ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 16

March 10, 2026

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Course Logistics

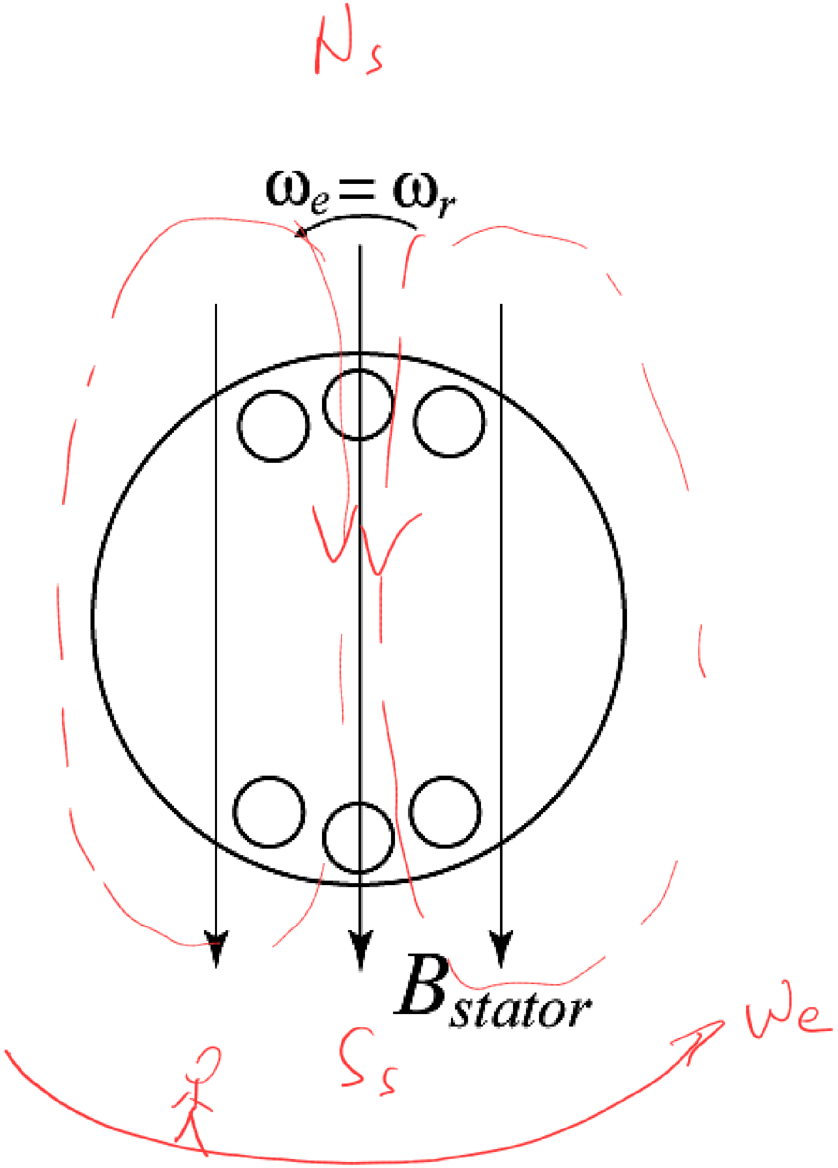
- Tutorial resumes this week
- Lab 3 takes place this week
- Assignment 5 posted
- Midterm available to pick up
- Quizzes
  - Quiz 3: March 20 at the beginning of the tutorial
  - Quiz 4: April 2 at the beginning of the lecture

Principle of Operation (Idle mode)

a) Assume synchronous speed $\omega_r = \omega_e$

$$s = \frac{\omega_e - \omega_r}{\omega_e} = 0 \Rightarrow \omega_s = \phi$$

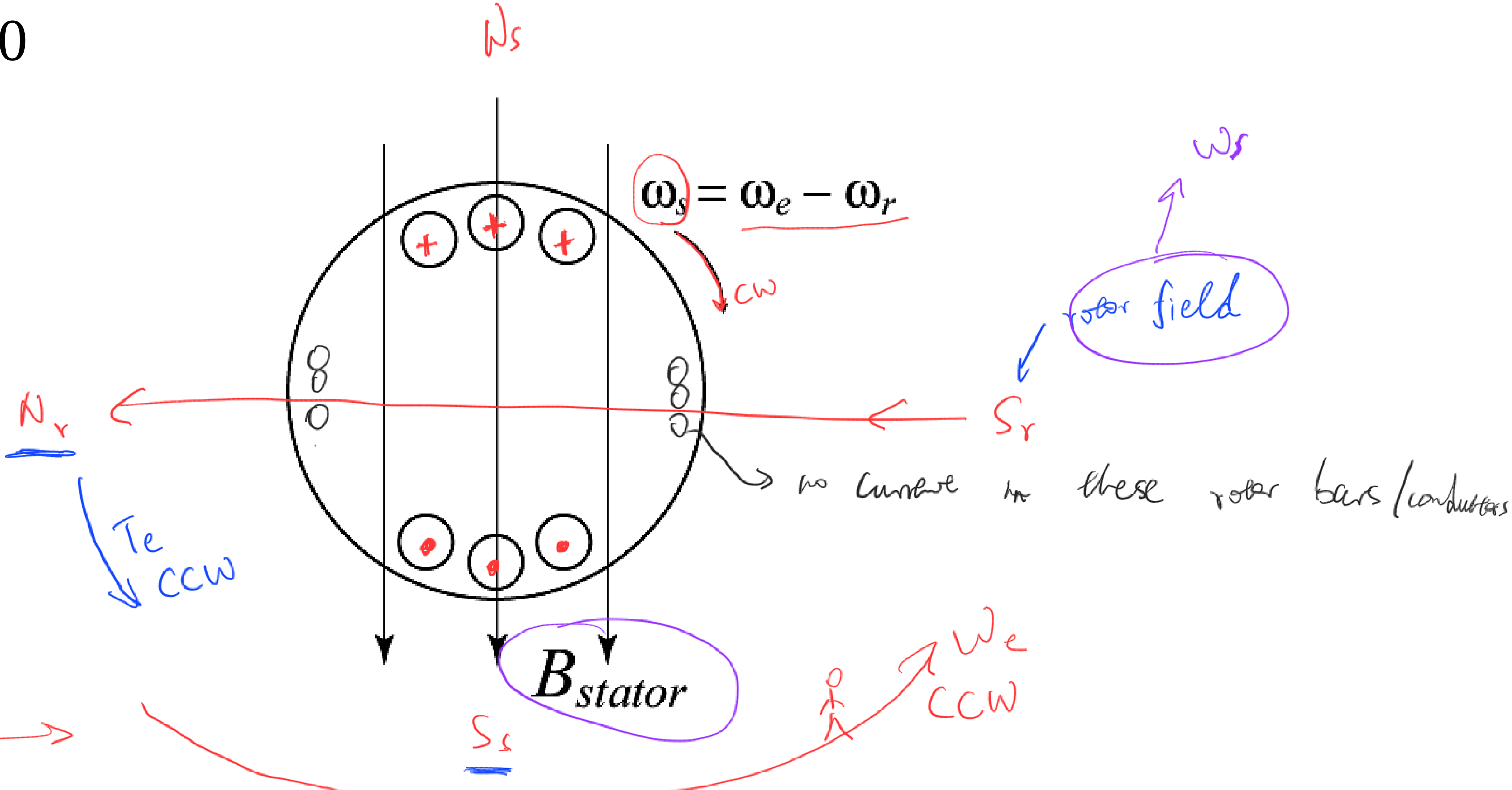
- Rotor bars do not cut magnetic flux - $\frac{d\lambda}{dt} = 0$
- No current induced
- No torque produced



Principle of Operation (motor)

b) Assume $T_m > 0$ and $\omega_r < \omega_e$

$$s = \frac{\omega_e - \omega_r}{\omega_e} > 0$$



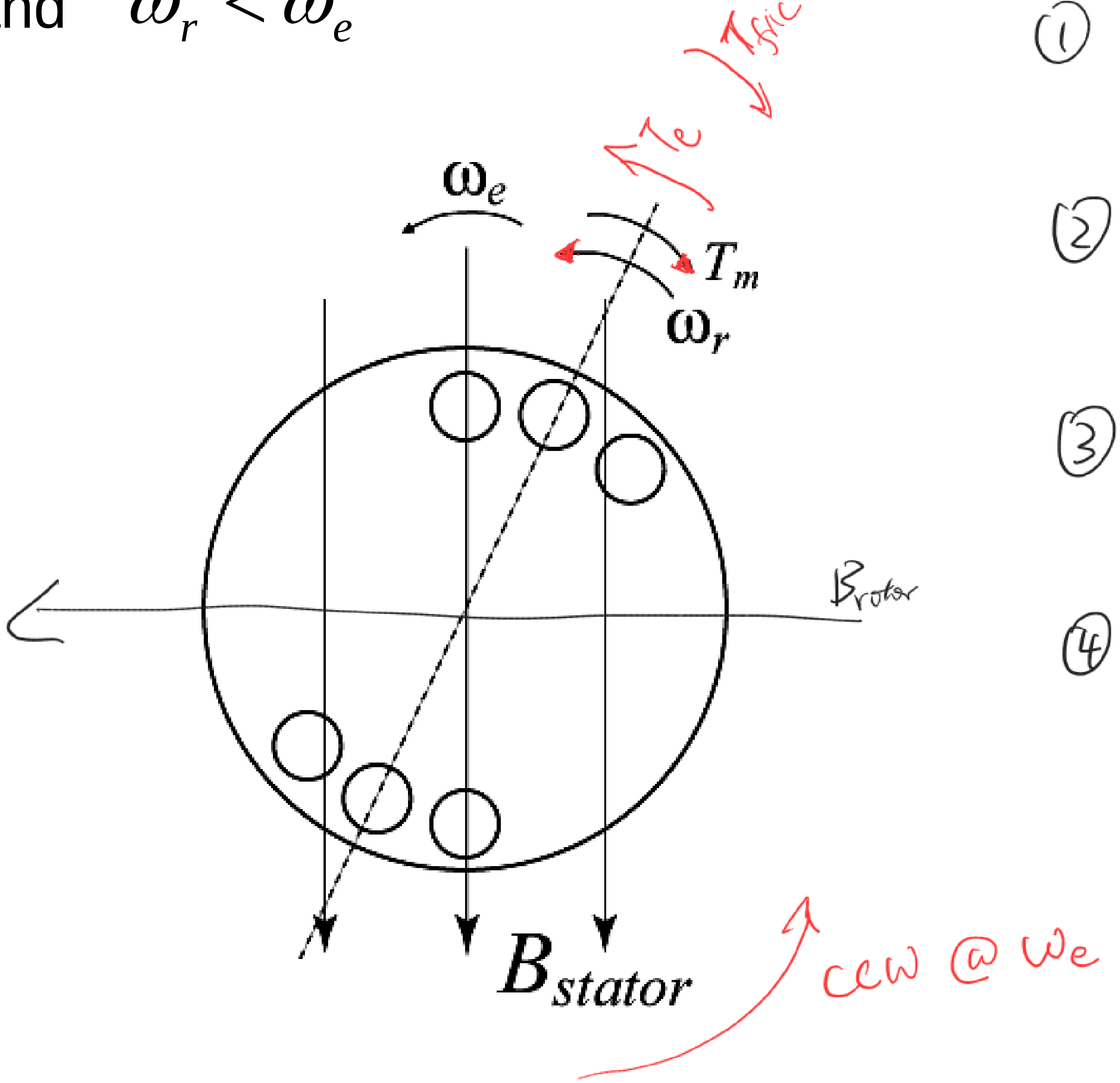
Rotating reference frame @ ω_e

Relative to the stator field

Principle of Operation (motor)

b) Assume $T_m > 0$ and $\omega_r < \omega_e$

$$s = \frac{\omega_e - \omega_r}{\omega_e} > 0$$



4 speeds:

① Stator speed (air gap)

② Rotor/shaft speed ω_r

③ Rotor current frequency ω_s

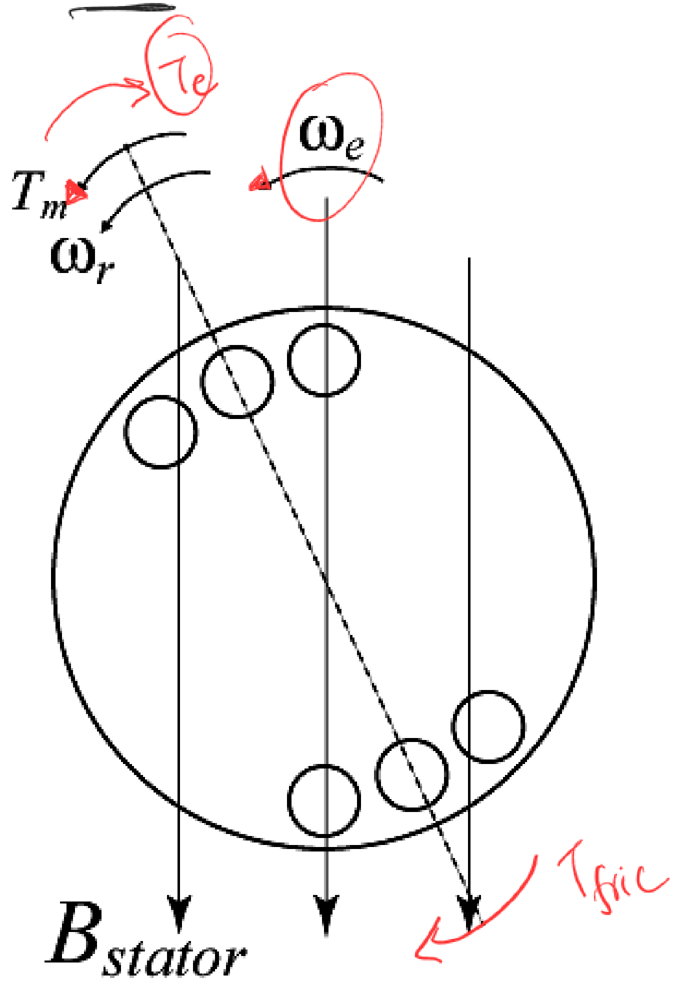
④ Rotor field speed
@ Rotation frame: ω_s
@ Stationary frame: $\omega_s + \omega_r = \omega_e$

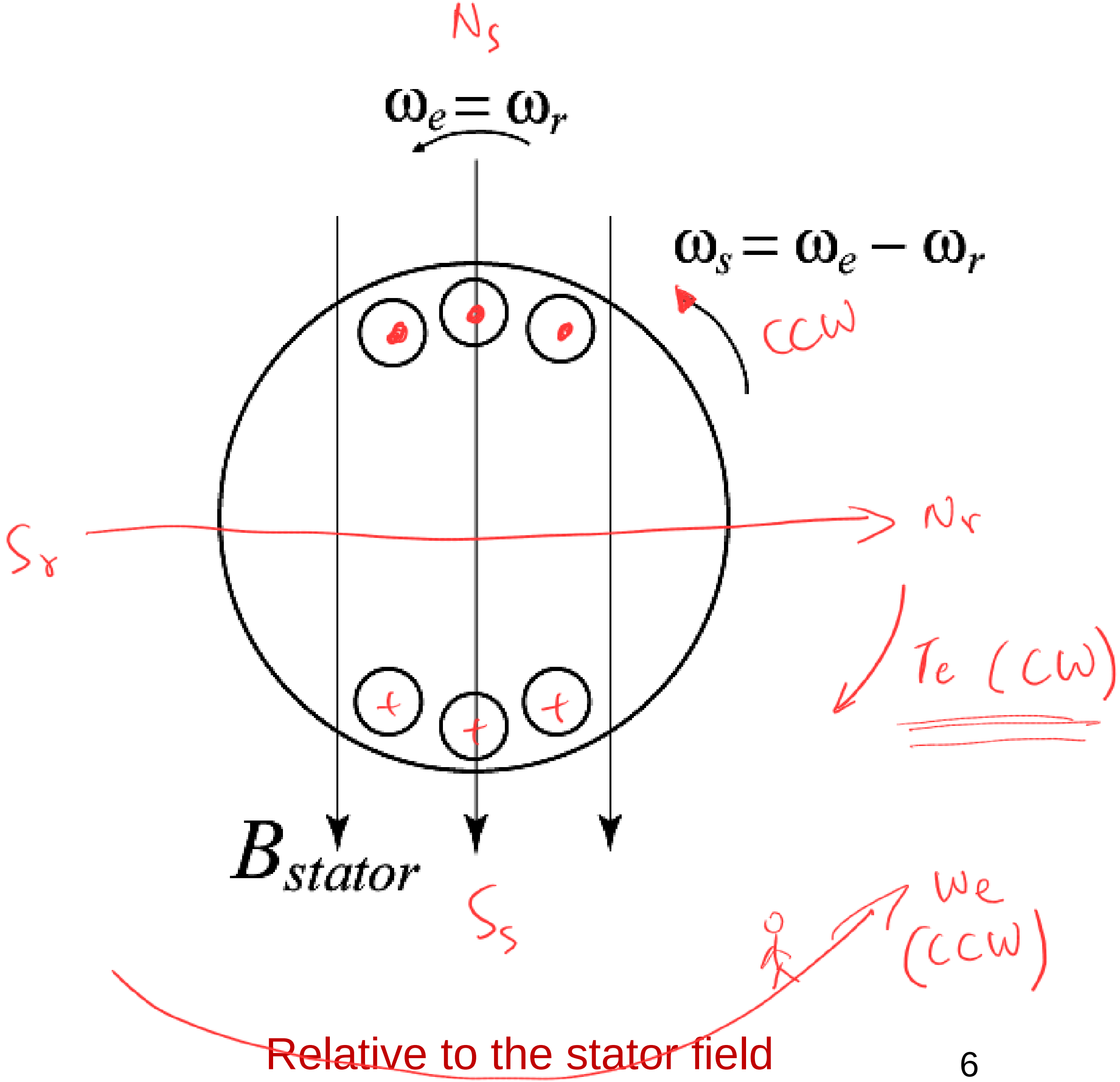
$\boxed{B_r \text{ and } B_s \text{ are } 90^\circ \text{ apart}}$

Principle of Operation (generator)

c) Assume $T_m < 0$ and $\omega_r > \omega_e$

$$s = \frac{\omega_e - \omega_r}{\omega_e} < 0$$





Speeds in Induction Machines

❖**Example 1:** A 3-phase, 60 Hz, four-pole induction generator operates at slip of 5%. Determine:

Slip > 0 (motor)
slip < 0 (gen)

(a) Synchronous speed $n_{sync.} = 60 \times 60 \times \frac{2}{p} = 1800 rpm \; (CCW)$

(b) Shaft speed $n_r = 1800(1+0.05) = 1890 \; rpm \; (CCW)$

(c) Speed of the rotating air-gap field 1800 rpm

(d) Frequency of the rotor currents $f_r = s \cdot f_s = 5\% \times 60 = 3 \; Hz$

(e) Slip in rpm $n_{slip} = 90 \; rpm \quad (1800 rpm \times 5\%)$

(f) Speed of the rotor field relative to:

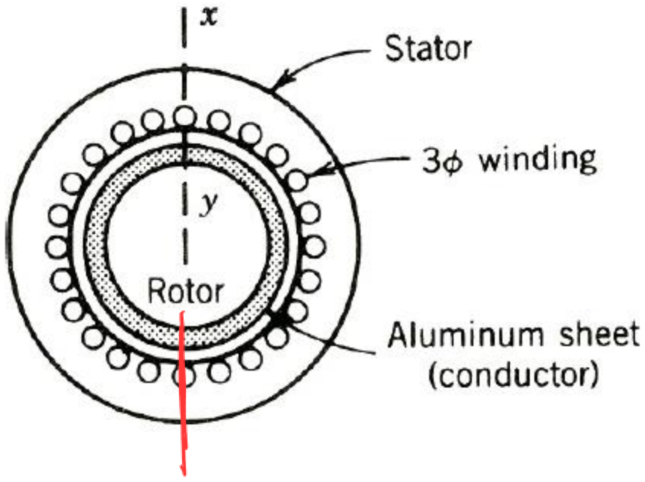
Stand on rotor (i) rotor body (you see rotor current freq: $\omega_s \Rightarrow B_r \Rightarrow \omega_s$) $\Rightarrow$ 90 rpm (CW)

Stand outside (ii) stator body 1800 rpm $\Rightarrow \omega_r + \omega_s = 1890 - 90 = 1800$ rpm

Stand on B_s (iii) stator rotating field $\emptyset$ rpm

Linear Induction Motors (LIM)

✓ Running Magnetic Field

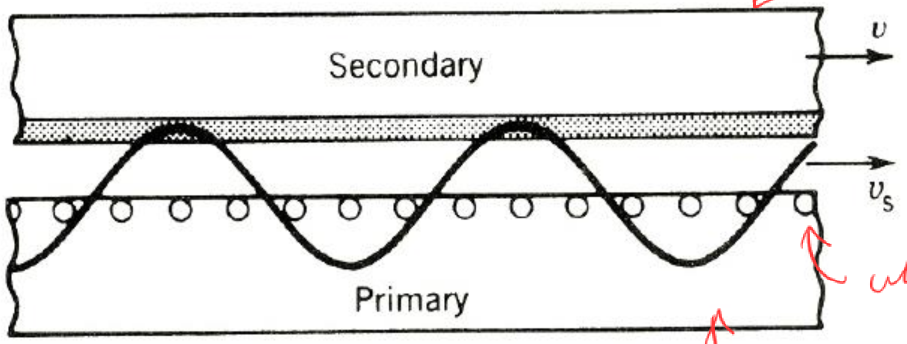


Cut open

unroll the rotary machine

force

rotor

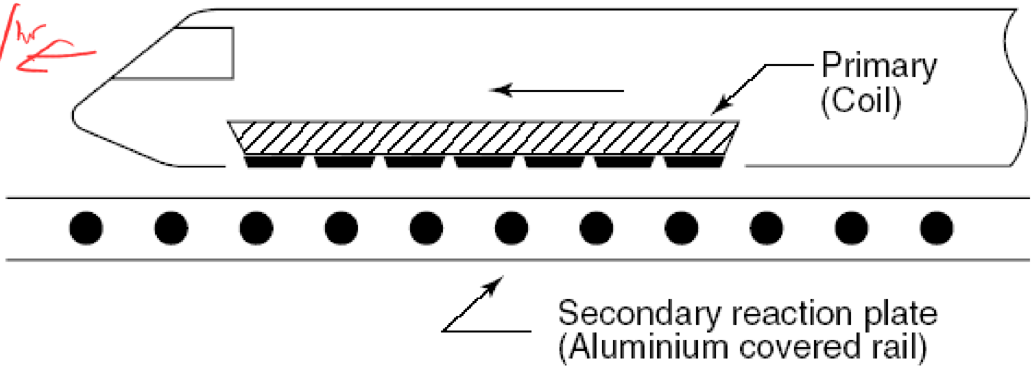


windings

Stator

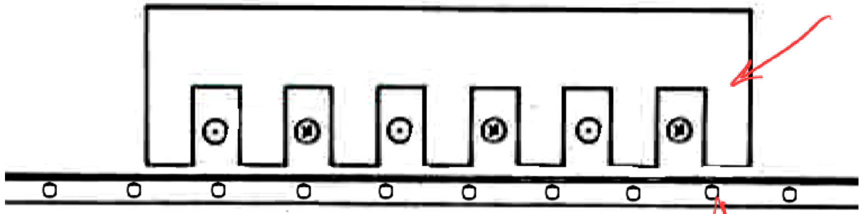
✓ Train Propulsion System

40 km/hr



Movable platform – what used to be “Stator”

windings



Stationary rail – what used to be “Rotor”

short circuited conductors

Linear Induction Motors (LIM)

Millennium Line

reaction rails (short circuited conductors)

- Propulsion System
- Motors react with aluminum-capped steel rail on the guideway
- LIM provide both propulsion and regenerative braking
- No moving parts
- Reduced maintenance and risk of mechanical failure

They can change # of poles and We on the fly (power electronics → audible noise)

Basic Analysis: Example 2

Consider a 25hp, 3-phase, 220V, 60Hz Ind. Motor. At full load we have:

$$P_{in} = 20.8\text{kW}; \quad P_{out} = 25\text{hp}; \quad I_1 = 64\text{A}; \quad n = 830\text{rpm};$$

Find: number of poles p, slip s, frequency f_2, power factor pf, torque T_m, and efficiency η

rotor current

Number of poles P

$n_{syn} = \frac{120}{p} \cdot f_e$; $p = \frac{120 f_e}{n_{syn}} = \left\lfloor \frac{120 f_e}{n} \right\rfloor$ floor operator

$p = \left\lfloor \frac{120 \cdot 60}{830} \right\rfloor = \lfloor 8.67 \rfloor = 8$; $n_{syn} = 900\text{rpm}$

P	n_{sync}
2	3600
4	1800
6	1200
8	900
10	...

Slip $S = \frac{n_{syn} - n}{n_{syn}} = \frac{900 - 830}{900} = 0.0778 = 7.78\%$

Frequency of rotor currents f_2

$f_2 = S \cdot f_e = 0.0778 \cdot 60 = 4.67\text{Hz}$

Power factor $pf = \cos\varphi = \frac{P_{in}}{\sqrt{3} \cdot V_L \cdot I_L} = \frac{20.8\text{kW}}{\sqrt{3} \cdot 220 \cdot 64} = 0.853$ → S

Torque $T_m = \frac{P_{out}}{\omega_{rm}} = \frac{30}{\pi} \cdot \frac{25 \cdot 746}{830} = 214\ \text{N}\cdot\text{m}$

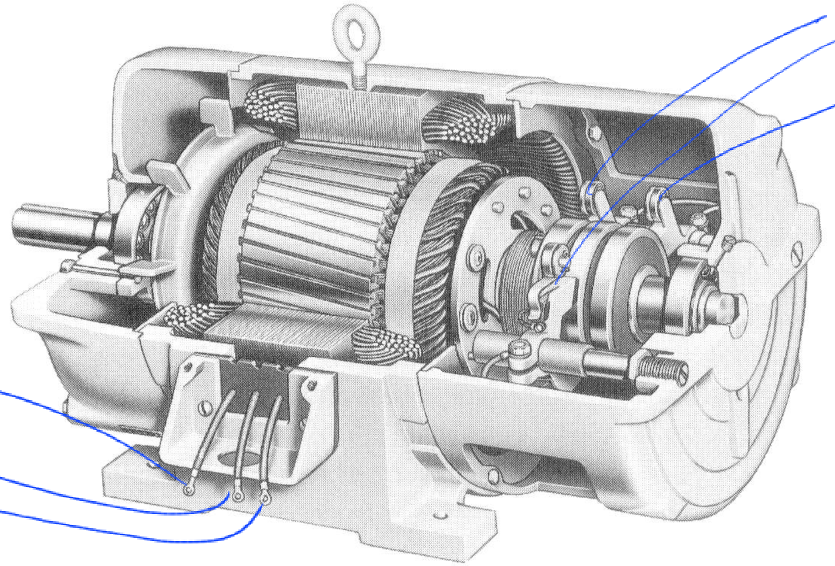
Efficiency $\eta = \frac{P_{out}}{P_{in}} = \frac{25 \cdot 746}{20.8\text{kW}} = 0.8966 = 89.66\%$

Steady-State Equivalent Circuit

- Assume rotor has the same number of phases as stator (3-ph)

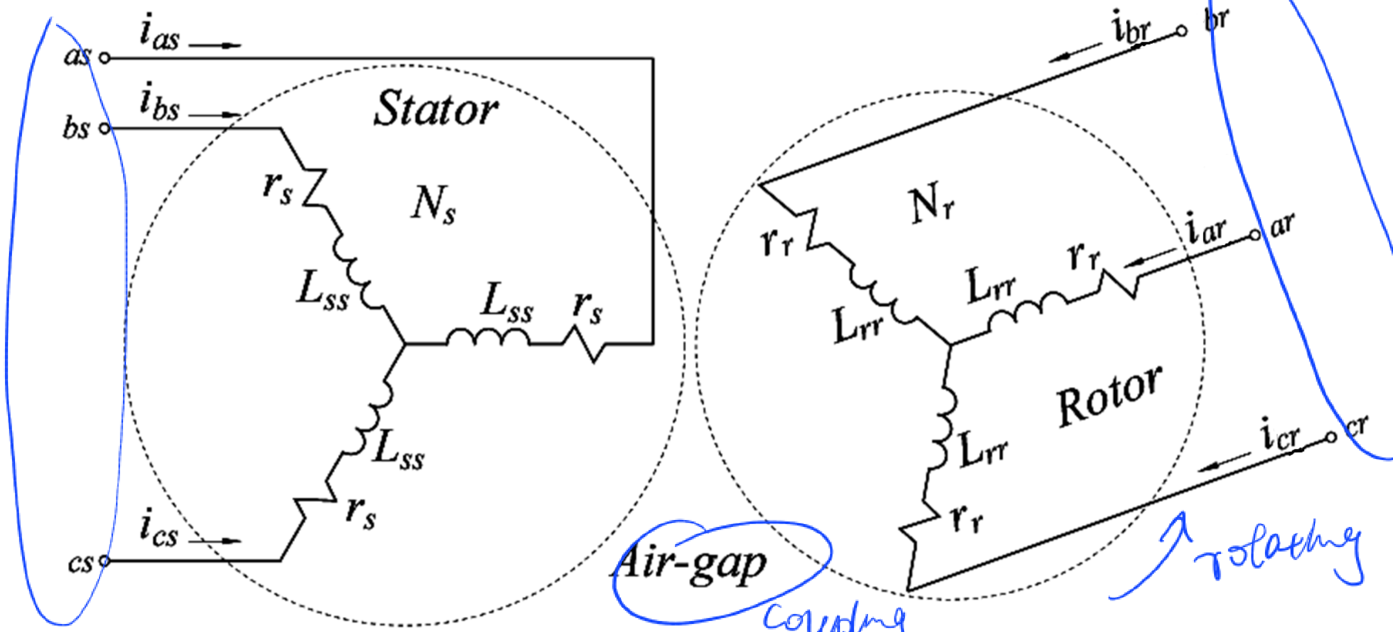
IM is NOT a three-phase transformer

rotor
a_r b_r c_r



Stator
a_s
b_s
c_s

3-phase transformer

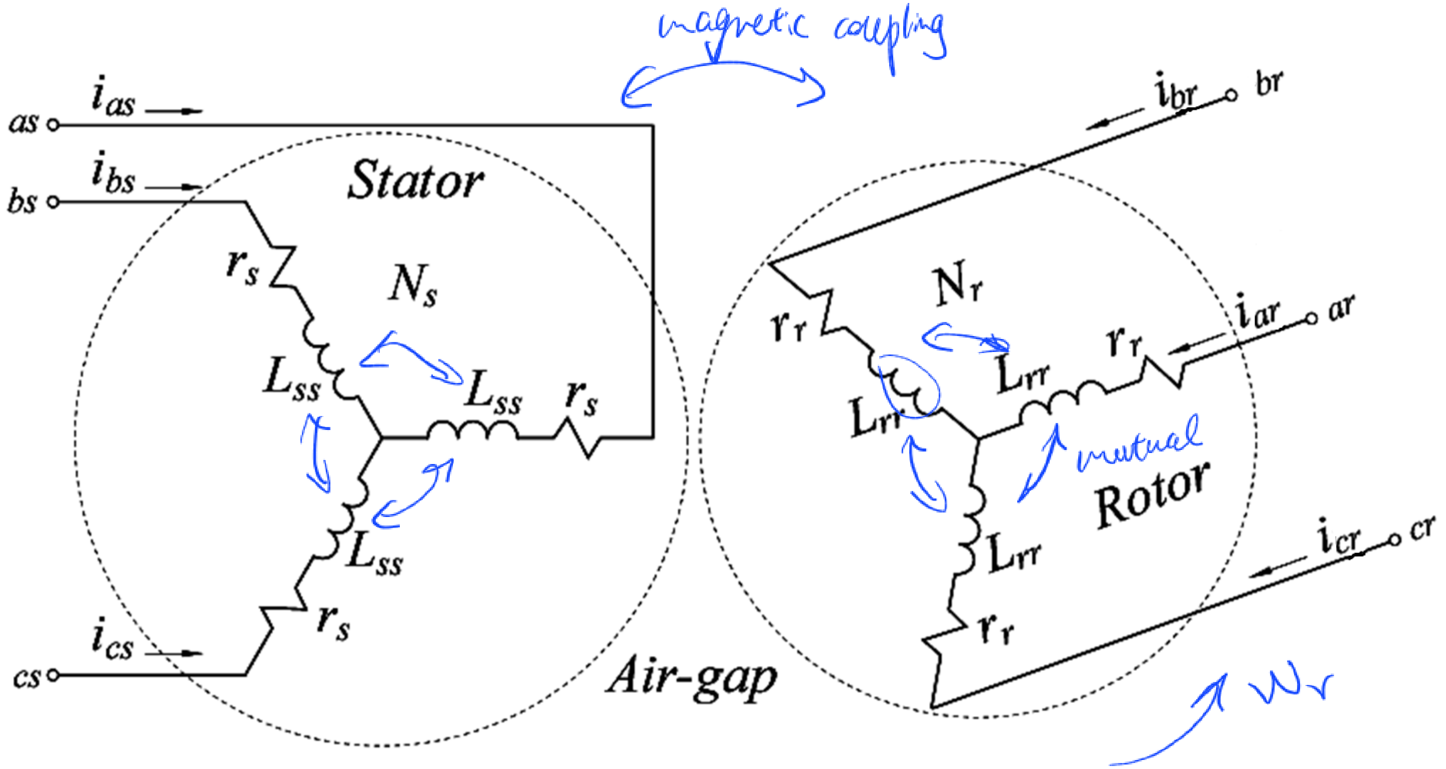


coupling

rotating

Steady-State Equivalent Circuit

- Assume rotor has the same number of phases as stator (3-ph)

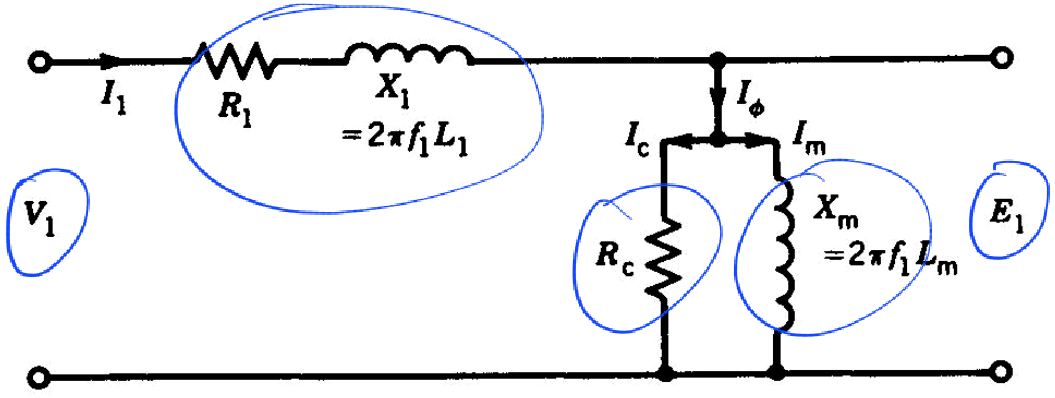


- Rotor winding (circuit) operates at different frequency $f_r = s \cdot f_e$
- Rotor circuit is typically shorted (squirrel-cage)
- Motor is like a 3-phase transformer (But it is NOT a 3-φ TF)
- Develop equivalent circuit per-phase
- Extend the results to 3-phase

Stator Equivalent Circuit (Primary)

In order to simplify notation we can assume

- Rotor has the same number of phases as stator (3-ph)
- Rotor winding has the same number of turns as stator



f_s

Secondary side:

Rotor circuit

ω_r

$V_1 =$ per-phase terminal voltage

$R_1 =$ per-phase stator winding resistance

$X_1 =$ per-phase stator winding leakage reactance

$X_m =$ per-phase magnetizing reactance

$R_c =$ per-phase core losses equivalent resistance

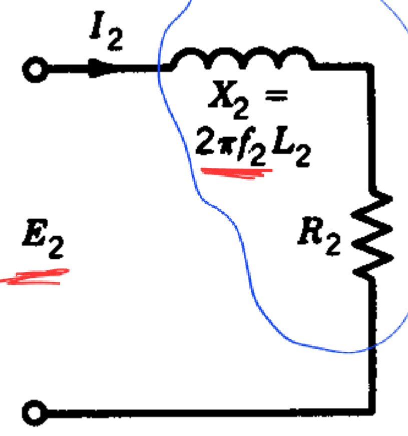
Goal: develop a T-circuit

Rotor Equivalent Circuit (secondary)

→ $S = 1$ (rotor freq and stator freq are same)

At stall, $\omega_r = 0$

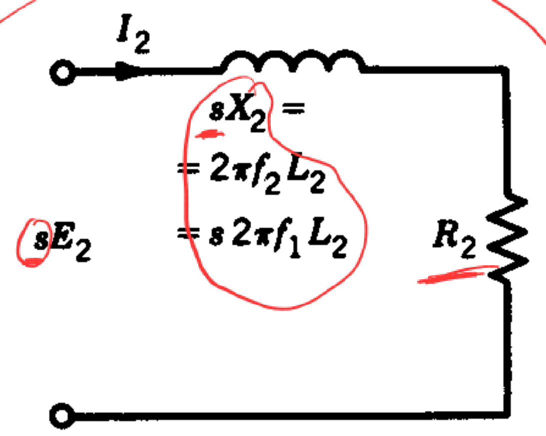
Ready to connect to stator



→ f_s

→ Rotor freq $= s \cdot f_s$

When rotor moves, ω_r



Slip-dependent circuit

Rotor voltage $\propto \frac{d\lambda}{dt}$

↓

scaled by frequency

→ $s \cdot f_s$

↓

to convert to stator circuit freq (f_s), divide everything by s

$E_2 =$ per-phase induced voltage in the rotor at $\omega_r = 0$

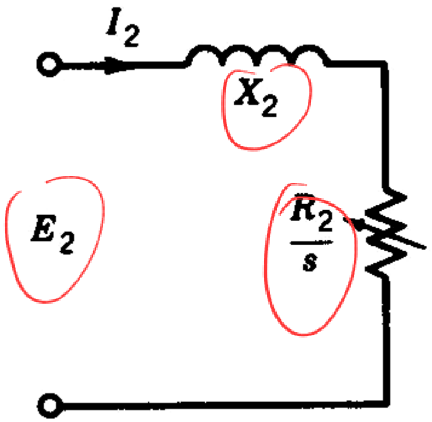
$R_2 =$ per-phase rotor winding resistance

$L_2 =$ per-phase rotor winding leakage inductance

$X_2 =$ per-phase rotor winding leakage reactance

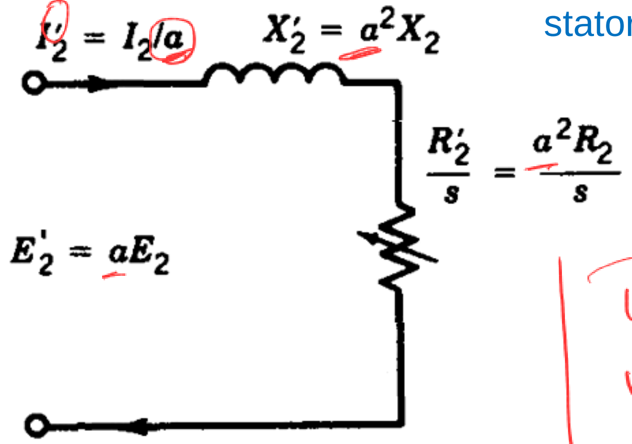
Overall Equivalent Circuit

Assume that the rotor rotates with speed ω_r



Normalized by the slip *s*

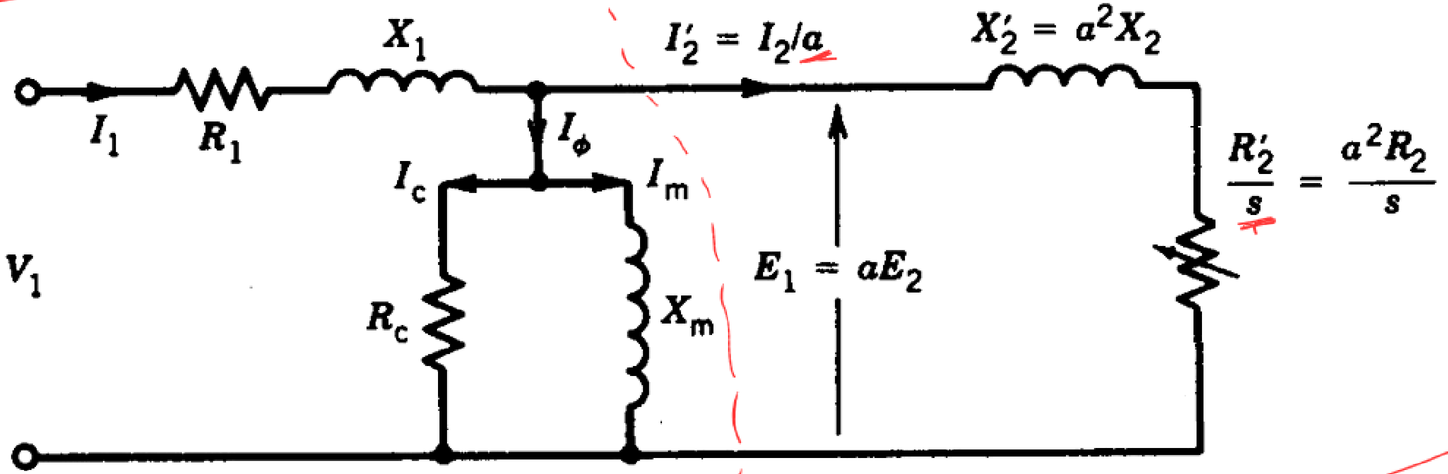
Prime ⇒ turns ratio conversion



Re-scale the rotor circuit to the stator side using the equivalent stator-to-rotor turns-ratio $a = N_s/N_r$

$$\boxed{\begin{aligned} \omega_r = 0 &\Rightarrow s = 1 \\ \omega_r = \omega_e &\Rightarrow s = \phi \end{aligned}}$$

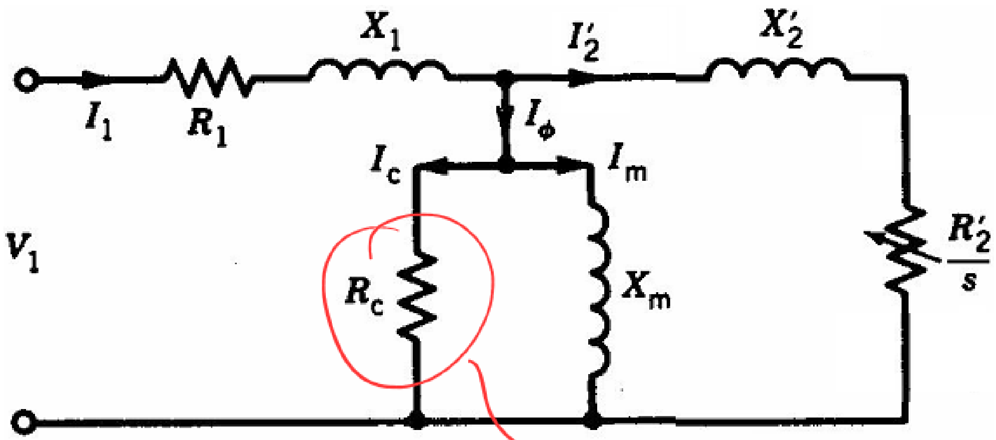
Stator ← → rotor



T-equivalent of IM per phase

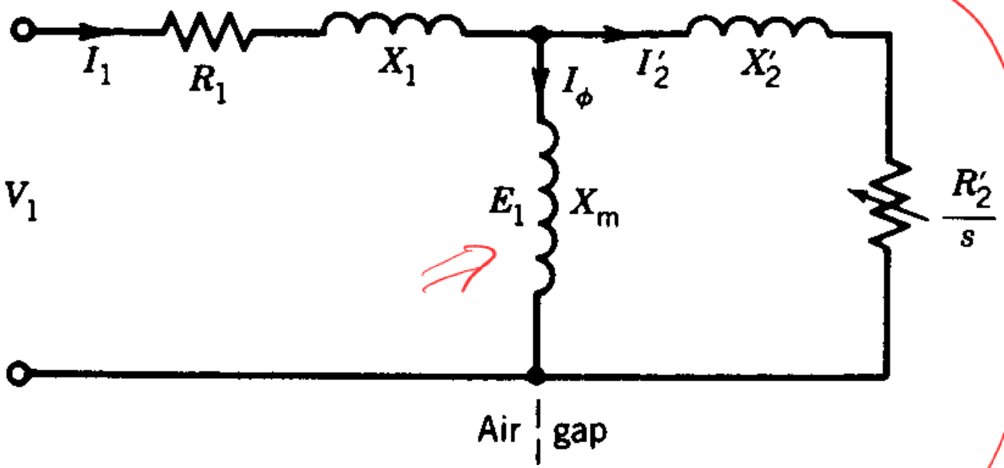
Combine with the stator part of the equivalent circuit

Practical Equivalent Circuits



Real power consumed by the rotor (part of which is converted to creating torque, and part is dissipated as rotor winding loss)

Lumped with T_{fric}



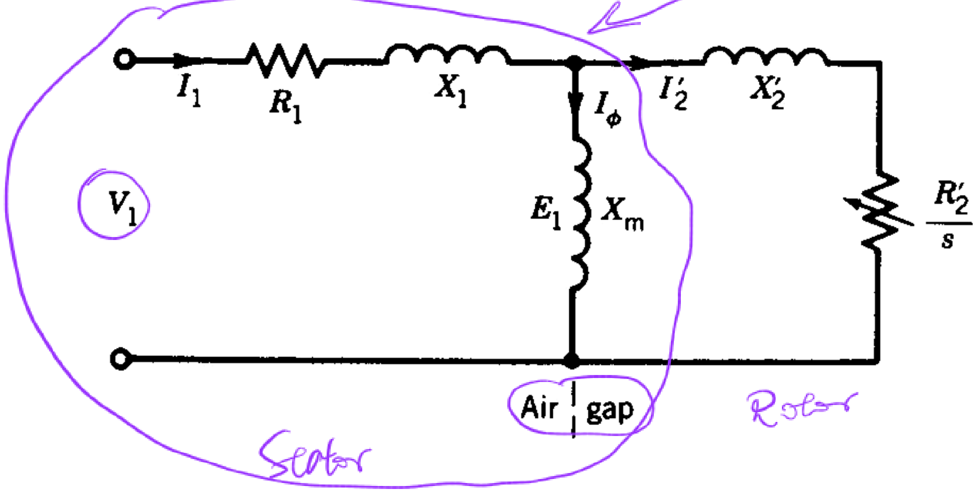
Commonly used (IEEE recommended) **T-equivalent circuit** (looks just like a transformer with shorted secondary side)

without R_c

Core losses are combined with rotational losses

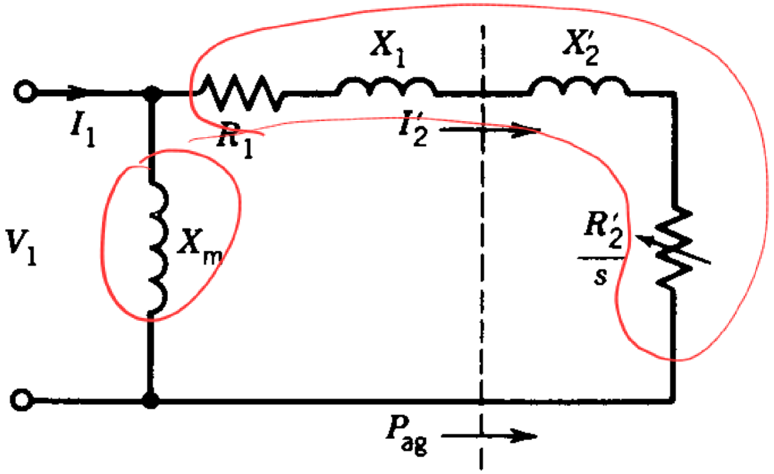
Thevenin Equivalent and Approximate Circuits

Full equivalent circuit



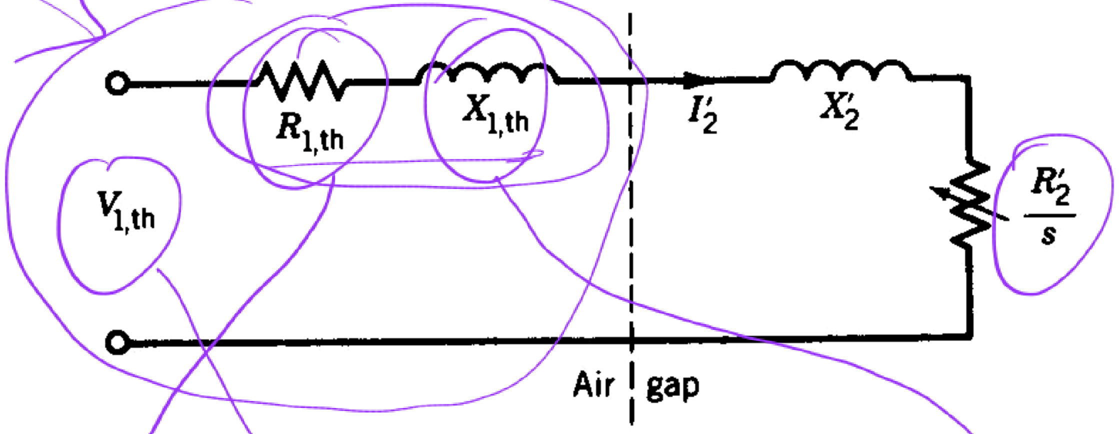
Approximate equivalent circuit

Move magnetizing branch to input terminals



Thevenin equivalent circuit

Recalculate parameters to remove magnetizing branch



$$V_{1,th} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X'_m)^2}} V_1$$

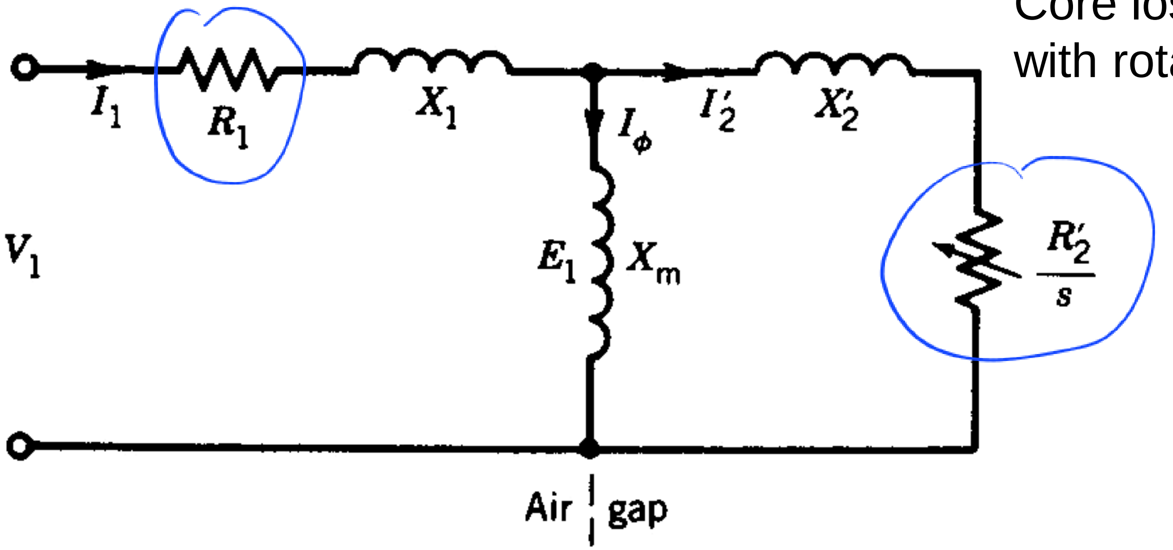
$$Z_{1,th} = \frac{jX_m\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_m\right)}$$

$$= \alpha + j\beta$$

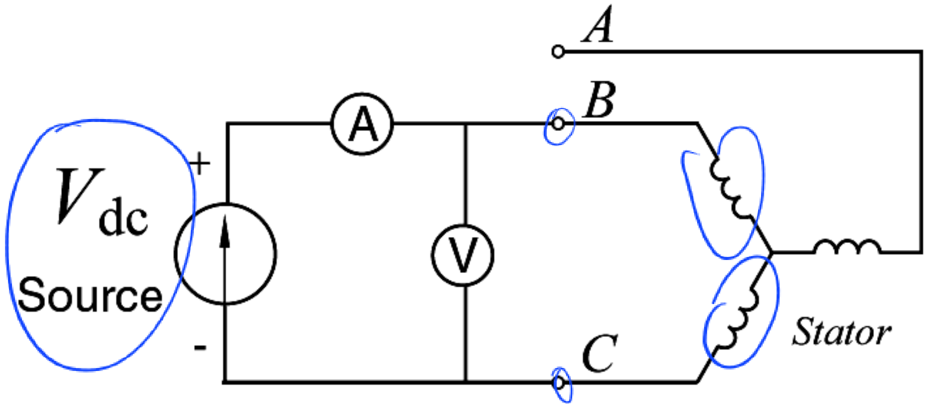
$$= R_{1,th} + jX_{1,th}$$

Determining Machine Parameters → Lab 4

Consider Full Equivalent Circuit



Core losses are combined with rotational losses



multimeter test

Tests to determine circuit parameters:

1. **DC Measurement of winding resistance**
2. No Load Test (like Open-Circuit in transformer)
3. Blocked Rotor Test (like Short-Circuit in transformer)

$$R_1 = \frac{V_{dc}}{2I_{dc}}$$

For Wound-Rotor IM, repeat the test on the rotor

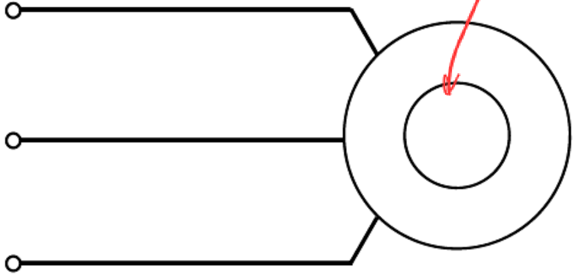
Determining Machine Parameters

open circuit test

2. No-Load (NL) Test

Apply rated voltage $V_{ph,NL}$

rotor spins ~ $\omega_{sync.}$



Measure

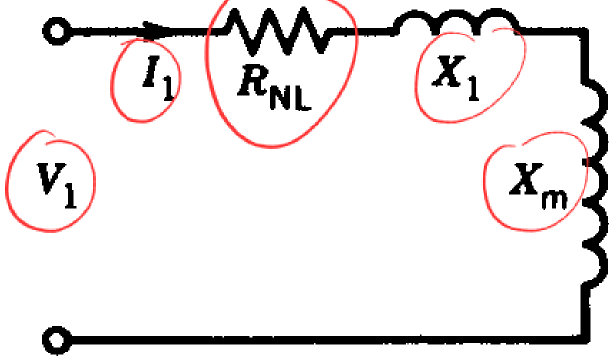
$$V_{ph}, P_{nl}, I_{nl}$$

Test 1

Rotational losses $$P_{rotational} = P_{NL} - 3R_1 I_{NL}^2$$

→ 3-φ

Assumed Equivalent Circuit



Combined reactance $$X_l + X_m = X_{NL} = \frac{Q_{NL}}{3I_{NL}^2}$$

= per-phase

where reactive power is

$$Q_{NL} = \sqrt{S_{NL}^2 - P_{NL}^2} = \sqrt{(3I_{NL}V_1)^2 - P_{NL}^2}$$

Can't separate these two

Stator ($s = \phi$) → no rotor

OR : $Z_{nl} = \frac{V_{ph,nl}}{I_{ph,nl}} \Rightarrow X_{nl} = X_1 + X_m = \sqrt{Z_{nl}^2 - R_1^2}$

Determining Machine Parameters

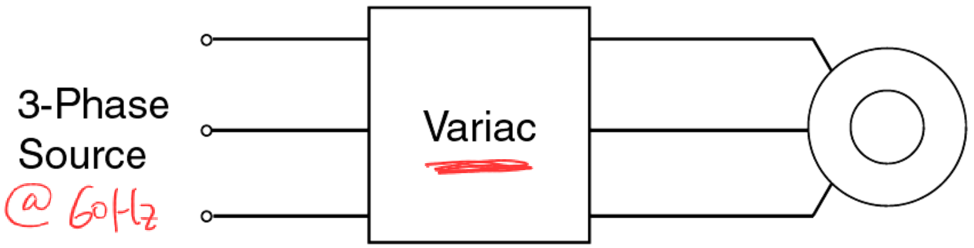
ramp up voltage slowly until you reach nominal current

3. Blocked-Rotor (BR) Test

Short circuit test $(s = 1)$

No rated voltage

Apply reduced voltage $V_{ph,BR}$
(this test may be performed at reduced frequency)



3-φ

Measure $V_{ph-BR}, P_{BR}, I_{BR}$ Test 1

Combined resistance $R_1 + R_2' = R_{BR} = \dfrac{P_{BR}}{3I_{BR}^2}$

Combined reactance

$$X_{BR} = \sqrt{Z_{BR}^2 - R_{BR}^2} = \sqrt{(V_1 / I_{BR})^2 - R_{BR}^2} = X_1 + X_2'$$

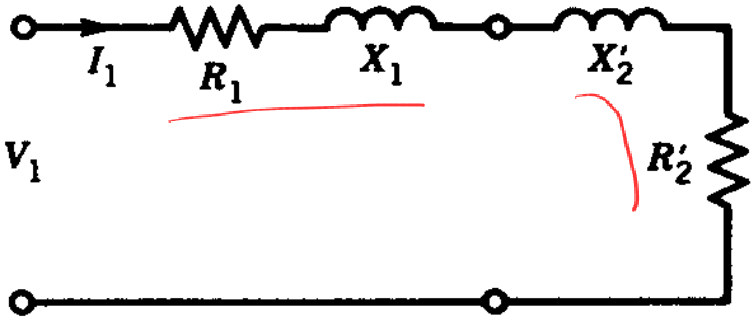
assume same

Leakage reactance $X_1 \approx X_2' = \dfrac{X_{BR}}{2}$

Magnetizing reactance $X_m = X_{NL} - X_1$

Rotor resistance $R_2' = R_{BR} - R_1$

Assumed Equivalent Circuit



Looking ahead…

- Continue **Module 5**