ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 17

March 12, 2026

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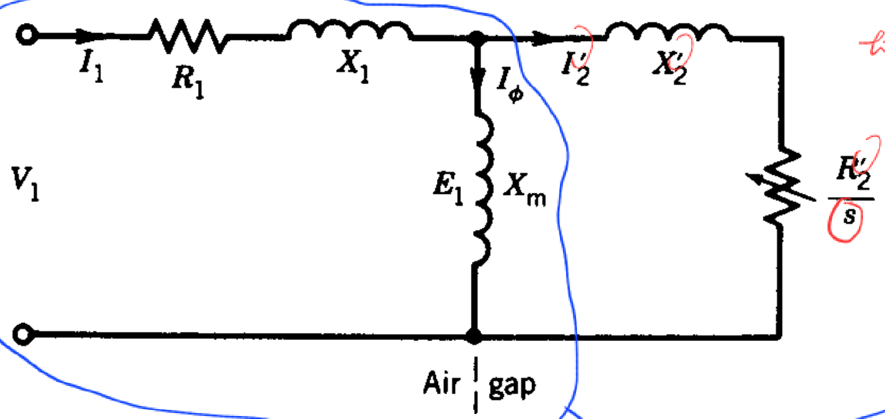
Course Logistics

- Tutorial resumes this week
- Lab 3 takes place this week
- Assignment 5 posted
- Midterm available to pick up
- Quizzes
  - Quiz 3: March 20 at the beginning of the tutorial
  - Quiz 4: April 2 at the beginning of the lecture

Thevenin Equivalent and Approximate Circuits

Stator → f_s Rotor → sf_s

Full equivalent circuit

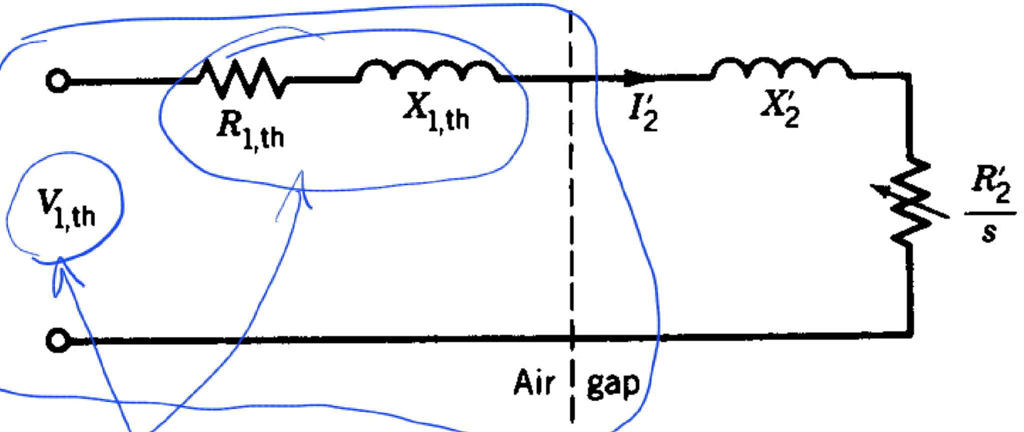


Prime notation: secondary (rotor) side referred to primary (stator) side

Thevenin

Thevenin equivalent circuit

Recalculate parameters to remove magnetizing branch



$$V_{1,th} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X'_m)^2}} V_1$$

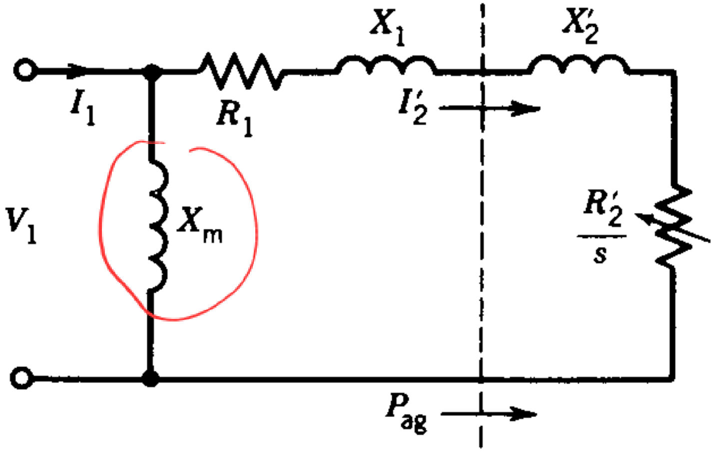
$$Z_{1,th} = \frac{jX_m\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_m\right)}$$

$$= \alpha + j\beta$$

$$= R_{1,th} + jX_{1,th}$$

Approximate equivalent circuit

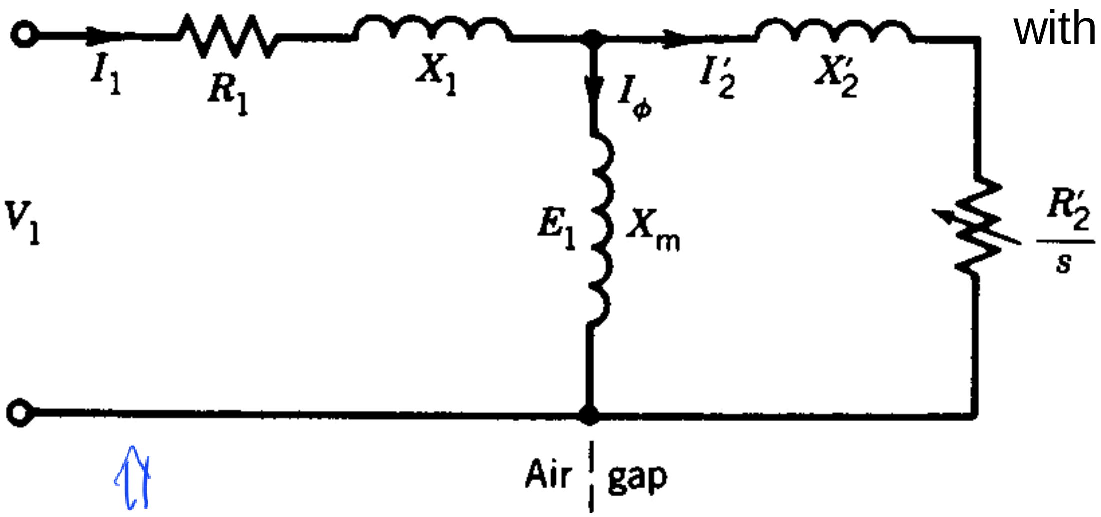
Move magnetizing branch to input terminals



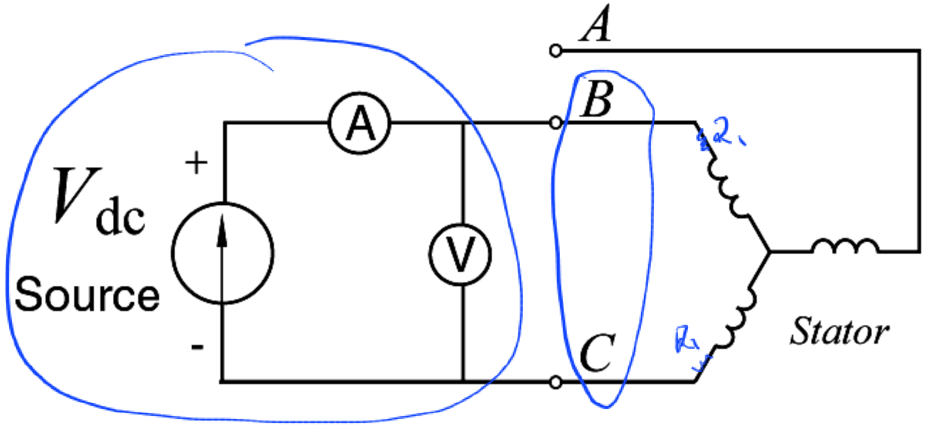
Determining Machine Parameters

Consider Full Equivalent Circuit

Core losses are combined with rotational losses



per-phase



Tests to determine circuit parameters:

1. **DC Measurement of winding resistance**
2. No Load Test (like Open-Circuit in transformer)
3. Blocked Rotor Test (like Short-Circuit in transformer)

$$R_1 = \frac{V_{dc}}{2 I_{dc}}$$

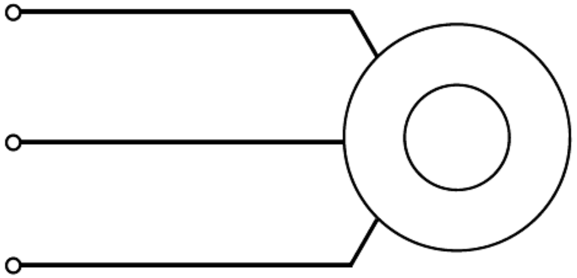
For Wound-Rotor IM, repeat the test on the rotor

Determining Machine Parameters

→ Rotor rotates $\sim \omega_{sync.} \Rightarrow s = 0$

2. No-Load (NL) Test

Apply rated voltage $V_{ph,NL}$



Measure

V_{ph}, P_{nl}, I_{nl}

P_{nl} → 3-phase

V_{ph} → per-phase

I_{nl} → per-phase

Rotational losses $P_{rotational} = P_{NL} - 3R_1 I_{NL}^2$

Combined reactance $X_l + X_m = X_{NL} = \dfrac{Q_{NL}}{3I_{NL}^2}$

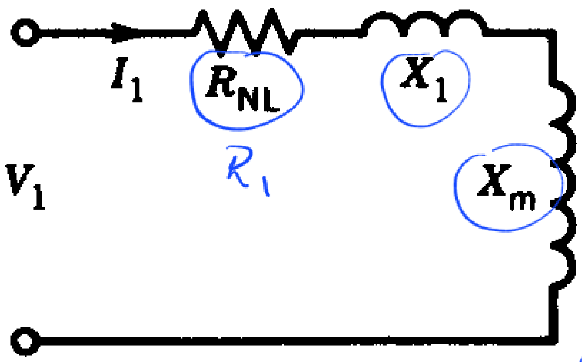
can't separate

where reactive power is

$$Q_{NL} = \sqrt{S_{NL}^2 - P_{NL}^2} = \sqrt{(3I_{NL}V_1)^2 - P_{NL}^2}$$

Or: $Z_{nl} = \dfrac{V_{ph,nl}}{I_{ph,nl}} \Rightarrow X_{nl} = \sqrt{Z_{nl}^2 - R_1^2}$

Assumed Equivalent Circuit

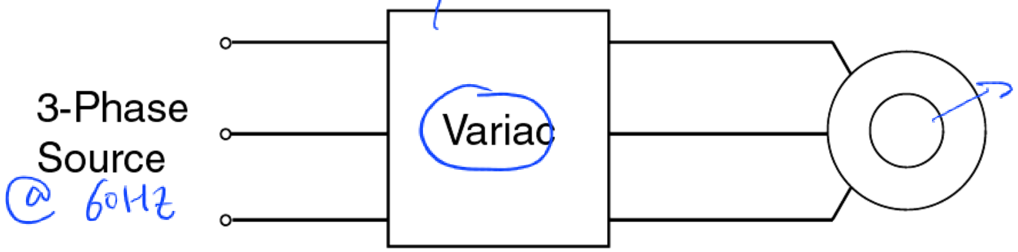


Rotor disconnected

Determining Machine Parameters

3. Blocked-Rotor (BR) Test → $s = 1$

Ramp up voltage slowly until you see rated current



@ 60Hz

Rotor locked

Apply reduced voltage $V_{ph,BR}$
(this test may be performed at reduced frequency)

Measure $V_{ph-BR}, P_{BR}, I_{BR}$

Combined resistance $R_1 + R_2' = R_{BR} = \dfrac{P_{BR}}{3I_{BR}^2}$

Combined reactance

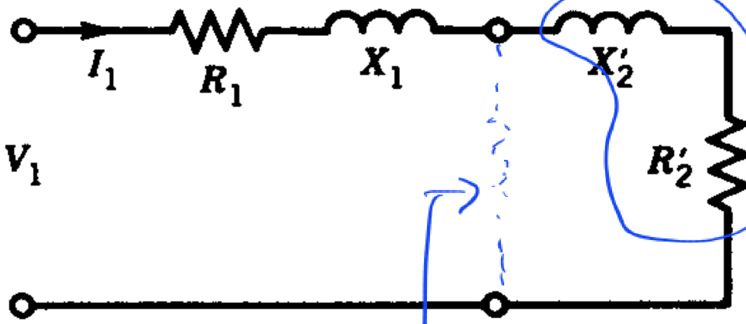
$$X_{BR} = \sqrt{Z_{BR}^2 - R_{BR}^2} = \sqrt{(V_1 / I_{BR})^2 - R_{BR}^2}$$

Leakage reactance $X_1 \approx X_2' = \dfrac{X_{BR}}{2}$ → $X_1 \Rightarrow X_m = X_{NL} - X_1$

Magnetizing reactance $X_m = X_{NL} - X_1$

Rotor resistance $R_2' = R_{BR} - R_1$

Assumed Equivalent Circuit

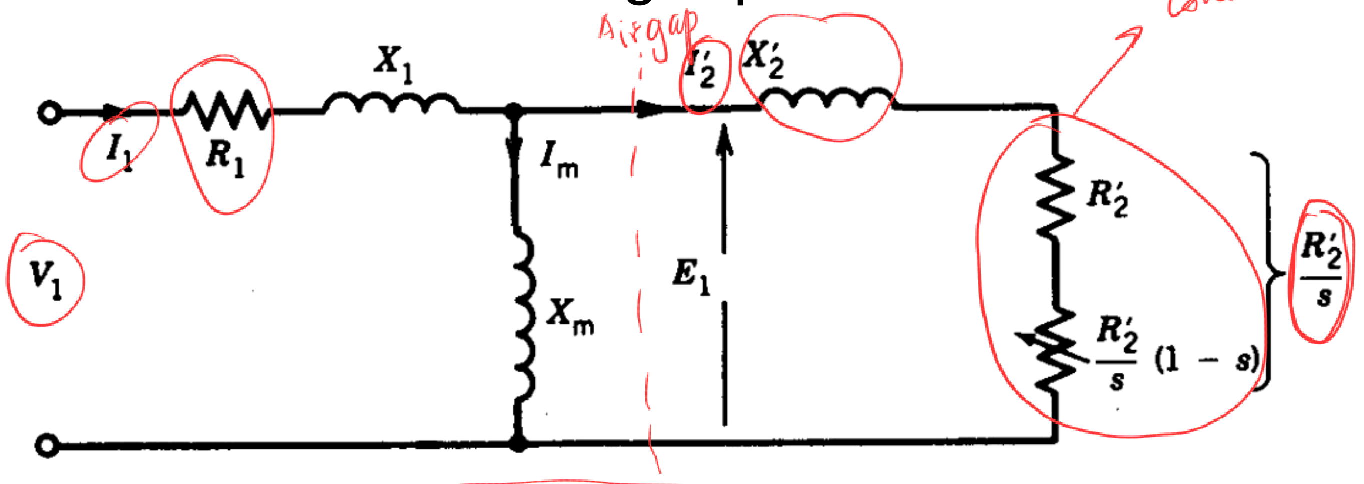


Shunt branch ignored

Power Conversion & Torque

Consider the following equivalent circuit

Consumes real power from air gap



Separate the rotor winding resistance and the remaining part that represent conversion of real power into torque

$$\frac{R_2'}{s} = \frac{s-s+1}{s}R_2' = \underbrace{R_2'}_{\text{winding losses in rotor}} + \underbrace{\frac{1-s}{s}R_2'}_{\text{Creating } T_e}$$

$\frac{1}{s}$

Input Power $P_{in} = 3V_1I_1\cos(\varphi)$

Stator copper loss $P_{SCL} = 3I_1^2R_1$

Air-gap power $P_{ag} = 3I_2'^2\frac{R_2'}{s}$

$P_{ag} = P_e + P_{rotor\ loss}$ → R_2'

Conversion power (for creating T_e)

$$P_e = 3I_2'^2\underbrace{R_2'\left(\frac{1-s}{s}\right)}_{T_e} = (1-s)P_{ag}$$

Electromagnetic torque

$$T_e = \frac{P_e}{\omega_{rm}} = \frac{(1-s)P_{ag}}{\omega_{rm}} = \frac{P_{ag}}{\omega_{syn}} = 3\frac{I_2'^2R_2'}{\omega_{syn}s} = \boxed{3\left(\frac{P}{2}\right)\frac{I_2'^2R_2'}{\omega_e s}}$$

pole

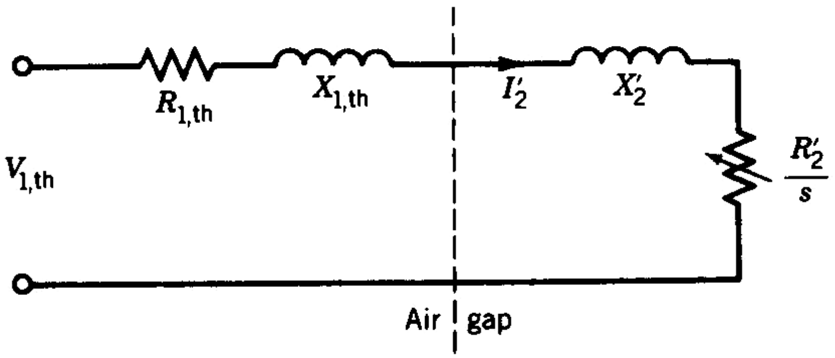
electrical freq.

→ Express in terms of V_1
↳ I_2' is hard to determine

Torque-Speed Characteristic

Curve more complicated than DCM

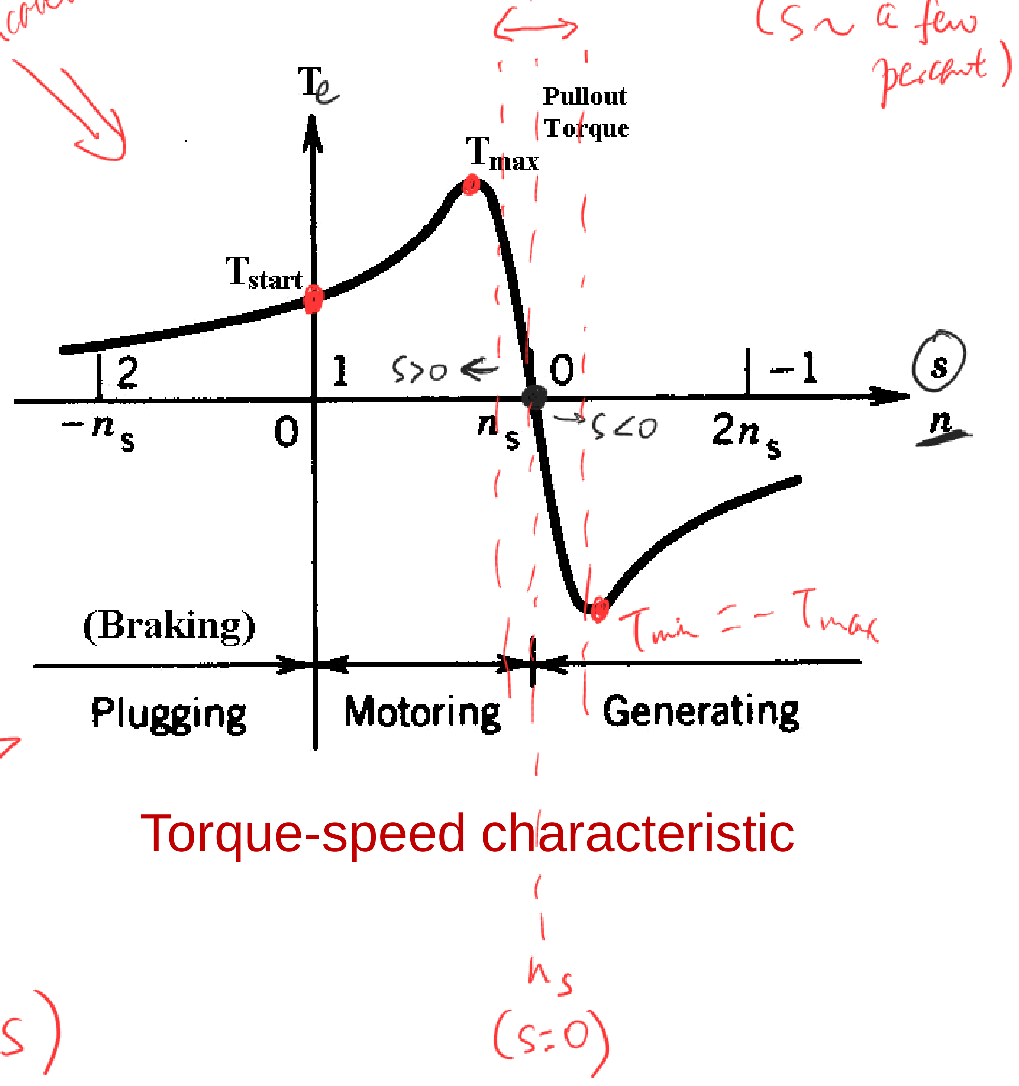
1) Using Thevenin Equivalent Circuit



Recall $$T_e = \frac{P_e}{\omega_{rm}} = 3\frac{P}{2}\frac{I'^2_2 R'_2}{\omega_e s}$$

$$I'_2 = \frac{V_{1,th}}{(R_{1,th} + R'_2/s) + j(X_{1,th} + X'_2)}$$

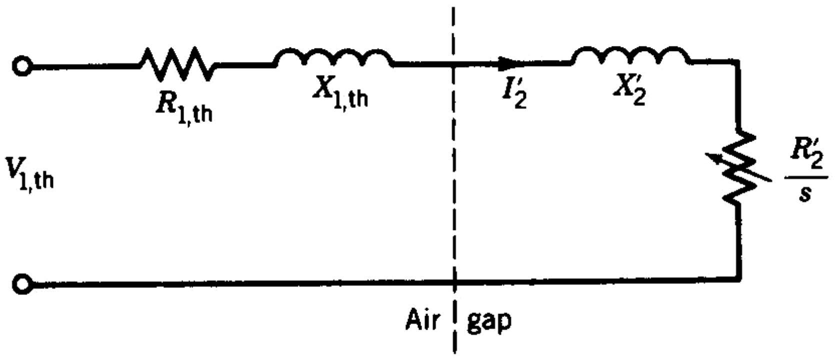
$$\boxed{T_e = 3\frac{P}{2}\frac{V^2_{1,th}}{\omega_e} \cdot \frac{R'_2/s}{(R_{1,th} + R'_2/s)^2 + (X_{1,th} + X'_2)^2}} = f(V_{1,th}, s)$$



Torque-speed characteristic

Torque-Speed Characteristic

1) Using Thevenin Equivalent Circuit

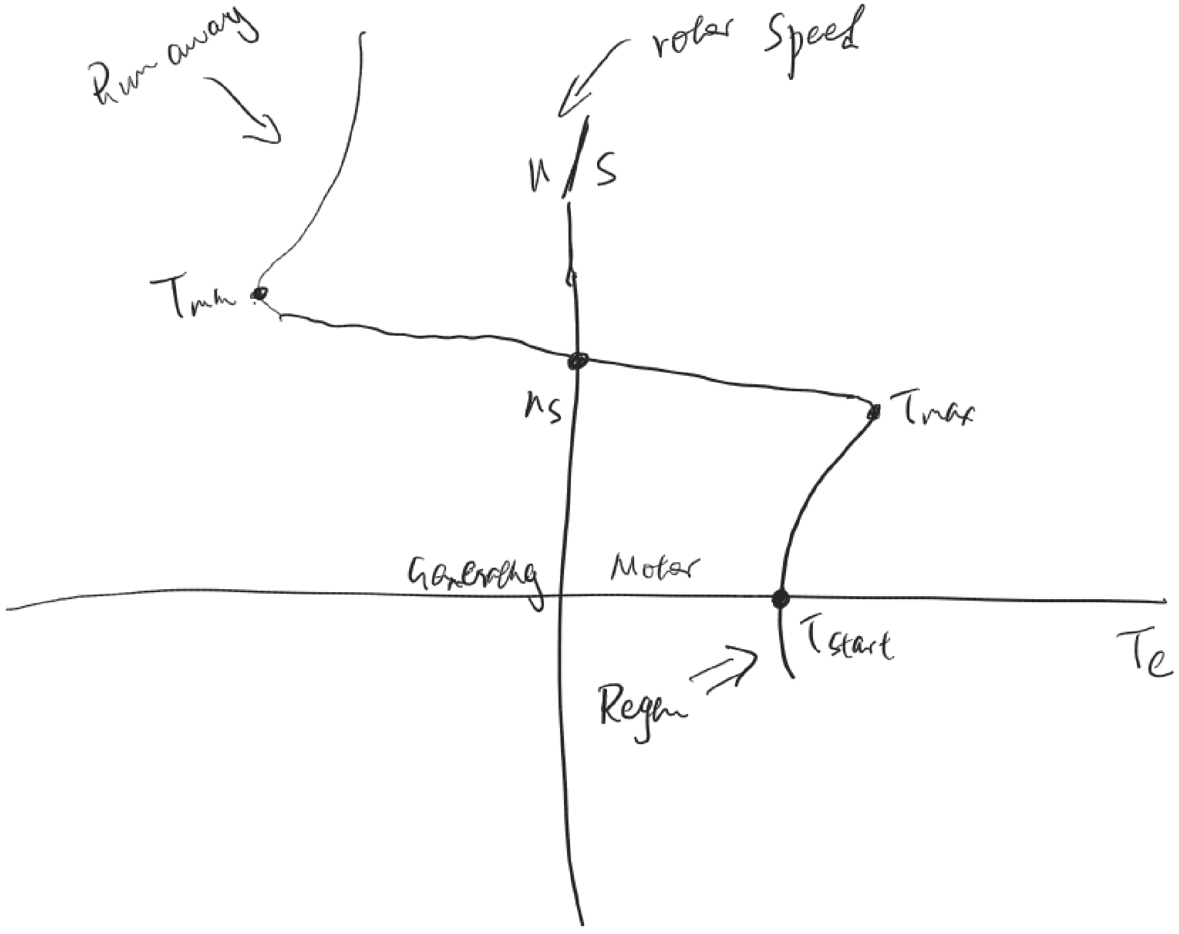


Recall $$T_e = \frac{P_e}{\omega_{rm}} = 3\frac{P}{2}\frac{I_2'^2 R_2'}{\omega_e s}$$

$$I_2' = \frac{V_{1,th}}{(R_{1,th} + R_2'/s) + j(X_{1,th} + X_2')}$$

$$T_e = 3\frac{P}{2}\frac{V_{1,th}^2}{\omega_e} \cdot \frac{R_2'/s}{(R_{1,th} + R_2'/s)^2 + (X_{1,th} + X_2')^2}$$

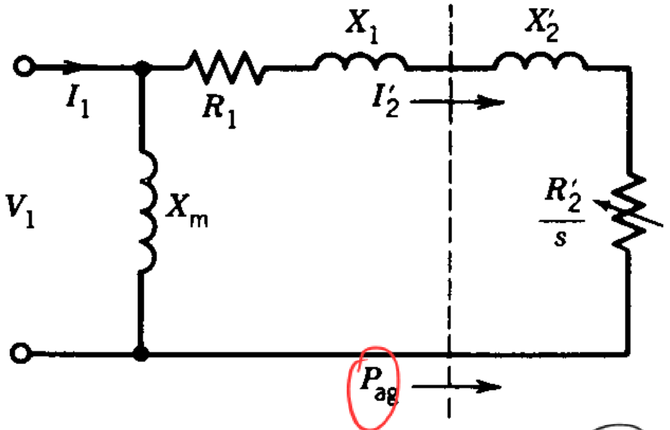
→ Provide formula on test / Exam



Speed-torque characteristic

Torque-Speed Characteristic

2) Using Approximate Equivalent Circuit

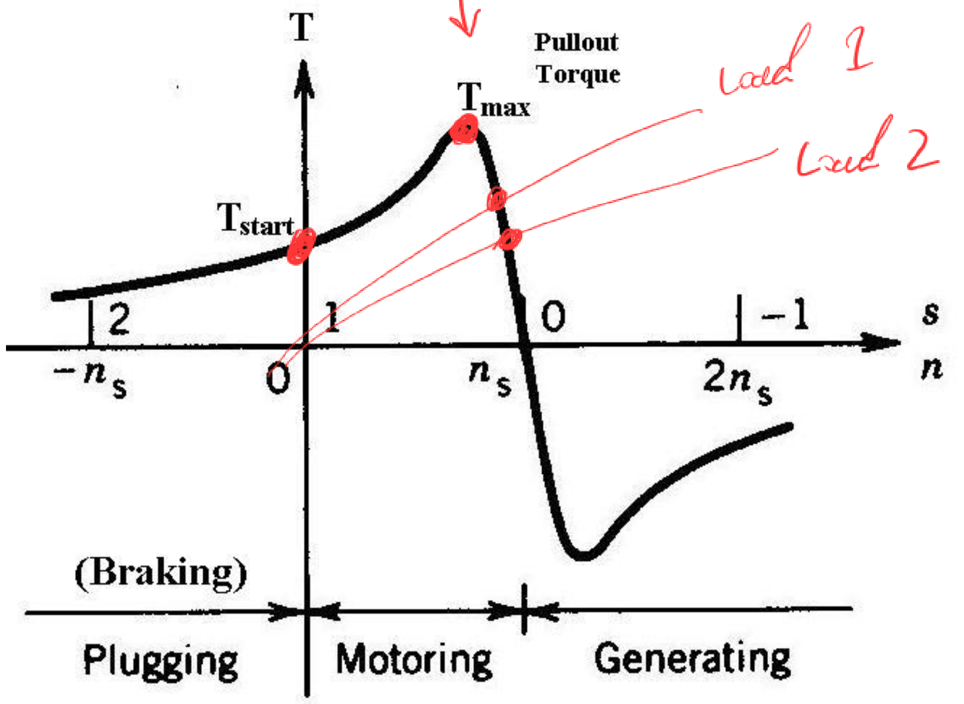


Recall $$T_e = \frac{P_e}{\omega_{rm}} = 3\frac{P}{2}\frac{I_2'^2 R_2'}{\omega_e s}$$

$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')}$$

$$T_e = 3\frac{P}{2}\frac{V_1^2}{\omega_e}\cdot\frac{R_2'/s}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} = f(V_1, s)$$

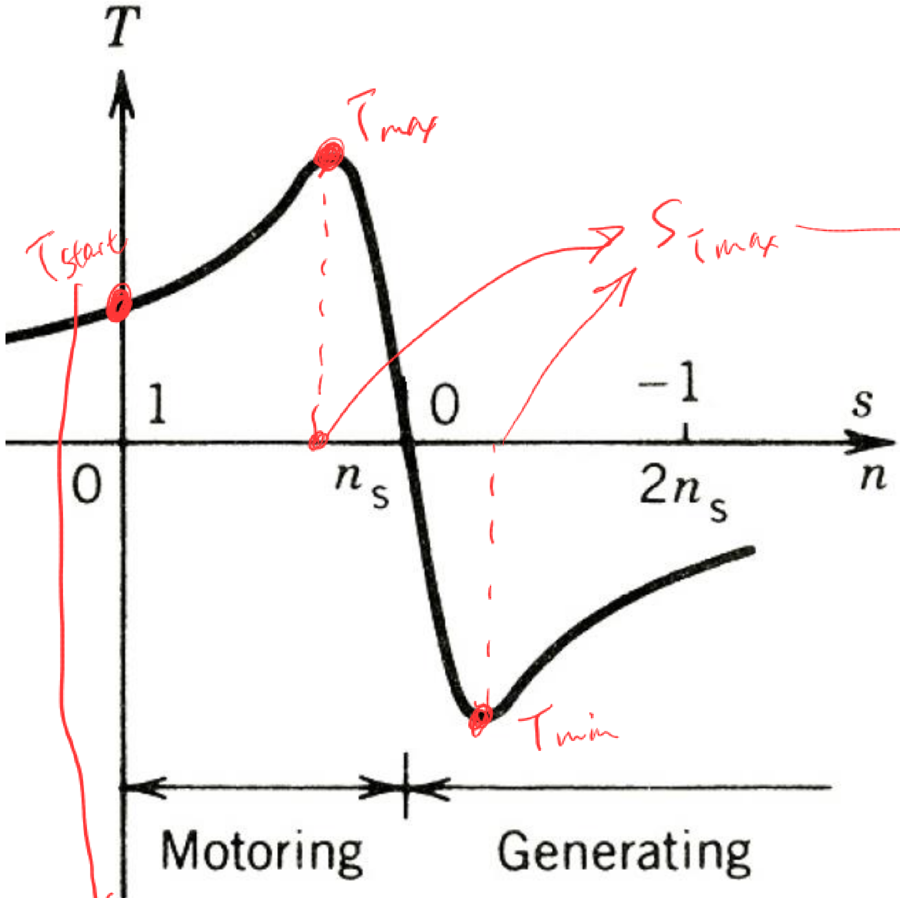
Optimization problem



Torque-speed characteristic

Torque-Speed Characteristic

$$T_e = 3\frac{P}{2}\frac{V_{1,th}^2}{\omega_e}\cdot\frac{R_2'/s}{(R_{1,th}+R_2'/s)^2+(X_{1,th}+X_2')^2}$$



Starting torque/current

set slip s = 1

Maximum torque

$$\frac{\partial T}{\partial s} = 0$$

rotor winding resistance

$$s_{T\max} = \pm\frac{R_2'}{\sqrt{R_{1,th}^2+\left(X_{1,th}+X_2'\right)^2}}$$

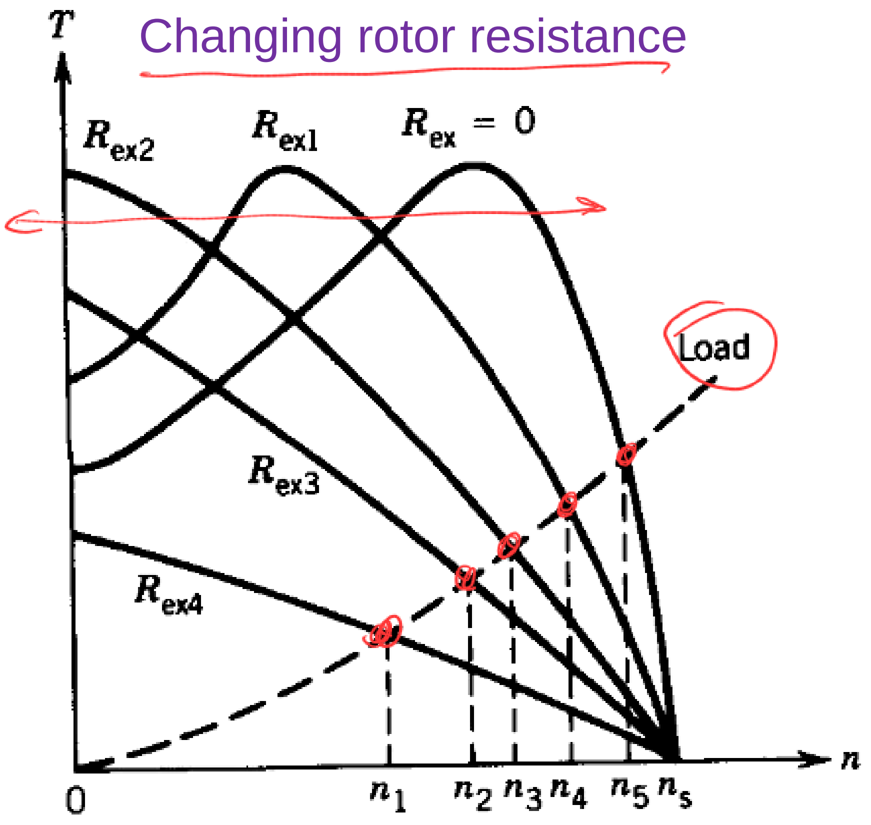
$$T_{\max} = 3\frac{P}{2}\frac{V_{1,th}^2}{2\omega_e}\cdot\frac{1}{R_{1,th}+\sqrt{R_{1,th}^2+(X_{1,th}+X_2')^2}}$$

$$T_{\min} = 3\frac{P}{2}\frac{V_{1,th}^2}{2\omega_e}\cdot\frac{1}{\sqrt{R_{1,th}^2+(X_{1,th}+X_2')^2}-R_{1,th}}$$

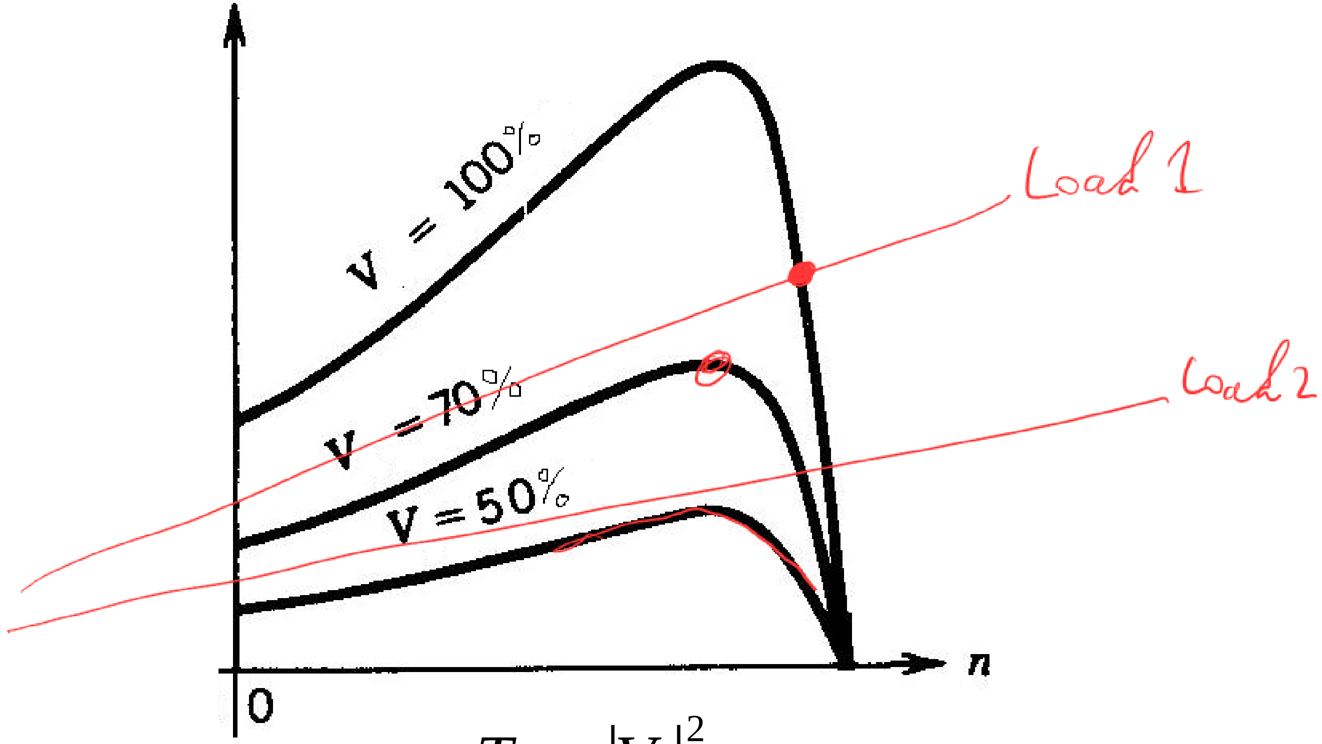
Does not depend on R_2'

Torque-Speed Characteristic

Changing rotor resistance

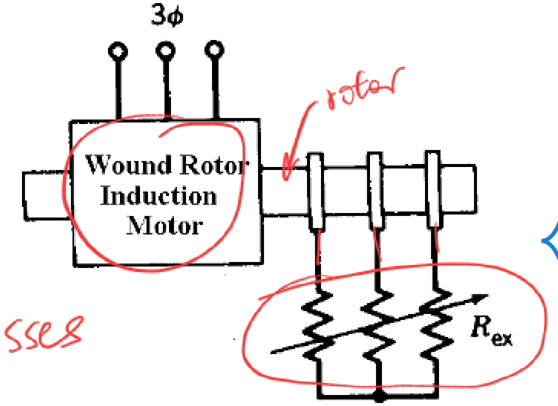


Changing input voltage



$$T_e \propto |V_1|^2$$

Make R_2' very large at the start-up to have a high start-up torque
↓
remove to reduce losses at steady state



$$s_{T\max} \propto R_2'$$

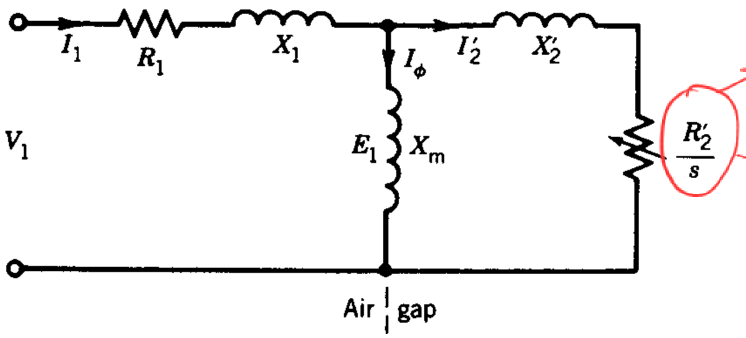
$T_{\max}$ independent of rotor resistance

Changing V_1 is not a good way to control IM

Starting Current

→ at steady-state → $s \approx 0.03$

Recommended T-Equivalent Circuit



→ large at steady-state

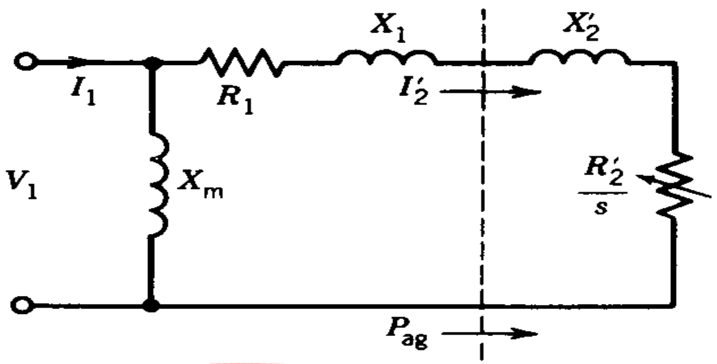
→ small at start-up

high current

$$Z_{eq,start} = R_1 + jX_1 + \frac{jX_m\left(R_2' + jX_2'\right)}{R_2' + j\left(X_m + X_2'\right)}$$

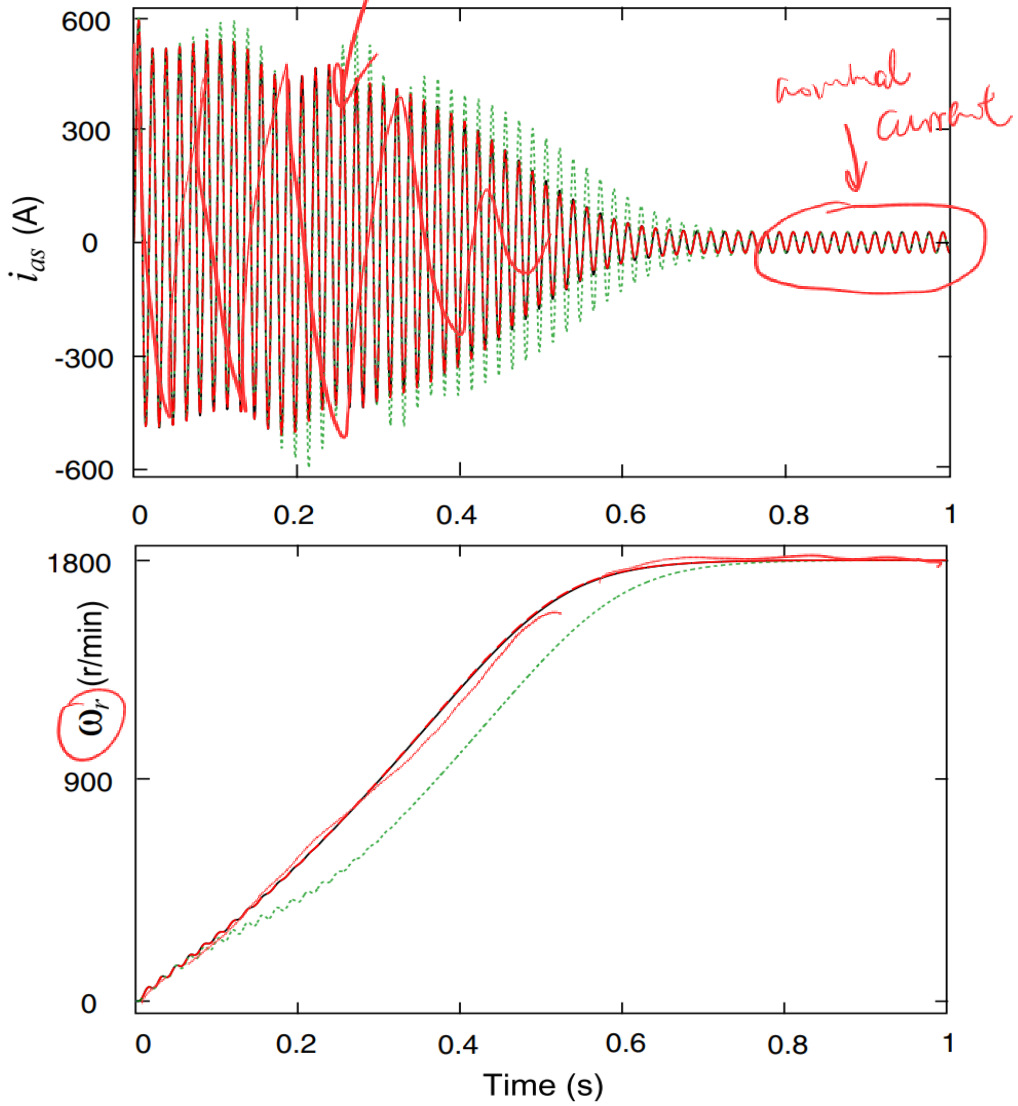
$$I_{1,start} = \frac{V_1}{Z_{eq,start}}$$

Approximate Equivalent Circuit

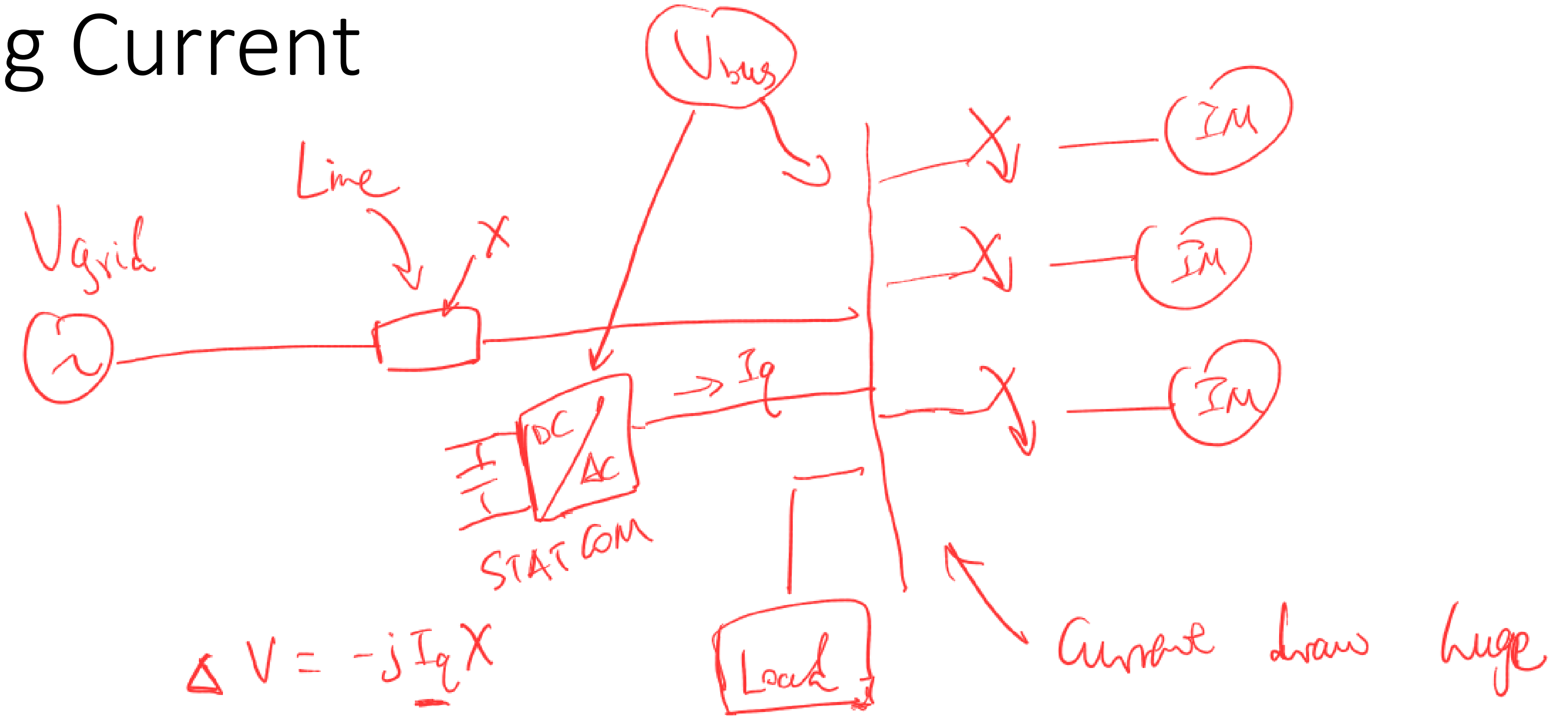


well-known IM behavior

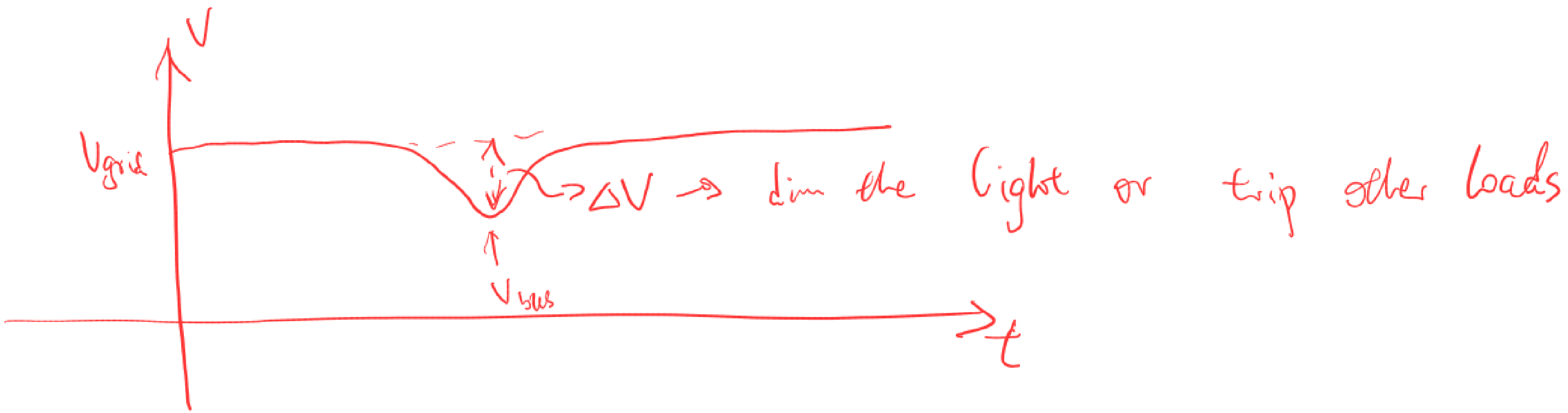
$$I_{1,start} = I_m + I_2' = \frac{V_1}{jX_m} + \frac{V_1}{(R_1 + R_2') + j(X_1 + X_2')}$$



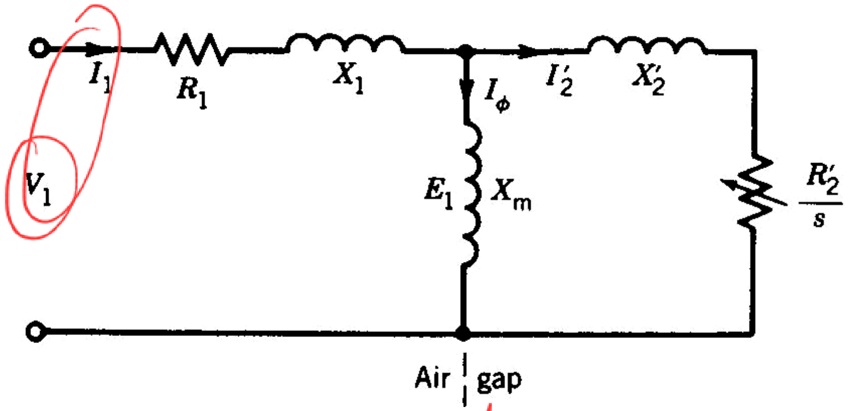
Starting Current



$\Delta V = -j \underline{I_q} X$



Power Flow & Efficiency

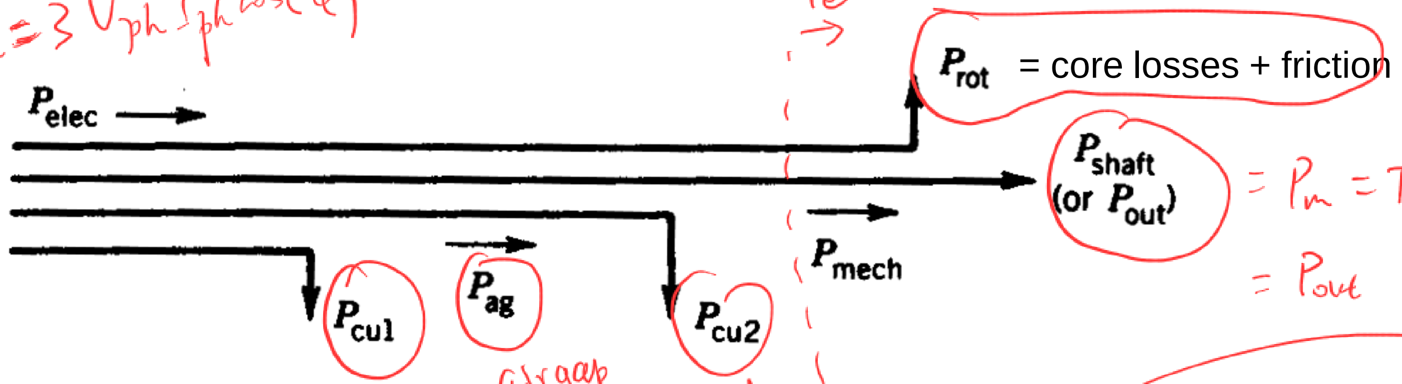


use full T-equivalent circuit

Motor Mode

$P_{in} = 3 V_{ph} I_{ph} \cos(\varphi)$

P_e



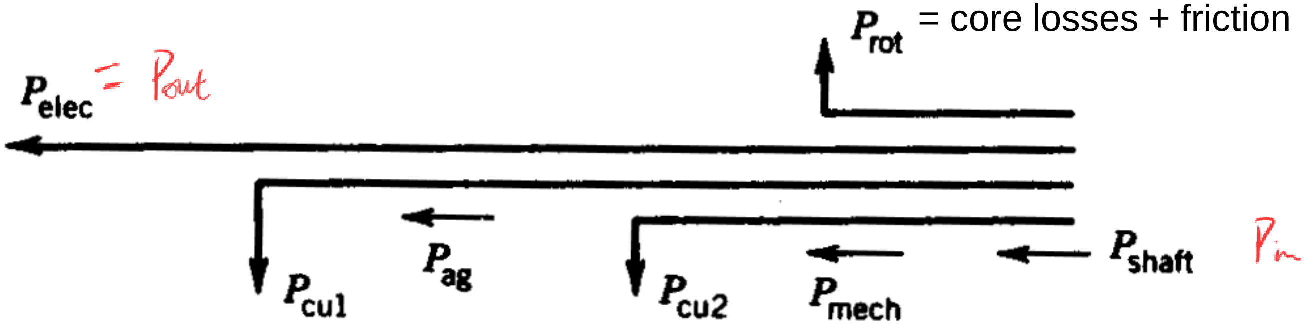
$= P_m = T_m \cdot \omega_{rm}$

$= P_{out}$

R_1 airgap R'_2

$$\boxed{\eta = \frac{P_{out}}{P_{in}}}$$

Generator Mode

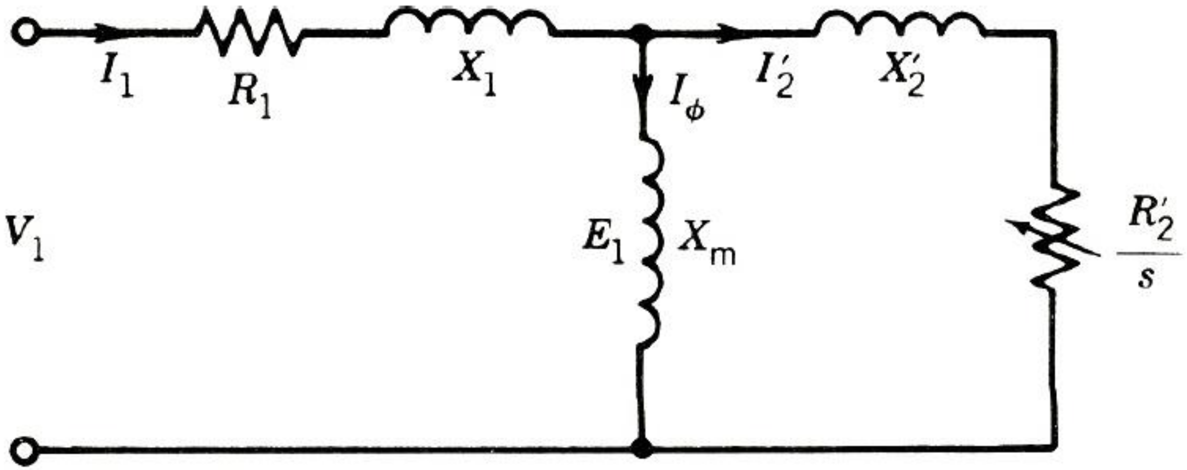


$P_{elec} = P_{out}$

$P_m = T_m \cdot \omega_m$

Analysis of Induction Machines (using Equivalent Circuit)

❖**Typical Question:** A 3-phase, 460 V, 1740 rpm, 60 Hz, four-pole induction motor has the following parameters per-phase:



$$\begin{cases} R_1 = 0.25\ \Omega \\ R_2' = 0.20\ \Omega \\ X_1 = X_2' = 0.5\ \Omega \\ X_m = 30\ \Omega \end{cases}$$

Most questions just involve solving circuit
↓
Te equation will be provided to you

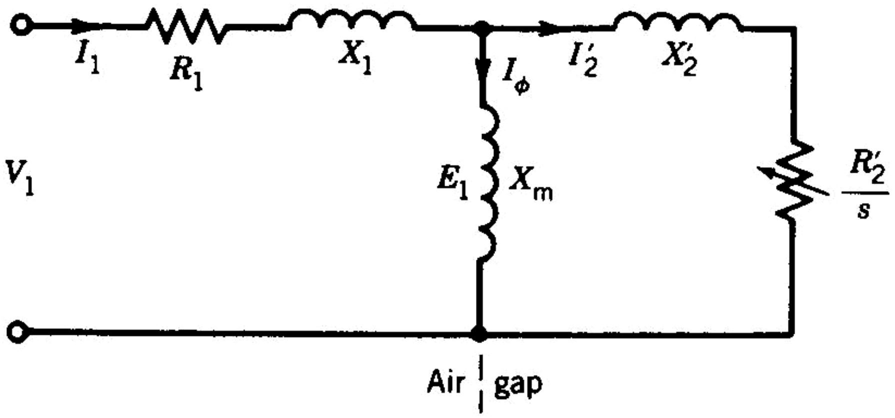
Rotational losses are constant 1700 W. Calculate:

(a) Starting current & torque
(b) Under given load (defined either by speed or torque), find current & torque & power factor & efficiency
(c) Maximum torque & corresponding slip
(d) For DFIM, find external resistance needed to get max torque at start-up

Example 3

we need phase voltage

Consider a Y-connected 7.5kW, 220V (line-line), 60Hz, 6-pole, Squirrel-cage motor with the following parameters:
R1 = 0.294Ω, R’2 = 0.144Ω, X1 = 0.503Ω, X’2 = 0.209Ω, Xm = 13.5Ω



Using original T-Equivalent Circuit

a) Compute starting current I_{1start}

$\rightarrow s = 1$

$$V_1 = \frac{V_{\ell-\ell}}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127\,V_{rms}$$

$$Z_{eq,st} = R_1 + jX_1 + \frac{jX_m(jX_2 + R_2)}{jX_m + jX_2 + R_2} =$$

$$= 0.4336 + j\,0.7103\ \Omega$$

Sanity check: should be lagging

$$I_{1,st} = \frac{V_1}{Z_{eq,st}} = \frac{127}{0.4336 + j0.7103} = 79.5 - j130.3\ A$$

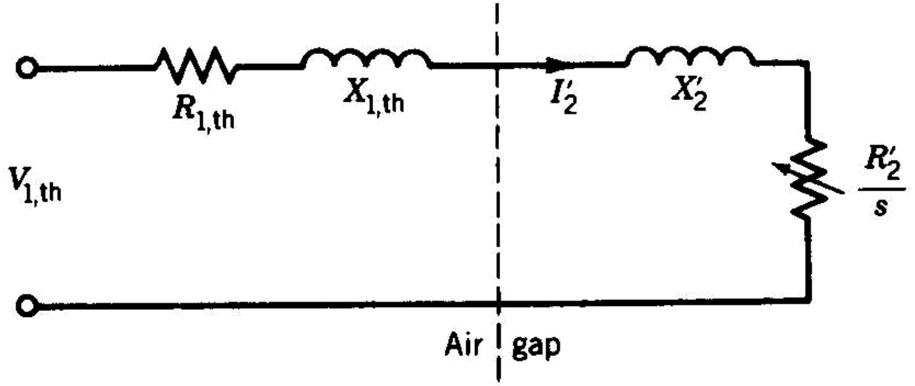
$$= 152.63\ A(rms)$$

Example 3

Consider a Y-connected 7.5kW, 220V (line-line), 60Hz, 6-pole, Squirrel-cage motor with the following parameters:
R1 = 0.294Ω, R'2 = 0.144Ω, X1 = 0.503Ω, X'2 = 0.209Ω, Xm = 13.5Ω

Using Thevenin Equivalent Circuit

b) Compute the starting torque T_{start} $\Rightarrow s = 1$



$$\omega_e = 2\cdot\pi\cdot f_e = 2\cdot\pi\cdot 60 = 376.99 \text{ rad/sec}$$

$$V_{1,th} = V_1 \cdot \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} = 122.43 \text{ V(rms)}$$

$$Z_{th} = \frac{jX_m(R_1 + X_1)}{(jX_m + R_1 + jX_1)} = \underbrace{0.2731}_{R_{1,th}} + j\underbrace{0.491}_{X_{1,th}} \ \Omega$$

Convert to Thevenin circuit

$$T_{st} = 3 \cdot \frac{P}{2} \cdot \frac{V_{1,th}^2}{\omega_e} \cdot \frac{R_2}{(R_{1,th} + R_2)^2 + (X_{1,th} + X_2)^2} =$$

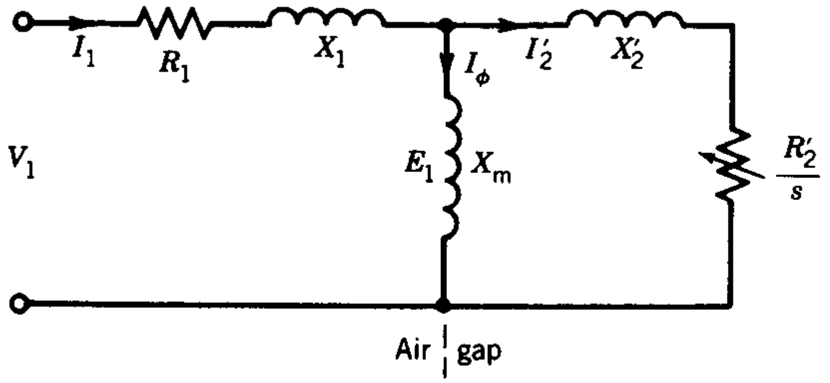
$$= 77.65 \text{ N-m}$$

Example 3

Consider a Y-connected 7.5kW, 220V (line-line), 60Hz, 6-pole, Squirrel-cage motor with the following parameters: R1 = 0.294Ω, R'2 = 0.144Ω, X1 = 0.503Ω, X'2 = 0.209Ω, Xm = 13.5Ω. Assume motor operates under load with slip s = 0.03 (3%).

Assume T-Equivalent Circuit

c) Compute stator current I_1 ⇐ Steady-state at $s = 0.03$



$$Z_{eq} = R_1 + jX_1 + \frac{jX_m \cdot (jX_2 + R_2'/s)}{jX_m + jX_2 + R_2/s} =$$

$$= 4.44 + j2.16$$

lagging

$$I_1 = \frac{V_1}{Z_{eq}} = \frac{127}{4.44 + j2.16} = 23.13 - j11.23 \text{ A}$$

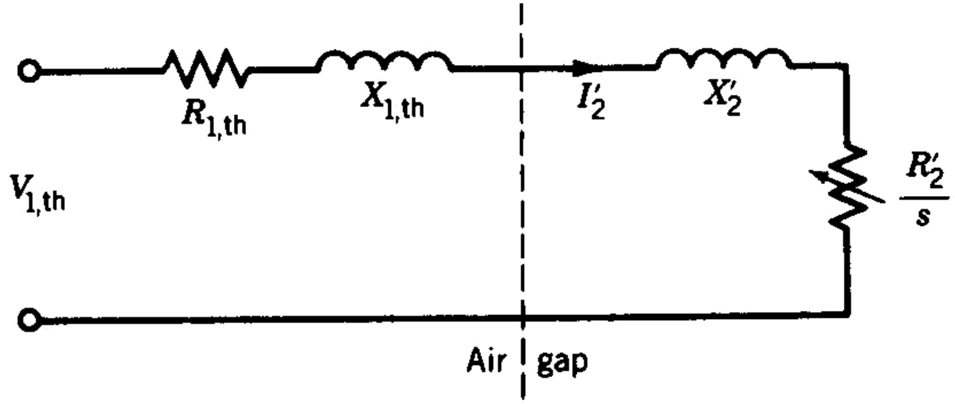
$$= 25.72 \text{ A(rms)} \quad \ll I_{start-up}$$

Example 3

Consider a Y-connected 7.5kW, 220V (line-line), 60Hz, 6-pole, Squirrel-cage motor with the following parameters: R1 = 0.294Ω, R'2 = 0.144Ω, X1 = 0.503Ω, X'2 = 0.209Ω, Xm = 13.5Ω. Assume motor operates under load with slip s = 0.03 (3%).

Using Thevenin Equivalent Circuit

d) Compute the torque T_e



$$T_{e,th} = 3 \cdot \left(\frac{P}{2}\right) \cdot \frac{V_{1,th}^2}{\omega_e} \cdot \frac{R_2'/s}{(R_{1,th} + R_2'/s)^2 + (X_{1,th} + X_2')^2}$$

$$= 3 \cdot \left(\frac{6}{2}\right) \cdot \frac{122.4^2}{377} \cdot \frac{0.144/0.03}{\left(0.2731 + \frac{0.144}{0.03}\right)^2 + (0.4907 + 0.209)^2} =$$

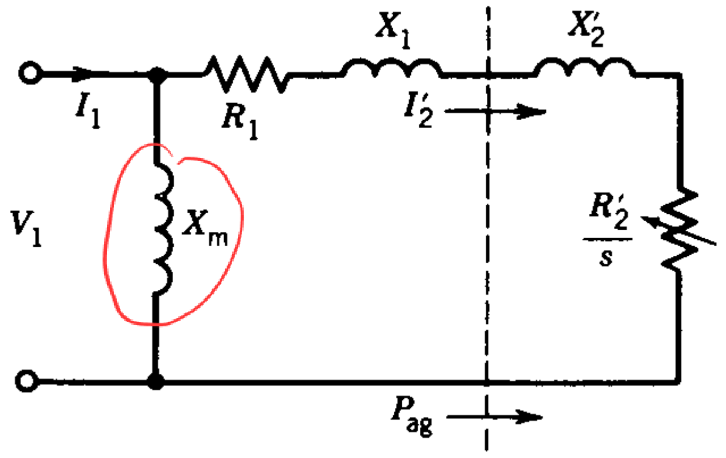
$= 65.5\ N \cdot m$ at $s = 3\%$ → one point

Lab 4: do multiple points → T_e calculations can be simplified using a script
→ get $T_e - s$ curve

Example 3

Results of Thevenin = Results of Full ≠ Results of Approximate model

Consider a Y-connected 7.5kW, 220V (line-line), 60Hz, 6-pole, Squirrel-cage motor with the following parameters:
R1 = 0.294Ω, R'2 = 0.144Ω, X1 = 0.503Ω, X'2 = 0.209Ω, Xm = 13.5Ω



Using Approximate Equivalent Circuit

a) Compute starting current I_{1start}

$$Z_{12,st} = R_1 + jX_1 + R_2 + jX_2 = 0.438 + j0.712\ \Omega$$

$$Z_{eq,st,ap} = \frac{jX_m \cdot Z_{12,st}}{jX_m + Z_{12,st}} = 0.3948 + j0.6885\ \Omega$$

$$I_{1,st,ap} = \frac{V_1}{Z_{eq,st,ap}} = \frac{127}{0.3948 + j0.6885} = 79.6 - j138.8\ A$$

$$= 160.04\ A\,(rms)$$

(152.63 A)

losing accuracy

b) Compute the starting torque T_{start}

$$T_{st,ap} = 3 \cdot \frac{P}{2} \cdot \frac{V_1^2}{\omega_e} \cdot \frac{R_2}{(R_1+R_2)^2 + (X_1+X_2)^2} =$$

$$= 79.37\ N \cdot m$$

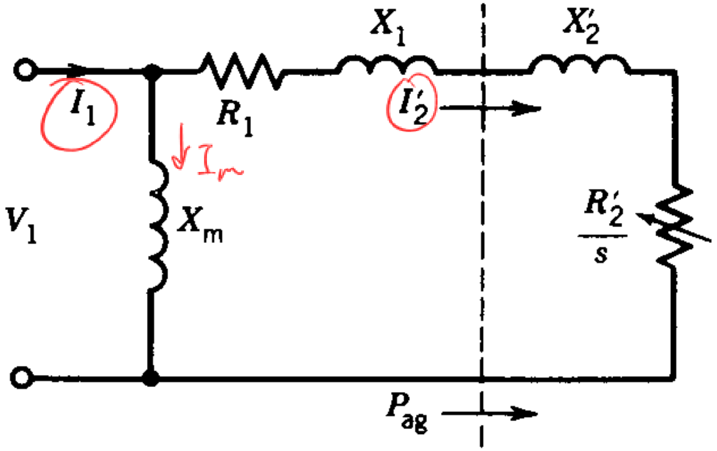
(77.65 N·m)

Example 3

Consider a Y-connected 7.5kW, 220V (line-line), 60Hz, 6-pole, Squirrel-cage motor with the following parameters: R1 = 0.294Ω, R'2 = 0.144Ω, X1 = 0.503Ω, X'2 = 0.209Ω, Xm = 13.5Ω. Assume motor operates under load with slip s = 0.03 (3%).

Using Approximate Equivalent Circuit

c) Compute stator current I_1



①

$$I_2' = \frac{V_1}{R_1 + jX_1 + R_2/s + jX_2} = 24.46 - j\,3.42\ A$$

$$I_m = \frac{V_1}{jX_m} = -j\,9.41\ A$$

$$I_{1,ap} = I_m + I_2' = 24.46 - j\,12.83\ A$$

$$= 27.62\ A\ (rms) \neq (25.72\ A)$$

②

$$Z_{eq,ap} = \frac{jX_m(jX_1 + R_1 + jX_2 + R_2/s)}{jX_m + R_1 + jX_1 + jX_2 + R_2/s} = 4.07 + j\,2.14\ \Omega$$

$$I_{1,ap} = \frac{V_1}{Z_{eq,ap}} = \frac{127}{4.07 + j\,2.14} = 24.46 - j\,12.83\ A$$

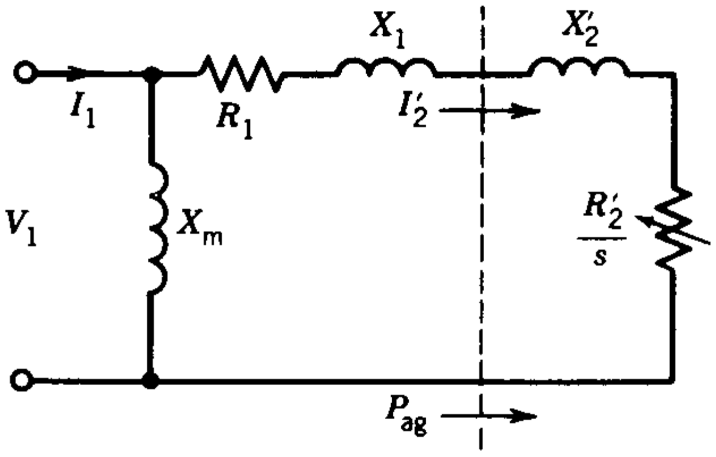
$$= 27.62\ A\ (rms)$$

Example 3

Consider a Y-connected 7.5kW, 220V (line-line), 60Hz, 6-pole, Squirrel-cage motor with the following parameters: R1 = 0.294Ω, R'2 = 0.144Ω, X1 = 0.503Ω, X'2 = 0.209Ω, Xm = 13.5Ω. Assume motor operates under load with slip s = 0.03 (3%).

Using Approximate Equivalent Circuit

d) Compute the torque T_e



$$T_{e,ap} = 3 \cdot \left(\frac{P}{2}\right) \cdot \frac{V_1^2}{\omega_e} \cdot \frac{R_2'/s}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2}$$

$$= 3 \cdot \frac{6}{2} \cdot \frac{127^2}{377} \cdot \frac{0.144/0.03}{\left(0.294 + \frac{0.144}{0.03}\right)^2 + (0.503 + 0.209)^2}$$

$$= 69.88 \text{ N}\cdot\text{m} \neq 65.5 \text{ N}\cdot\text{m}$$

Example 3

Consider a Y-connected 7.5kW, 220V (line-line), 60Hz, 6-pole, Squirrel-cage motor with the following parameters: R1 = 0.294Ω, R'2 = 0.144Ω, X1 = 0.503Ω, X'2 = 0.209Ω, Xm = 13.5Ω. Assume motor operates under load with slip s = 0.03 (3%).

e) Compute mechanical speed and power

Mechanical speed under load

$$n_{syn} = \frac{2}{p} \cdot 60 \cdot f_e = \frac{120}{6} \cdot 60 = 1200 \text{ rpm}$$

$$n = n_{syn}(1-s) = 1200 \cdot 0.97 = 1164 \text{ rpm}$$

shaft speed

$$\omega_{rm} = \omega_{syn} \cdot (1-s) = 125.66 \cdot 0.97 = 121.89 \text{ r/s}$$

$$P_m = T_m \cdot \omega_{rm}$$

Assume No Friction $T_{fric} = 0 \Rightarrow T_e = T_m$

$$P_m = 65.41 \cdot 121.89 = 7.9 \text{ kW} = \frac{7.9\text{kW}}{746\text{W}} = 10.69 \text{ Hp}$$

no need to convert to Horsepower

Looking ahead…

- Continue **Module 5**