THE UNIVERSITY OF BRITISH COLUMBIA

Department of Electrical and Computer Engineering

ELEC 342: Electromechanical Energy Conversion and Transmission

Midterm Exam
Spring Term 2026
February 26

Topics Covered: AC Circuits, Magnetic Circuits, AC Transformers, Electromechanical Energy Conversion, and DC Machines.

Surname:____________________________________

First Name:__________________________________

Student ID:__________________________________

Solution

I understand the principles of academic integrity, and I will not cheat on this exam.

Signature:___________________________________

Problem	Points	Max.
1		25
2		25
3		25
4		25
Total		100

- **Close notes and books.** No formula sheets.
- You are allowed to have only a **calculator** and a **pen/pencil**.
- Show your work, including **derivations**, **comments**, **assumptions**, and **units** wherever appropriate.
- Use the back side of each page or ask for additional pages if you need extra space to write your answers.
- Exams suspected of cheating and/or turned in late will not be marked – **failed exam**.
- You have **80** minutes to answer the following questions.
- Good luck!

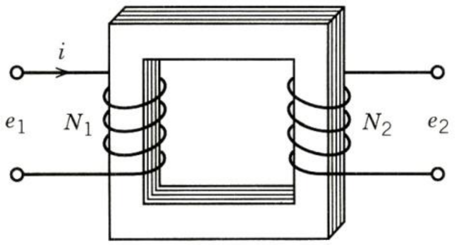
Problem 1 (25pts)

Figure below shows a two-winding transformer with a core. The first winding with $N_1 = 200$ turns is connected to a voltage to produce a flux density in the core $B = 1.2\sin(377t)\ Wb/m^2$. The second winding, with $N_2 = 400$ turns, is left open-circuited. The cross-sectional area of the core is $23.75\ cm^2$, and μ_r for the core is 10,000. The core length is $l_c = 90\ cm$. For reference, $\mu_0 = 4\pi \times 10^{-7}\ H/m$.

(a) Determine the RMS value of the applied voltage e_1 (corresponding to first winding).

(b) Determine the current in the first winding (note that we are asking for its full expression, not just RMS value. Also, it is not zero).

(c) Determine the RMS value of the voltage induced in the second winding.



$N_1 = 200$, $N_2 = 400$, $B = 1.2\sin(377t)$

$A_c = 23.75\ cm^2 = 0.002375\ m^2$ ← unit conversion

$\mu_r = 10000$, $l_c = 90\ cm = 0.9\ m$ ← unit conversion

(a) $\Phi = BA_c = 1.2\sin(377t) \times 0.002375$

$= 0.00285\sin(377t)$

$e_1 = N_1 \frac{d\Phi}{dt} = 200 \times 0.00285 \times 377\cos(377t)$

$= 214.89\cos(377t)$

$e_{1_{rms}} = \frac{214.89}{\sqrt{2}} = \boxed{151.95}$

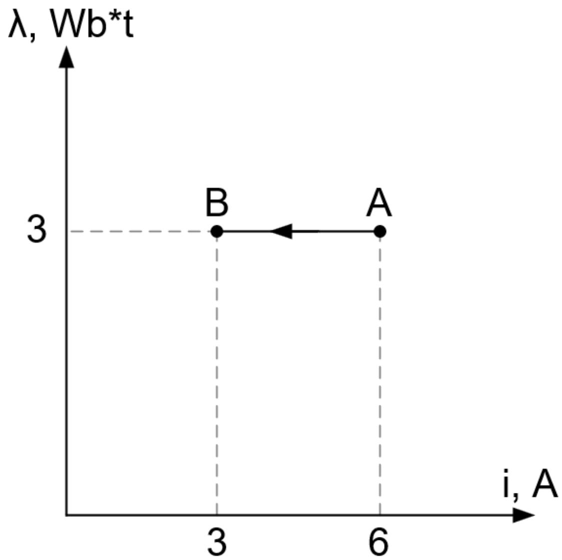
(b) $H_c = \frac{B}{\mu_0 \mu_r} = 95.49\sin(377t)$

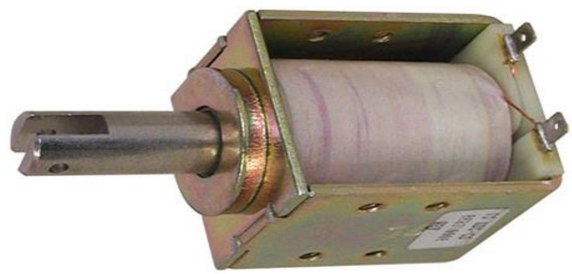
$N_1 i_1 = H_c l_c \Rightarrow i_1 = \frac{H_c l_c}{N_1} = \boxed{0.4297\sin(377t)}$

(c) $e_{2_{rms}} = \frac{N_2}{N_1} e_{1_{rms}} = 2 \times 151.95 = \boxed{303.9}$

Problem 2 (25pts):

Assume an electromechanical solenoid with a movable plunger. The system may be assumed **magnetically linear**. The system's state is shown in the $\lambda - i$ figure below, wherein the system has moved from point A to point B, at constant flux. Using the numerical values given on this figure, **calculate the change in** W_f, W_c, W_e, and W_m. In other words, complete the Table given below. **Be sure to include the units and check the energy conversion balance.**





Transition	From A to B
(3pts) Change in coupling field energy, ΔW_f	$4.5 - 9 = -4.5$ J
(3pts) Change in co-energy, ΔW_c	-4.5 J
(3pts) Change in electrical input, ΔW_e	0 J
(3pts) Change in mechanical input, ΔW_m	4.5 J
(4pts) The energy was taken from **(and how much)**	4.5 J was taken from coupling field
(4pts) The energy was supplied to **(and how much)**	4.5 J was supplied to mechanical system
(5pts) Write and check the energy balance using ΔW_e, ΔW_f, and ΔW_m, and conclude: Was the plunger pulled IN or OUT?	$\Delta W_e = \Delta W_f + \Delta W_m$ $0 = -4.5 + 4.5$ $\Rightarrow$ Proved Since $\Delta W_m > 0$,

plunger is pulled IN (work supplied to mechanical system, which naturally seeks to minimize reluctance)

$\} \; \Delta W_f = \Delta W_c$

$\rightarrow \int i \, d\lambda = 0$

Problem 3 (25pts):

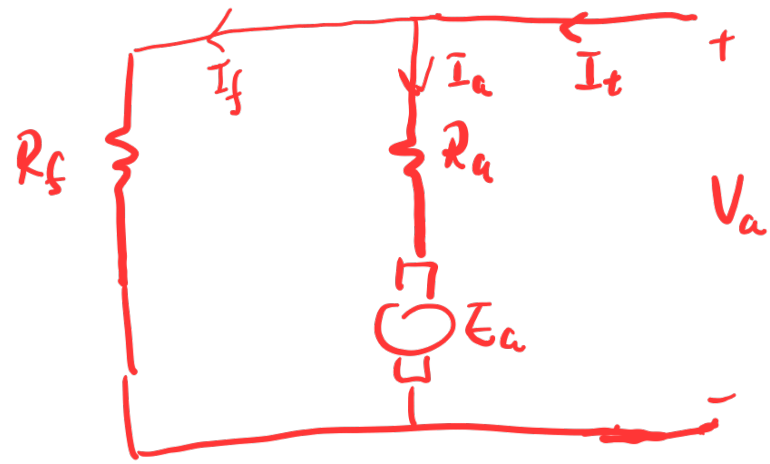
Consider a 250 V **Shunt DC Motor** with armature resistance $R_a = 0.1\ \Omega$ and field winding resistance $R_f = 25\ \Omega$. The motor operates a mechanical load at a nominal speed of 1800 rpm, drawing the armature current of 20 A. Friction and rotational losses constitute 60 W.

(a) Calculate the induced back emf E_a

(b) Determine the useful mechanical load torque T_m

(c) Determine the efficiency of the motor η

Shunt motor Schematic (inductance ignored):



We are given:

$V_a = 250V$

$R_a = 0.1\ \Omega$

$R_f = 25\ \Omega$

$w_r = 1800$ rpm

$I_a = 20A$

$P_{loss} = 60W$

(a) $V_a - R_a I_a = E_a = 250 - 0.1 \times 20 = \boxed{248\ V}$

(b) $P_e = E_a I_a = 248 \times 20 = 4960\ W$

$P_m = P_e - P_{loss} = 4960 - 60 = 4900W$

$W_r = 1800\ rpm = \frac{1800}{60} \times 2\pi = 188.5\ rad/s$

$T_m = \frac{P_m}{W_r} = \boxed{25.995\ N\cdot m}$

(c) $P_{in} = V_a \times I_t = V_a \times (I_f + I_a)$

$= V_a \times \left(\frac{V_a}{R_f} + 20\right) = 250\left(\frac{250}{25} + 20\right)$

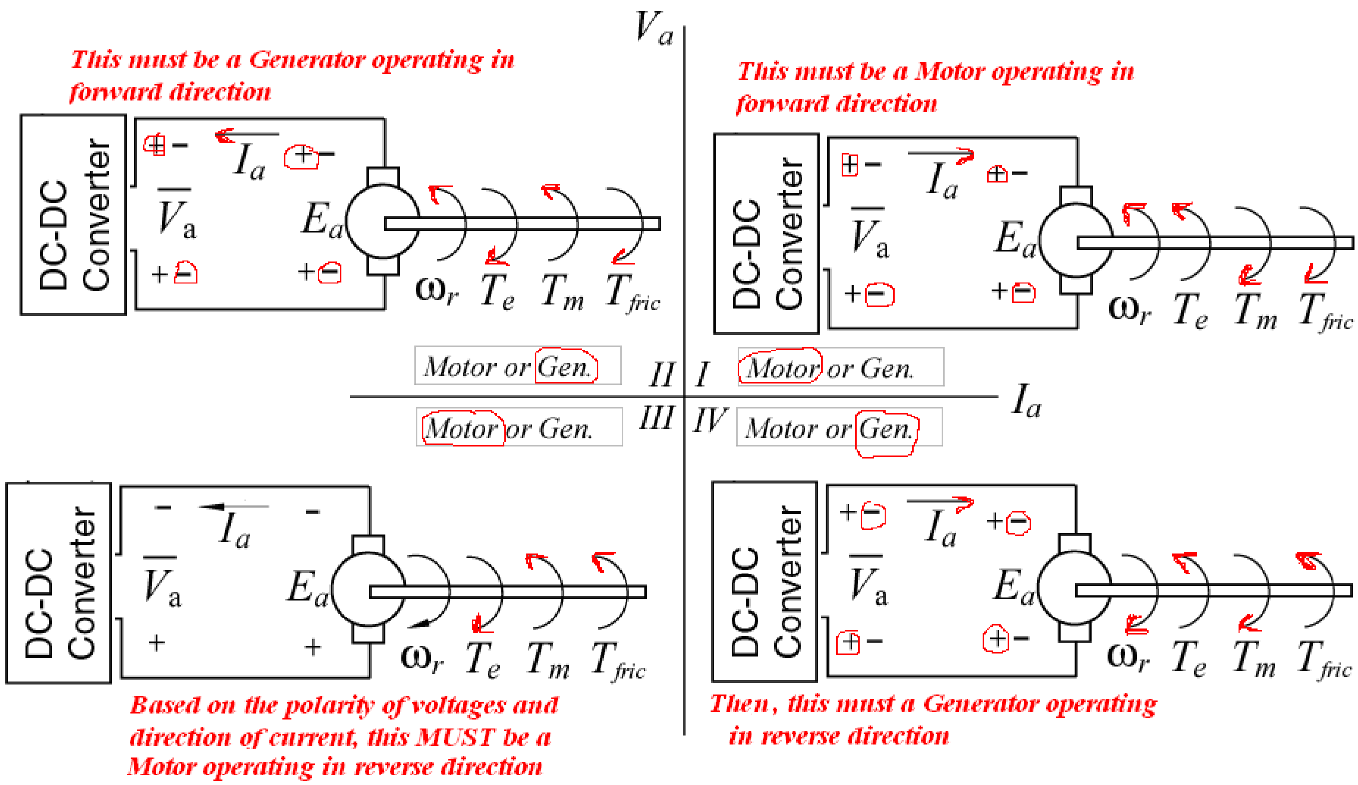
$= 7500W$

$P_{out} = P_m = 4900\ W$

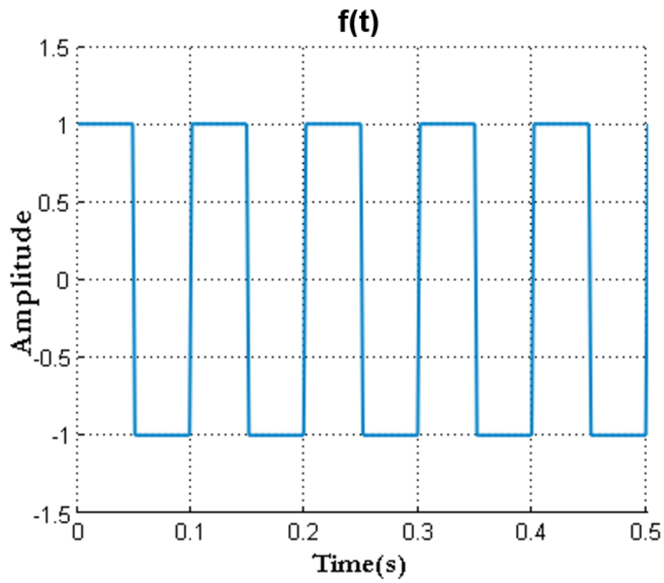
$\eta = \frac{P_{out}}{P_{in}} = \frac{4900}{7500} = 0.65 = \boxed{65\%}$

Problem 4 (25pts):

a) (15pts): Assume a PM DC machine operating in a steady-state from the DC-DC Converter, as we discussed in class. Based on the information already given for **Quadrant III** (directions of speed, voltages, and current) in the chart below, first identify the direction of torques in **Quadrant III** and whether it is a Motor or a Generator (circle the correct answer). Then complete the chart for **all other Quadrants,** circling the correct answers for all voltage polarities, back emf polarities, drawing the arrows for torques, currents, and speeds, and circle whether it is a Motor or a Generator.



b) (10pts): Given the waveform $f(t)$ below, select the following **statements that are true** by putting a cross or check mark in the box on the left and **provide a brief explanation**. Note that you may select more than one statements.



- ✓ $f(t)$ can represent voltage across an inductor
- ☐ $f(t)$ can represent current through an inductor
- ☐ $f(t)$ can represent voltage across a capacitor
- ✓ $f(t)$ can represent current through a capacitor
- ✓ $f(t)$ can represent voltage across a resistor
- ✓ $f(t)$ can represent current through a resistor

<u>Explain your choice(s):</u>

You cannot change inductor current and capacitor voltage instantaneously (as discussed in class).

Extra Page (if needed)