ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 21

March 26, 2026

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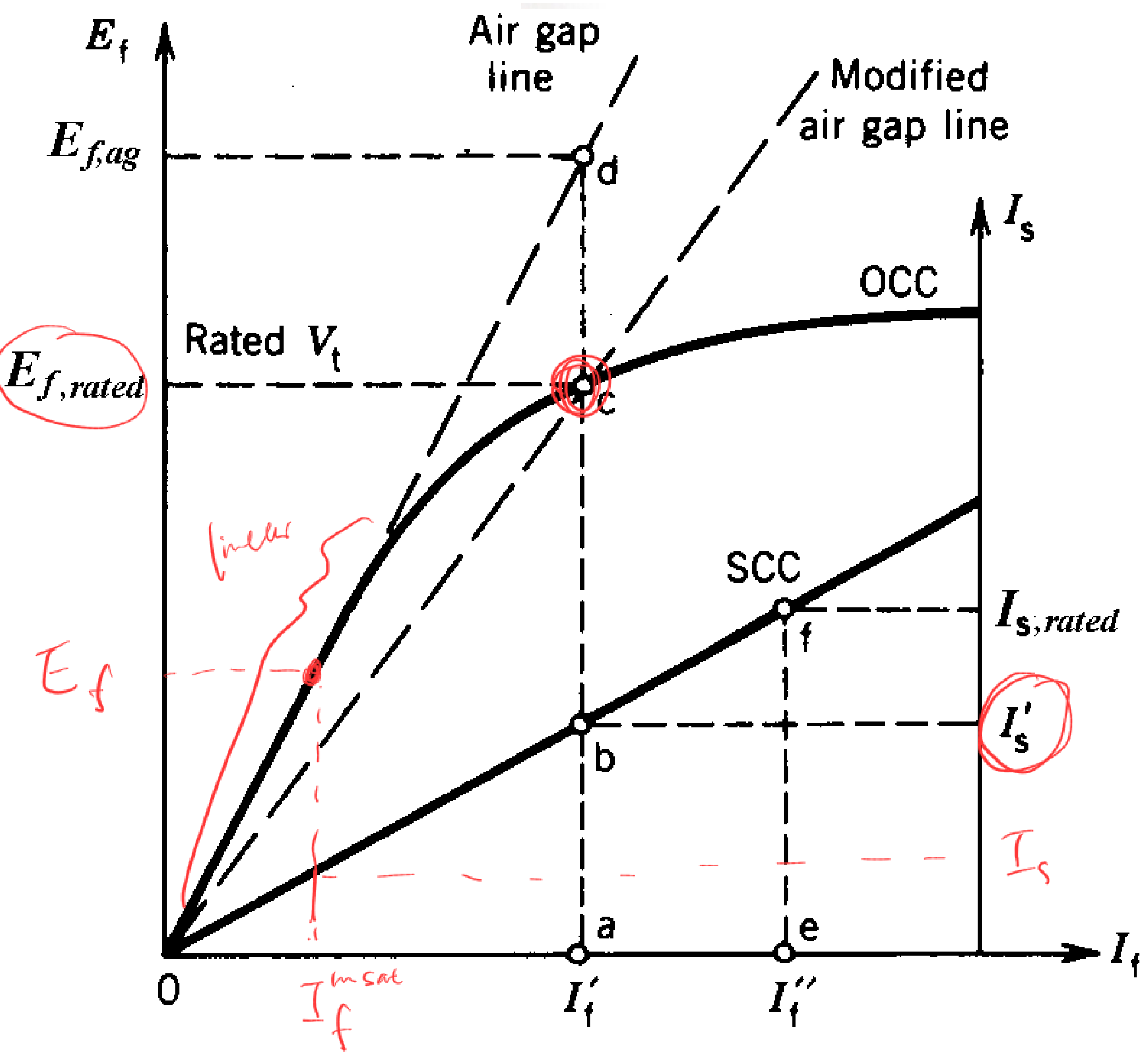
Course Logistics

- Lab 4 takes place this week
- Lab 5 Monday section moved to Wednesday (Apr. 8) at 10:30 AM
  - Only Monday section is modified, Friday sections are not impacted
- Quiz 4 takes place at the beginning of lecture on Apr. 2
- Assignment 6 is posted

Synchronous Reactance



$$E_f = (R_s + jX_s) I_s = Z_s I_s$$

$$\approx X_s I_s \quad \text{(in terms of magnitude)}$$

Unsaturated Syn. Reactance:

$$Z_{s,unsat} = \frac{E_{f,ag}}{I_s'}$$

$$X_{s,unsat} = \sqrt{Z_{s,unsat}^2 - R_s^2}$$

Saturated Syn. Reactance:

$$Z_{s,sat} = \frac{E_{f,rated}}{I_s'}$$

$$X_{s,sat} = \sqrt{Z_{s,sat}^2 - R_s^2}$$

Example 1:

Consider Synchronous Machine: S = 45kVA, 6-pole, 60Hz, 220V (line-to-line), Y-connected. Rated armature (stator) current Is = 118A.

OC Test

V = 202V and If = 2.2A (before saturation, Air-gap line)

V = 220V and If = 2.84A (nominal, saturated)

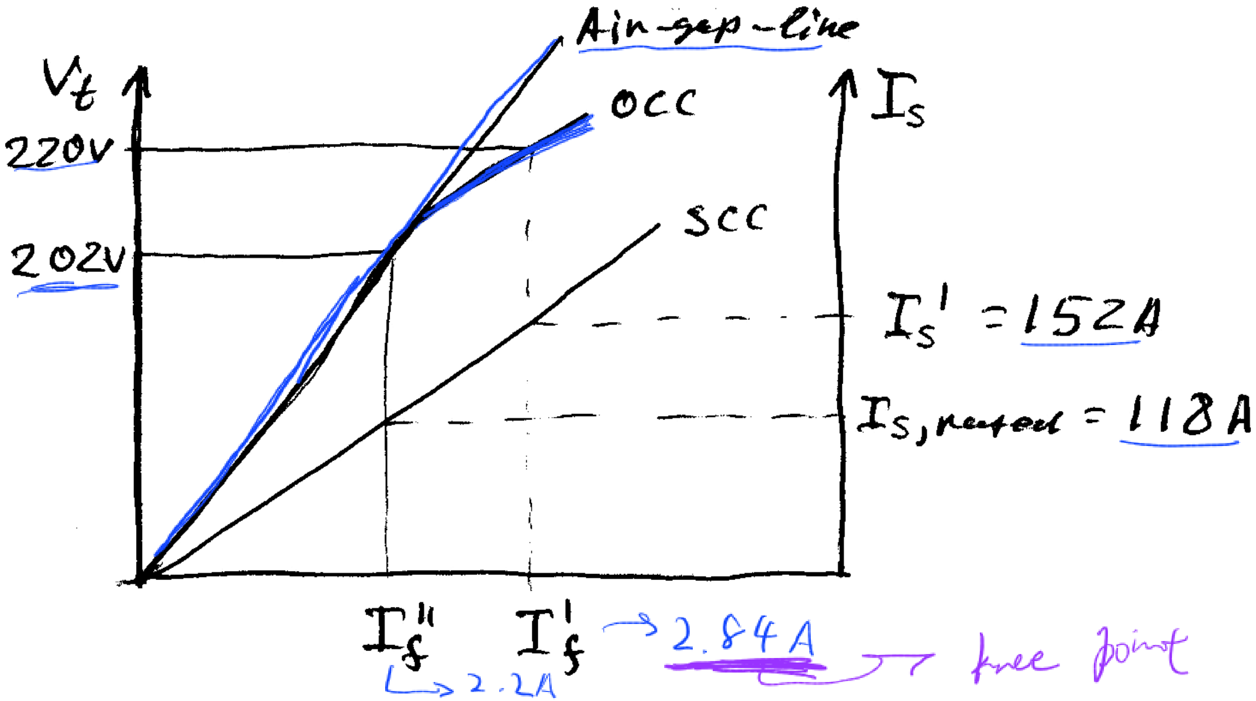
voltage

SC Test

Is = 118A and If = 2.2A (rated armature current is reached)

Is = 152A and If = 2.84A (field current corresponds to rated OC voltage)

Compute: speed n (in rpm), Xs,u (unsaturated), Xs (saturated), and SCR



Assumption: $X >> R$

neglected

Example 1:

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OC Test

V = 202V and If = 2.2A (before saturation, Air-gap line)

V = 220 and If = 2.84A (nominal, saturated)

SC Test

Is = 118A and If = 2.2A (rated armature current is reached)

Is = 152A and If = 2.84A (field current corresponds to rated OC voltage)

Compute: speed n (in rpm), Xs,u (unsaturated), Xs (saturated), and SCR

1) Synchronous Speed

$$n = \frac{120}{p} \cdot f_e = \frac{120}{6} \cdot 60 = \boxed{1200\,\text{rpm}}$$

$X >> R$

2) In Ohms per-phase:

$$V_{s,ag} = \frac{202}{\sqrt{3}} = 116.7\,V; \quad V_{s,rated} = \frac{220}{\sqrt{3}} = 127\,V$$

$$X_{s,u} = \frac{V_{s,ag}}{I_{s,rated}} = \frac{116.7}{118} = 0.987\,\Omega$$

$$X_s = \frac{V_{s,rated}}{I_s'} = \frac{127}{152} = 0.836\,\Omega$$

$$X_s < X_{s,u}$$

$$\text{Short-Circuit-Ratio } SCR = \frac{I_f'}{I_f''} = \frac{2.84}{2.2} = 1.29$$

Example 1:

Consider Synchronous Machine: S = 45kVA, 6-pole, 60Hz, 220V (line-to-line), Y-connected. Rated armature (stator) current Is = 118A.

OC Test

V = 202V and If = 2.2A (before saturation, Air-gap line)

V = 220 and If = 2.84A (nominal, saturated)

SC Test

Is = 118A and If = 2.2A (rated armature current is reached)

Is = 152A and If = 2.84A (field current corresponds to rated OC voltage)

Compute: **SCR in pu**

3) In pu Select base values:

$V_b = 220\text{ V}, \; I_b = 118\text{ A}$

$V_{s,ag} = \frac{202}{V_b} = \frac{202}{220} = 0.92\,pu$

$X_{s,u} = \frac{V_{s,ag}}{I_{s,rated}} = \frac{0.92\,pu}{1\,pu} = 0.92\,pu$

$I_s' = \frac{152\text{ A}}{I_b} = \frac{152}{118} = 1.29\,pu$

$X_s = \frac{V_{s,rated}}{I_s'} = \frac{1\,pu}{1.29\,pu} = 0.775\,pu$

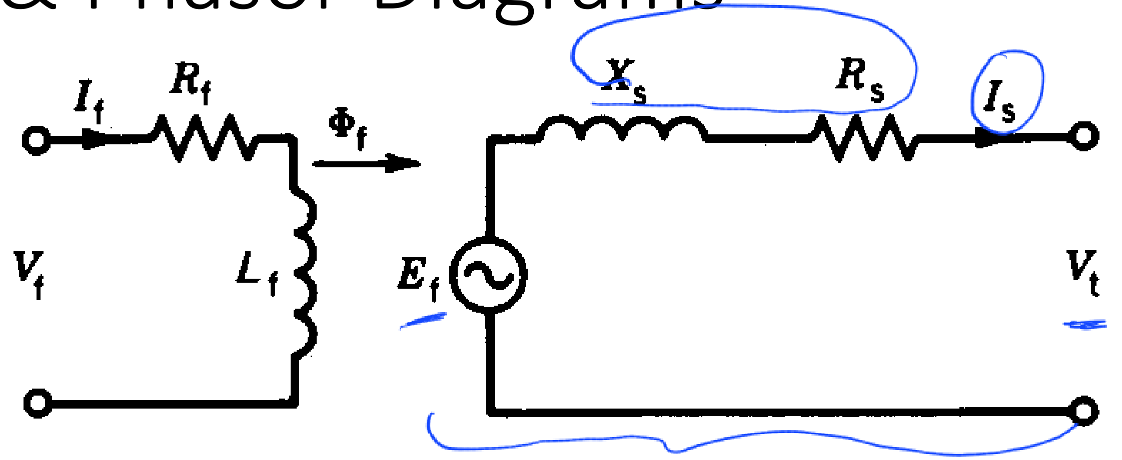
Saturated impedance

$SCR = \frac{1}{X_s} = \frac{1}{0.775} = 1.29$

$SCR = \frac{1}{X_{pu}}$

Equivalent Circuit & Phasor Diagrams

Field (excitation) winding



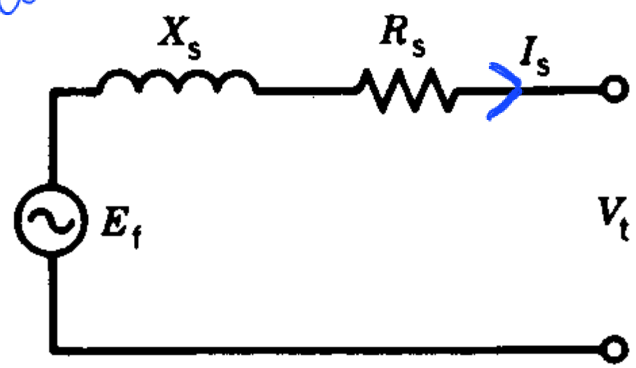
Stator (Armature) winding

Stator Voltage Equation $V_t = \pm(R_s + jX_s)I_s + E_f$

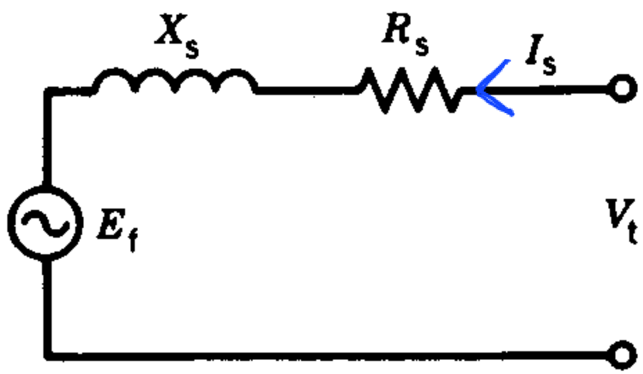
depends on motor or generator

Generator

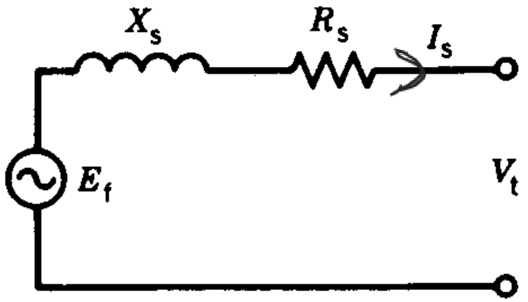
coupling field



Motor



Basic Phasor Diagram - Generator



$$V_t = E_f - (R_s + jX_s)I_s$$

$E_f = V_t + (R_s + jX_s) I_s$

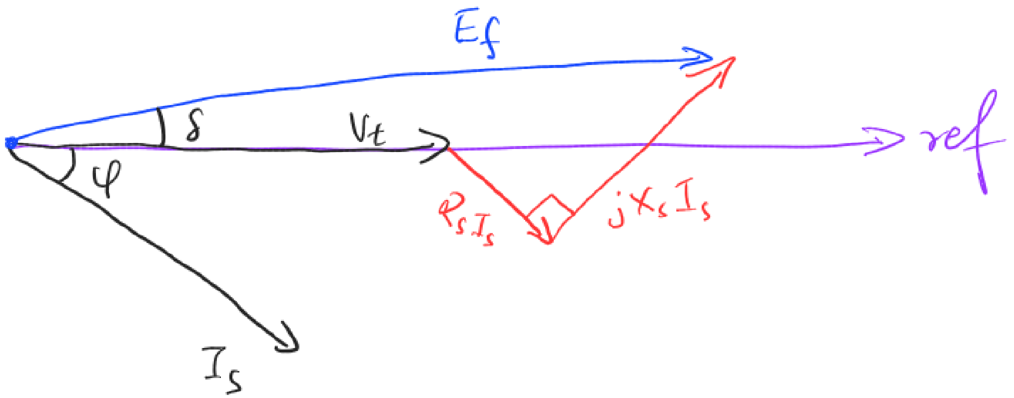
Not the same

ϕ - Power factor angle, angle between V_t and I_s

δ - Angle between V_t and E_f

$\delta = \theta_r - \theta_e$

Assume **lagging** pf



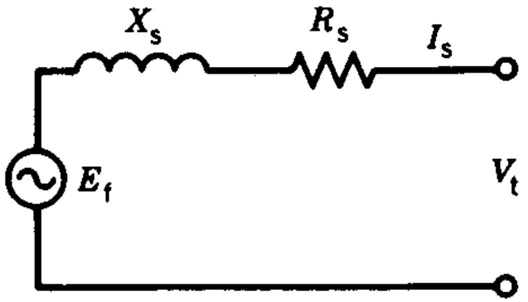
φ

① I_s lags V_t

② E_f leads V_t

δ rotor stator

Basic Phasor Diagram - Generator



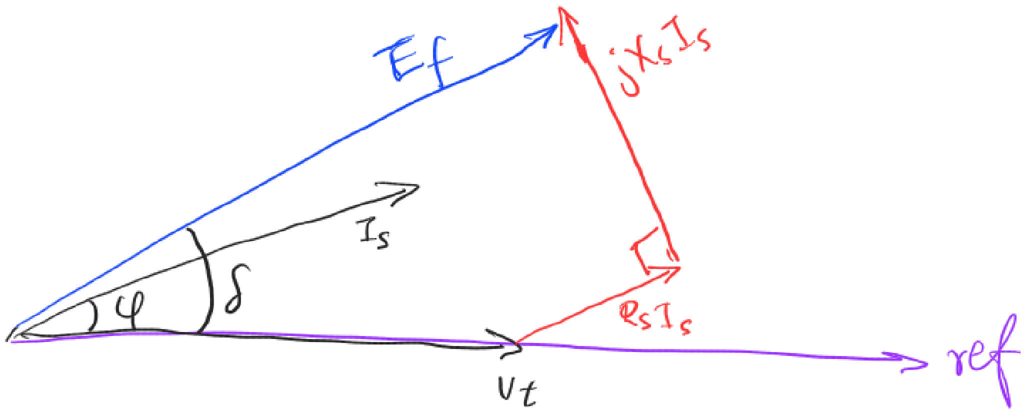
$$V_t = E_f - (R_s + jX_s)I_s$$

$$E_f = V_t + (R_s + jX_s)I_s$$

ϕ - Power factor angle, angle between V_t and I_s

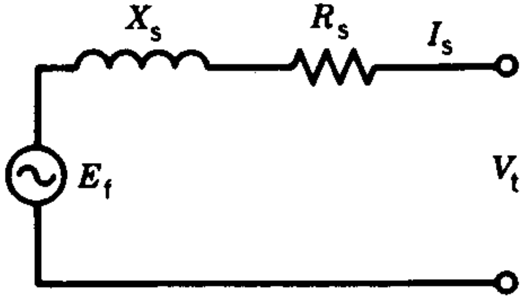
δ - Angle between V_t and E_f

Assume **leading** pf



Basic Phasor Diagram - Motor

Ef lags Vt

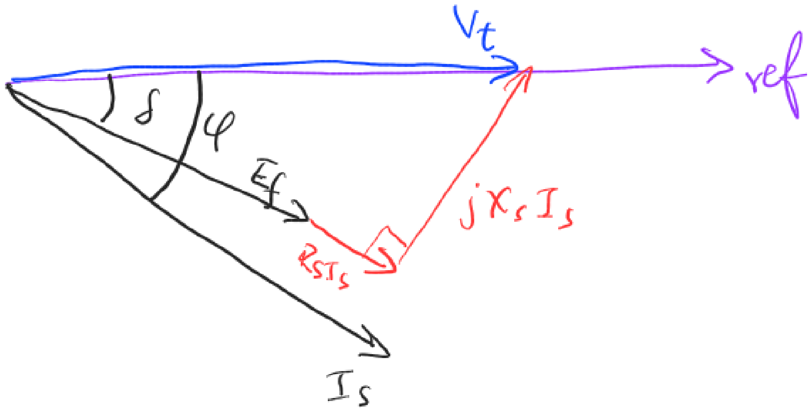


$$V_t = (R_s + jX_s)I_s + E_f$$

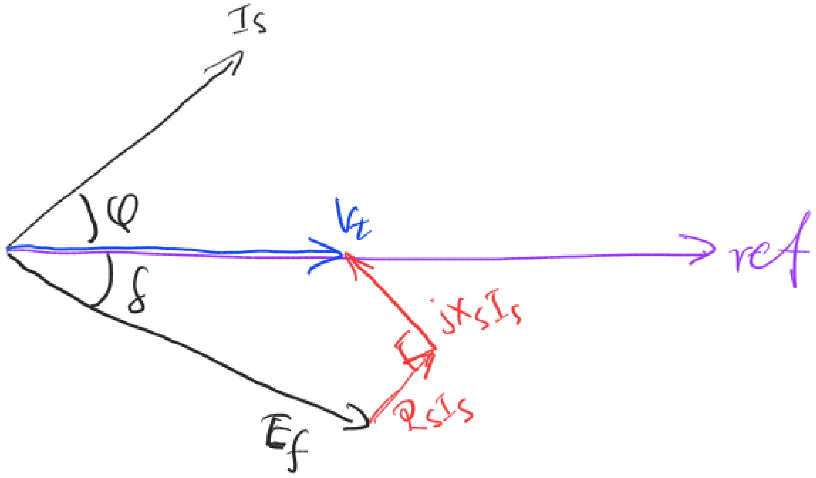
ϕ - Power factor angle, angle between V_t and I_s

δ - Angle between V_t and E_f

Assume **lagging** pf



Assume **leading** pf



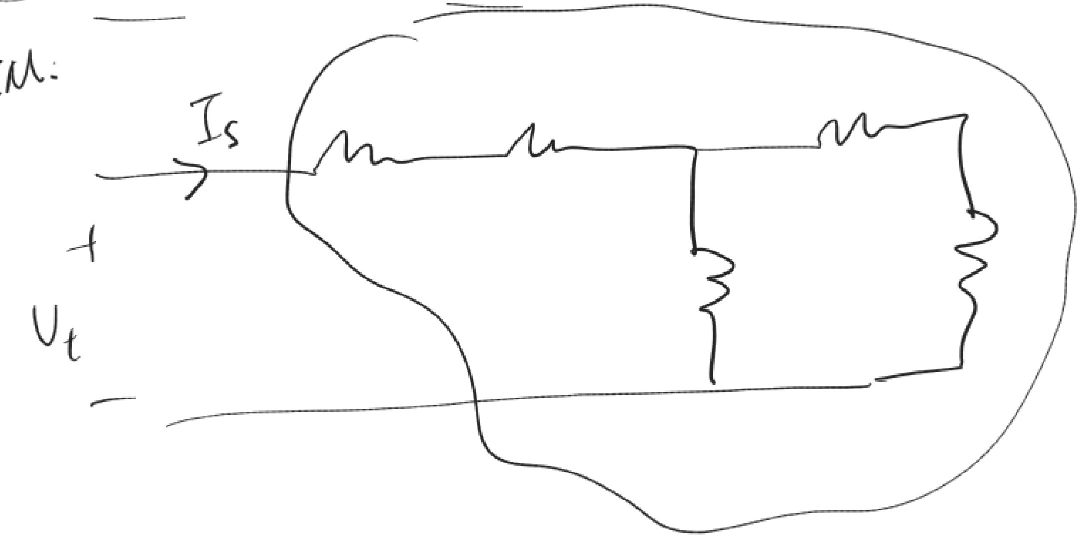
Summary of phasor diagrams

(1) I_s and V_t : $\varphi \longrightarrow$ inductive or capacitive behaviour

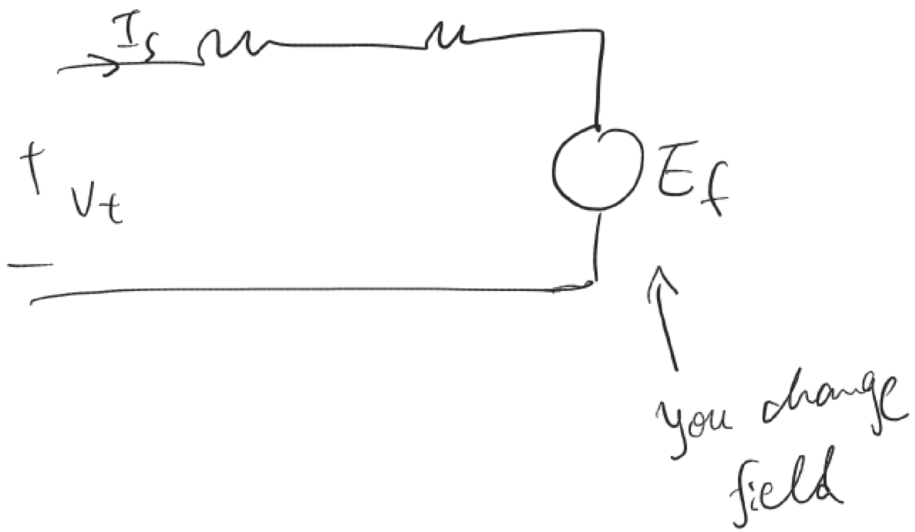
(2) E_f and V_t : δ

Independent control of E_f

IM:

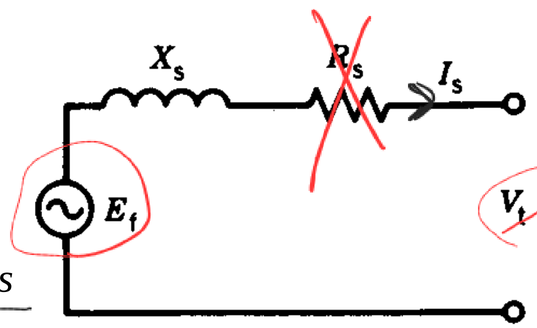


SM:



Real Power & Torque

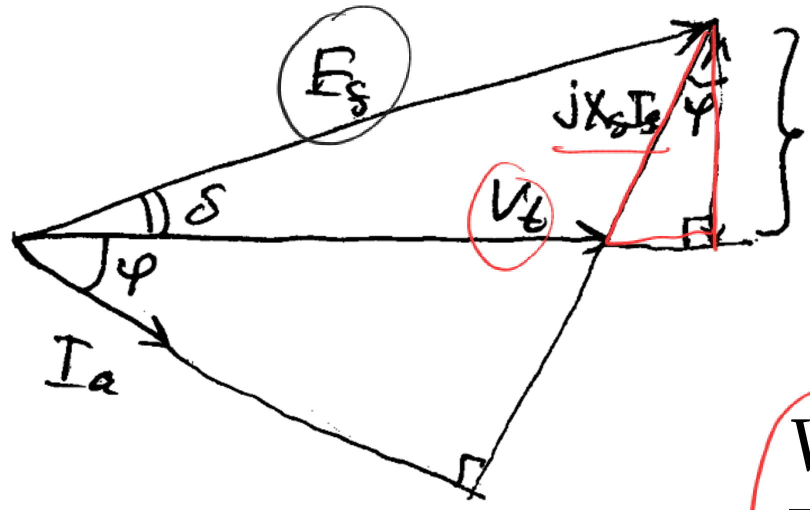
- Assume generator with lagging pf
- Neglect R_s



$$E_f = V_t + jX_s I_s$$

$P = VI\cos(\varphi)$

Generator with lagging pf →



Consider magnitudes, rms values

$$E_f \cdot \sin\delta = X_s \cdot I_s \cdot \cos\varphi$$ ← V_t on both sides

$$V_t E_f \sin(\delta) = V_t X_s I_s \cos(\varphi)$$ → P

$$\frac{V_t E_f}{X_s}\sin(\delta) = V_t I_s \cos(\varphi) = P_{per-phase}$$

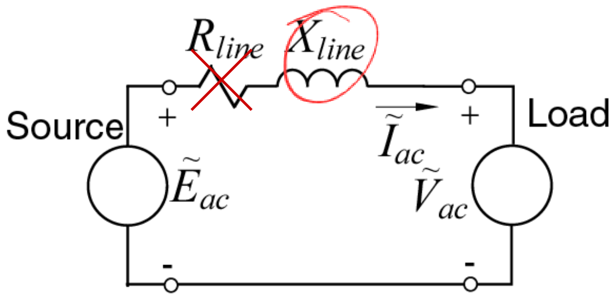
→ rotor and stator misalignment

Three-phase power

$$P_{3\phi} = 3\frac{V_t E_f}{X_s}\sin(\delta)$$

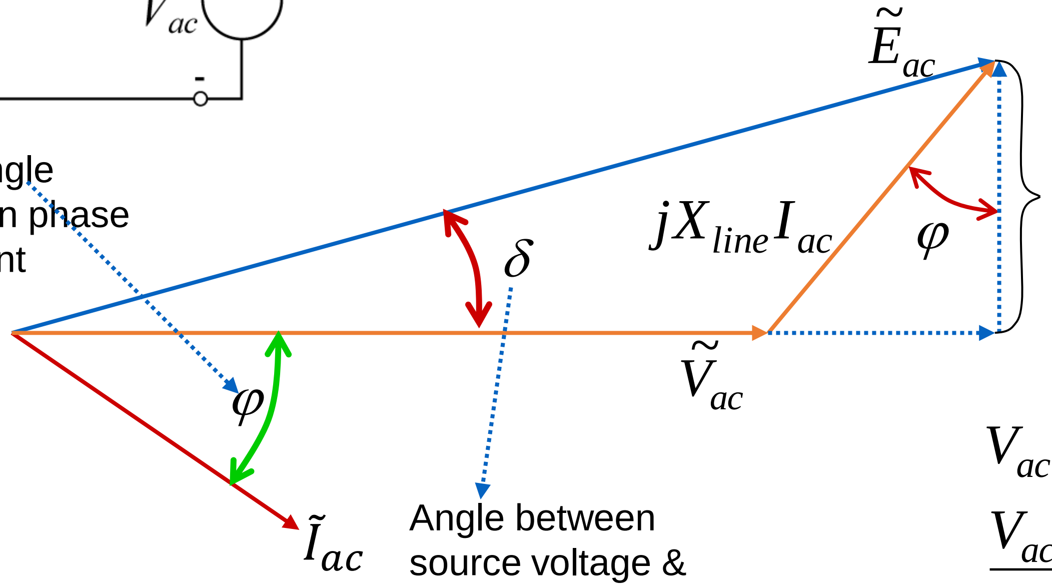
From Lecture 3: How to transmit power

- **For AC System**
  - Lines are mostly inductive $X_{line} >> R_{line}$
  - Assume lagging power factor (inductive load)



Voltage Equation & Phasor Diagram

$$E_{ac} \approx V_{ac} + jX_{line}I_{ac}$$



Power factor angle = angle between phase voltage & current

Angle between source voltage & load voltage

$$E_{ac}\sin(\delta) = X_{line}I_{ac}\cos(\phi)$$

How do we get an expression of real power from this?

$$V_{ac}E_{ac}\sin(\delta) = V_{ac}X_{line}I_{ac}\cos(\varphi)$$

$$\frac{V_{ac}E_{ac}}{X_{line}}\sin(\delta) = V_{ac}I_{ac}\cos(\varphi) = P_{transfer}$$

We can reverse the flow of real power from “Load” to “Source” by making the Vac lead Eac

$$P_{transfer} = \frac{V_{ac}E_{ac}}{X_{line}}\sin(\delta)$$

Torque (Power) – Angle Characteristic

- Assume $|V_t| = const,\ |E_f| = const,\ \omega_r = \omega_e = const$
- Neglect losses

$$P_{3\phi} = 3\frac{V_t E_f}{X_s}\sin(\delta) = \omega_{syn} T_e$$

$$T_e = \frac{3}{\omega_{syn}} \cdot \frac{V_t E_f}{X_s}\sin(\delta)$$

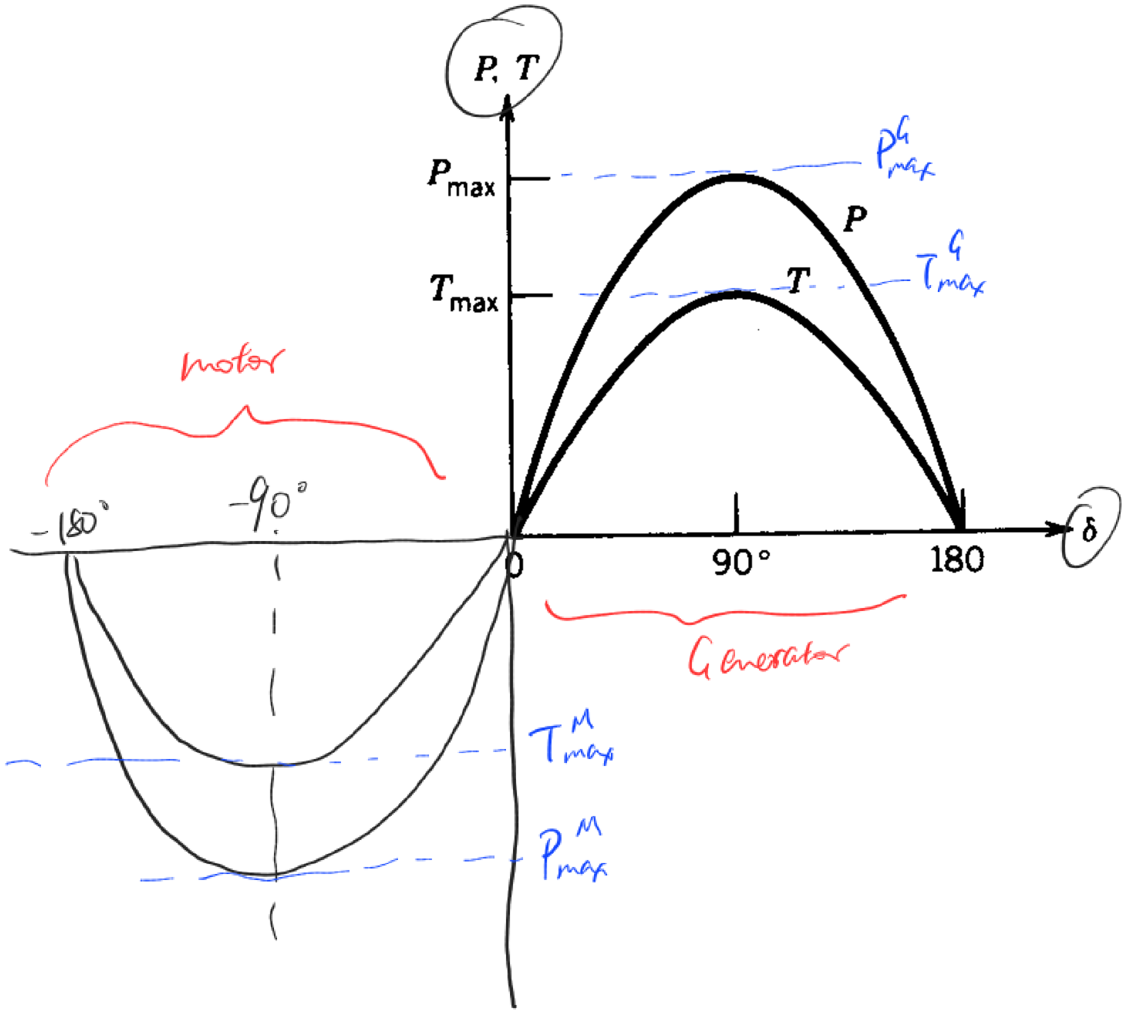
P

$$= \boxed{\left(\frac{P}{2}\right)\left(\frac{3}{\omega_e}\right) \cdot \frac{V_t E_f}{X_s}\sin(\delta)}$$

# of poles

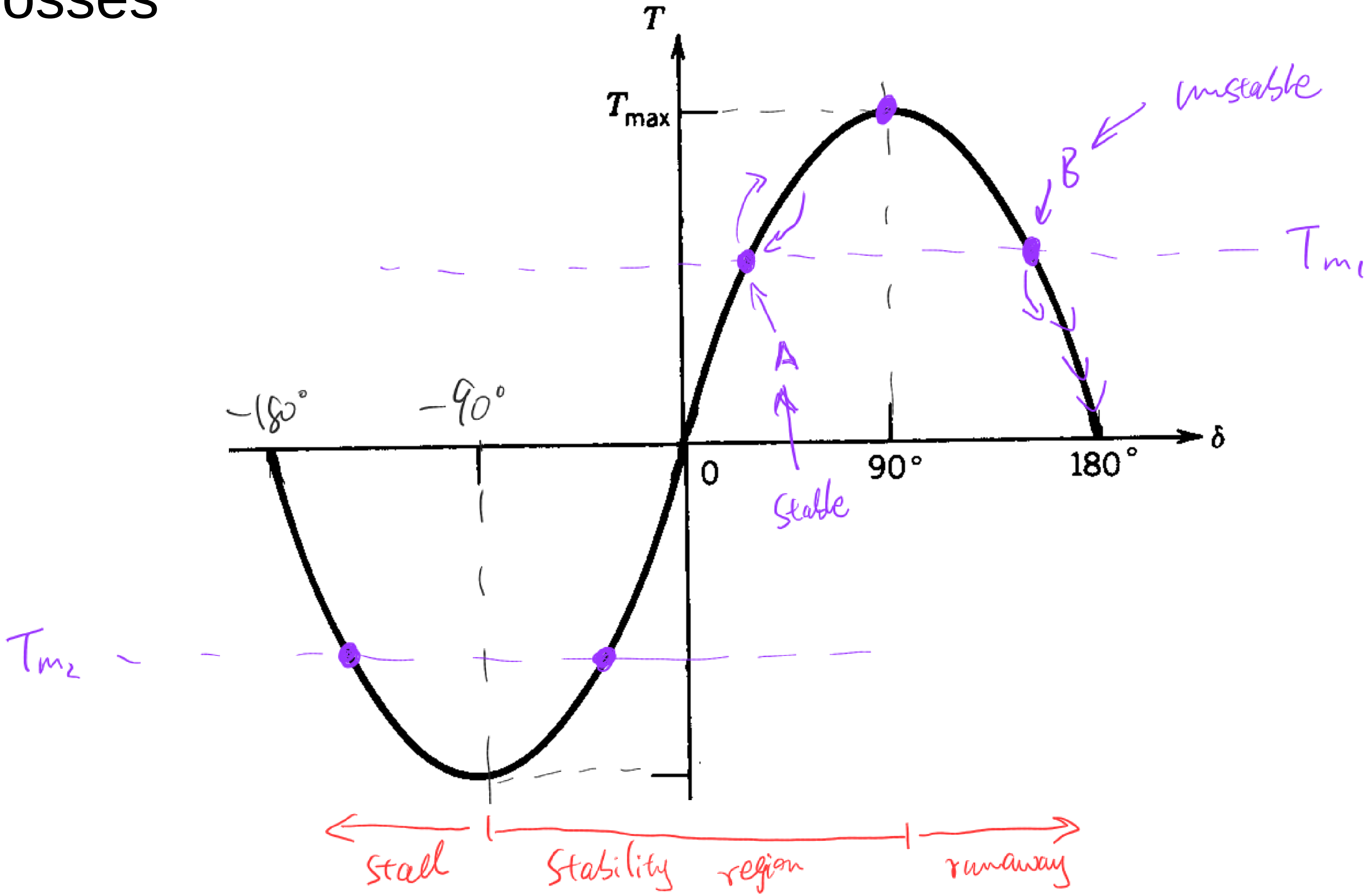
$$\Rightarrow P_{\max} = 3\frac{V_t E_f}{X_s}$$

$$\Rightarrow T_{\max} = \frac{P_{\max}}{\omega_{syn}}$$



Torque (Power) – Stability Limits

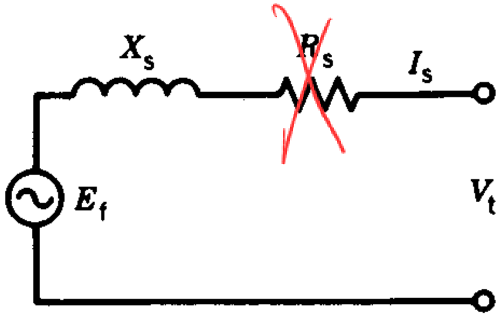
- Assume $|V_t| = const,\ |E_f| = const,\ \omega_r = \omega_e = const$
- Neglect losses



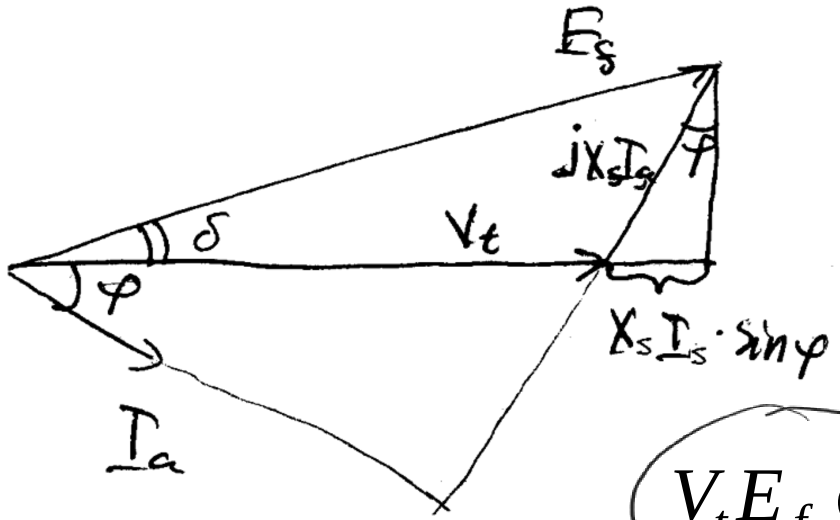
Reactive Power

- Assume generator with lagging pf
- Neglect $\cancel{R_s}$

$Q = VI\sin(\varphi)$



$$E_f = V_t + jX_sI_s$$



Consider magnitudes, rms values

$E_f \cdot \cos\delta = V_t + X_s I_s \cdot \sin\varphi$

$$V_tE_f\cos(\delta) = V_t^2 + V_tX_sI_s\sin(\varphi)$$

Q is not related to torque

real power P is dictated by $T_m \omega_{sync}$

$$\frac{V_tE_f\cos(\delta) - V_t^2}{X_s} = V_tI_s\sin(\varphi) = Q_{per-phase}$$

Three-phase reactive power $$Q_{3\phi} = 3\frac{V_tE_f\cos(\delta) - V_t^2}{X_s}$$

controlling reactive power

Reactive Power Control

- Assume inductive load, lagging pf:

$$Q_{per-phase} = \frac{V_t E_f \cos(\delta) - V_t^2}{X_s} \qquad P_{per-phase} = V_t I_s \cos(\varphi) = const$$

Control I_f one can achieve:

$Q > 0$ Reactive power was supplied to the grid (i.e. to inductive load with lagging pf)

$Q < 0$ Reactive power was absorbed from the grid (i.e. to supply capacitive load with leading pf)

- Assume $P_{per-phase} = V_t I_s \cos(\varphi) = 0$

=> Synchronous Machine with no mechanical load, idle mode

$\delta \approx 0$ and $\cos(\delta) \approx 1$

=> Synchronous Condenser → act as inductor or capacitor in grid

Synchronous Condenser $\delta = \phi \Rightarrow P = \phi$

- Assume $V_t = E_f + jX_sI_s$ (stator resistance is neglected)

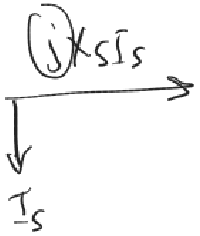
$\hookrightarrow$ constant

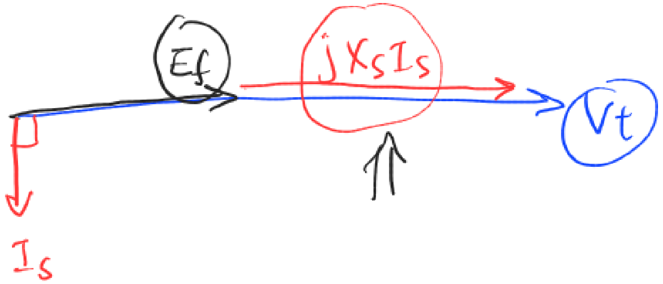
Under-Excited

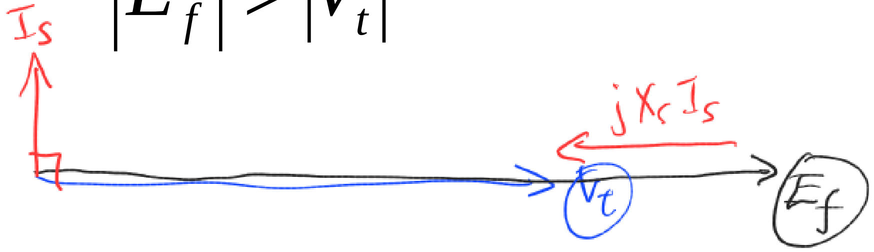
$$|E_f| < |V_t|$$

Over-Excited

$$|E_f| > |V_t|$$





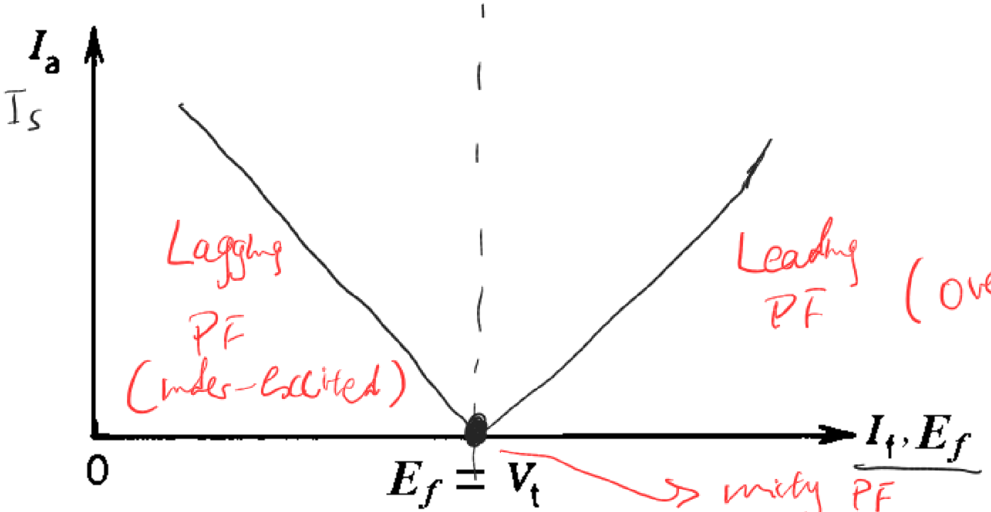


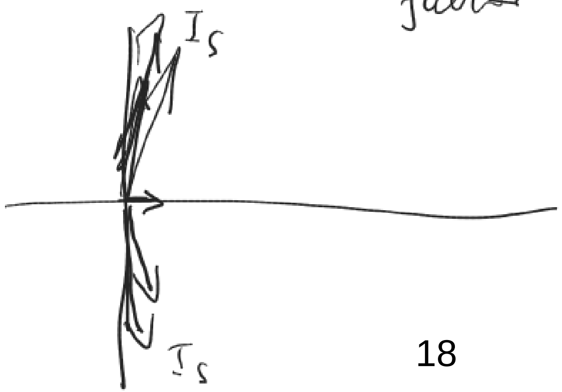
V – Characteristic of the Synchronous Condenser

shape of the curve

$E_f = V_t$

$\hookrightarrow$ unity power factor





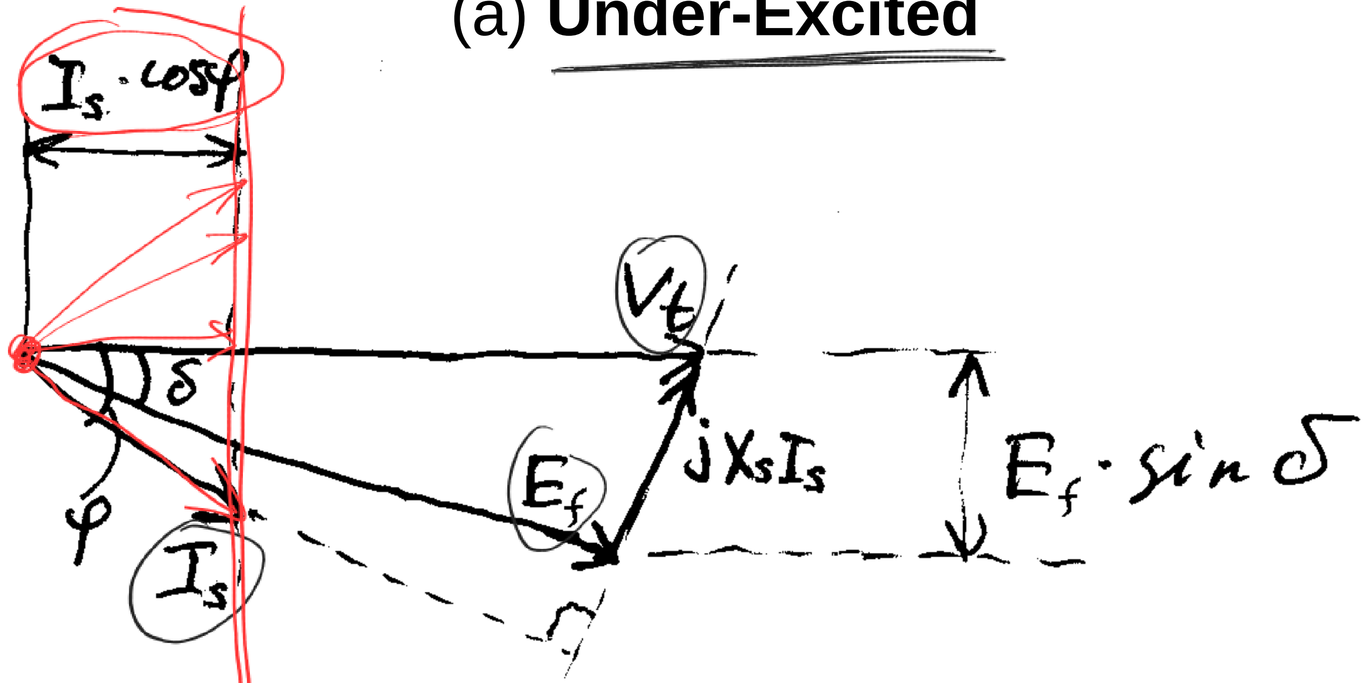
Synchronous Motor – PF Control

- Assume $V_t = E_f + jX_s I_s$ (stator resistance is neglected)
- Assume constant mechanical load $P_{1\phi} = \dfrac{V_t E_f}{X_s}\sin(\delta) = const$

=> $V_t E_f \sin(\delta) = const$

$V_t I_s \cos(\varphi) = const$

(a) **Under-Excited**

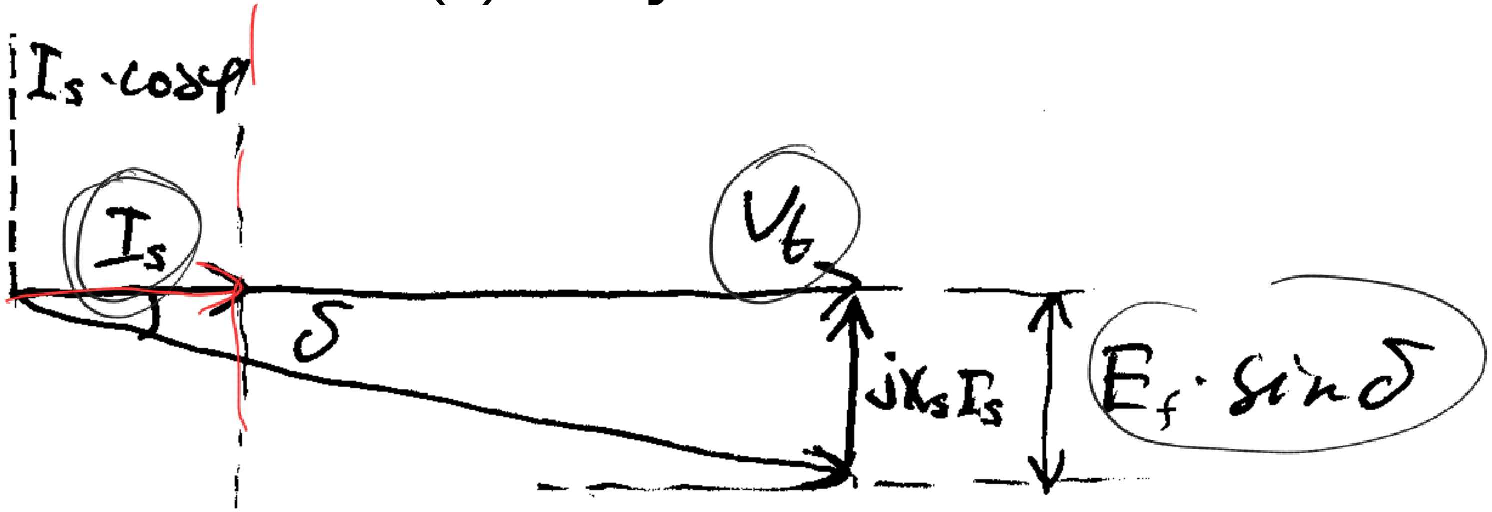


Synchronous Motor – PF Control

- Assume $V_t = E_f + jX_s I_s$ (stator resistance is neglected)
- Assume constant mechanical load $P_{1\phi} = \dfrac{V_t E_f}{X_s}\sin(\delta) = const$

$$\Rightarrow \begin{aligned} V_t E_f \sin(\delta) &= const \\ V_t I_s \cos(\varphi) &= const \end{aligned}$$

(b) **Unity Power Factor**

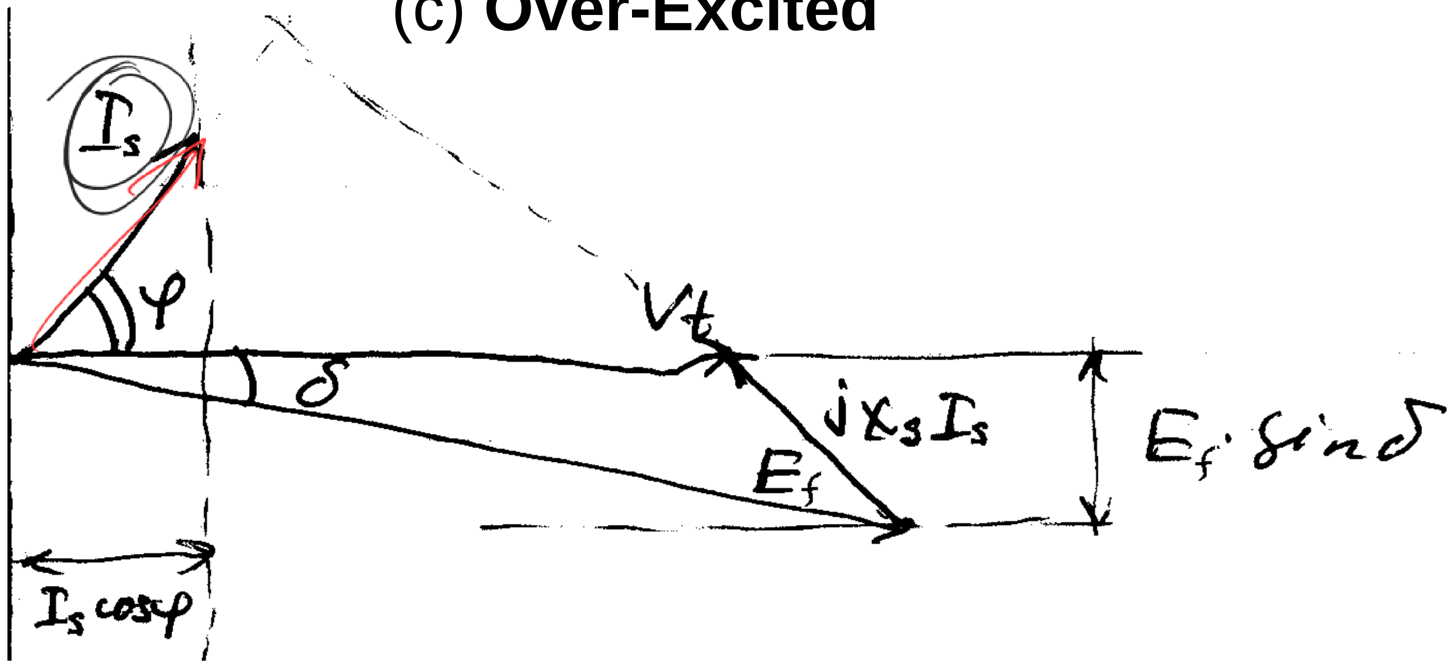


Synchronous Motor – PF Control

- Assume $V_t = E_f + jX_s I_s$ (stator resistance is neglected)
- Assume constant mechanical load $P_{1\phi} = \frac{V_t E_s}{X_s}\sin(\delta) = const$

$$\Rightarrow \begin{array}{l} V_t E_f \sin(\delta) = const \\ V_t I_s \cos(\varphi) = const \end{array}$$

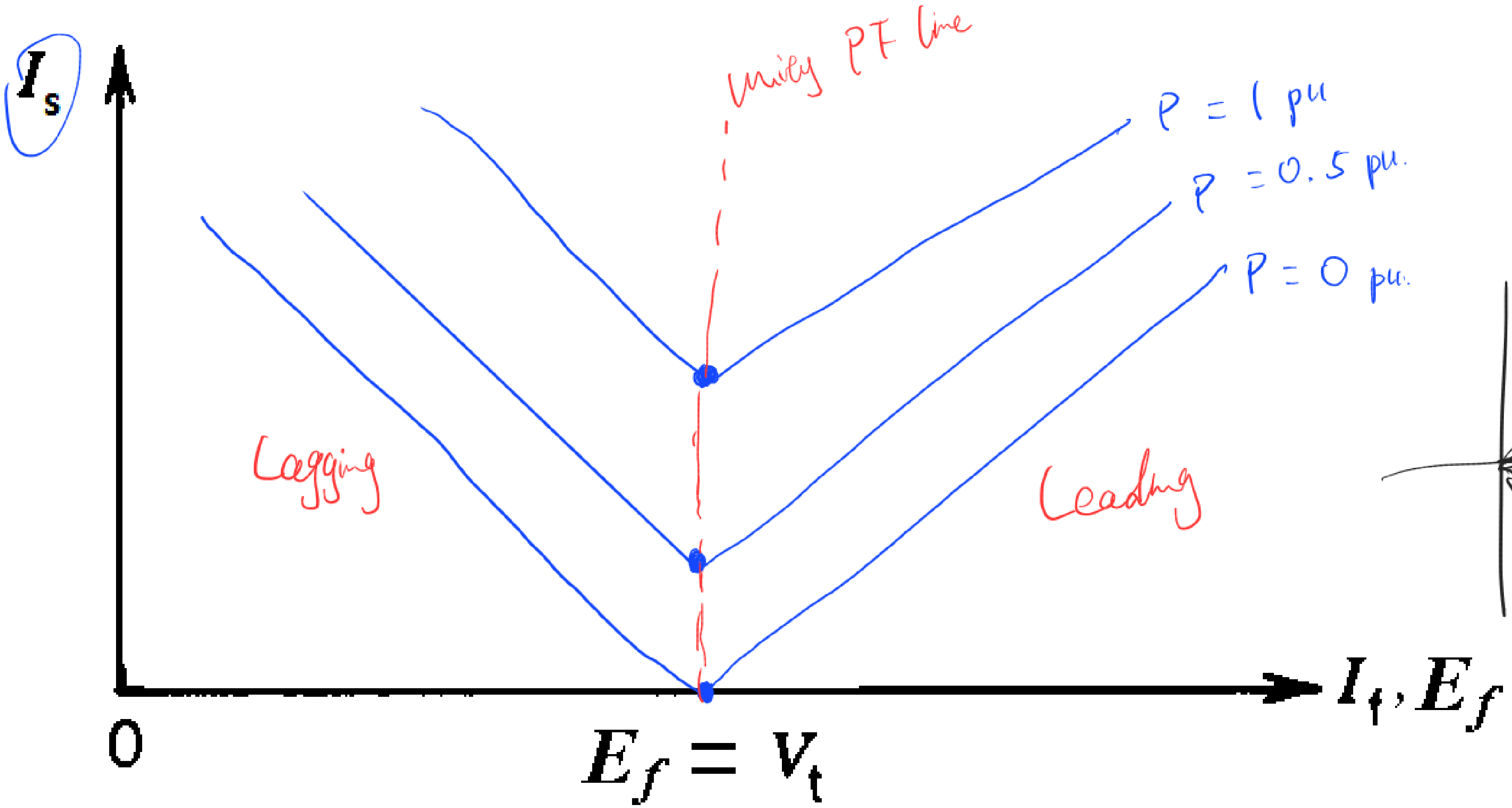
(c) **Over-Excited**

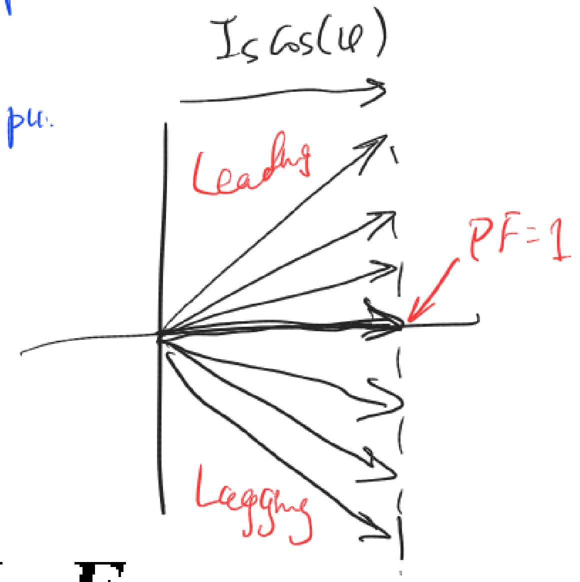


Synchronous Motor – PF Control

- Assume $V_t = E_f + jX_s I_s$ and $P_{1\phi} = \frac{V_t E_f}{X_s}\sin(\delta) = const$

V – Characteristic

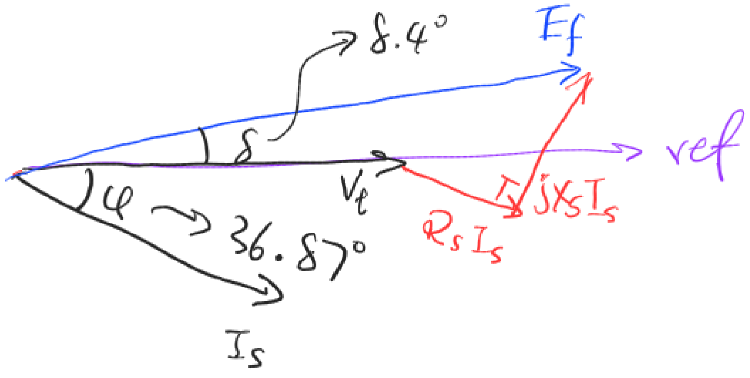




Example 2:

Three-Phase Synchronous Generator: S = 1250kVA, 10-pole, 60Hz, 4.16kV (line-to-line), Y-connected, and Rs = 0.123Ω, Xs = 3Ω (per-phase)

Find generated voltage Ef at full load and 0.8 lagging pf



Step 1: phasor diagram

Step2:

$$\underline{I_s} = \frac{S}{\sqrt{3}V_L} = \frac{1250{,}000}{\sqrt{3}\cdot 4160} = 173.48\,A$$

$$\cos\varphi = 0.8 = pf \Rightarrow \varphi = a\cos(0.8) = 36.87^\circ$$

Assume $V_t = \frac{V_L}{\sqrt{3}}\angle 0^\circ$; $I_s = 173.48\angle -36.87^\circ$

$$\underline{Z_s} = R_s + jX_s = 0.123 + j3 = 3.0026\angle 87.6^\circ$$

$$E_f = V_t + \underline{I_s \cdot Z_s} = \frac{4160}{\sqrt{3}} + (173.48\angle -36.87^\circ)\times$$

$$\times(3.0026\angle 87.6^\circ) = \underline{2761.14\angle 8.4^\circ}$$

$|E_f| = \underline{2761.14\,V\,(rms)}$ $\underline{\delta = 8.4^\circ}$

Looking ahead…

- Continue **Module 6**