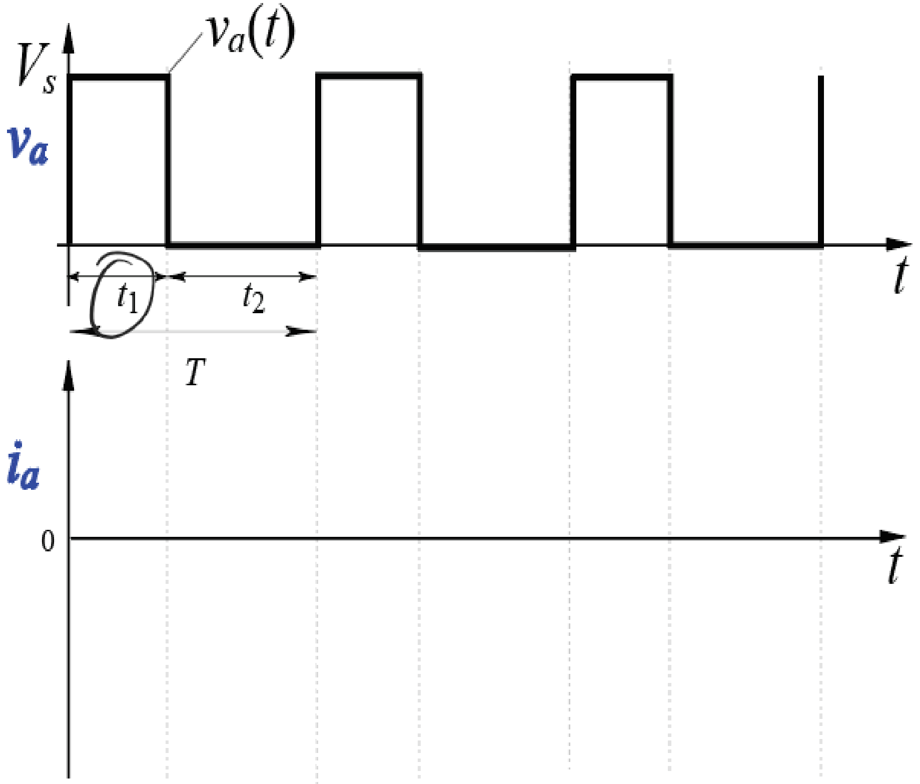
- **Labs:**
  - **Please be on time and come prepared**
  - **Each person needs to complete prelab and submit as a part of the final lab report**
  - **You need to submit all five (5) lab reports in order to write the final exam**

- Quiz 2 will take place on Feb. 13

- Midterm will take place on Feb. 26

Two-Quadrant DC-DC Converter



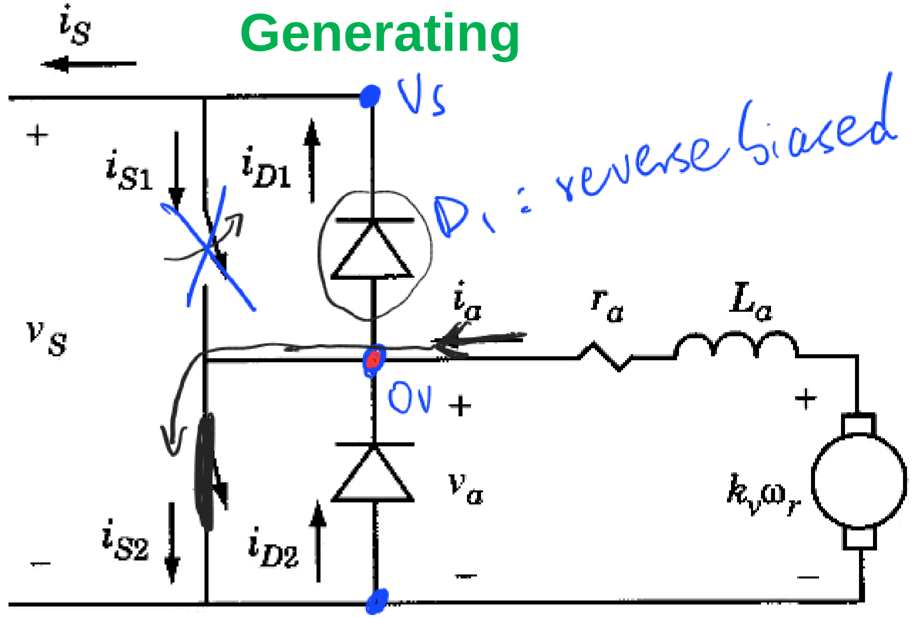


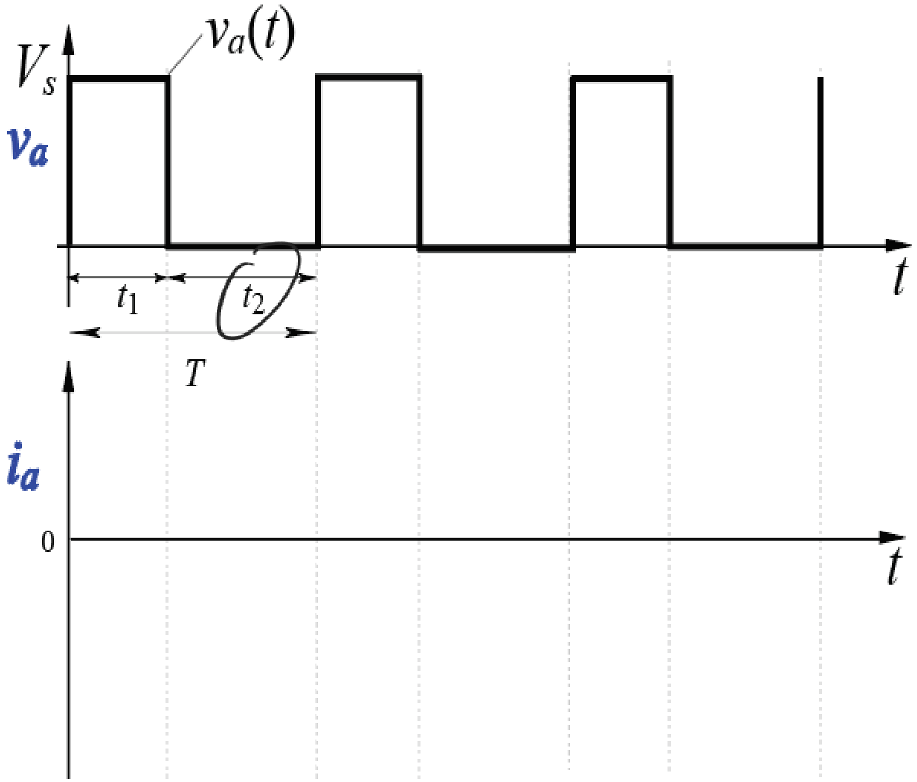
Pulse Width Modulation (**PWM**)

Assume complementary switching

Duty cycle

Two-Quadrant DC-DC Converter





Pulse Width Modulation (**PWM**)

Assume complementary switching

Duty cycle

State Equations for DC Motor with Field Winding

State Equations -> ODEs

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{di_f}{dt} = -\frac{r_f}{L_f} i_f + \frac{1}{L_f} v_f$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J} \omega_r + \frac{1}{J} (T_e - T_m)$$

Coupling Terms

$$e_a = \omega_r L_{af} i_f$$

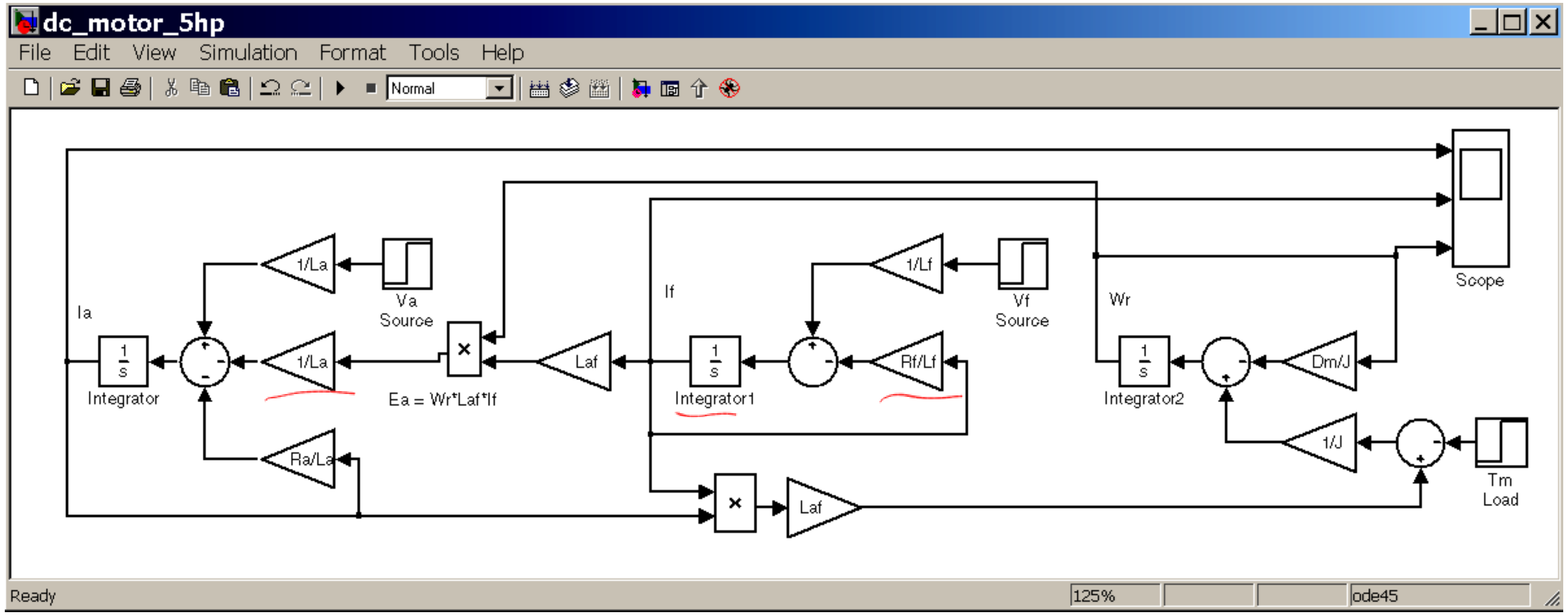
$$T_e = L_{af} i_f i_a$$

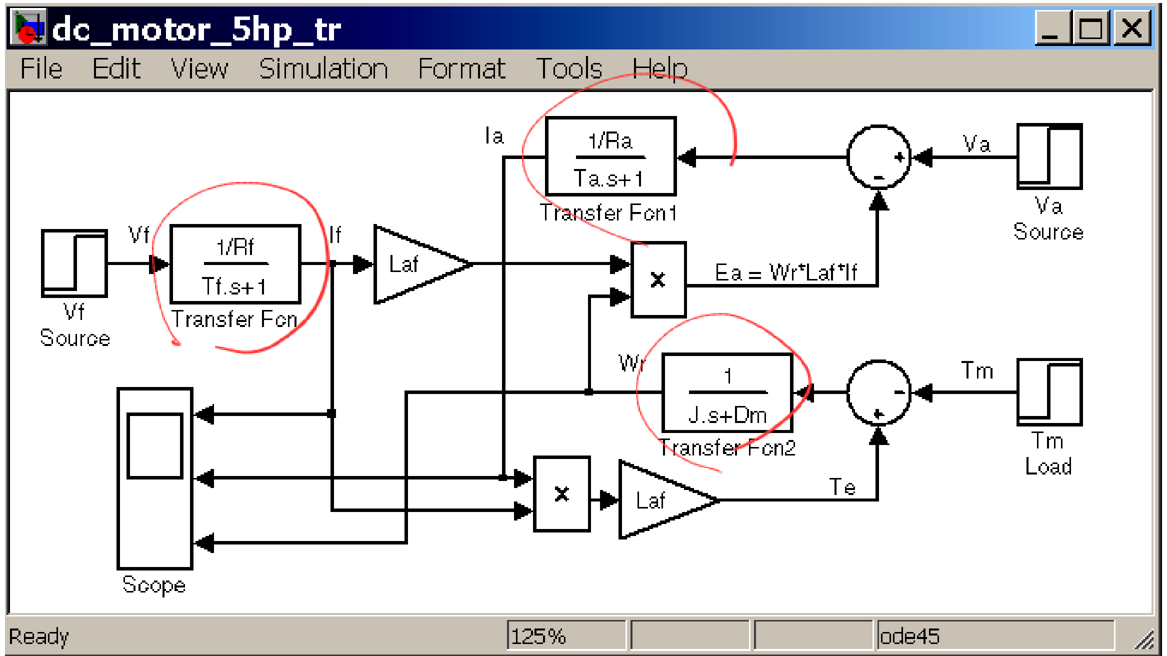
Are these equations coupled or decoupled ? Coupled

Are these equations linear or non-linear ? Nonlinear

How do we solve these equations ? Difficult to solve analytically
Numerical methods need to be used

5-HP DC Motor (With Field Winding) Simulink Model





Try to do it yourself

- Both models are absolutely equivalent
- Models behave like “virtual” DC machine

Dynamic Response of a 5-HP DC Motor

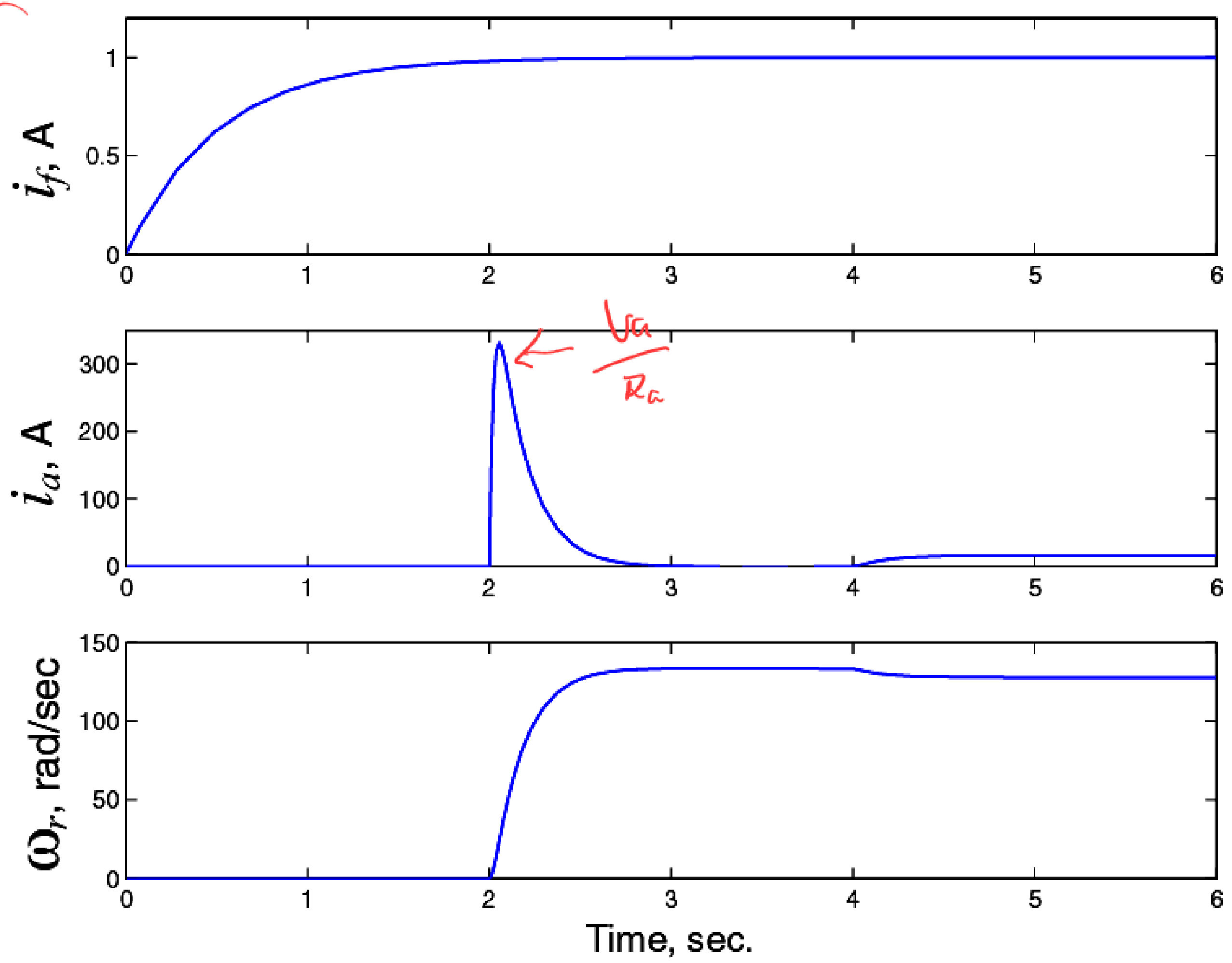
Example: Motor parameters:

Va = Vf = 240 V, Rf = 240 Ω, Lf = 120 H, Ra = 0.6 Ω, La = 0.012 H, Laf = 1.8 H, J = 1 kg*m^2, Dm = 1e-4 N*m*s, Tm = 29.2 N*m

In this computer study:

Zero initial conditions assumed

Step-1: field winding is energized
Step-2: armature is energized
Step-3: load torque is applied



Dynamic Modeling of PM DC Machine → simpler

State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}\left(T_e - T_m\right)$$

Coupling Terms

$$e_a = k_v \omega_r \qquad T_e = k_v i_a$$

Are these equations coupled or decoupled ? Couple

Are these equations linear or non-linear ? Linear

How do we solve these equations ? Analytically possible

Standard State-Space Form

State variables

input variable

$\dot{x} = Ax + Bu$

Lots of analytical tools

$$\frac{d}{dt}\underbrace{\begin{bmatrix} i_a \\ \omega_r \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_v}{J} & -\frac{D_m}{J} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_a \\ \omega_r \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \frac{1}{L_a} & \\ & -\frac{1}{J} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} v_a \\ T_m \end{bmatrix}}_{u}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

Implementation of the state equations

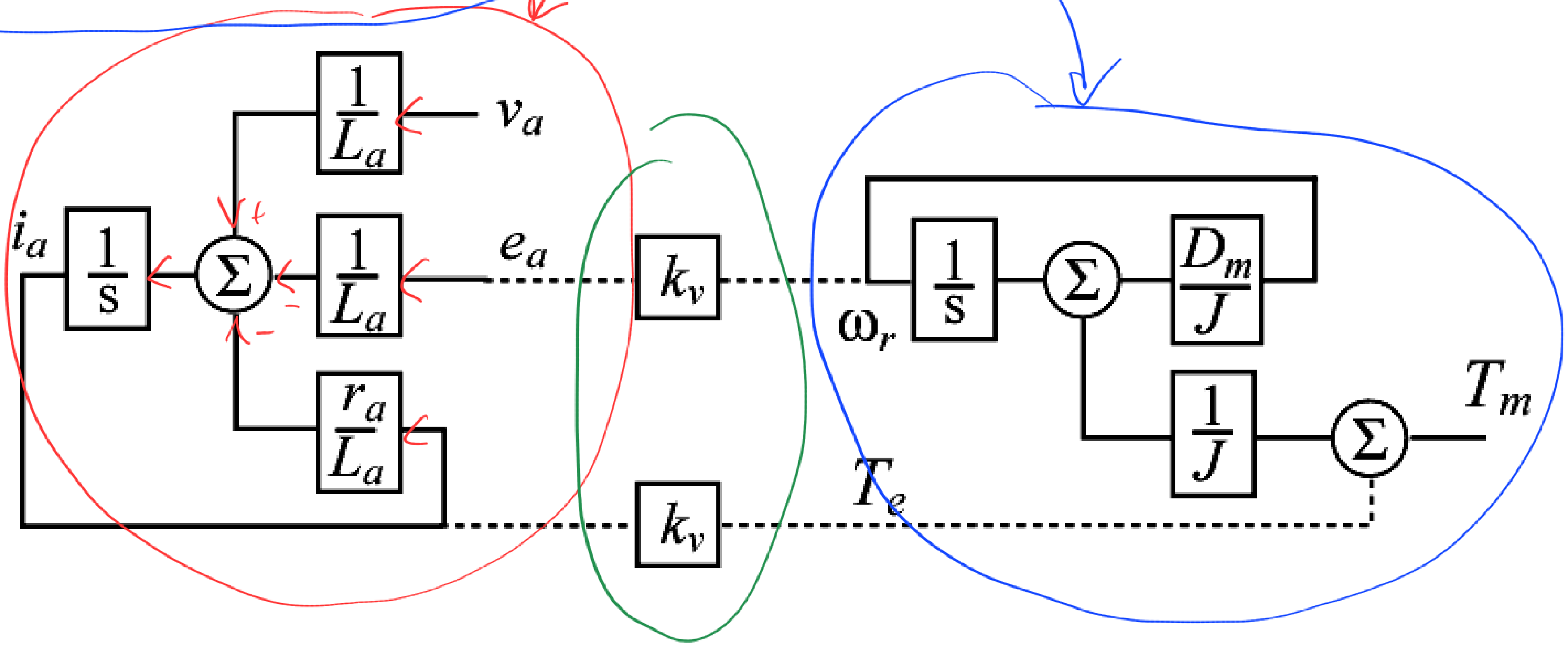
State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

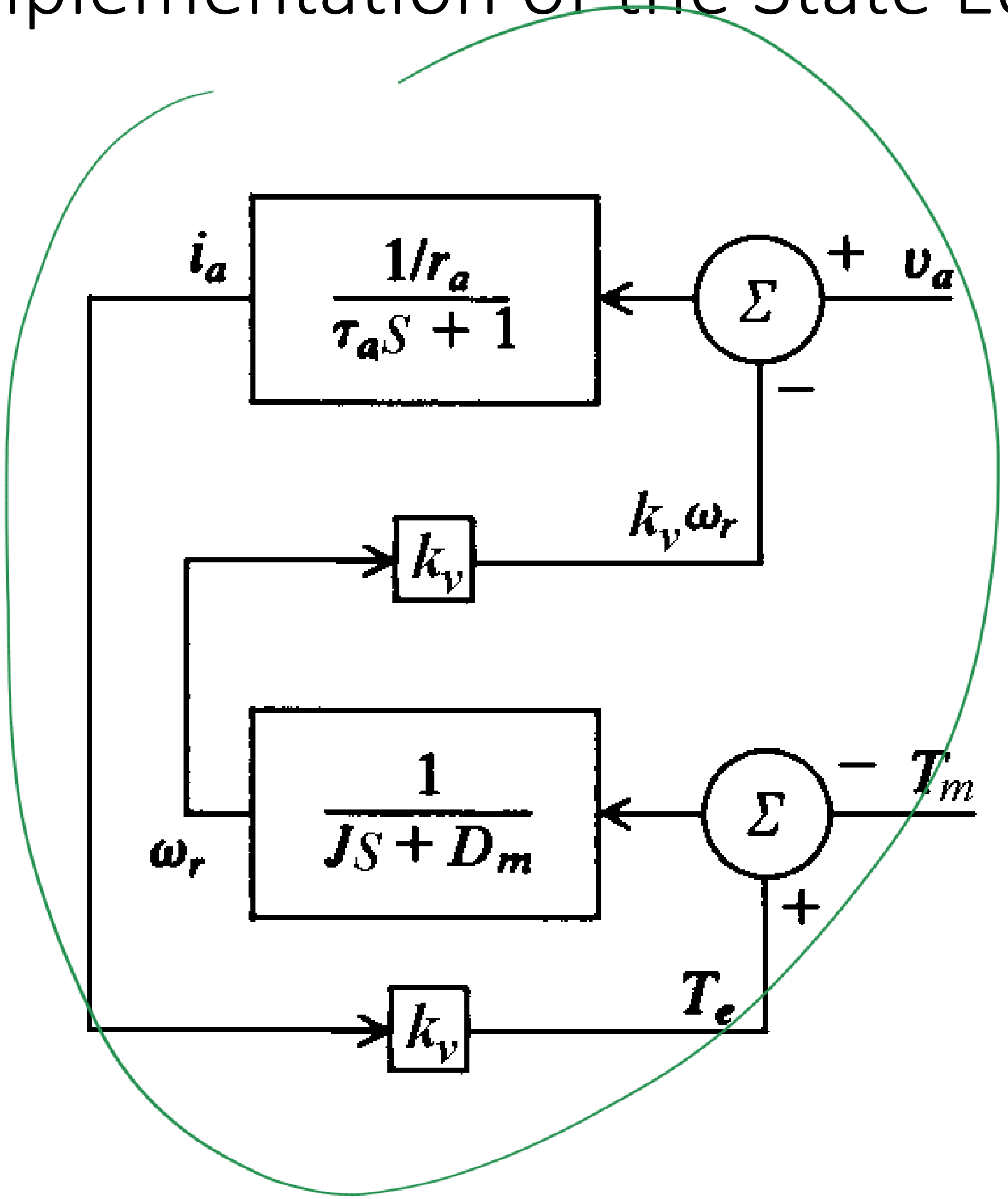
$$\frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}(T_e - T_m)$$

Coupling Terms

$$e_a = k_v \omega_r \qquad T_e = k_v i_a$$



Implementation of the State Equations



Re-write the equations in *s*-domain

$$v_a = r_a (1 + \tau_a s) i_a + e_a$$

$$e_a = k_v \omega_r$$

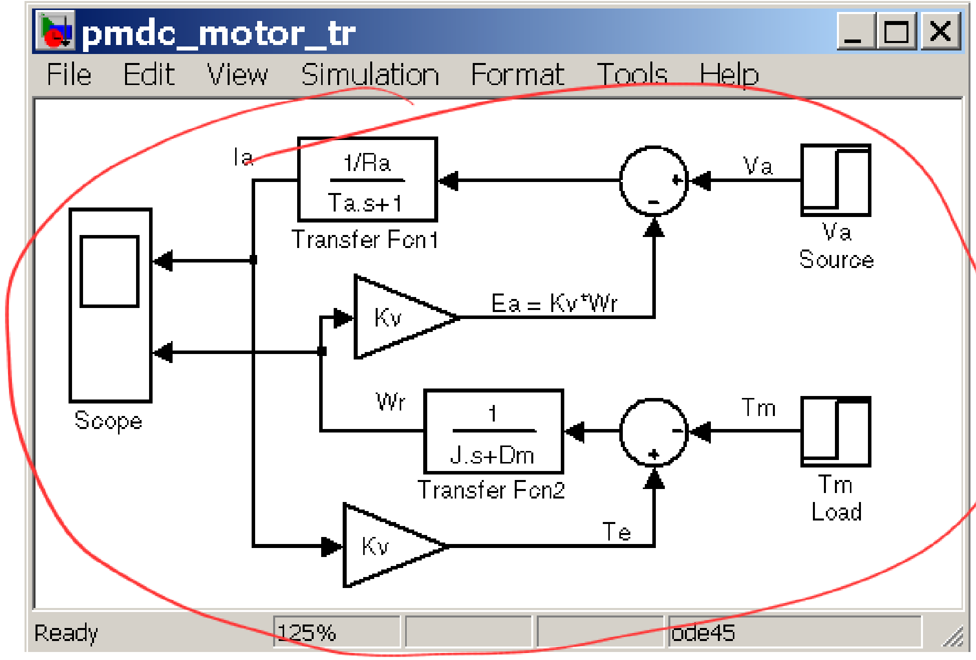
$$T_e - T_m = (D_m + Js)\omega_r$$

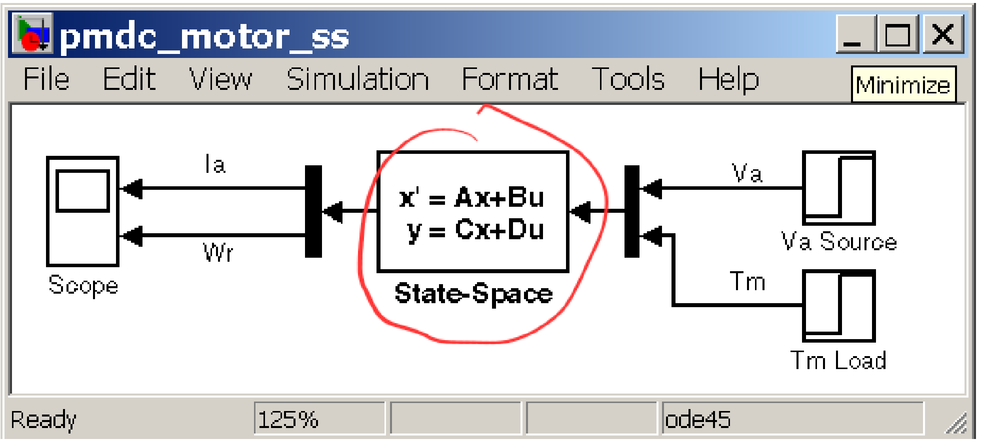
$$T_e = k_v i_a$$

Define time constant

$$\tau_a = \frac{L_a}{r_a}$$

Simulink Model of a PM DC Motor





$$i_a = \frac{1}{r_a(1+\tau_a s)}(v_a - e_a)$$

$$e_a = k_v \omega_r$$

$$\omega_r = \frac{1}{D_m + Js}(T_e - T_m)$$

$$T_e = k_v i_a$$

State equation:

$$\frac{d}{dt}\underbrace{\begin{bmatrix} i_a \\ \omega_r \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_v}{J} & -\frac{D_m}{J} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_a \\ \omega_r \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \frac{1}{L_a} & \\ & -\frac{1}{J} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} v_a \\ T_m \end{bmatrix}}_{u}$$

Output equation: $y = Cx + Du$

$$\overset{y}{\begin{bmatrix} i_a \\ \omega_r \\ T_e \end{bmatrix}} = \overset{C}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ k_v & 0 \end{bmatrix}} \overset{x}{\begin{bmatrix} i_a \\ \omega_r \end{bmatrix}} + \overset{D}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}} \overset{u}{\begin{bmatrix} v_a \\ T_m \end{bmatrix}}$$

Dynamic Response of a 6V PM DC Motor

Example: Motor parameters:

Va = 6 V, Ra = 7 Ω, La = 120 mH, Kv = 0.0141 N*A/m, J = 1.06e-6 kg*m^2, Dm = 6.01e-6 N*m*s, Tm = 3.53e-3 N*m

$\frac{V_a}{R_a} \sim \frac{6}{7} \sim 0.85A$

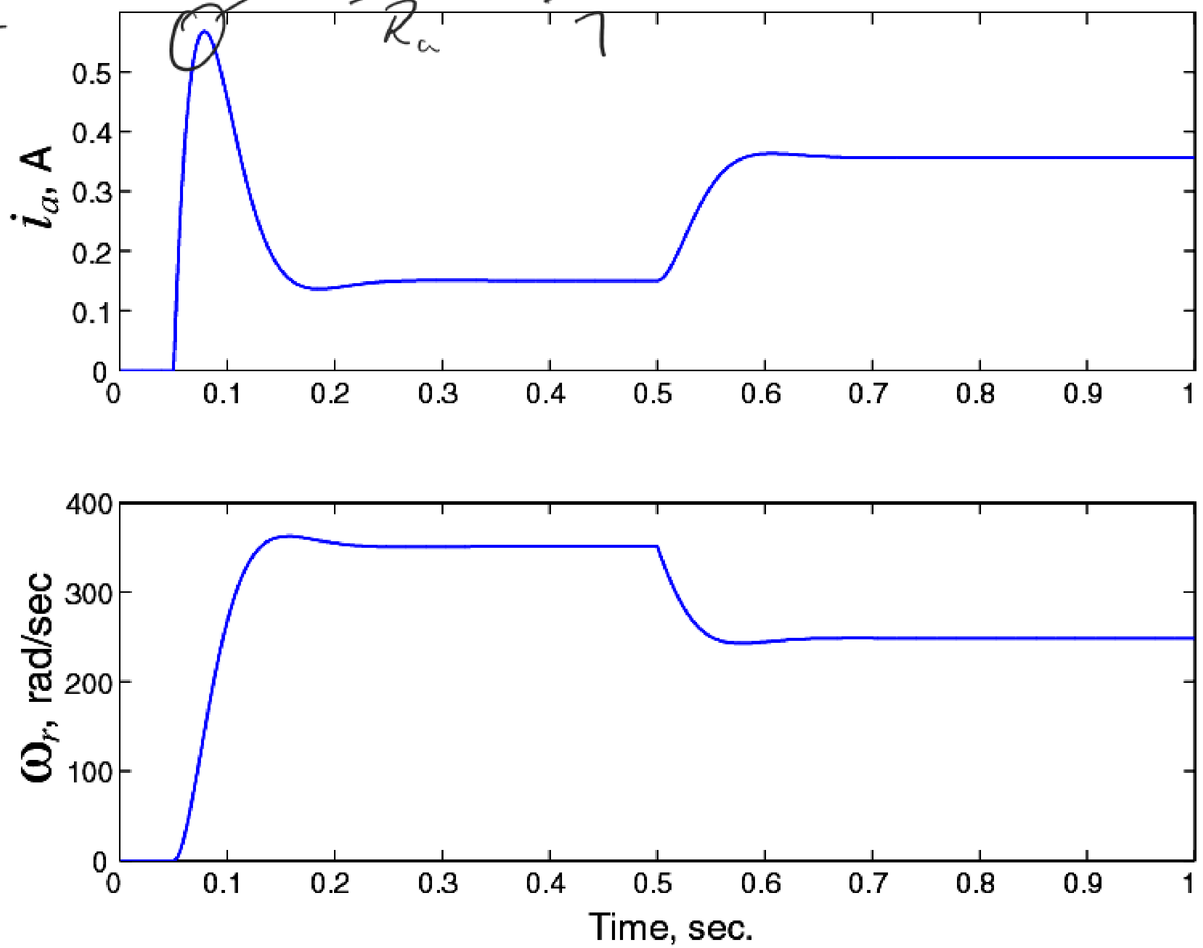
In this computer study:

Zero initial conditions assumed

Step-1: voltage is applied

Step-2: load torque is applied

Understand Simulink files
↓
need for Lab 3 and Hwk 4

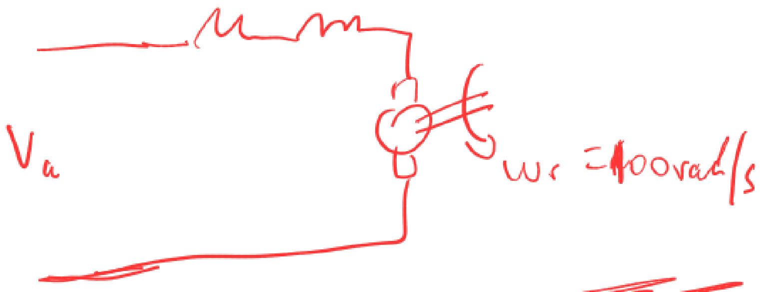


Example 1

ω_r

SP3.3-1 When a 12-V, permanent-magnet dc motor is driven at 100 rad/s, the open-circuit voltage is 10 V. Calculate k_v. [$k_v = 0.1\ \mathrm{V \cdot s/rad}$]

V_a



$i_a = 0\ \mathrm{A}$

$V_a = \cancel{i_a R_a} + E_a$

$V_a = E_a = k_v \omega_r$

$$\boxed{k_v = \frac{V_a}{\omega_r} = \frac{10\ \mathrm{V}}{100\ \mathrm{rad/s}} = 0.1\ \mathrm{V \cdot s/rad}}$$

$k_v = k_t$

Example 2

SP3.3-3 Calculate the $\boxed{\text{no-load speed } (T_L = 0)}$ for the permanent-magnet dc motor in SP3.3-1 when rated voltage (12 V) is applied to the armature. $[\omega_r = 120 \text{ rad/s}]$

$T_e = 0$

$k_v = k_t = 0.1$

$V_a = i_a R_a + E_a = i_a R_a + k_v \omega_r$, $T_e = 0 = k_t i_a \Rightarrow i_a = 0$

$V_a = k_v \omega_r$

$12V = 0.1 \omega_r \Rightarrow \boxed{\omega_r = 120 \text{ rad/s}}$

Example 3

SP3.4-1 A 12-V, permanent-magnet dc motor has an armature resistance of 12 Ω and $k_v = 0.01$ V · s/rad. Calculate the steady-state stall torque (T_e with $\omega_r = 0$). [$T_e = 0.01$ N · m]

$k_v = 0.01\,V/rpm \rightarrow$ needs conversion to $V \cdot s/rad$

$E_a = k_v \omega_r = 0V$

$V_a = R_a i_a + \cancel{E_a}^{\,0} \Rightarrow 12V = 12\Omega \cdot i_a \Rightarrow i_a = 1A$

$T_e = k_t i_a = 0.01 \cdot 1 = 0.01\ N{\cdot}m$

$k_t = k_v$

- DC Generator: $V_a + R_a i_a = E_a$
- DC motor: $V_a = R_a i_a + E_a$

Example 4

A permanent magnet dc motor drives a mechanical load requiring a constant torque of 25 N · m. The motor produces 10 N · m with an armature current of 10 A. The resistance of the armature circuit is 0.2 Ω. A 200 V dc supply is applied to the armature terminals. Determine the speed of the motor.

T_m

Find k_t : $T_e = k_t i_a \Rightarrow 10\,N\cdot m = k_t \cdot 10A \Rightarrow k_t = 1\ V\cdot s/rad$

$R_a = 0.2\,\Omega$

$V_a = 200V$

$T_m = T_e = 25\ N\cdot m$ (ignore friction) $\rightarrow T_e = k_t i_a$

$\omega_r = ?$

$\hookrightarrow E_a = k_v \cdot \omega_r$

① $i_a = \frac{T_e}{k_t} = \frac{25\ N\cdot m}{1} = 25\ A$

② $V_a = R_a i_a + E_a$

$200V = 0.2 \cdot 25A + k_v \cdot \omega_r$

$\omega_r = \frac{200 - 0.2 \times 25}{1} = 195\ rad/s$

Example 5

E_a V_a I_a ω_r needs conversion

A dc machine is connected across a 240-volt line. It rotates at 1200 rpm and is generating 230 volts. The armature current is 40 amps.

(a) Is the machine functioning as a generator or as a motor?

(b) Determine the resistance of the armature circuit. $\rightarrow R_a$

(c) Determine power loss in the armature circuit resistance and the electro-magnetic power. $\rightarrow P_e$

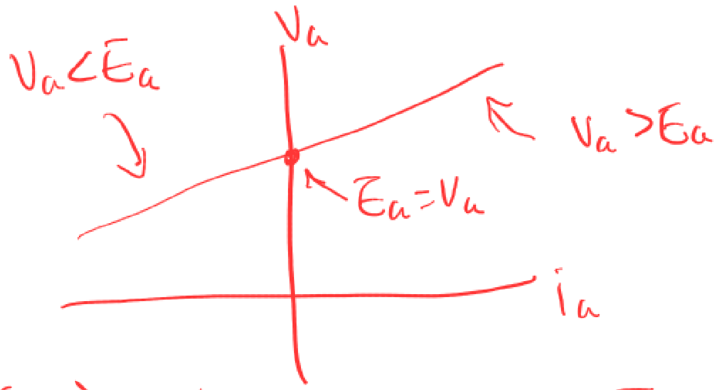
$\rightarrow \frac{V^2}{R} \rightarrow I^2R$

(d) Determine the electromagnetic torque in newton-meters. $\rightarrow T_e$

(e) If the load is thrown off, what will the generated voltage and the rpm of the machine be

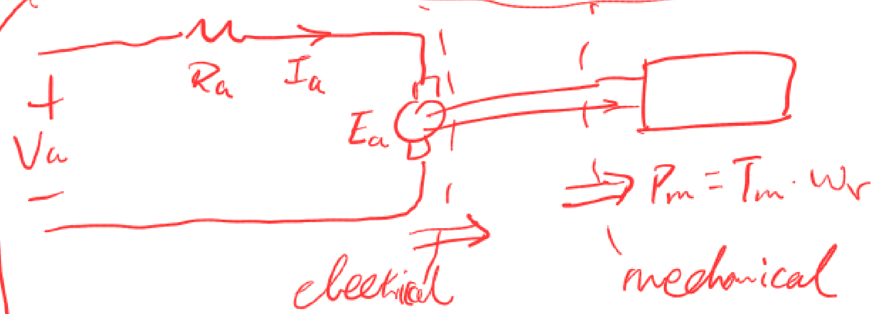
Example 5

(a) $V_a = 240\,V$, $E_a = 230\,V$, $I_a = 40\,A$, $\underline{\omega_r} = 1200$ rpm



$V_a > E_a \Rightarrow$ Motoring

(b) $V_a = R_a i_a + E_a$ (motoring) $\Rightarrow 240 = R_a \cdot 40 + 230 \Rightarrow R_a = 0.25\,\Omega$



(c) Power Loss: $R_a I_a^2 = 40^2 \times 0.25 = 400\,W$

$P_e = E_a \cdot I_a = 230\,V \times 40\,A = 9200\,W$

(d) $T_e = \frac{P_e}{\omega_r} = \frac{9200\,W}{125.7}$

$= 73.19\ N\cdot m$

$\omega_r = 1200 \frac{rev}{min}$

$= \frac{1200\ \cancel{rev}}{60\ sec/\cancel{min} \times \cancel{min}} \times 2\pi \frac{rad}{\cancel{rev}}$

$= \frac{1200}{60} \times 2\pi = 125.7$ rad/sec

$P_e = E_a I_a$

$T_e = T_m$

or $T_e = T_m + T_{fric}$

(e) You need k_t

Looking ahead…

- Start **Module 5**