ELEC 342
Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 22

March 31, 2026

Zhi Jin (Justin) Zhang

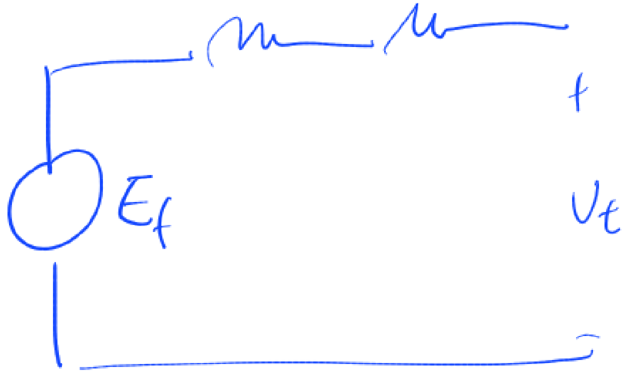
Course Logistics

- Lab 5 Monday section moved to Wednesday (Apr. 8) at 10:30 AM
  - Only Monday section is modified, Friday sections are not impacted
- Quiz 4 takes place at the beginning of lecture on Apr. 2
  - You can leave after quiz
- Assignment 6 is posted
- SEI survey: March 28 – April 13
- Final exam: Apr. 19 @ 12 PM in IRC 4

Synchronous Motor – PF Control

Constant V_t and P

- Assume $V_t = E_f + jX_s I_s$ (stator resistance is neglected)
- Assume constant mechanical load $P_{1\phi} = \dfrac{V_t E_f}{X_s}\sin(\delta) = const$

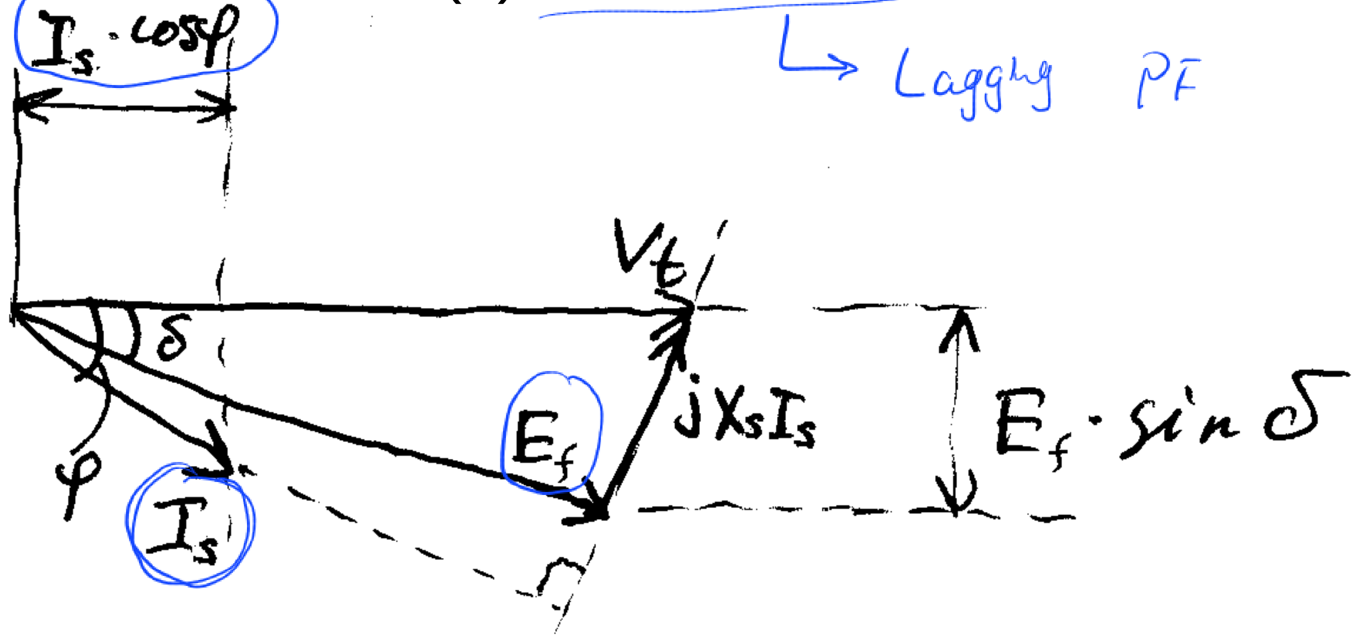


$$\Rightarrow \begin{array}{l} V_t E_f \sin(\delta) = const \\ V_t I_s \cos(\varphi) = const \end{array}$$

$P = VI\cos(\varphi)$

(a) **Under-Excited**

↳ Lagging PF



Synchronous Motor – PF Control

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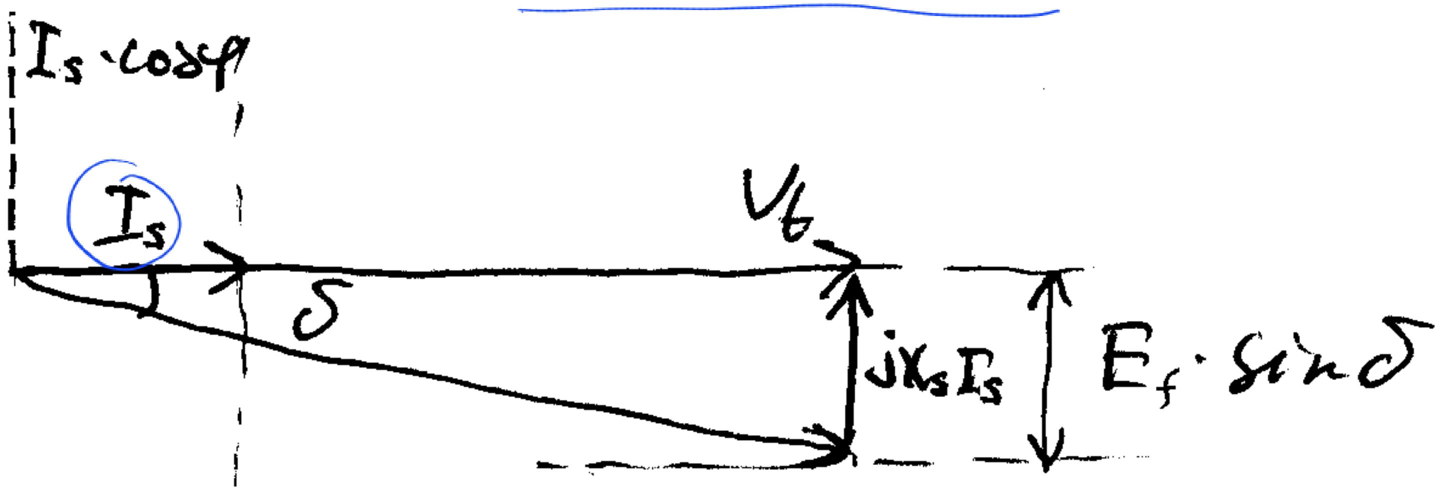
=> $V_t E_f \sin(\delta) = const$

$V_t I_s \cos(\varphi) = const$

(b) **Unity Power Factor**



Synchronous Motor – PF Control

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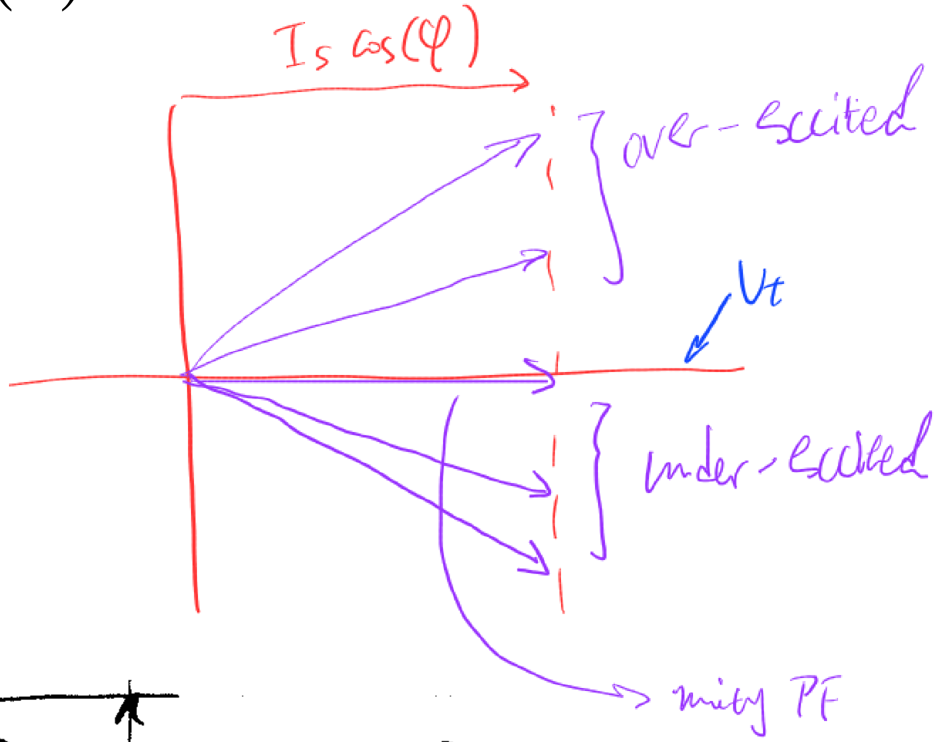
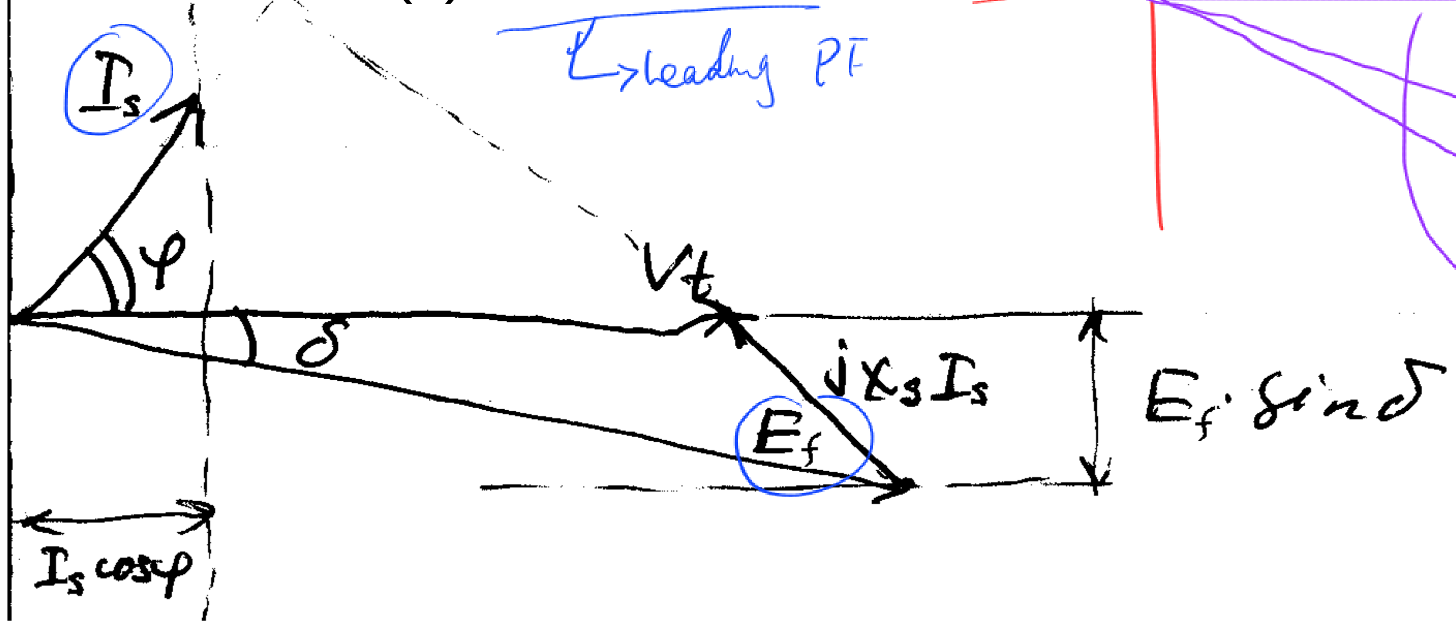
=> $V_t E_f \sin(\delta) = const$

$V_t I_s \cos(\varphi) = const$

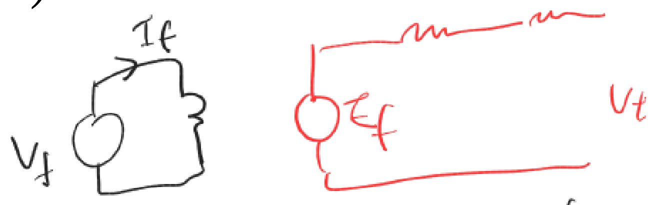


(c) **Over-Excited**



Synchronous Motor – PF Control

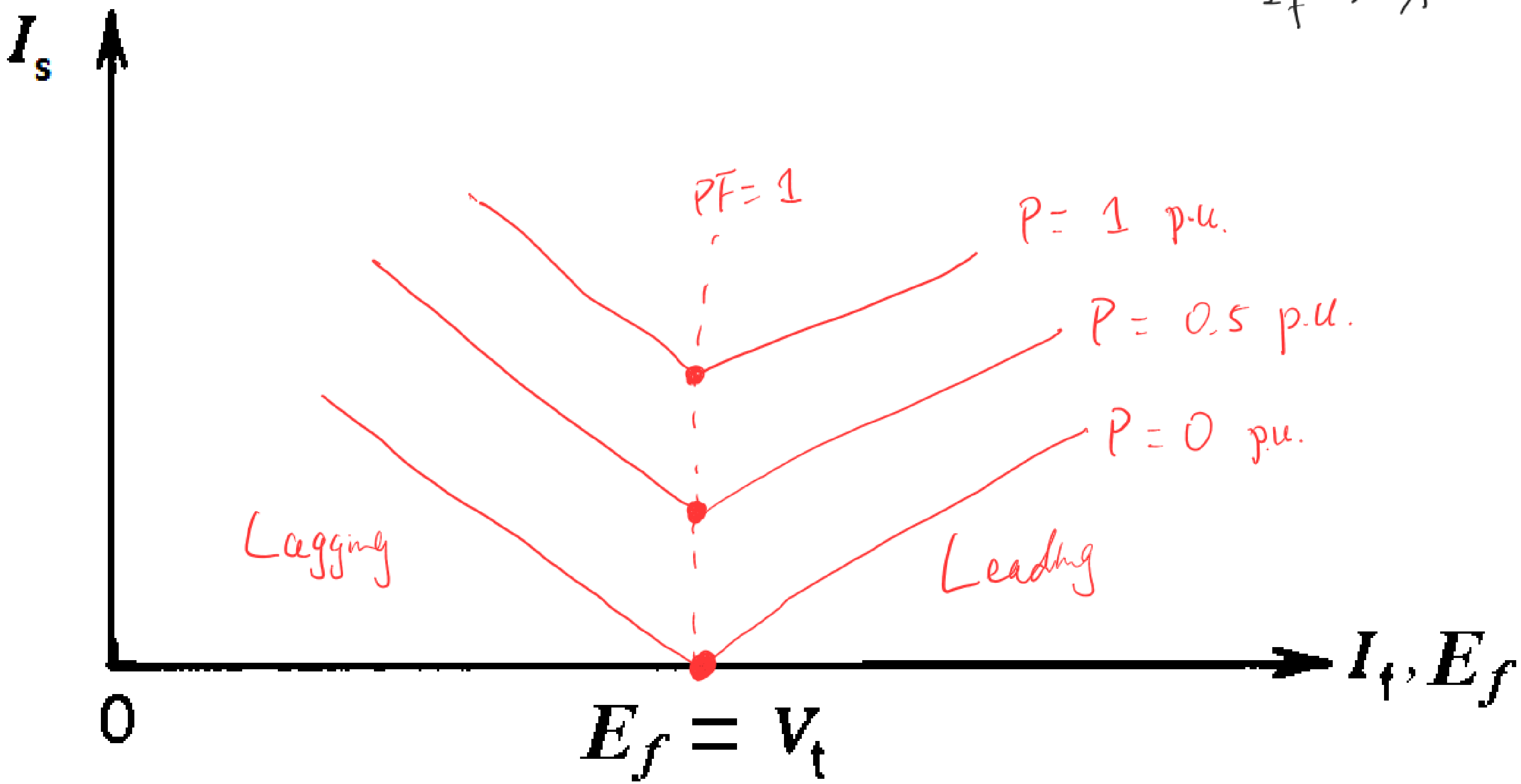
- Assume $V_t = E_f + jX_s I_s$ and $P_{1\phi} = \dfrac{V_t E_f}{X_s}\sin(\delta) = const$



$I_f \to \lambda \to \frac{d\lambda}{dt} = E_f$

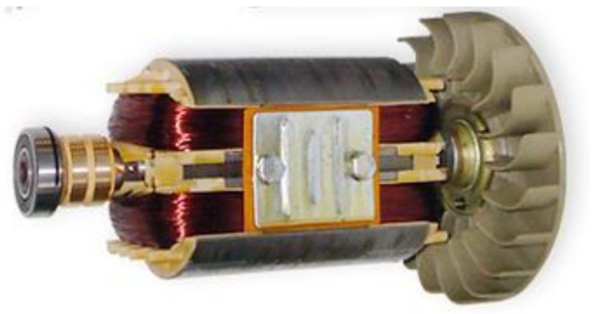
V – Characteristic

shape of curve

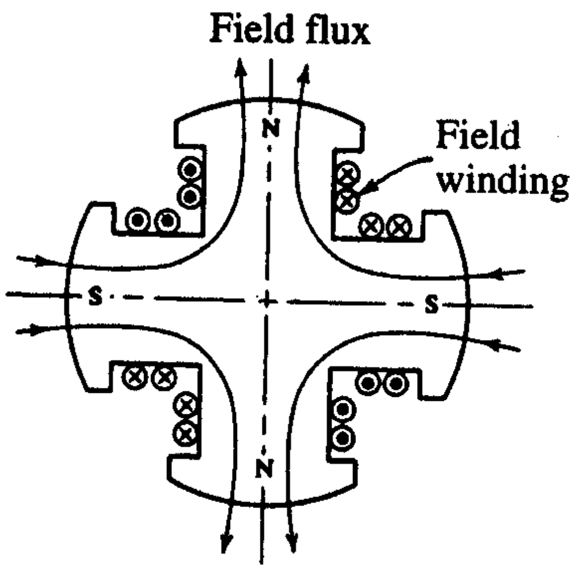


Salient Rotor Synchronous Machines

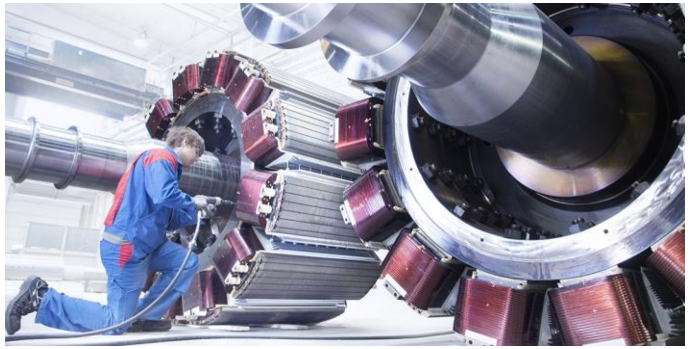
Most high-pole count rotors have salient structure



2-Pole Salient Rotor

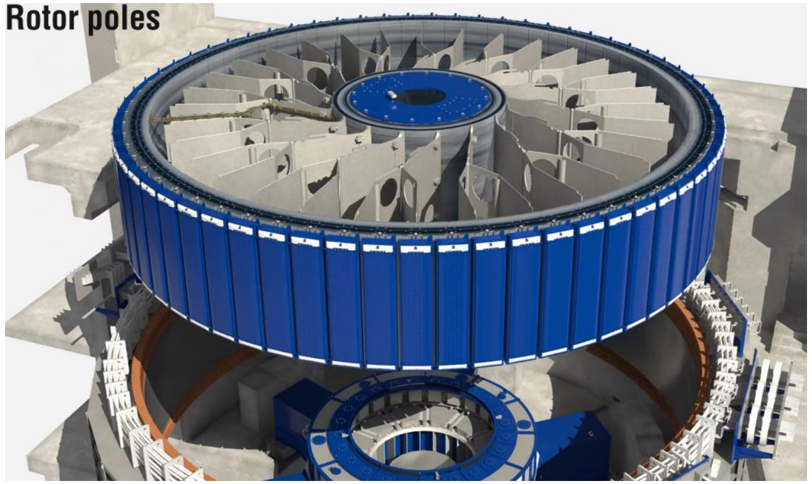


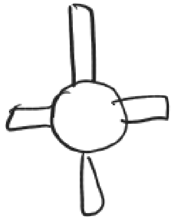
4-Pole Salient Rotor



12-Pole Salient Rotor

64-Pole Salient Rotor Hydro Generator





Salient SM: easy to construct
for large number of poles

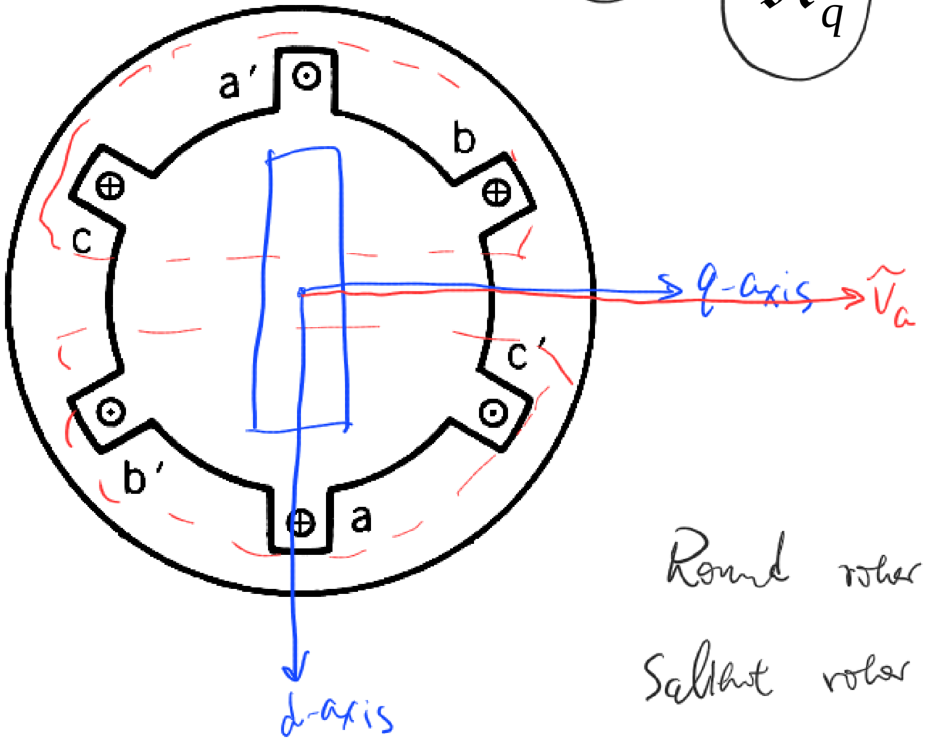
↳ Harder to analyze compared
to round rotor SM

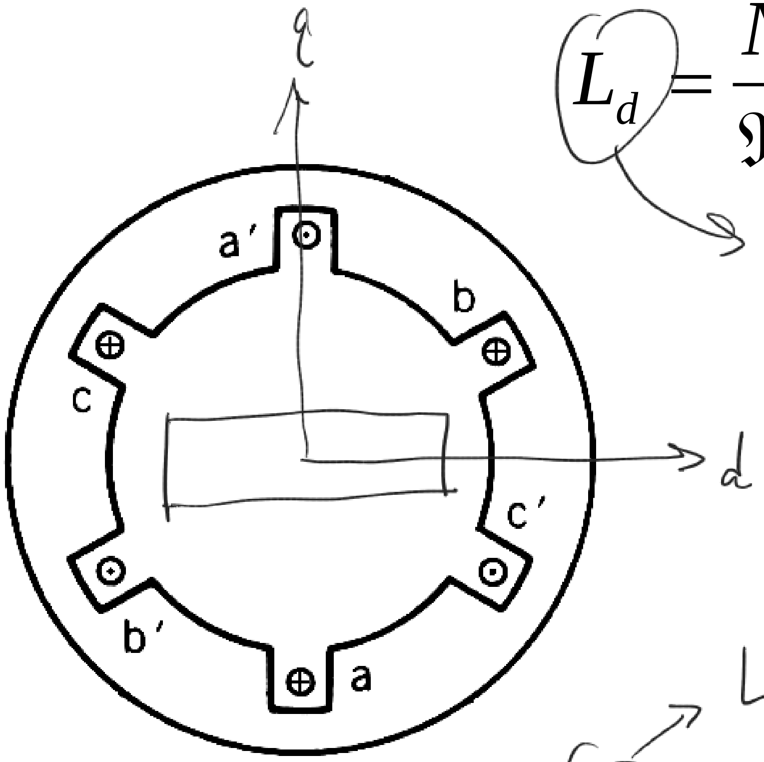
Salient-Pole Machines

⇝ L varies depending on rotor orientation

Smaller L_q ← $L_q = \frac{N^2}{\Re_q}$ → larger R_q due to larger airgap

$L_d = \frac{N^2}{\Re_d}$ → larger than L_q

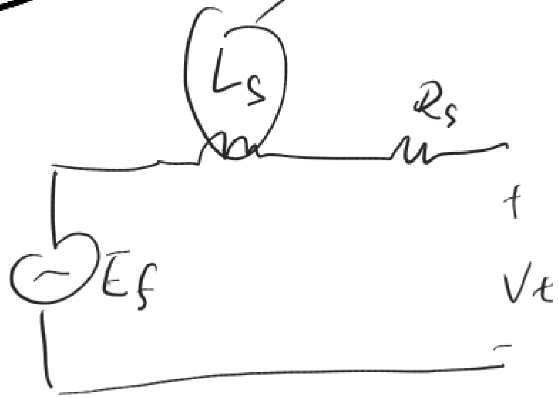




Round rotor : $L_d = L_q$

Salient rotor : $L_d > L_q$ (generally true)

$L_d = L_q$



$X_q = X_{mq} + X_{ls}$ - Quadrature axis synchronous reactance

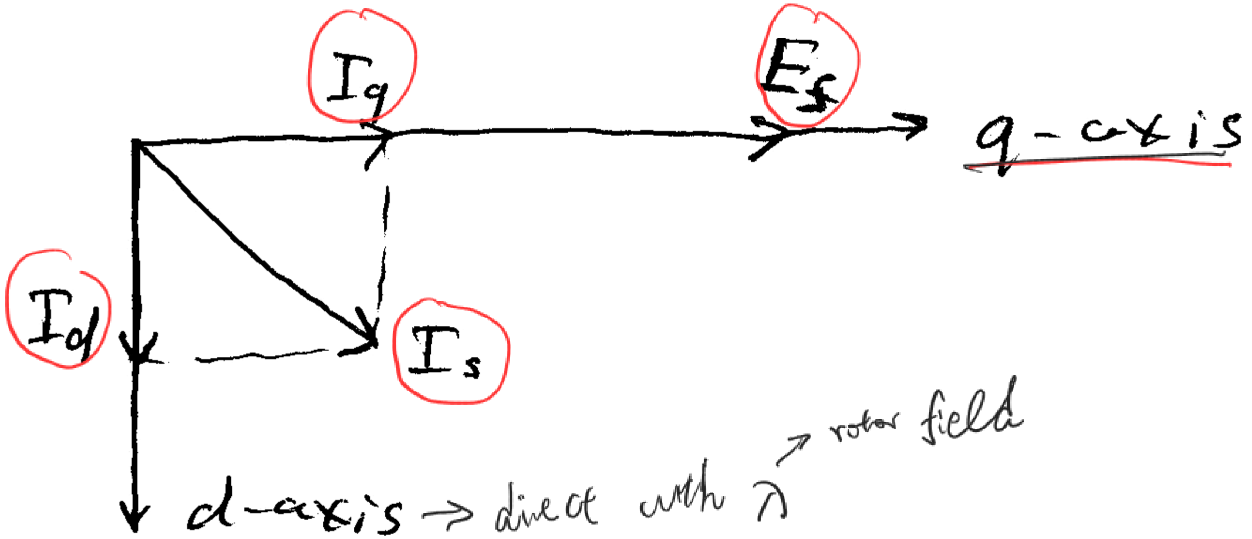
$X_d = X_{md} + X_{ls}$ - Direct axis synchronous reactance

Solution: separate X_d and X_q in circuit model

Current Components

$\hat{q}$: unit vector along q-axis
$\hat{d}$: unit vector along d-axis

Stator current $\tilde{I}_s = I_q\tilde{q} + I_d\tilde{d}$

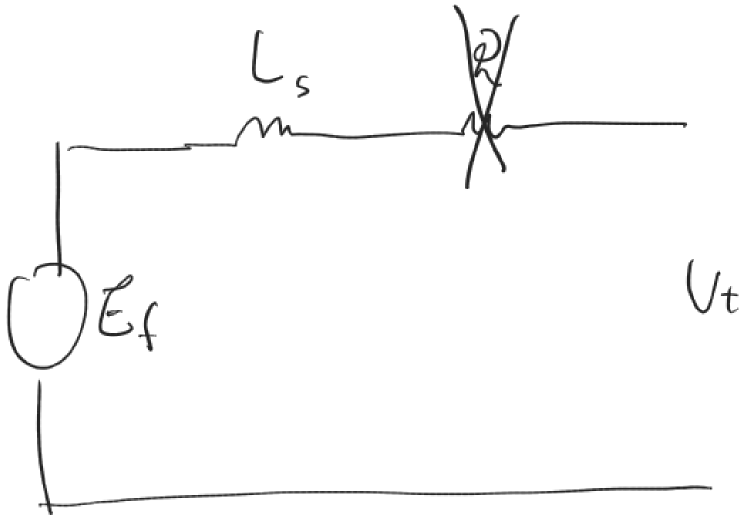


→ direct with λ → rotor field

$E_f = \frac{d\lambda}{dt} = s\lambda$

$s \rightarrow j\omega$

j → 90° lead



$$\Rightarrow \tilde{E}_f = \tilde{V}_t + jX_s\tilde{I}_s$$

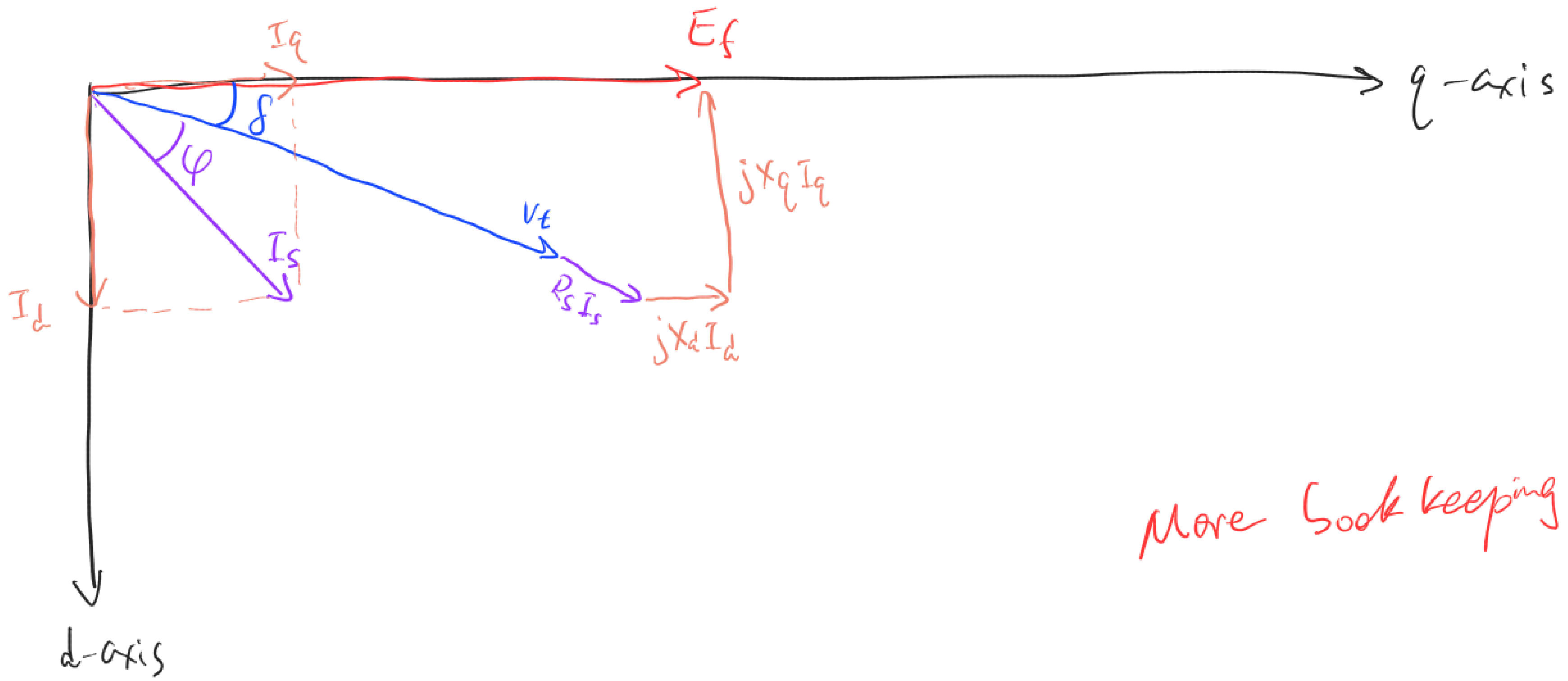
$$E_{fd}\hat{d} + E_{fq}\hat{q} = V_{td}\hat{d} + V_{tq}\hat{q} + j(X_s)(I_{sq}\hat{q} + I_{sd}\hat{d})$$

Phasor Diagram (generator)

Voltage equation $\boxed{\widetilde{E_f} = \widetilde{V_t} + R_s\widetilde{I_s} + jX_q I_q \tilde{q} + jX_d I_d \tilde{d}}$ → $jX\tilde{I}_s$

Also written as $E_f = V_t + R_s I_s + jX_q I_q + jX_d I_d$ for simplicity

Assume lagging pf



More book keeping

Phasor Diagram (generator)

Voltage equation $I_s = I_q + I_d \quad => \quad I_q = I_s - I_d$ last slide

purple phasor

$$\boxed{E_f = V_t + R_s I_s + jX_q I_q + jX_d I_d}$$

green phasors

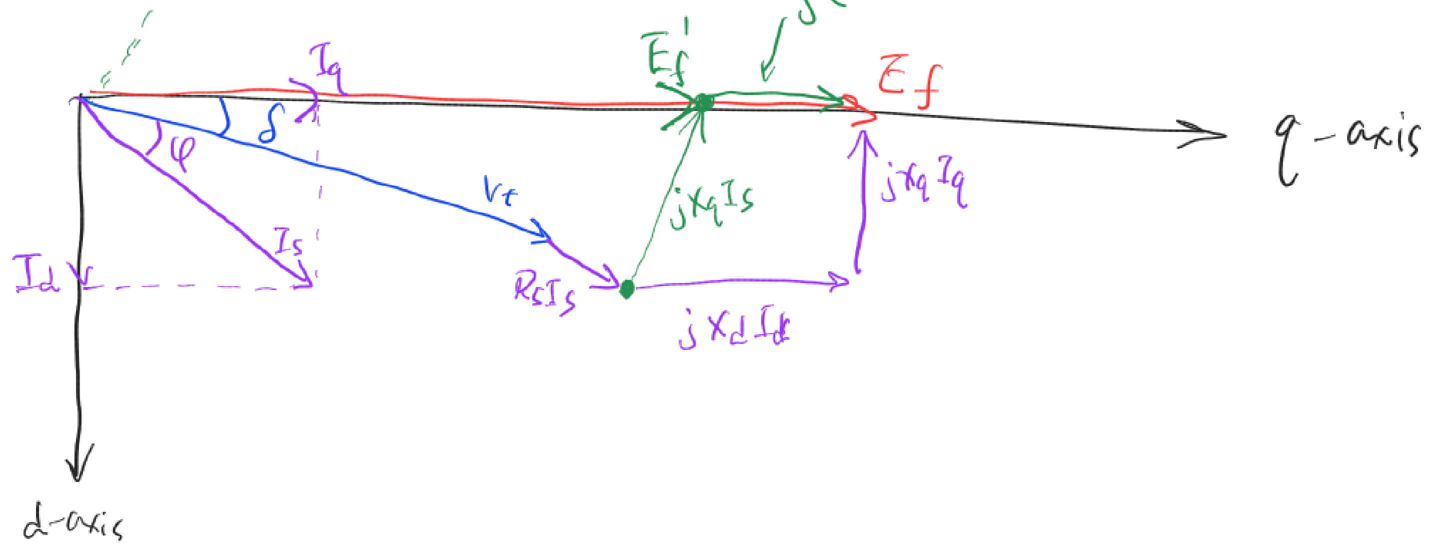
$$E_f = V_t + R_s I_s + jX_q I_s + j\left(X_d - X_q\right) I_d$$

$$E_f = E'_f + j\left(X_d - X_q\right) I_d$$

round rotor

Saliency

Salient SM is a generalization of round rotor



Round rotor: no need to define dq-coordinate

Salient rotor: define dq-coordinates

Example 3

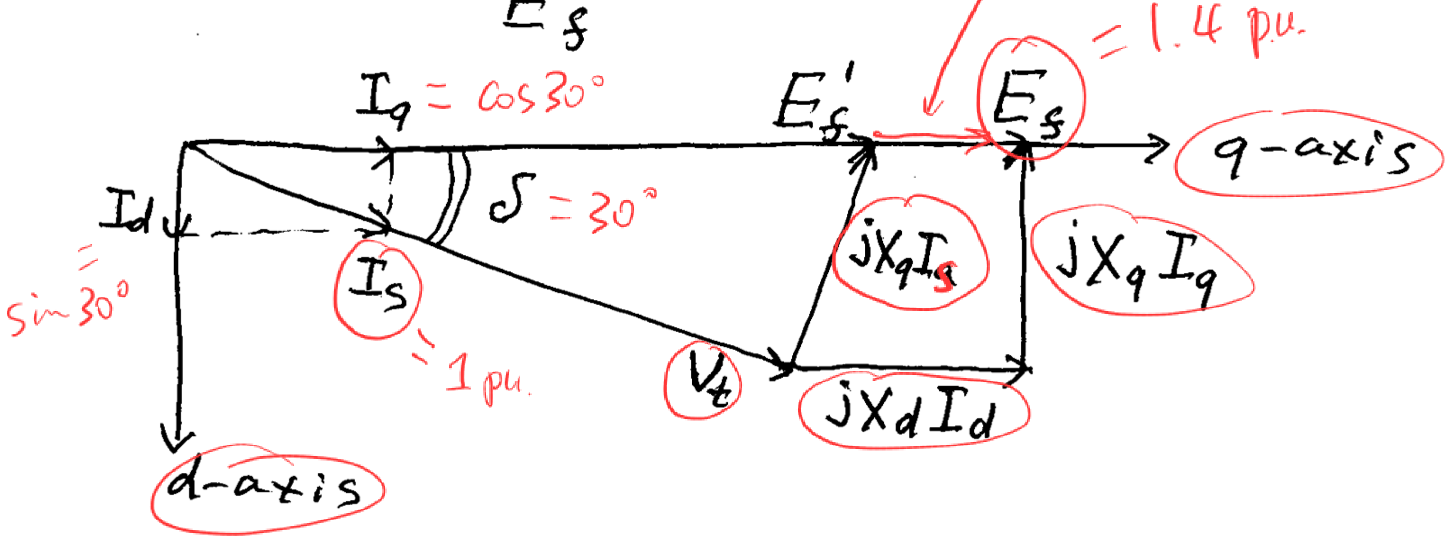
Salient-pole Syn. Gen delivers rated kVA power at unity PF and rated terminal voltage. The rotor angle $\delta = 30^\circ$. The open-circuit voltage is $E_f = 1.4\,pu$

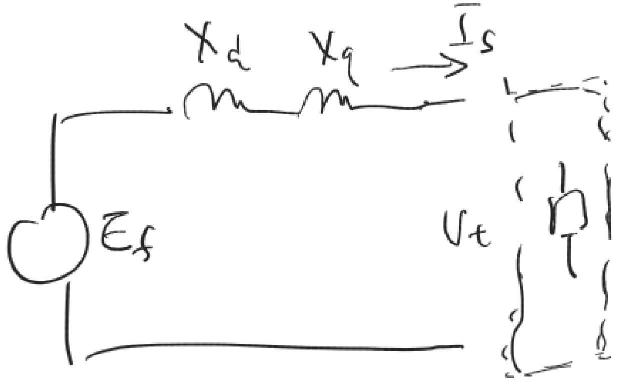
Winding resistance R_s is negligible.

(a) Sketch a phasor diagram and show relevant angles

$$E_f = V_t + jX_d \cdot I_d + jX_q \cdot I_q + \cancel{I_s R_s}$$

$$= \underbrace{V_t + jX_q I_s}_{E_f'} + j(X_d - X_q)I_d$$





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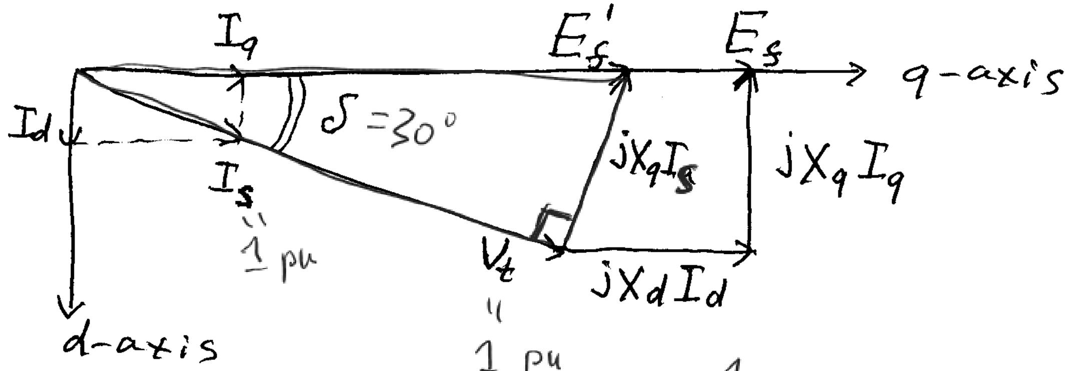
(b) Calculate reactances Xq and Xd in pu.



$$\tan(\delta) = \frac{X_q I_s}{V_t} = X_q$$

($I_s = 1$, $V_t = 1$)

$V_t = 1\,pu$, $I_s = 1\,pu$, $\delta = 30^\circ$

$$\tan(\delta) = \frac{X_q I_s}{V_t} = X_q$$

$$X_q = \tan(30^\circ) = 0.5774\,pu$$

$$E_f = V_t \cdot \cos(\delta) + X_d \cdot I_d \Rightarrow$$

$$X_d = \frac{E_f - V_t \cdot \cos(\delta)}{I_d} = \frac{E_f - V_t \cdot \cos(\delta)}{I_s \cdot \sin(\delta)}$$

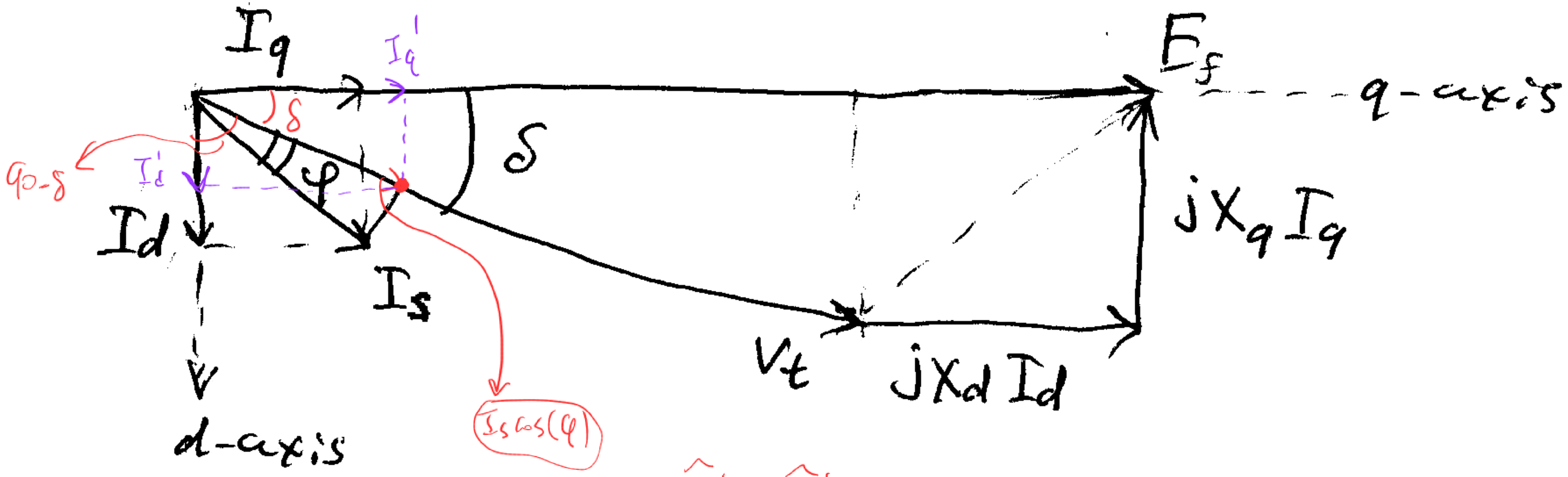
$$= \frac{1.4 - 1 \cdot \cos(30^\circ)}{1 \cdot \sin(30^\circ)} = 1.0679\,pu$$

$$X_d > X_q$$

Power & Torque (generator mode)

Assume $R_s << X_q, X_d \quad \Rightarrow \quad R_s$ can be neglected

Voltage equation $\quad E_f = V_t + jX_q I_q + jX_d I_d$



$I_s \cos(\varphi) = \hat{I}_q' + \hat{I}_d'$

$= I_q \cos(\delta) + I_d \sin(\delta)$ ⤳ real power

Real & Reactive Power (generator mode)

real

$$I_s \cos(\varphi) = I_q \cos(\delta) + I_d \sin(\delta)$$

$$I_s \sin(\varphi) = -I_q \sin(\delta) + I_d \cos(\delta)$$

reactive

multiply by V_t $\Rightarrow$

real power

$$V_t I_s \cos(\varphi) = V_t\left(I_q \cos(\delta) + I_d \sin(\delta)\right)$$

$$V_t I_s \sin(\varphi) = V_t\left(I_d \cos(\delta) - I_q \sin(\delta)\right)$$

reactive power

phasor diagram

$$E_f = V_t \cos(\delta) + X_d I_d$$

$$V_t \sin(\delta) = X_q I_q$$

$\Rightarrow$

$$I_d = \frac{E_f - V_t \cos(\delta)}{X_d}$$

$$I_q = \frac{V_t \sin(\delta)}{X_q}$$

Real Power (generator mode)

$$P_{1\phi} = V_t\left(\underbrace{\frac{V_t\sin(\delta)}{X_q}}_{\to I_q}\cos(\delta) + \underbrace{\frac{E_f - V_t\cos(\delta)}{X_d}}_{\to I_d}\sin(\delta)\right)$$

$$= \frac{V_t^2\sin(\delta)\cos(\delta)}{X_q} + \frac{V_t E_f\sin(\delta)}{X_d} - \frac{V_t^2\cos(\delta)\sin(\delta)}{X_d}$$

$$= \frac{V_t E_f}{X_d}\sin(\delta) + V_t^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin(\delta)\cos(\delta)$$

$$= \boxed{\frac{V_t E_f}{X_d}\sin(\delta)} + \boxed{\frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin(2\delta)}$$

round rotor
↳ P is changed by E_f

↳ saliency
↳ does not depend on E_f

$P_{3\varphi} = 3 \times P_{1\varphi}$

Summary: salient rotor SM has 2 mechanisms to produce torque / real power
↳ Excitation of rotor (same as round rotor)
↳ Saliency

So generally, salient rotor SM delivers more power / torque

Reactive Power (generator mode)

$$Q_{1\phi} = V_t\left(I_d\cos(\delta) - I_q\sin(\delta)\right)$$

$Q_{3\varphi} = 3 \times Q_{1\varphi}$

$$= \frac{V_t E_f - V_t^2\cos(\delta)}{X_d}\cos(\delta) - \frac{V_t^2\sin^2(\delta)}{X_q}$$

$$= \frac{V_t E_f}{X_d}\cos(\delta) - \frac{V_t^2\cos^2(\delta)}{X_d} - \frac{V_t^2\sin^2(\delta)}{X_q}$$

$$= \underbrace{\frac{V_t E_f\cos(\delta) - V_t^2}{X_d}}_{\text{round rotor}} + \underbrace{V_t^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin^2(\delta)}_{\text{saliency}}$$

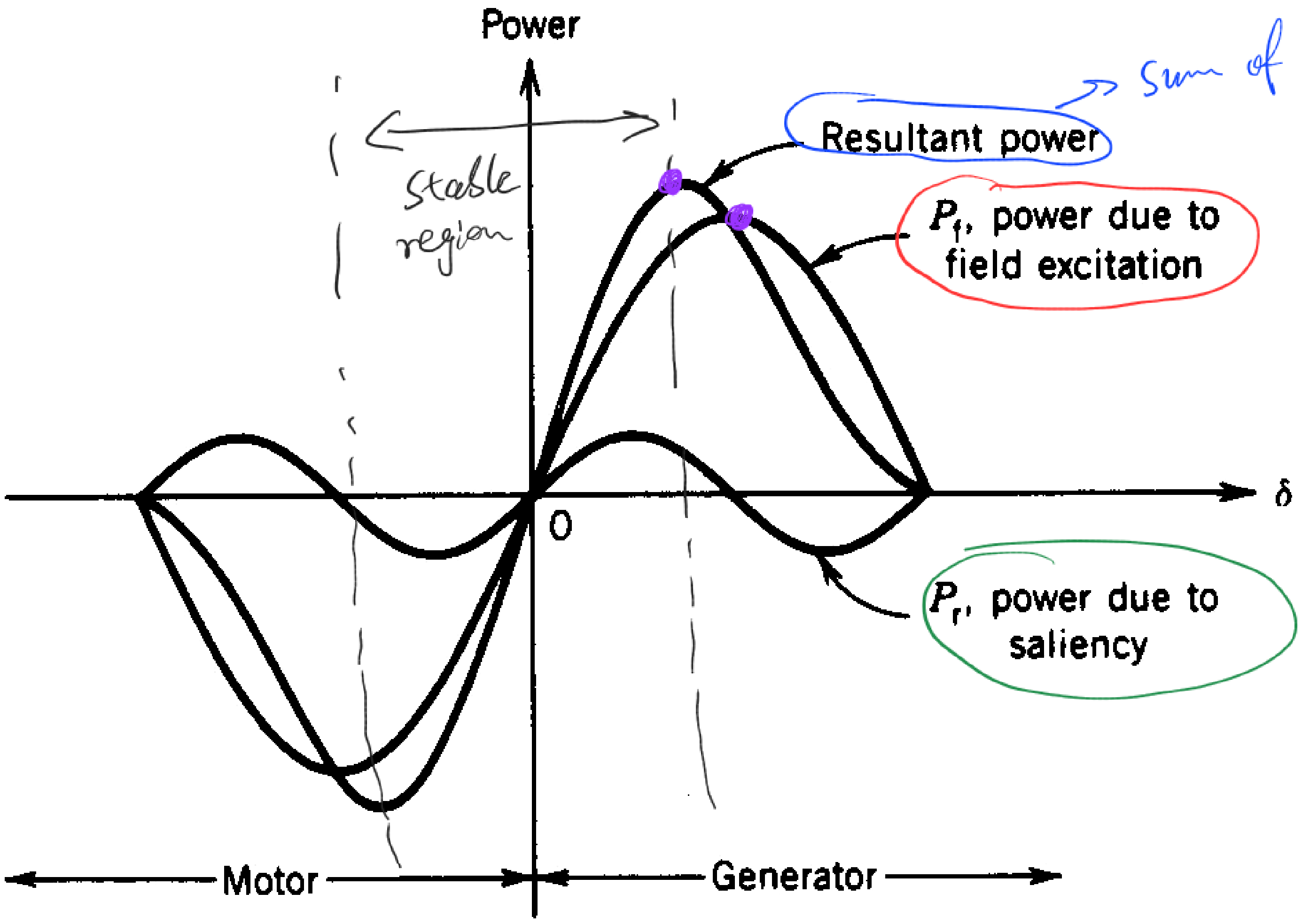
Power-Angle Characteristic (generator mode)

$$P_{3\phi} = 3P_{1\phi}$$

$$P_{3\phi} = 3\frac{V_t E_f}{X_d}\sin(\delta) + 3\frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin(2\delta)$$

$$P_{3\phi} = \omega_{syn} T_e = \frac{2}{P}\omega_r T_e$$

$$T_e = \frac{P_{3\phi}}{\omega_{syn}}$$



→ sum of the two

↓

Te is larger

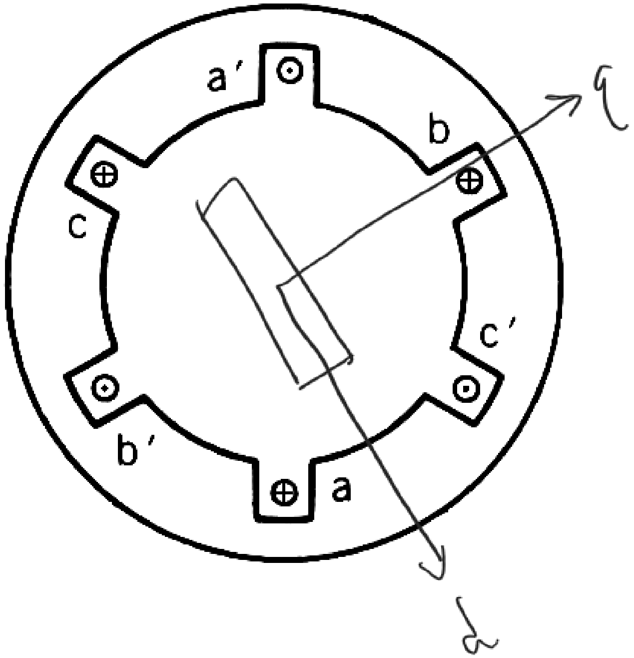
Principle of Reluctance Motor → SM → needs external starter

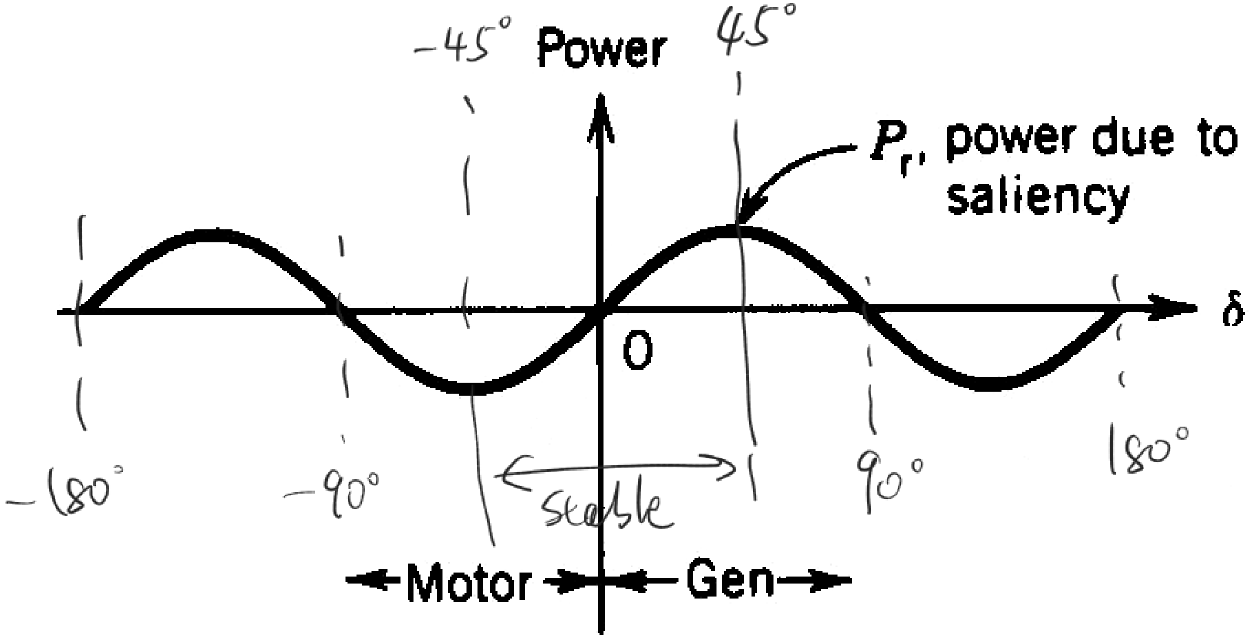
Use salient rotor with no winding

$$P_{3\phi} = 3\frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin(2\delta)$$

$$P_{3\phi} = 3P_{1\phi}$$

$$T_e = \frac{P_{3\phi}}{\omega_{syn}}$$

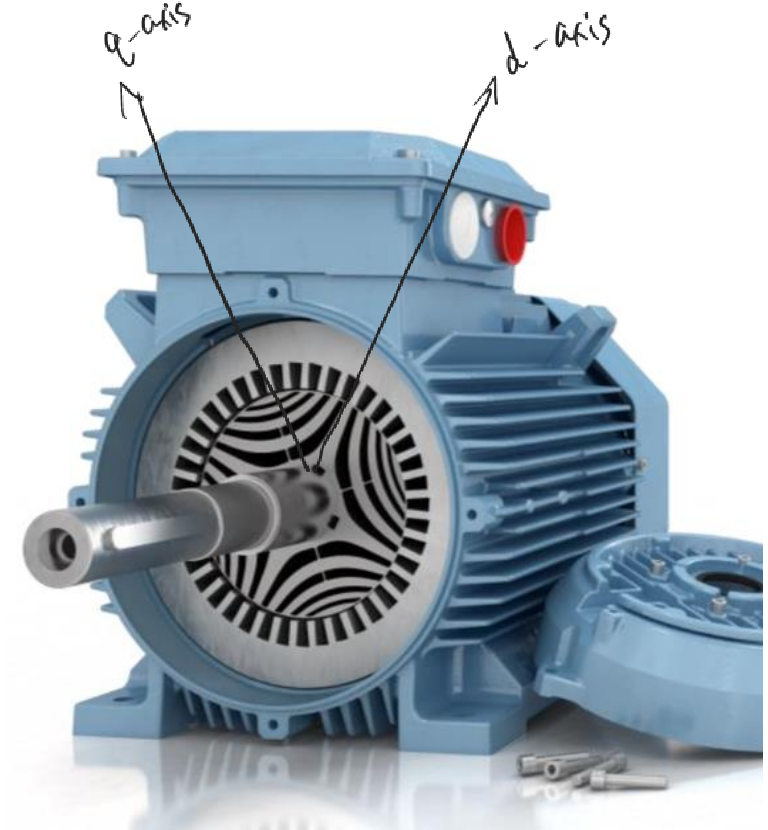




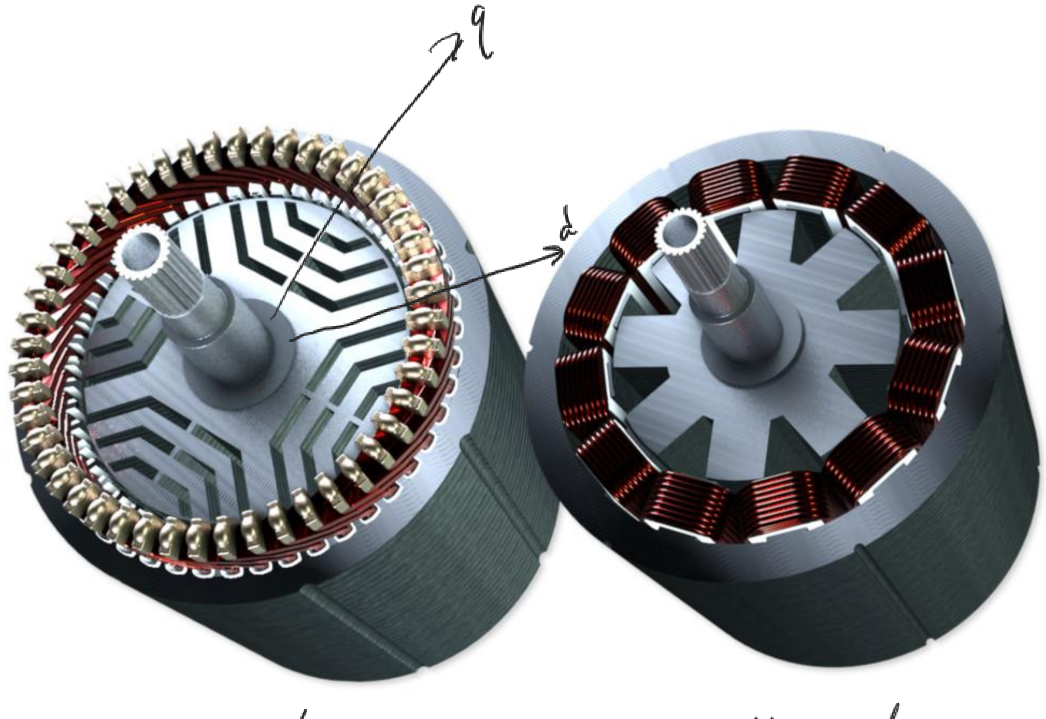
Principle of Reluctance Motor

Use salient rotor with no winding

$$P_{3\phi} = 3\frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin(2\delta) \qquad P_{3\phi} = 3P_{1\phi} \qquad T_e = \frac{P_{3\phi}}{\omega_{syn}}$$



4 - pole: dq 45° (mech) apart
90° (elec) apart



4-pole

Many poles

Principle of Reluctance Motor

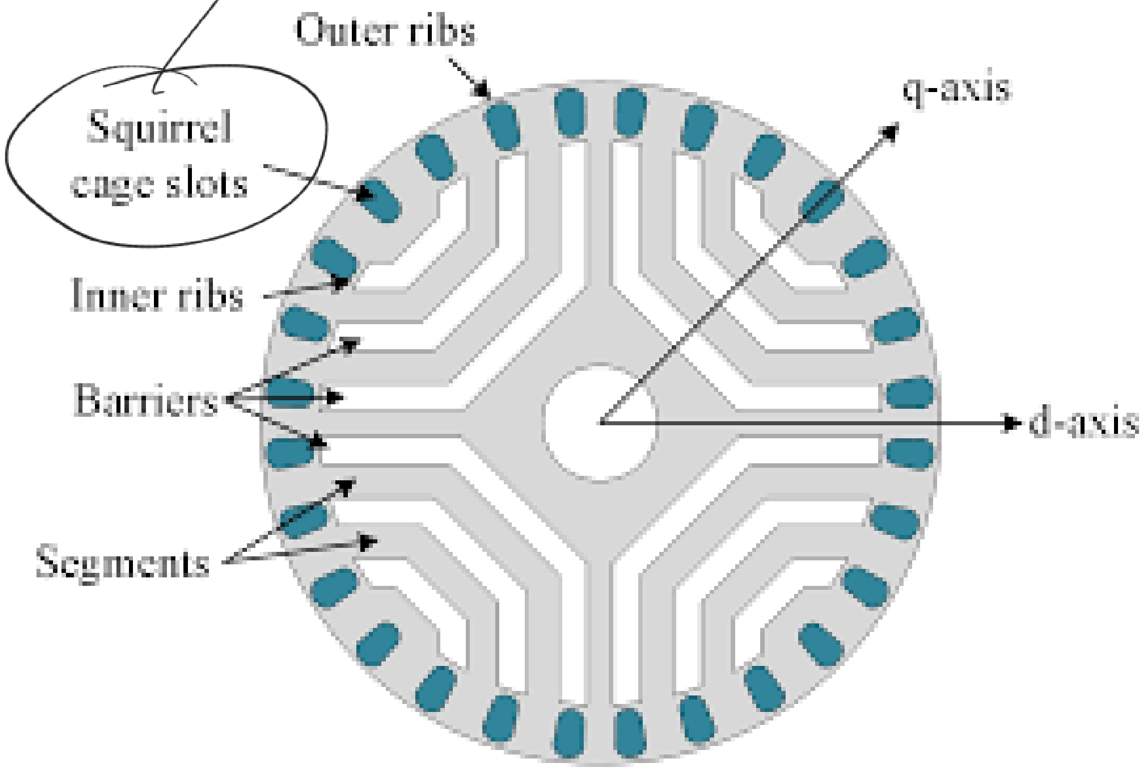
Use salient rotor with no winding

Start-up, doesn't do anything at sync. Speed

↳ Also acts as damper winding

↓

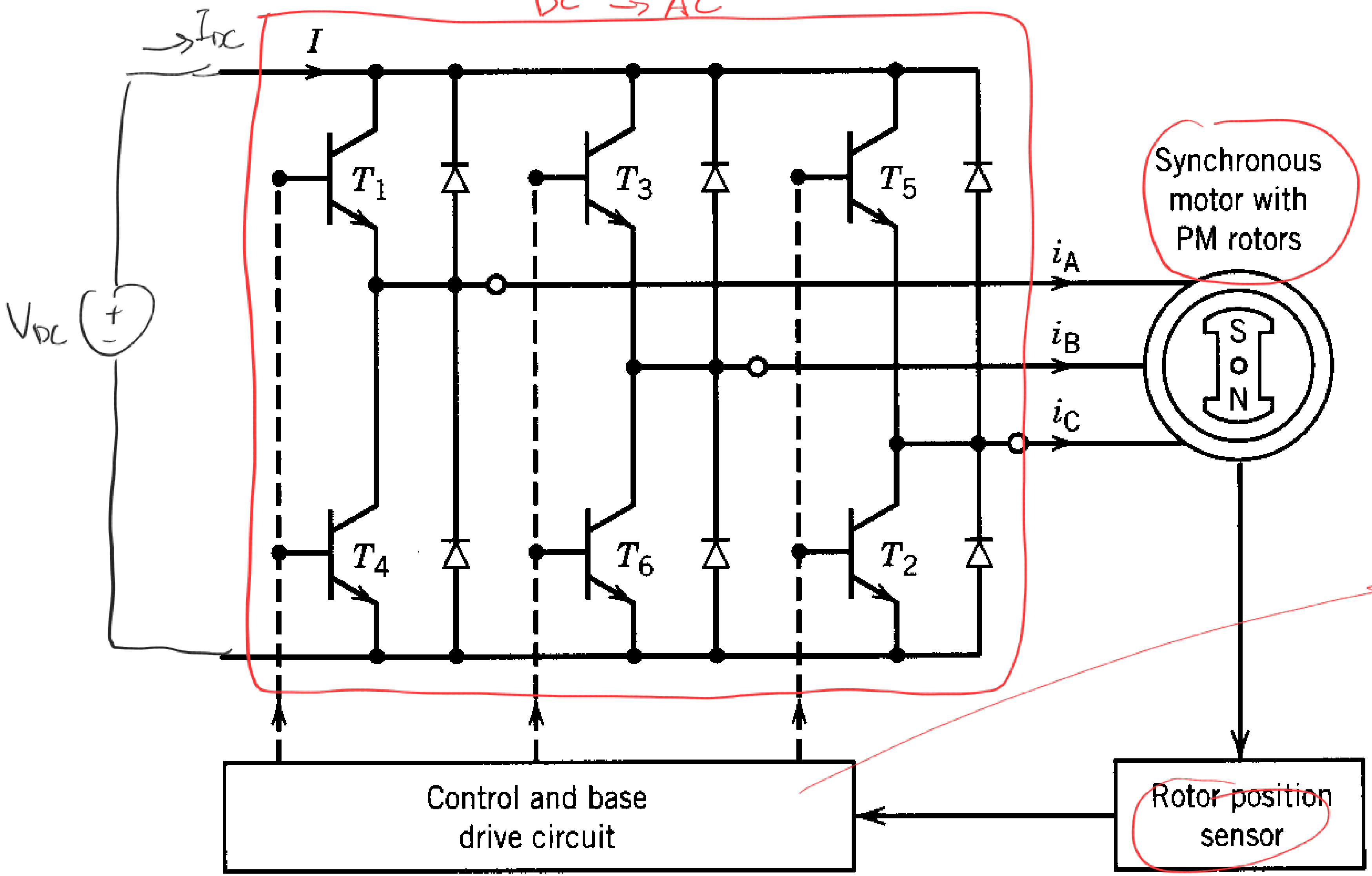
PD controllers



Brushless DC Machine (BLDC) → SM, not DCM

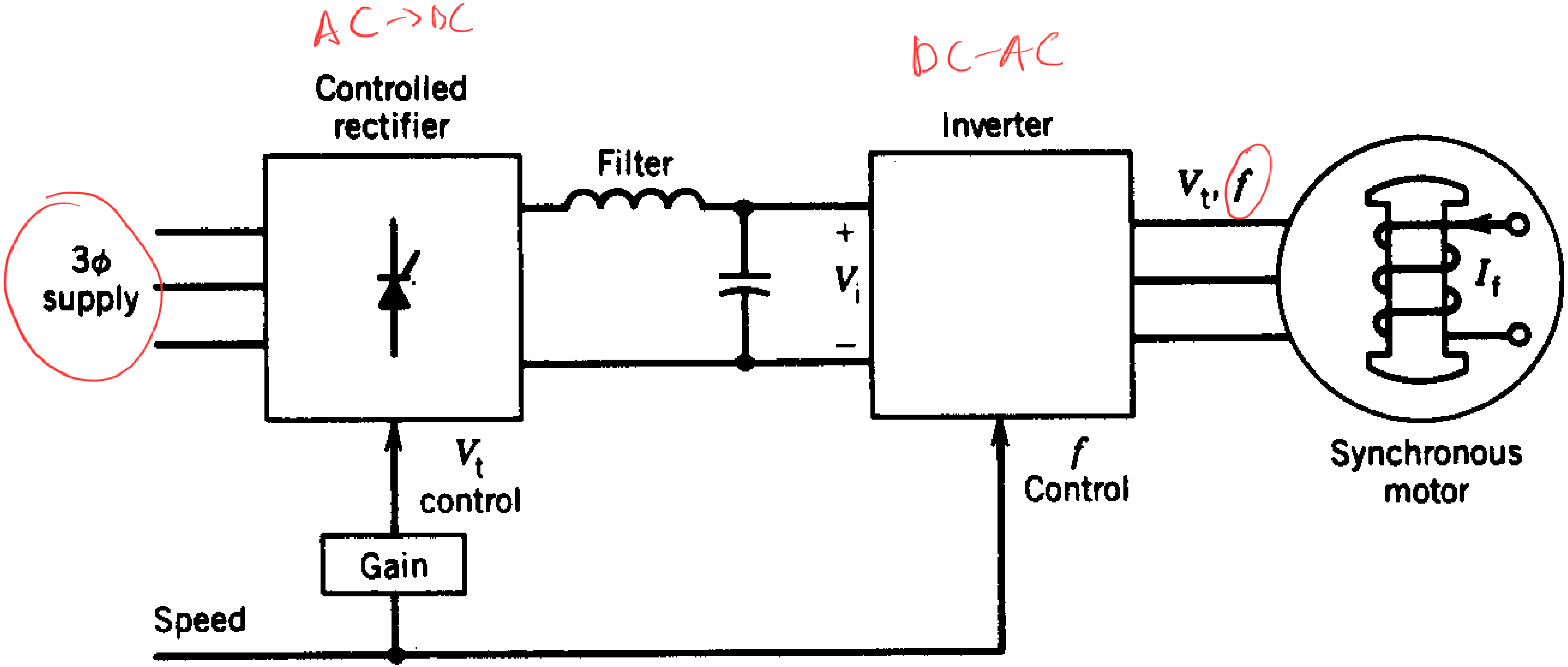
- Use PM Synchronous Machine & Variable Frequency Drive
- Use Hall-Effect or Electro-Optical sensors of the rotor position

Torque - Speed curve of BLDC looks same as DCM



→ Ensure stator and rotor are aligned

SM Control → VFD



In practice: large SM → prime mover
small SM → VFD

Looking ahead…

- Start **Module 7**