ELEC 342 Electromechanical Energy Conversion and Transmission

2025-26 Winter Term 2

Lecture 24

April 9, 2026

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Course Logistics

- Lab 5 this week
- No tutorial on Friday
- Office hour
  - My regular office hour on April 8 and 15 at 4 PM
  - The TA will hold an additional office hour on either April 13, 14, 16, or 17: **vote on Piazza**
- SEI survey: March 28 – April 13
- **Final exam: Apr. 19 @ 12 PM in IRC 4**

Example 1

Consider a 60Hz, 36V (line-to-line), Y-connected, NEMA Clsass B (Squirrel-Cage) Induction Motor with the following per-phase parameters: $R_1 = 0.1\,\Omega$, $R_2 = 0.2\,\Omega$, $X_1 = X_2 = 0.3\,\Omega$, and $X_m = 6\,\Omega$ (**all parameters and variables are referred to the stator**). The motor is supplied with the nominal (rated) voltage and is driving a mechanical load. The speed of the motor shaft is $n = 864$ rpm. You can neglect core and friction losses and use an approximate equivalent circuit. Recall that $T_e = 3\dfrac{1}{\omega_{syn}} \cdot (I_2)^2 \cdot \dfrac{R_s}{s}$. Determine the following:

(a) (10pts) Number of poles P and slip s

(b) (5pts) The input power factor PF and the total (3-phase) input real power

(c) (5pts) Total output mechanical power

(a) $n_{sync} = \frac{2}{P} n_e$

$n_e \rightarrow 60\,Hz \rightarrow 3600$ rpm

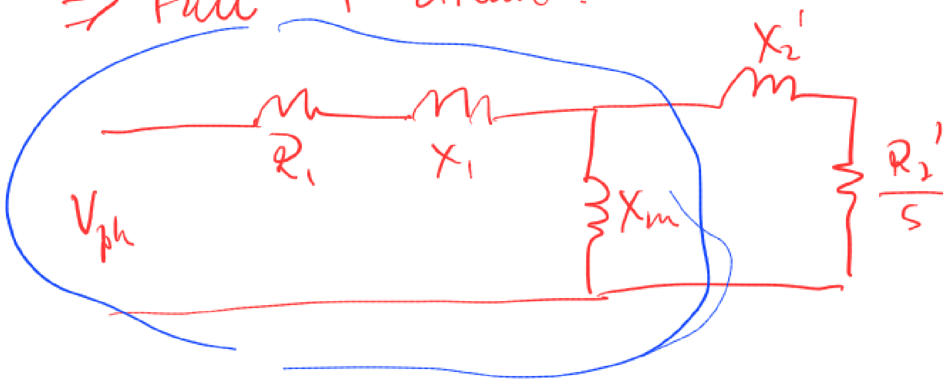
$n_{sync} > 864$ rpm

$$P = \frac{2 \times 3600}{864} = \lfloor 8.33 \rfloor \Rightarrow \boxed{8 \text{ poles}} \Rightarrow n_{sync} = 900 \text{ rpm}$$

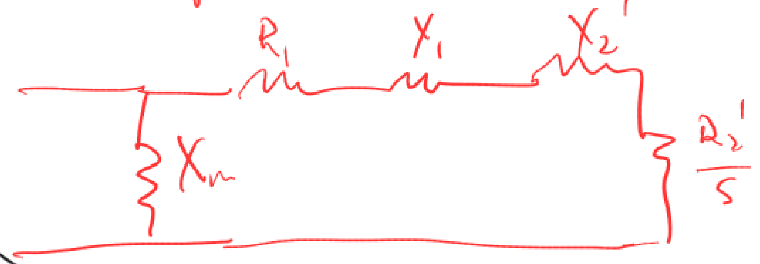
$$S = \frac{n_{sync} - n_{rotor}}{n_{sync}} = \frac{900 - 864}{900} = \boxed{0.04}$$

(b) $V_{LL} = 36\,V \rightarrow V_{ph} = \frac{36}{\sqrt{3}} = 20.7846 \angle 0^\circ\ V$

$\Rightarrow$ Full T-circuit :

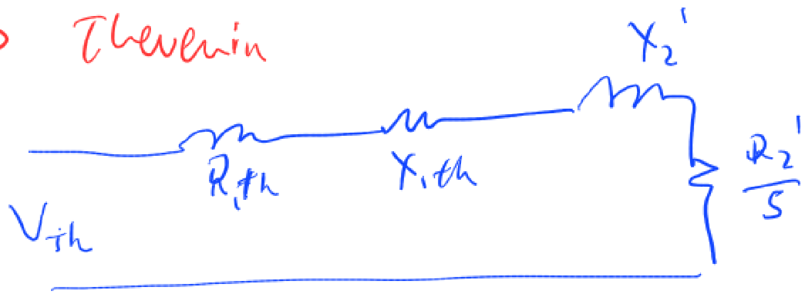


$\Rightarrow$ Simplified



Use this as instructed by question

$\Rightarrow$ Thevenin



$$\Downarrow$$

$$Z_{eq} = jX_m \parallel \left(R_1 + jX_1 + X_2' + \frac{R_2'}{s}\right)$$

$$= 2.6391 + j\,2.5847$$

$$I_{ph} = \frac{V_{ph}}{Z_{eq}} = 5.6266 \angle -0.775 \text{ A}$$

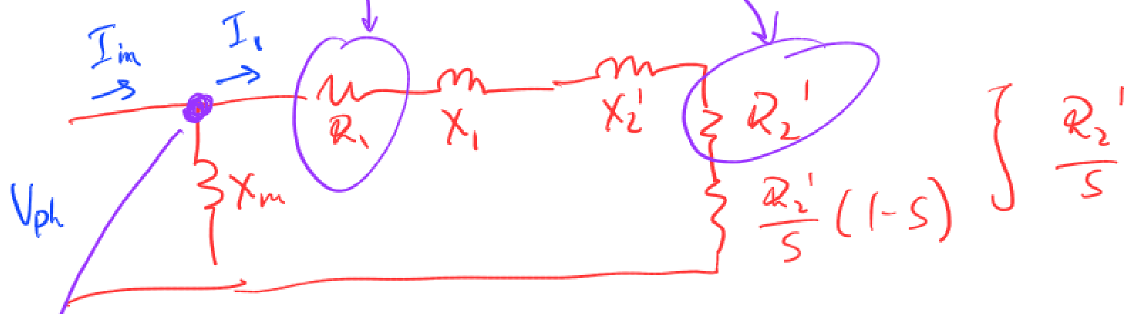
$$PF = \cos(\theta) = \cos(-0.775) = \boxed{0.7144}$$

$$P_{in} = 3\,V_{ph}\,I_{ph}\cos(\theta) = \boxed{250.6436 \text{ W}}$$

don't forget

(c) $P_{in} \xrightarrow{} P_{ag} \xrightarrow{} P_{mech} / P_e \xrightarrow{} P_{out}$

$R_1 \qquad R_2' \qquad$ Friction / rotation



$I_{in} = I_{ph} = 5.6266 \angle -0.795$ A

KCL: $I_1 = I_{ph} - I_{x_m} = I_{ph} - \frac{20.7846}{j6} = 4.0475 \angle -0.1171$ A

R_1 loss $= 3 \times I_1^2 \times R_1 = 4.9147$ W

$\Rightarrow P_{ag} = P_{in} - R_1 \text{ loss} = 250.6436 - 4.9147 = 245.7289$ W

$P_e = P_{ag} - 3I_1^2 R_2' = 3 I_1^2 R_2' \left(\frac{1-s}{s}\right) = (1-s) P_{ag}$

$P_e = (1-0.04)(245.7289 \text{ W}) = \boxed{235.8997 \text{ W}}$

Example 2

The following test results are obtained for a 3ϕ, 25 kV, 750 MVA, 60 Hz, 3600 rpm, star-connected synchronous machine at rated speed.

I_f (A)	V_{LL} (kV) open-circuit test	I_a (A) short-circuit test	V_{LL} (kV) air gap line
1500	25	10,000	30

(a) Determine the number of poles of the synchronous machine.

(b) Determine the unsaturated and saturated values of the synchronous reactance in ohms and per unit.

(c) The short-circuit test is performed at constant field current (1500 A) but at different speeds—1000 rpm, 2000 rpm, 3000 rpm, and 3600 rpm. Determine the short-circuit current at these speeds. → I_a

(d) Determine the field voltage if the synchronous machine delivers rated MVA to an infinite bus at 0.9 lagging power factor.

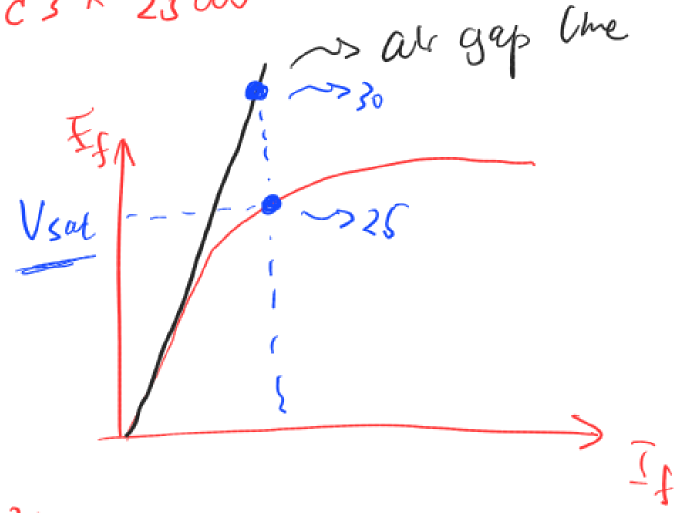
(a) $60\,Hz \Rightarrow 60 \times 60 \times \frac{2}{\underbrace{2}_{\text{pole}}} = 3600 \Rightarrow \boxed{P = 2}$

(b) $V_b = \frac{25}{\sqrt{3}} = 14.43\ kV \qquad I_b = \frac{S_{3p}}{\sqrt{3}\,V_{LL}} = \frac{250 \times 10^6}{\sqrt{3} \times 25000} = 17.32\ kA$

$Z_{base} = \frac{V_b}{I_b} = 0.8334\ \Omega$

$X_{m\,sat} = \frac{V_{unsat}}{I_a} = \frac{30\,kV/\sqrt{3}}{10\ kA} = 1.732\ \Omega$

$\Rightarrow \frac{1.732}{0.8334} = 2.0782\ p.u.$



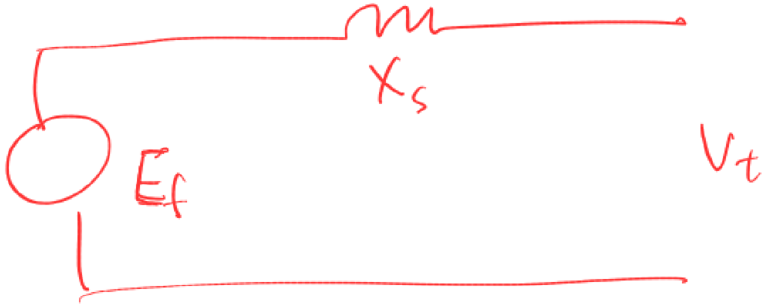
$X_{sat} = \frac{25\,kV/\sqrt{3}}{10\ kA} = 1.4434\ \Omega \Rightarrow \frac{1.4434}{0.8334} = 1.732\ p.u.$

(c) $E_f = L_f I_f \omega_r$

$$I_a = \frac{E_f}{X_s} = \frac{L_f I_f \cancel{\omega_r}}{\cancel{\omega_r} L_s} = \frac{L_f I_f}{L_s} = 10\ kA \quad \text{(constant)}$$

(d) $V_t = 1 \angle 0^\circ$ p.u.

$$I_a = 1 \text{ pu.} \angle -\cos^{-1}(0.9) = 1 \angle -25.84^\circ \text{ pu}$$



$$\cancel{E_f = V_t - I_a X_s}$$

$$E_f = V_t + I_a X_s \quad \text{(generator)}$$

$$E_f = 1 \angle 0^\circ + (1 \angle -25.84^\circ)(j1.732 \text{ pu.})$$
$$= 2.3473 \angle 41.6133^\circ \text{ pu.}$$

Example 3

A three-phase, 250 hp, 2300 V, 60 Hz, Y-connected nonsalient rotor synchronous motor has a synchronous reactance of 11 ohms per phase. When it draws 165.8 kW, the power angle is 15 electrical degrees. Neglect ohmic losses.

$\hookrightarrow P$ δ

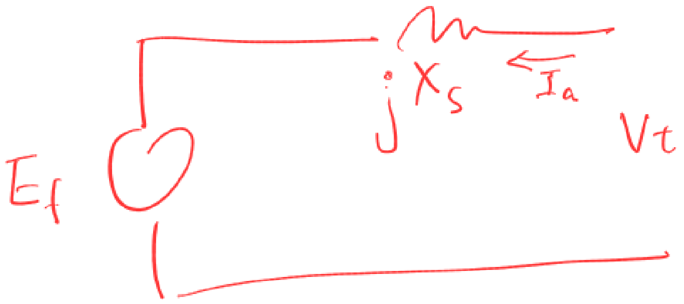
(a) Determine the excitation voltage per phase, E_f.

(b) Determine the supply line current, I_a.

(c) Determine the supply power factor.

(d) If the mechanical load is thrown off and all losses become negligible,

- **(i)** Determine the new line current and supply power factor.
- **(ii)** Draw the phasor diagram for the condition in (i).
- **(iii)** By what percent should the field current I_f be changed to minimize the line current?



(a) $V_t = \frac{2300}{\sqrt{3}} = 1328\,V \Rightarrow 1328\angle 0^\circ$

$$\underbrace{P_{3\phi}}_{165.8\,kW} = 3\,\frac{\overset{1328V}{V_t}\,E_f}{\underbrace{X_s}_{11}}\,\sin(\overset{15^\circ}{\delta}) \Rightarrow |E_f| = 1769\,V$$

$$\boxed{E_f = 1769\angle -15^\circ\ V}$$

(b) $$I_a = \frac{V_t - E_f}{jX_s} = \frac{1328\angle 0^\circ - 1769\angle -15^\circ}{11\angle 90^\circ} = 54.14\angle 39.8^\circ\,A$$

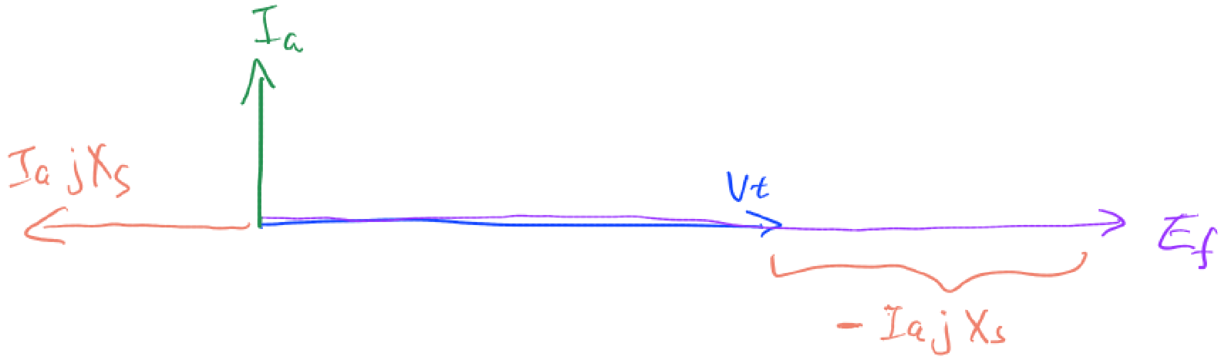
(c) $PF = \cos(39.8^\circ) = 0.77$ leading

(d) $P_{out} = 0^\circ = P_{in} \Rightarrow \delta = 0^\circ$

(i) $\underline{\underline{I_a}} = \dfrac{1328\angle 0^\circ - 1769\angle 0^\circ}{11\angle 90^\circ} = 40.1\angle 90^\circ \text{ A} = -40.1\angle -90^\circ \text{ A}$

$PF = \cos 90^\circ = 0$ leading

(ii)



(iii) $I_a = 0$ when $\underset{1769}{E_f} = \underset{1328}{V_t}$

$$I_{f,new} = \frac{1328}{1769} \times I_{f,old} = 0.75 \times I_{f,old}$$

Example 4

A 1 MVA, 3ϕ, 2300 V, 60 Hz, 10 pole, star-connected cylindrical-rotor synchronous motor is connected to an infinite bus. The synchronous reactance is 0.8 pu. All losses may be neglected. The synchronous motor delivers 1000 hp, and the motor operates at 0.85 power factor leading.

(a) Determine the excitation voltage E_f.

(b) Determine the maximum power and torque the motor can deliver for the excitation current of part (a).

(c) The power output is kept constant at 1000 hp, and the field current is decreased. By what factor can the field current of part (a) be reduced before synchronism is lost?

↳ 90°

(a) $V_t = \frac{2300}{\sqrt{3}} = 1327.9 \angle 0^\circ \text{ V} \Rightarrow V_b$

$P_{out} = 1000 \text{ hp} = 746 \text{ kW} \Rightarrow P_{1\varphi} = \frac{746}{3} \Rightarrow P_{1\varphi,pu} = \frac{P_{1\varphi}}{\frac{1\text{MVA}}{3}} = \underline{\underline{0.2487 \text{ pu.}}}$

$1\text{-}\varphi: P_{in} = P_{out} = 0.2487 \text{ pu.}$

$0.2487 = \underbrace{|V_t|}_{1 \text{ pu.}} |I_a| \underbrace{\cos\theta}_{0.85} = 1 \times |I_a| \times 0.85 \Rightarrow |I_a| = 0.2926 \text{ pu.}$

$\cos(\theta) = 0.85 \Rightarrow \theta = 31.8^\circ$ (leading)

$I_a = 0.2926 \text{ pu.} \angle 31.8^\circ \text{ A}$

$E_f = V_t - jI_aX_s = 1.1408 \angle -10.043^\circ \text{ pu.}$

(b) $P_{max} = \frac{V_t E_f}{X_s} = \frac{1 \times 1.1408}{0.8} = 1.426$ pu. $(1-\varphi)$

$T_{max} = \underset{\substack{\uparrow \\ 1 \text{ pu}}}{\omega_{sync}} \cdot P_{max} = 1 \times 1.426 = 1.426$ pu.

(c) Boundary $= 90^\circ \Rightarrow \sin 90^\circ = 1$

$1000\text{hp} = P = \frac{\overset{1}{V_t} E_f}{\underset{0.8}{X}} \underbrace{\sin(\delta)}_{1} \Rightarrow E_{f,new} = \frac{X_s \cdot \overset{0.2487}{P}}{V_t} = \frac{0.8 \times 0.2487}{1}$

$= 0.19896$ pu.

$\frac{I_{f,new}}{I_{f,old}} = \frac{E_{f,new}}{E_{f,old}} = \frac{0.19896}{1.1408} = \boxed{17.44\%}$

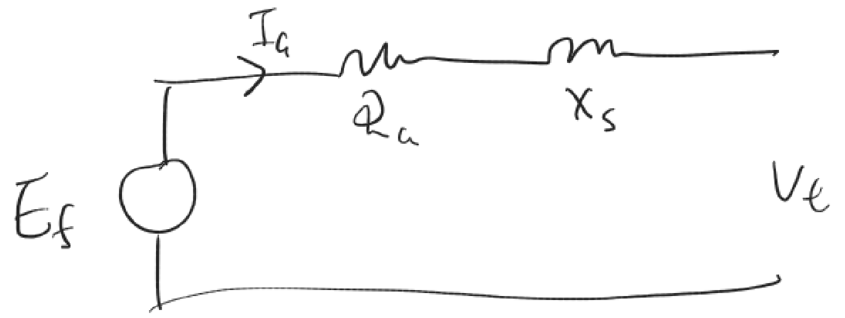
Example 5

Consider a 3-phase, 2-pole **Permanent Magnet Round-Rotor Synchronous Generator** with per-phase stator resistance and synchronous reactance $R_a = 0.1\,\Omega$ and $X_s = 2\ \Omega$, respectively. Assume that this generator operates at 60Hz, 24V (line-to-line), and outputs electric power $S_{3\phi} = 300VA$ to a load with 0.8 lagging power factor.

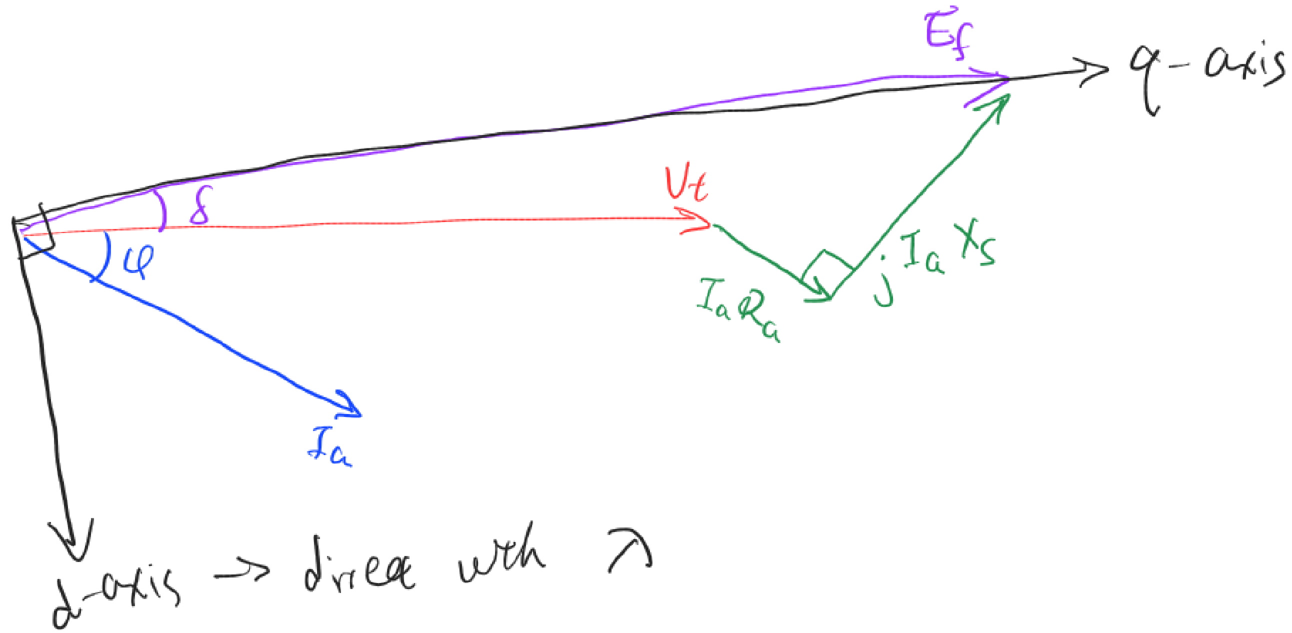
(a) (6pts) Sketch a phasor diagram and label the q- and d- axes, all relevant angles, and phasors

(b) (7pts) Calculate the induced voltage E_f and the rotor angle δ in degrees

(c) (7pts) Assume mechanical rotational losses $P_{mech\_loss} = 20W$, calculate the efficiency η



(a)



d: in line with λ

q: 90° wrt d

↓

$E_f = \frac{d\lambda}{dt}$ ⇒ E_f aligned with q-axis

(b) $V_{LL} = 24V$ ⇒ $V_t = 13.8564 \angle 0°$ V

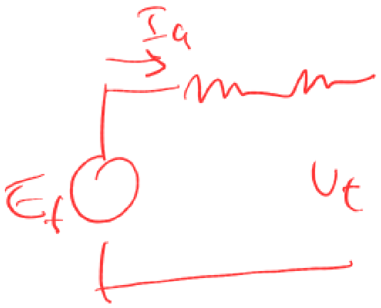
$R_a = 0.1\,\Omega$, $X_s = 2\,\Omega$, $S_{3\phi} = 300$ VA

$\cos\varphi = 0.8$ ⇒ $\varphi = 36.8699$ lagging

$$|I_a| = \frac{S_{3\varphi}}{3 \times V_t} = 7.2169\ A \Rightarrow I_a = 7.2169\angle -36.8699^\circ A$$

$$E_f = V_t + I_a R_a + I_a (jX_s) = \boxed{25.6292\angle 25.6992^\circ\ V}$$

$$\boxed{\delta = +25.6992^\circ}$$



(c) $P_{rot} = 20\ W$ (no field loss due to PM)

$P_{stator} = 3\,|\hat{I}_a|^2 \hat{R}_a$, $P_{out} = 300\ VA \times 0.8$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{rot} + P_{stator}} = \boxed{78.2\%}$$

The End

- I hope this class has taught you something about electromechanical energy conversion.

- Congratulations on finishing this class! It’s been a real pleasure.