ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 15

March 5, 2026

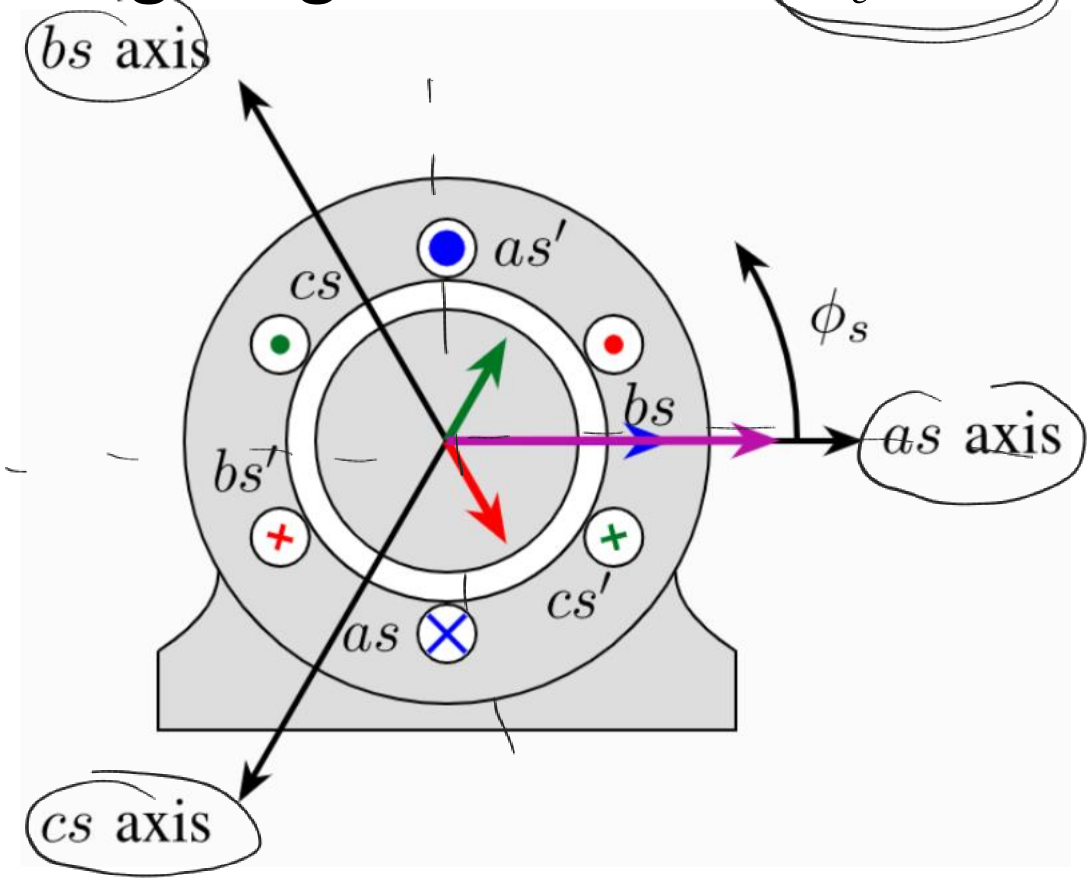
Zhi Jin (Justin) Zhang

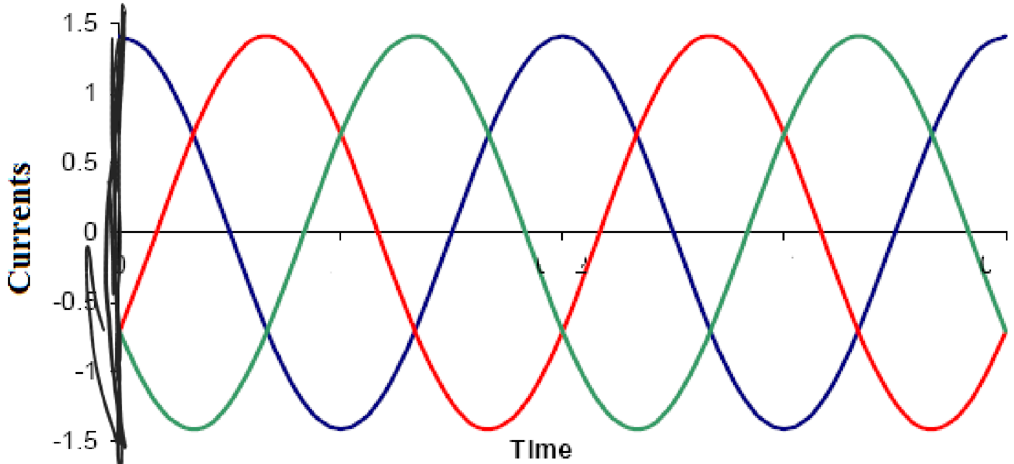
Course Logistics

- No tutorial this week

- Quizzes
    - Quiz 3: March 20 at the beginning of the tutorial
    - Quiz 4: April 2 at the beginning of the lecture

Rotating Magnetic Field at $\omega_e t = 0$





Apply balanced set of currents

$$\begin{cases} i_{as} = I_m \cos(\omega_e t) \\ i_{bs} = I_m \cos(\omega_e t - 120^\circ) \\ i_{cs} = I_m \cos(\omega_e t + 120^\circ) \end{cases}$$

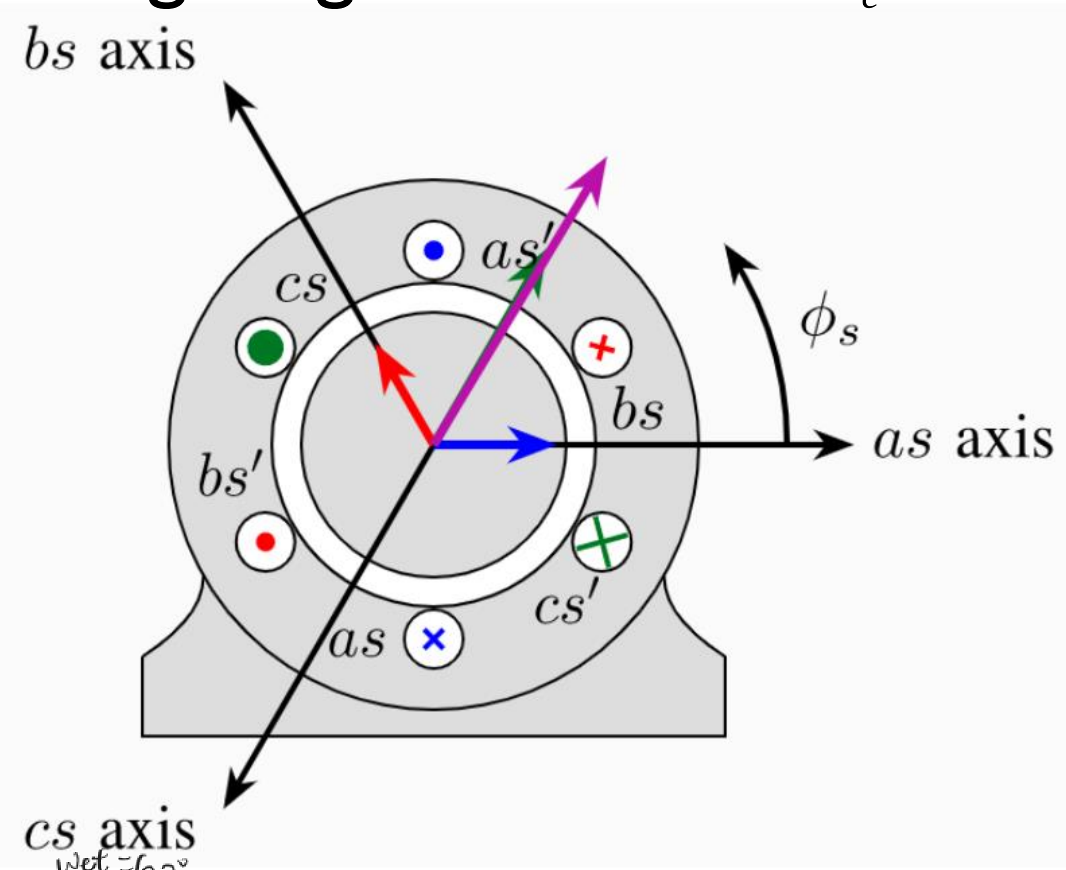
$$\begin{cases} F_{as} = F_m \rightarrow N I_m \\ F_{bs} = -\frac{1}{2} F_m \\ F_{cs} = -\frac{1}{2} F_m \end{cases}$$

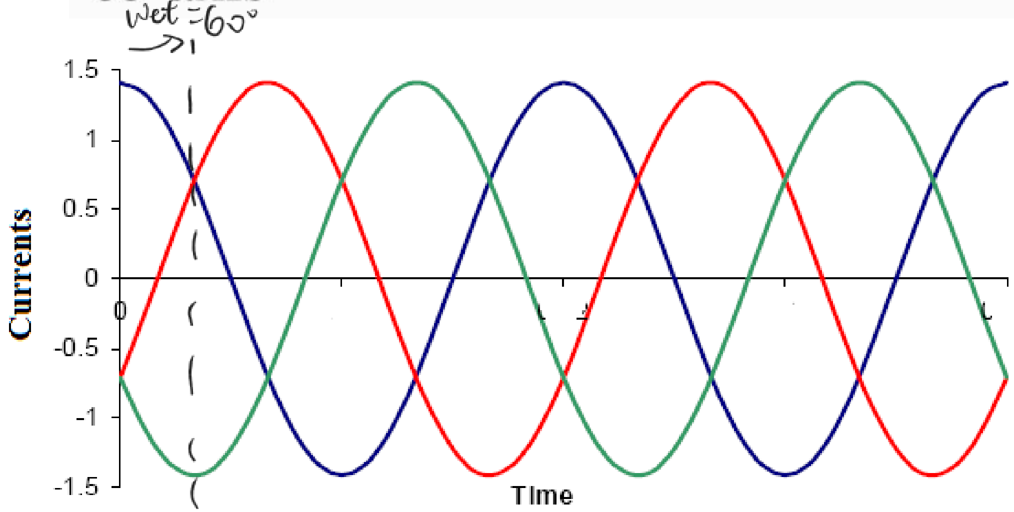
Resulting MMF

$$\mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs} + \mathbf{F}_{cs}$$

$$= \frac{3}{2} F_m \angle 0^\circ$$

Rotating Magnetic Field at $\omega_e t = 60^\circ$





Apply balanced set of currents

$$\begin{cases} i_{as} = I_m \cos(\omega_e t) \\ i_{bs} = I_m \cos(\omega_e t - 120^\circ) \\ i_{cs} = I_m \cos(\omega_e t + 120^\circ) \end{cases}$$

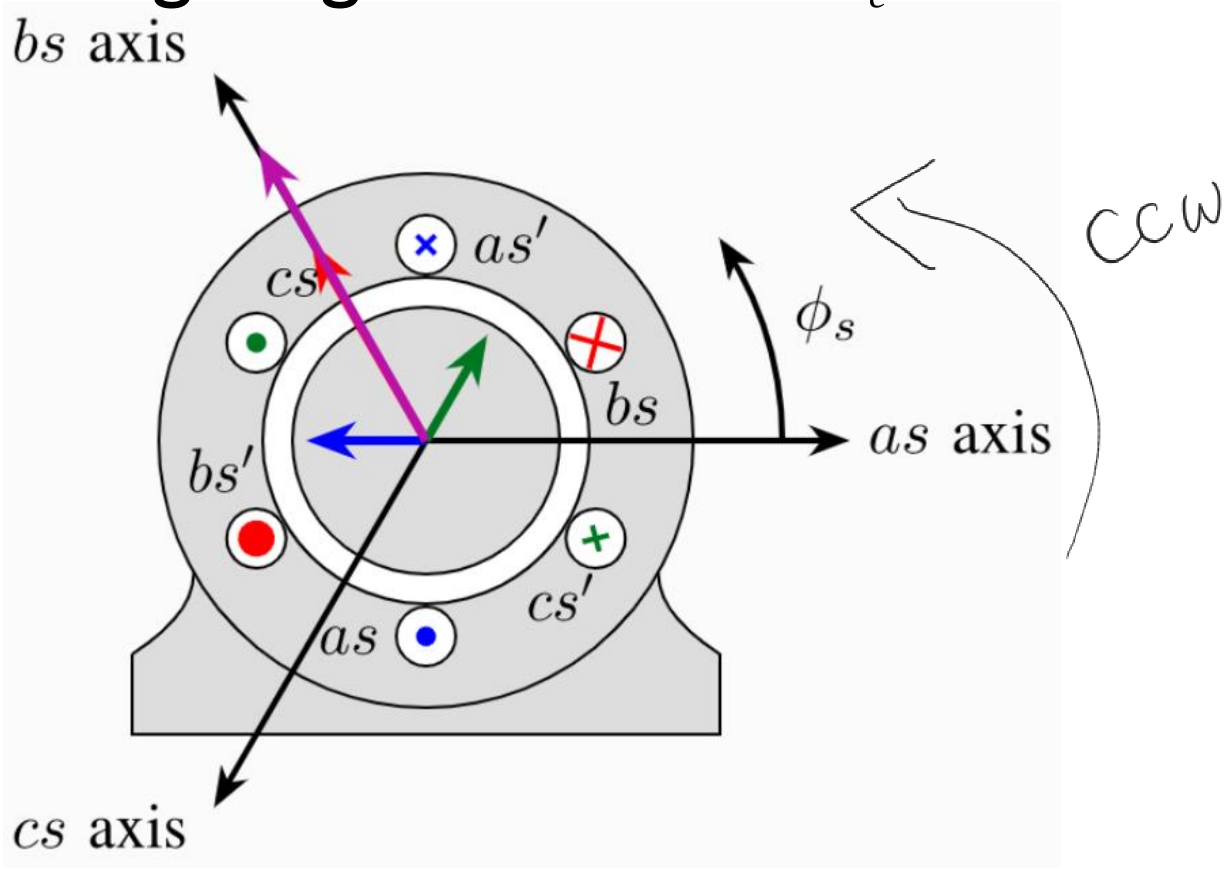
$$\begin{cases} F_{as} = \frac{1}{2} F_m \\ F_{bs} = \frac{1}{2} F_m \\ F_{cs} = -F_m \end{cases}$$

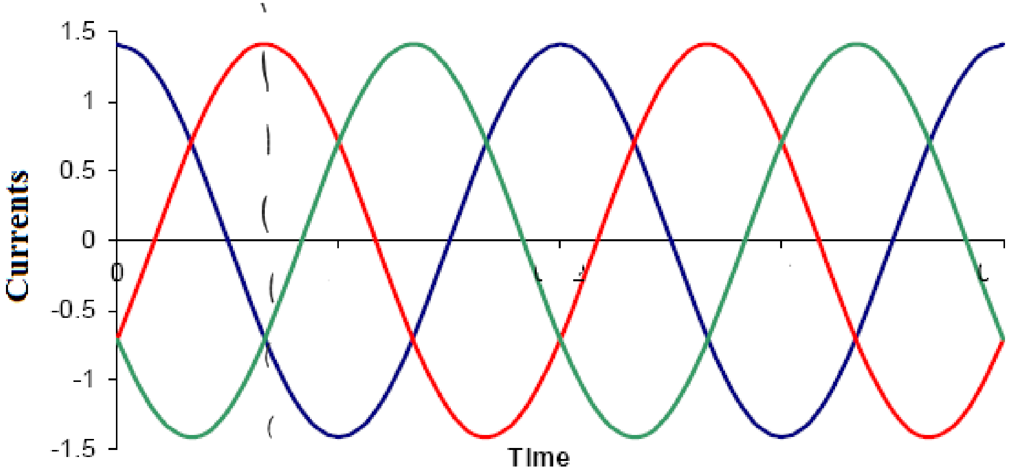
Resulting MMF

$$\mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs} + \mathbf{F}_{cs}$$

$$= \frac{3}{2} F_m \angle 60^\circ$$

Rotating Magnetic Field at $\omega_e t = 120^\circ$





Apply balanced set of currents

$$\begin{cases} i_{as} = I_m \cos(\omega_e t) \\ i_{bs} = I_m \cos(\omega_e t - 120^\circ) \\ i_{cs} = I_m \cos(\omega_e t + 120^\circ) \end{cases}$$

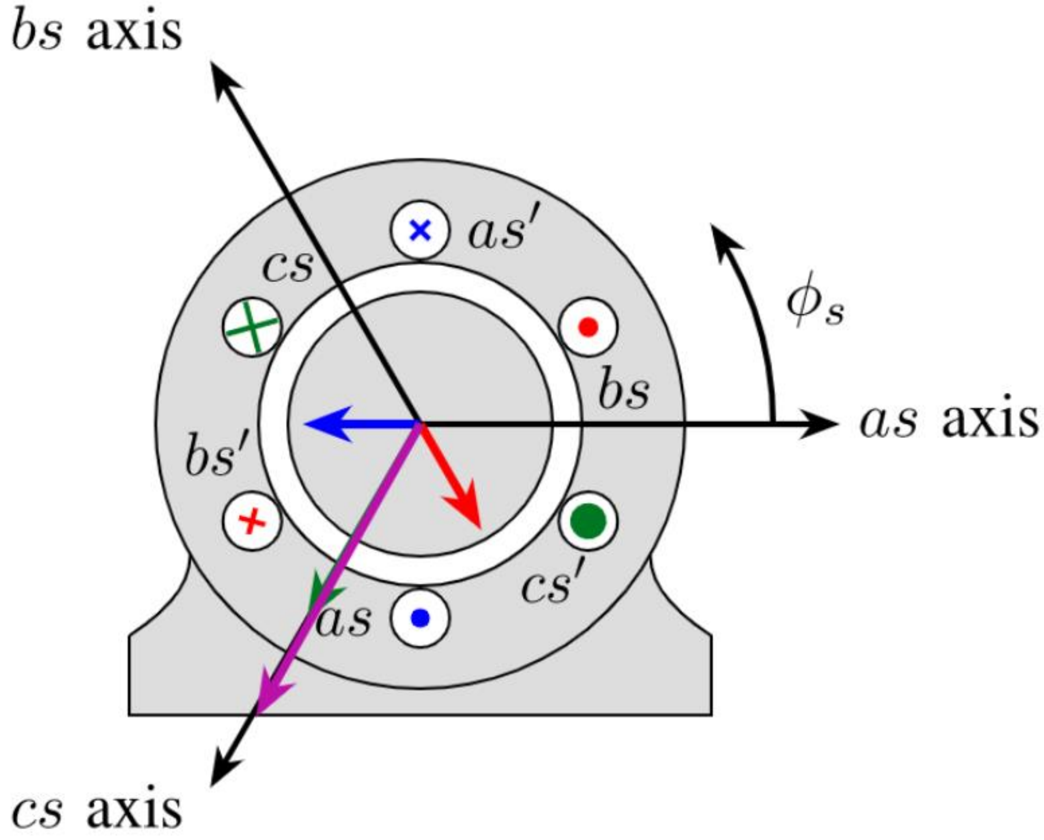
$$\begin{cases} F_{as} = -\frac{1}{2} F_m \\ F_{bs} = + F_m \\ F_{cs} = -\frac{1}{2} F_m \end{cases}$$

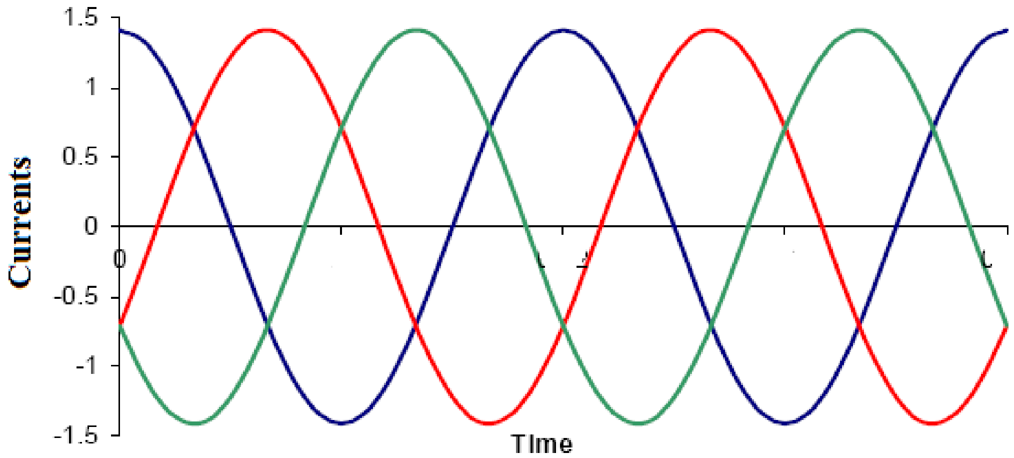
Resulting MMF

$$\mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs} + \mathbf{F}_{cs}$$

$$= \frac{3}{2} F_m \angle 120^\circ$$

Rotating Magnetic Field at $\omega_e t = 240^\circ$





Apply balanced set of currents

$$\begin{cases} i_{as} = I_m \cos(\omega_e t) \\ i_{bs} = I_m \cos(\omega_e t - 120^\circ) \\ i_{cs} = I_m \cos(\omega_e t + 120^\circ) \end{cases}$$

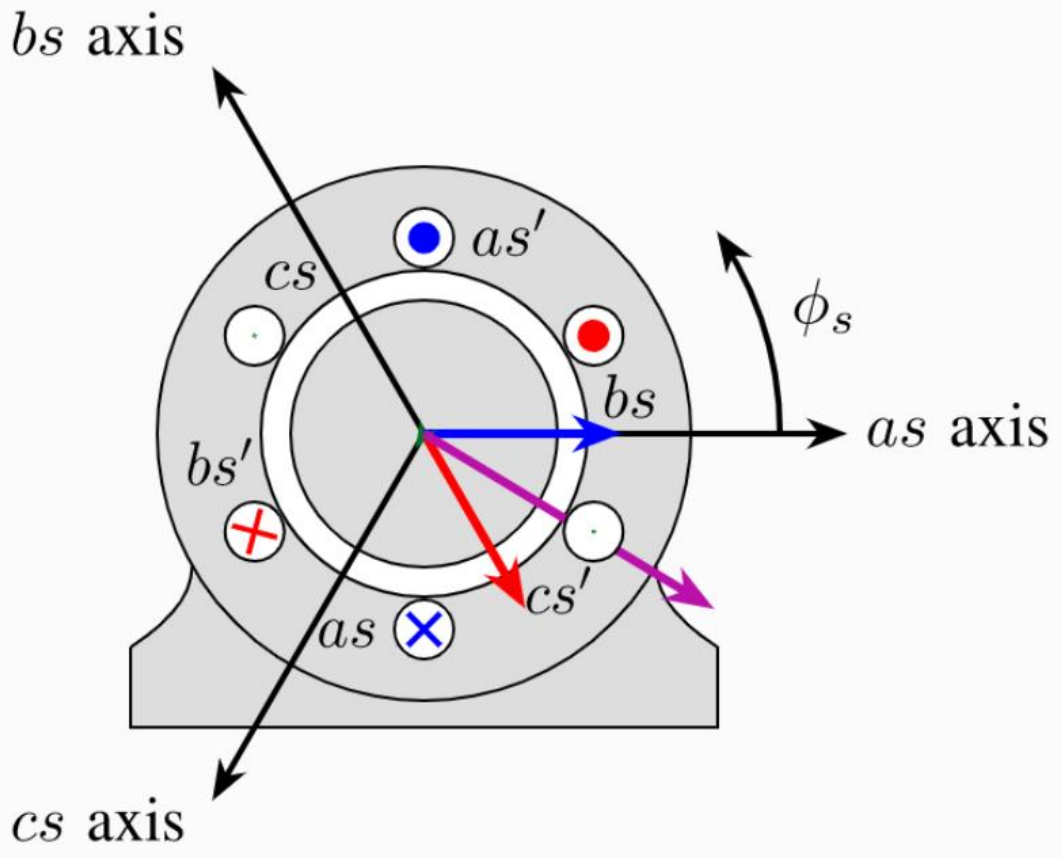
$$\begin{cases} F_{as} = -\frac{1}{2} F_m \\ F_{bs} = -\frac{1}{2} F_m \\ F_{cs} = +F_m \end{cases}$$

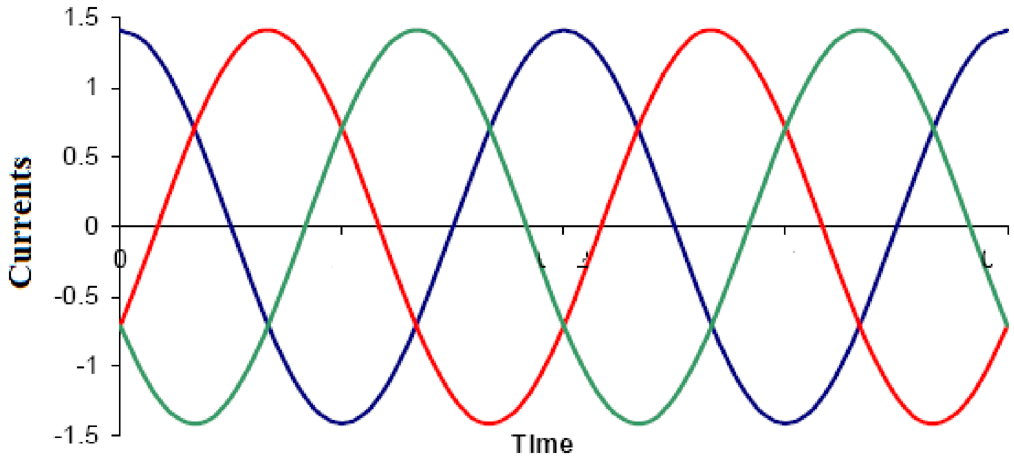
Resulting MMF

$$\mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs} + \mathbf{F}_{cs}$$

$$= \frac{3}{2} F_m \angle 240^\circ$$

Rotating Magnetic Field at $\omega_e t = 330^\circ$





Apply balanced set of currents

$$\begin{cases} i_{as} = I_m \cos(\omega_e t) \\ i_{bs} = I_m \cos(\omega_e t - 120^\circ) \\ i_{cs} = I_m \cos(\omega_e t + 120^\circ) \end{cases}$$

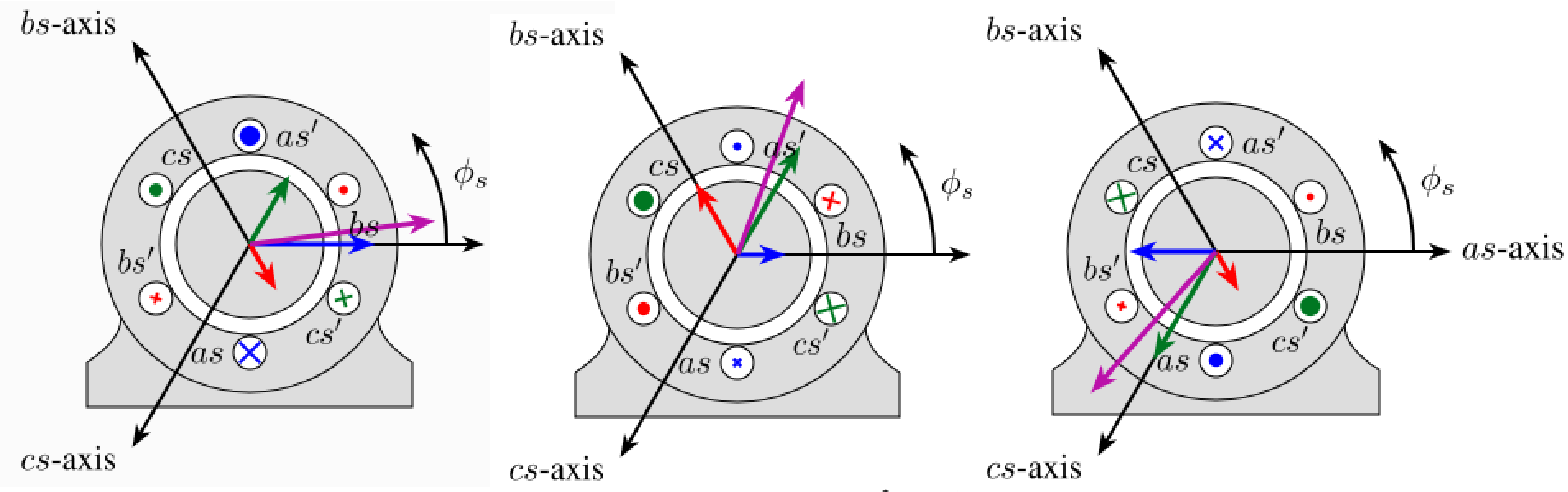
$$\begin{cases} F_{as} = \\ F_{bs} = \\ F_{cs} = \end{cases}$$

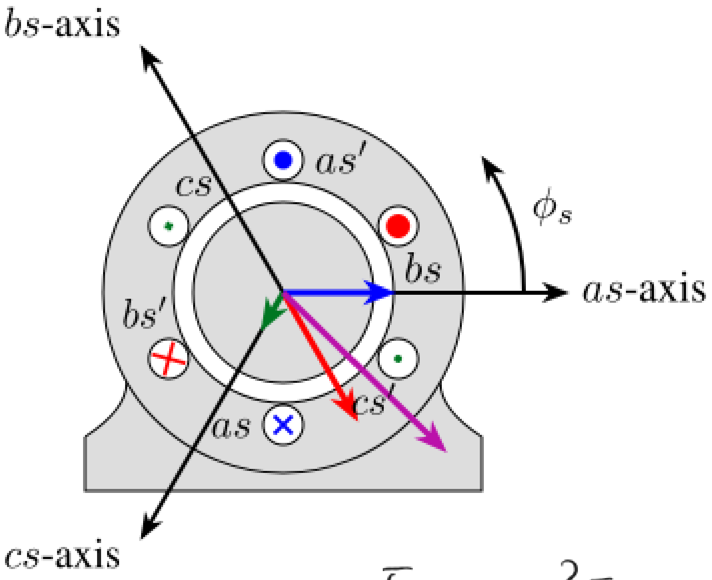
Resulting MMF

$$\mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs} + \mathbf{F}_{cs}$$

$= \frac{3}{2} F_m \angle 330^\circ$

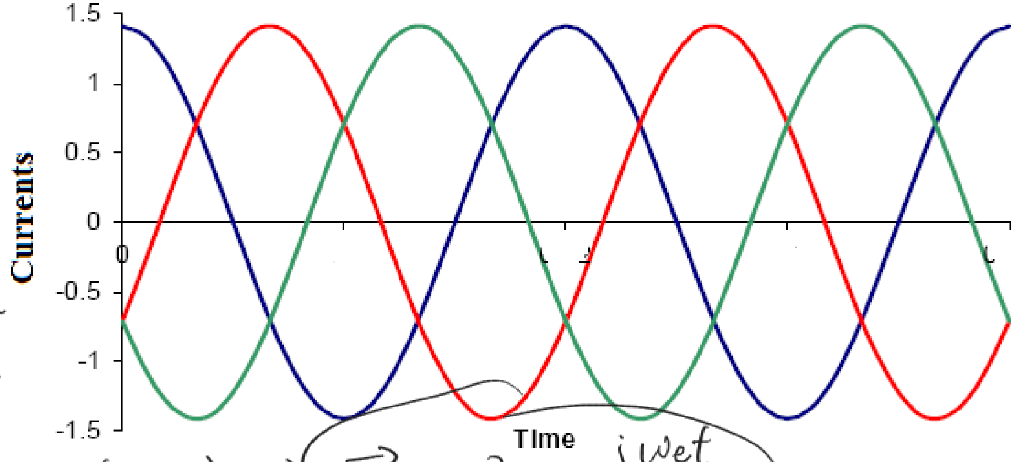
Rotating Magnetic Field





CCW (positive rotation)

$\vec{F}_{net} = \frac{3}{2} F_m \angle \underbrace{\omega_e t}_{\text{function of time}}$



$F_{net_x} = \frac{3}{2} F_m \cos(\omega_e t)$ $\quad$ $F_{net_y} = \frac{3}{2} F_m \sin(\omega_e t)$ $\Rightarrow$ $\vec{F}_{net} = \frac{3}{2} F_m e^{j\omega_e t}$

Producing Rotating Magnetic Field

1. Given a set of AC currents shifted in time
2. Apply these currents to shifted in space stator windings → Produce MMF vector $\mathbf{F}_s$ that:
   - Has constant magnitude
   - Rotates in space

2 phase

$$i_a = I_m \cos(\omega_e t)$$
$$i_b = I_m \cos(\omega_e t - 90^\circ)$$

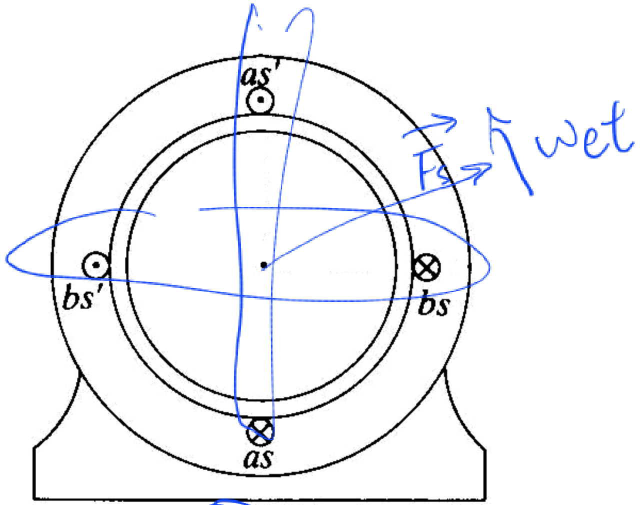
3 phase

$$i_a = I_m \cos(\omega_e t)$$
$$i_b = I_m \cos(\omega_e t - 120^\circ)$$
$$i_c = I_m \cos(\omega_e t + 120^\circ)$$

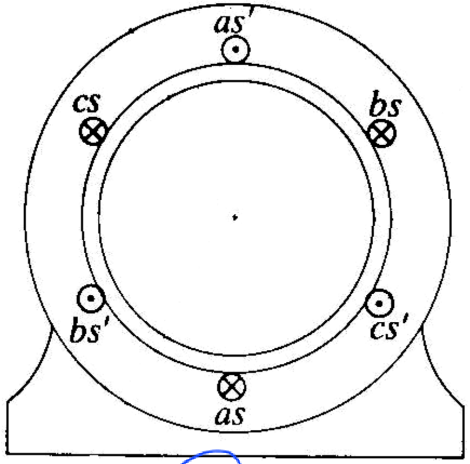
How many phases can you have ?

↳ min 2 phases

↳ multiples of 3



$$\mathbf{F}_s = \left(\frac{2}{2}\right) F_m \angle \theta_e$$



$$\mathbf{F}_s = \left(\frac{3}{2}\right) F_m \angle \theta_e$$

How do you change direction of rotation ?

↳ change sequence

↳ a → b → c → CCW

a → c → b → CW

More phases → Stronger MMF

↳ For a given MMF, more phases means smaller current needed

2-Pole Rotating Magnetic Field

N — S

For 2-pole Stator System

Let P – represent number of poles ($P = 2$)

$$\theta_e = \omega_e t$$ Electrical displacement

$$\theta_e = 0 \rightarrow 2\pi$$ One cycle of currents = one complete revolution of magnetic poles

Synchronous speed is the speed at which stator magnetic poles rotate

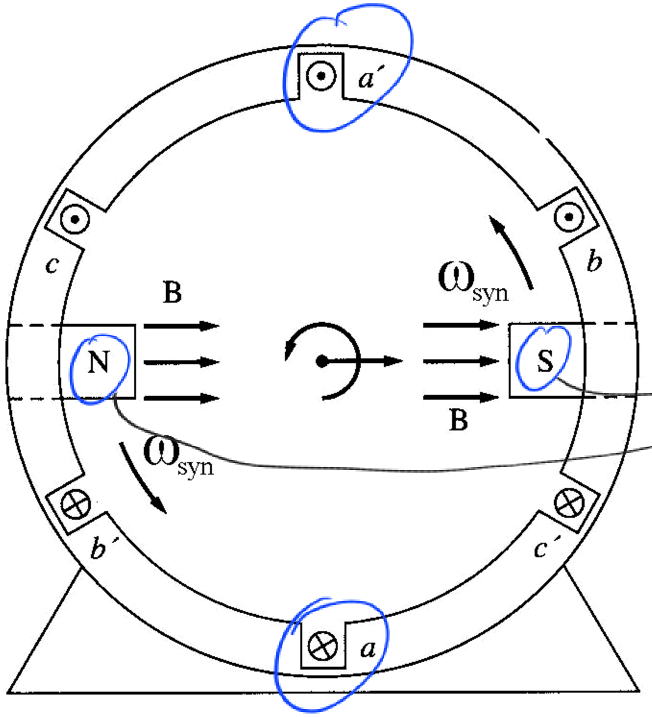
In this case, this is the same as electrical frequency / speed

magnetic pole rotation speed

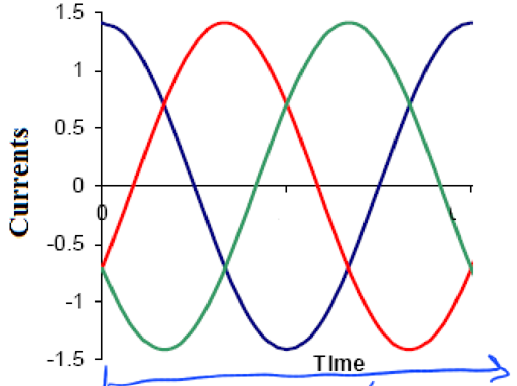
electrical freq.

$$\omega_{syn} = \omega_e$$

$$n_{syn} = 60 \cdot f_e \quad [\text{rpm}]$$

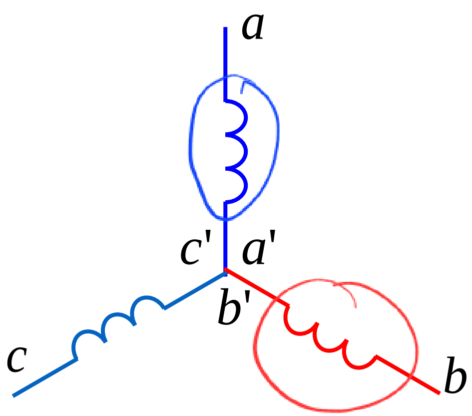


2 poles per phase



1 cycle

To return to the original magnetic pole position



4-Pole Rotating Magnetic Field

For 4-pole Stator System

Let P – represent number of poles (P = 4)

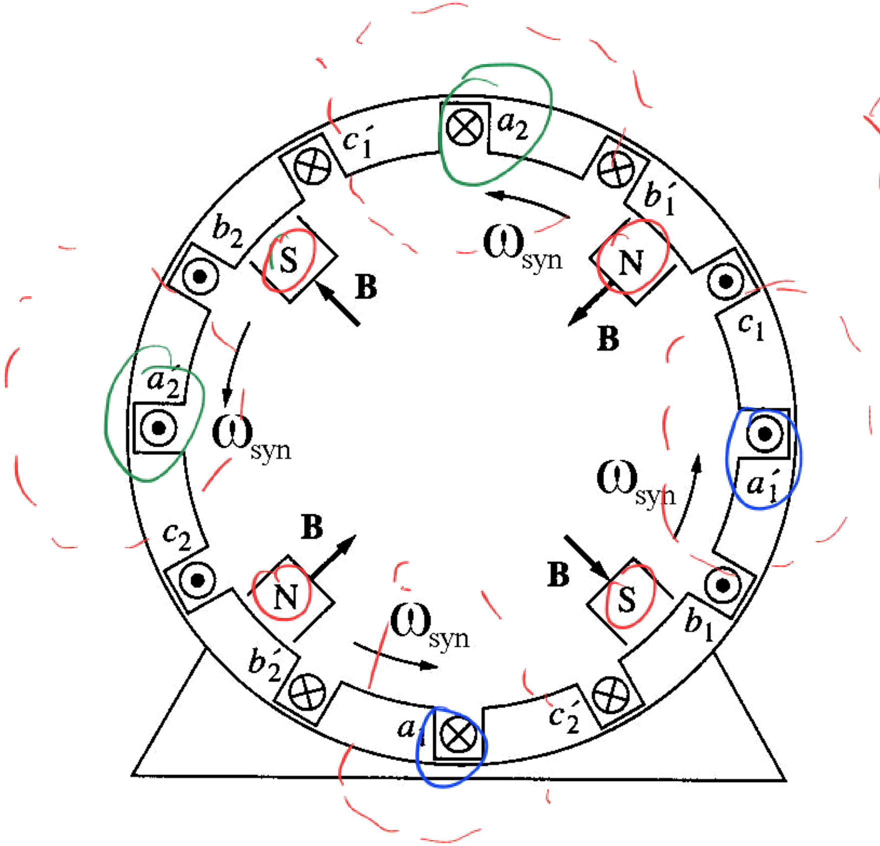
$$\theta_e = \omega_e t$$ Electrical displacement

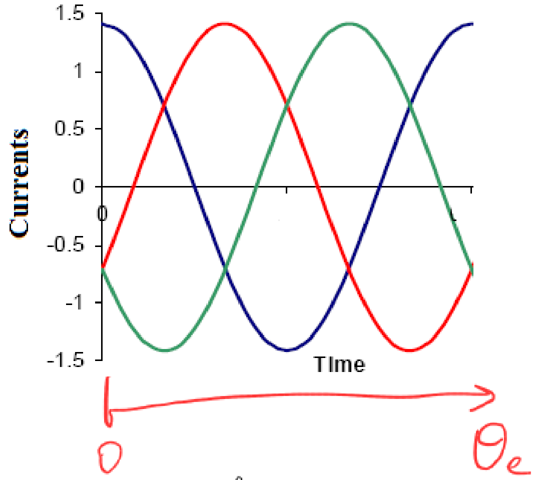
$$\theta_e = 0 \rightarrow 2\pi$$ One cycle of currents = half revolution of magnetic poles

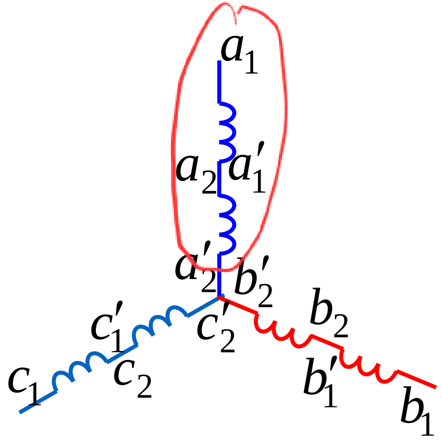
Synchronous speed is the speed at which stator magnetic poles rotate
In this case, this is half of the electrical frequency / speed

$$\omega_{syn} = \frac{1}{2}\omega_e$$

$$n_{syn} = 30 \cdot f_e \quad [\text{rpm}]$$







Returning to original position only needs 180° cycle

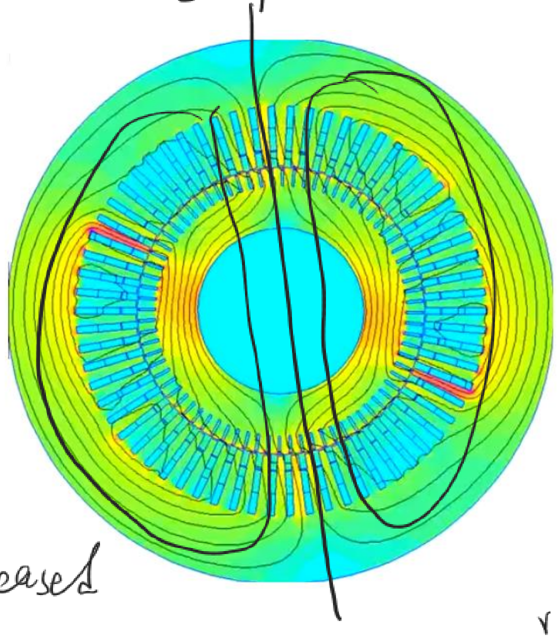
Number of Magnetic Poles

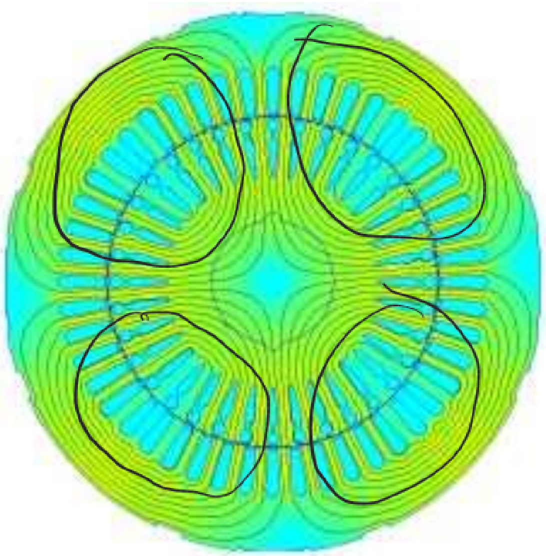
(1) You can have as many poles as you want

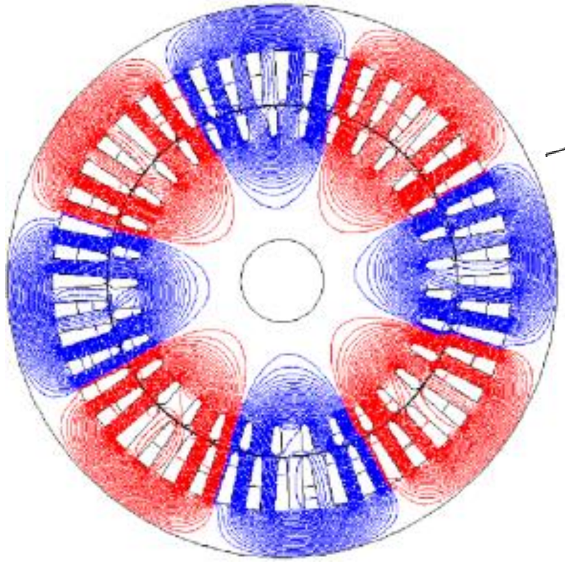
↓

must be even

(2) Increase number of poles means reduced speed but increased torque







rpm

Speed $n_{syn} \propto \frac{2}{P}$ Torque $T_e \propto \frac{P}{2}$

Mechanical and Electrical speed and position

$$\omega_{rm} = \frac{2}{P}\omega_r$$

$$\underbrace{\theta_{rm}}_{mech} = \frac{2}{P}\underbrace{\theta_r}_{elec}$$

Speed of a *P*-Pole Motor → IM: asynchronous machine → rotor and stator not same speed

***P* – denoted number of poles (*P* = 2, 4, 6, …)**

$$\theta_e = \omega_e t$$ Electrical displacement

Electrical (stator) speed for $f_e = 60$ Hz

$$\omega_e = 2\pi f_e \approx 377 \text{ [r/s]}$$

Synchronous speed is 2/*P* of the electrical speed

$$\omega_{syn} = \left(\frac{2}{P}\right)\omega_e \text{ [r/s]}$$

$$n_{syn} = \frac{2}{P} 60 f_e \quad \text{[rpm]}$$

$f_e = 60$ Hz

P	ω_{syn} rad/sec	n_{syn} rpm	Typical speed, n rpm
2	120π	3600	3520
4	60π	1800	1750
6	40π	1200	1120
8	30π	900	860

We can always "convert" a *P*-pole motor into an equivalent 2-pole motor using the *P*/2 factor ! → put into a common reference frame

Equivalent 2-pole motor seed $\quad \omega_r = \frac{P}{2}\omega_{rm}$ Actual mechanical speed

Actual mechanical speed is not constant and depends on the mechanical load!

Speed of a *P*-Pole Motor

P – denoted number of poles (P = 2, 4, 6, …)

$$\theta_e = \omega_e t$$ Electrical displacement

Electrical (stator) speed for $f_e = 50$ Hz

$$\omega_e = 2\pi f_e \approx 314 \left[\text{r/s}\right]$$

Synchronous speed is 2/*P* of the electrical speed

$$\omega_{syn} = \frac{2}{P}\omega_e \left[\text{r/s}\right]$$

$$n_{syn} = \frac{2}{P}\cdot 60 f_e \quad [\text{rpm}]$$

P	ω_{syn} rad/sec	n_{syn} rpm	Typical speed, n rpm
2	100π	3000	2920
4	50π	1500	1420
6	33.33π	1000	980
8	25π	750	710

We can always "convert" a *P*-pole motor into an equivalent 2-pole motor using the *P*/2 factor !

Equivalent 2-pole motor seed $$\omega_r = \frac{P}{2}\omega_{rm}$$ Actual mechanical speed

Actual mechanical speed is not constant and depends on the mechanical load!

Slip & Asynchronous Speed

- Induction Machine = Asynchronous Machine

→ Speed of the rotor is different from the speed of the stator magnetic field / poles

→ **Slip** – difference between the speed of synchronous airgap field and the rotor speed

→ Stator

Fractional Slip: $$s = \frac{n_{syn} - n}{n_{syn}} = \frac{\omega_{syn} - \omega_{rm}}{\omega_{syn}} = \frac{\omega_e - \omega_r}{\omega_e}$$

→ rotor mech speed

→ rotor elec speed (referred to the equivalent 2-pole machine)

→ Stator mech/sync. speed

$\omega_r = \frac{P}{2}\omega_{rm}$

Percent Slip: $$s = \frac{n_{syn} - n}{n_{syn}} 100\%$$

~ 1.5 – 8%

Rotor speed relative to the stator magnetic field (poles) →

Slip frequency: $$\omega_s = \omega_e - \omega_r = s \cdot \omega_e$$

Rotor mechanical speed: $$\omega_{rm} = (1 - s)\,\omega_{syn} \qquad n = (1 - s)\,n_{syn}$$

Principle of Operation (Idle mode)

no mech. load

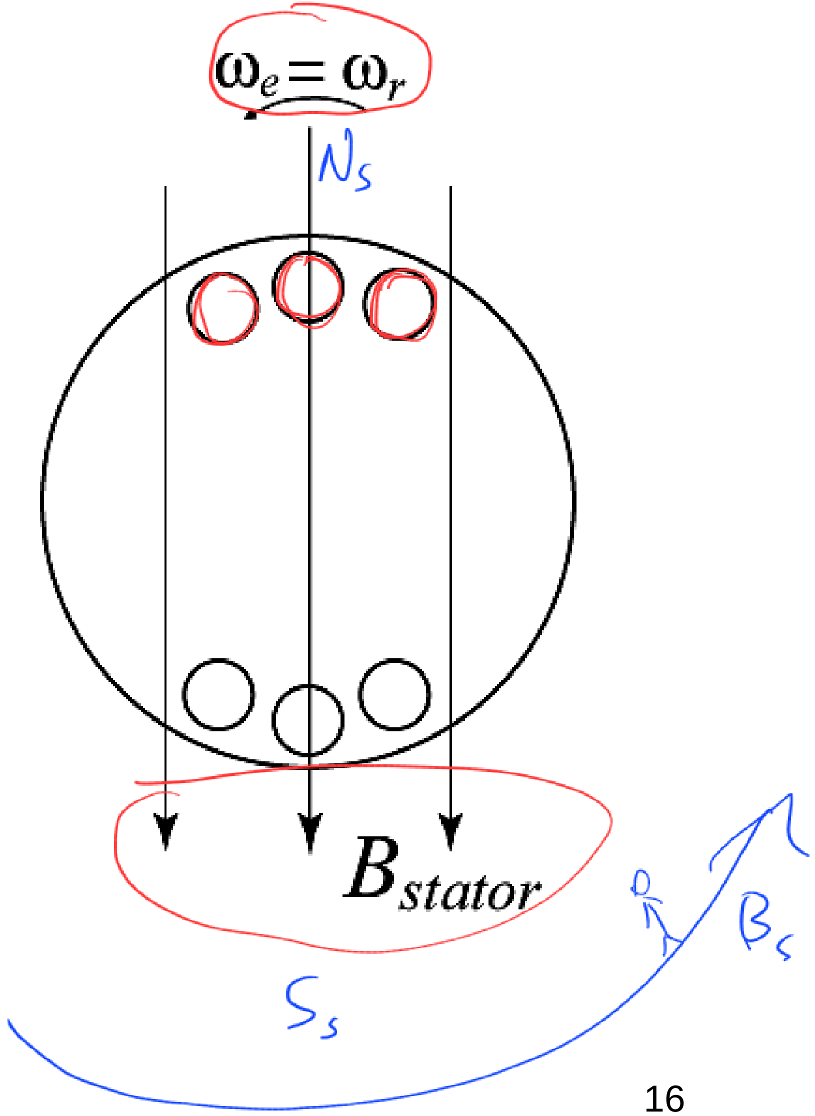
a) Assume synchronous speed $\omega_r = \omega_e$

$$s = \frac{\omega_e - \omega_r}{\omega_e} = 0$$

$w_s = \emptyset$

- Rotor bars do not cut magnetic flux - $\frac{d\lambda}{dt} = 0$
- No current induced
- No torque produced

Rotor speed same as stator
↳ So from B_s ref. frame
↳ Rotor is not moving
↳ No flux crosses rotor bars



Principle of Operation (motor)

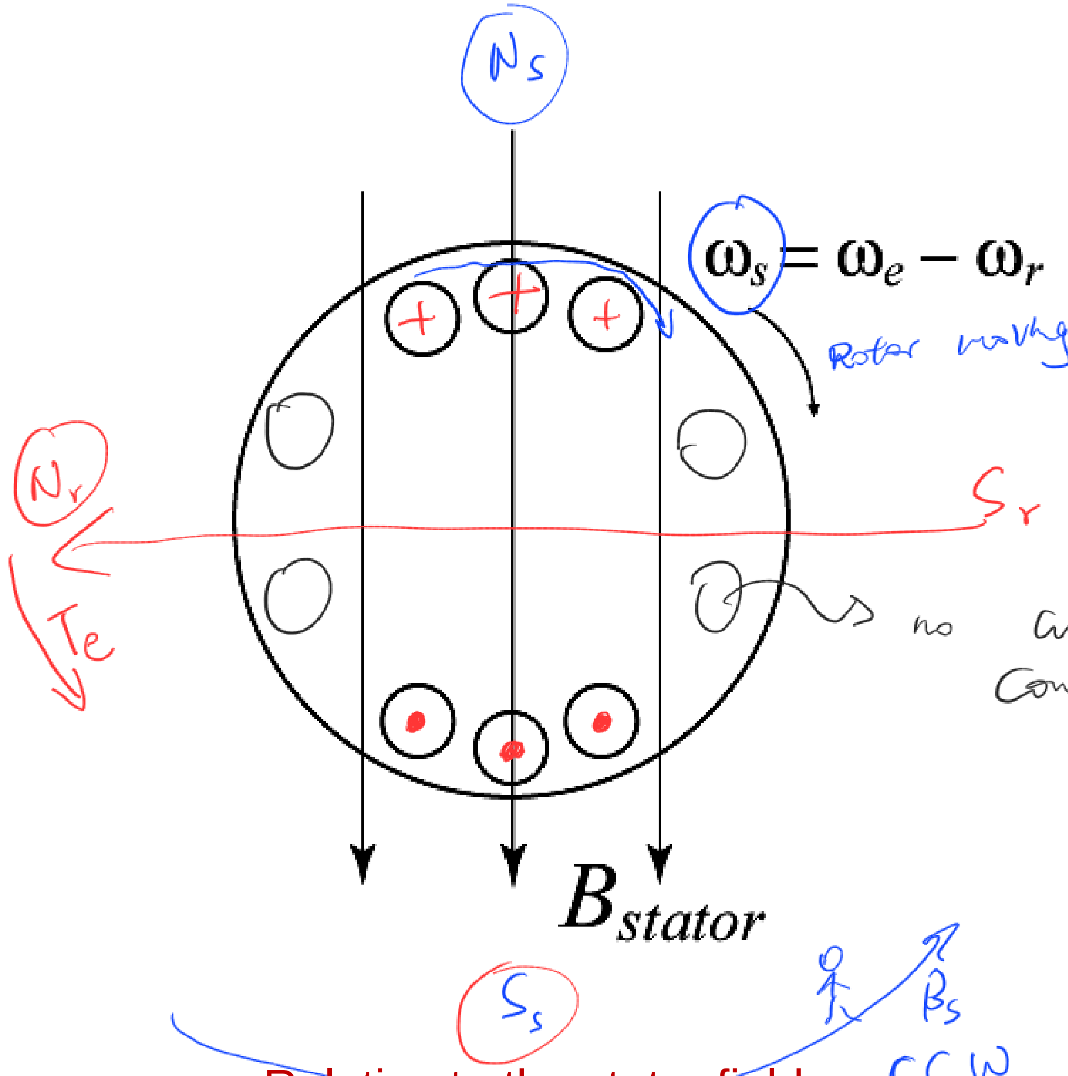
→ Apply T_m, rotor slows down

$F = v \times B$
Right hand rule

North: exit
South: enter

b) Assume $T_m > 0$ and $\omega_r < \omega_e$

$$s = \frac{\omega_e - \omega_r}{\omega_e} > 0$$

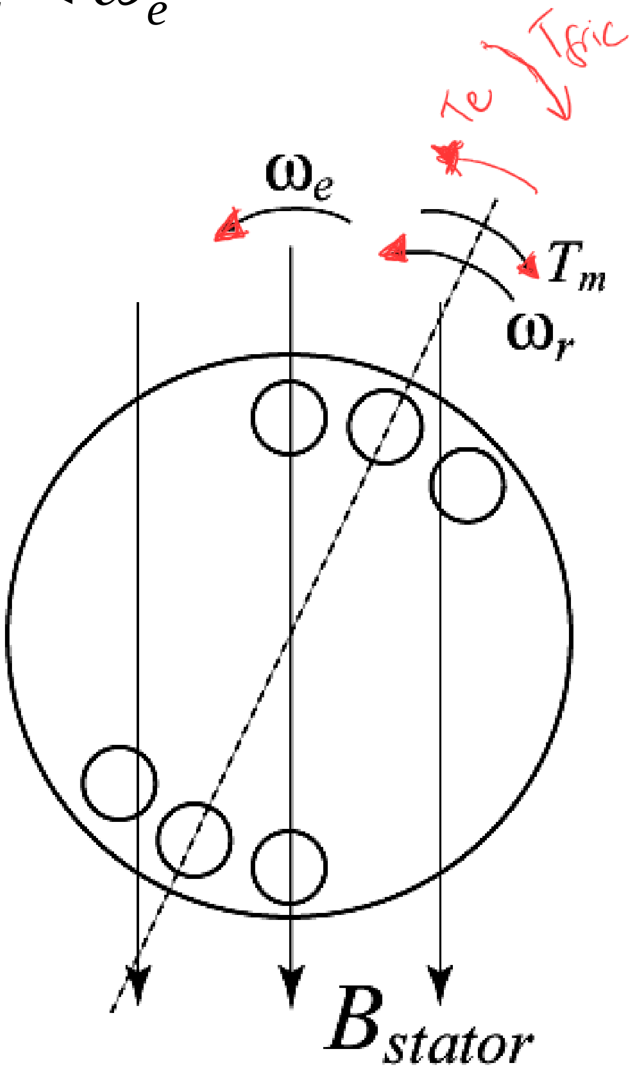


Relative to the stator field

Principle of Operation (motor)

b) Assume $T_m > 0$ and $\omega_r < \omega_e$

$$s = \frac{\omega_e - \omega_r}{\omega_e} > 0$$



4 Speed:

① Stator field (airgap)
↳ ω_e

② Rotor mechanical speed ⇝ ω_r

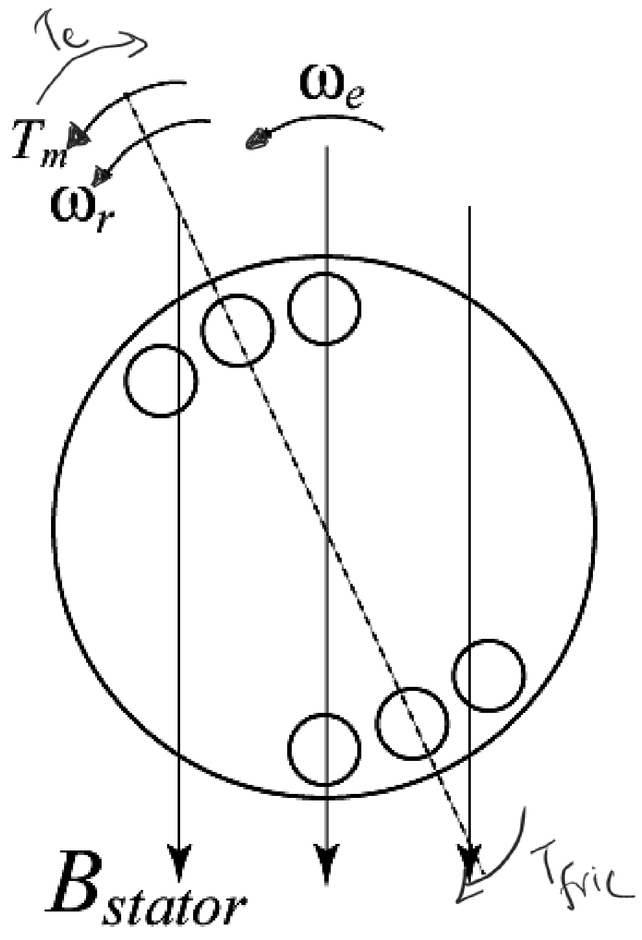
③ Frequency of rotor current ⇝ ω_s
↓
what you see on scope

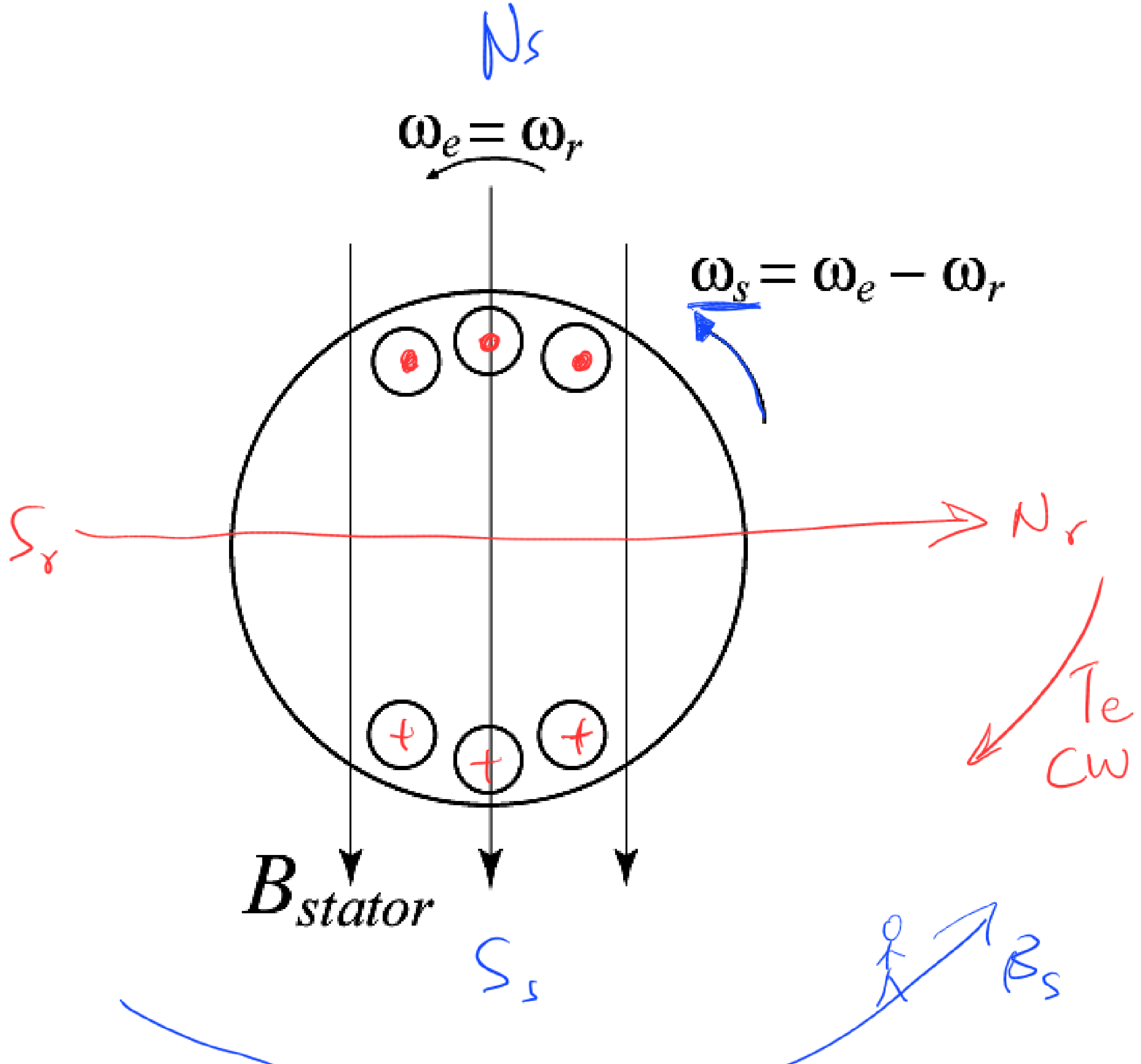
④ Speed of rotor field
↳ $\omega_r + \omega_s = \omega_e$

Principle of Operation (generator)

c) Assume $T_m < 0$ and $\omega_r > \omega_e$

$$s = \frac{\omega_e - \omega_r}{\omega_e} < 0$$





Relative to the stator field

Frequency of Rotor Currents

Slip frequency $\omega_s = \omega_e - \omega_r = s \cdot \omega_e$ Rotor electrical speed relative to the stator magnetic field

$$n_{syn} = \frac{120}{P} f_e$$

$$f_r = \frac{\omega_s}{2\pi} = \frac{s\omega_e}{2\pi} = s \cdot f_e$$

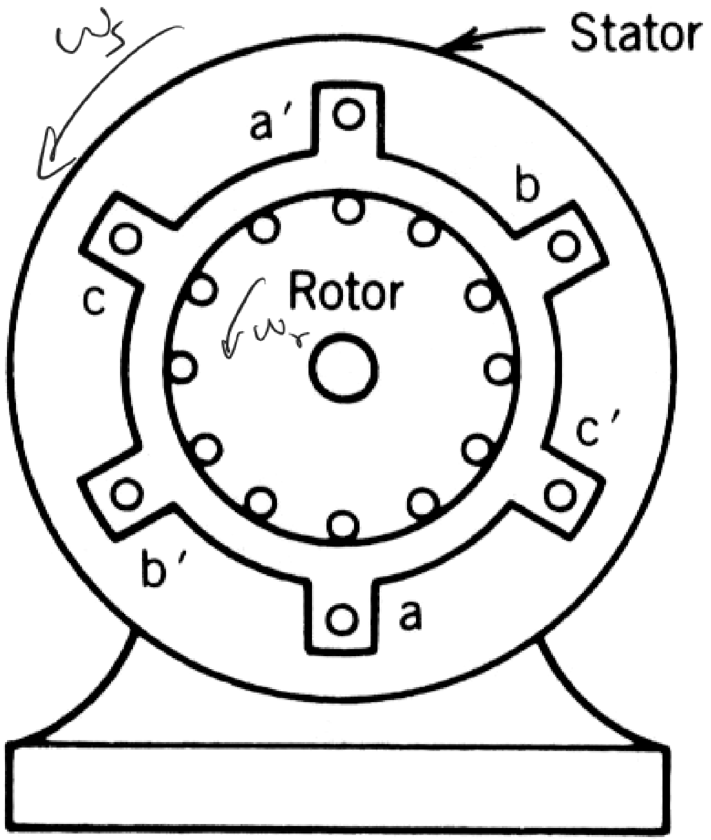
slip

(frequency of rotor current)

$f_e = 60\,Hz$

$s = 5\%$

$f_r = 5\% \times 60 = 3\ Hz$



- ✓ Rotor winding (circuit) operates at different frequency
- ✓ This frequency is proportional to slip *s*

Speeds in Induction Machines

❖**Example 1:** A 3-phase, 60 Hz, four-pole induction motor operates at slip of 5%. Determine:

$P=4$

sec/min

(a) Synchronous speed $= 60\,\text{Hz} \times 60 \times \frac{2}{P} = 1800$ rpm

(b) Shaft speed $n_{rm} = 1800\,(1-0.05) = 1710$ rpm

(c) Speed of the rotating air-gap field $= 1800$ rpm

(d) Frequency of the rotor currents $f_r = S \cdot f_s = 0.05 \times 60 = 3$ Hz

(e) Slip in rpm $n_{slip} = 1800 \cdot S = 90$ rpm

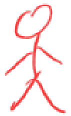
(f) Speed of the rotor field relative to:

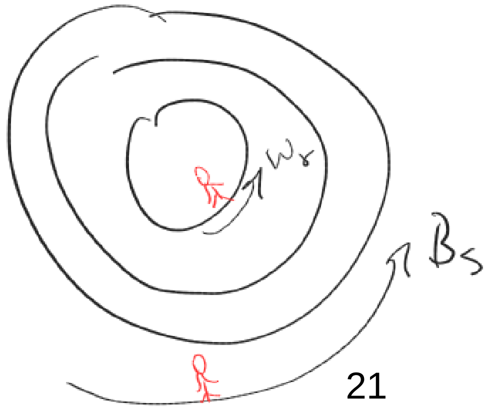
Stand on rotor → (i) rotor body 90 rpm (CCW) → forward

Stand outside → (ii) stator body 1800 rpm

→ (iii) stator rotating field 0 rpm

Stand on stator field





Looking ahead…

- Continue **Module 5**