ELEC 342: Electromechanical Energy Conversion and Transmission

Homework 1

- ✓ Examples (questions with full answers)
- ✓ **Assignments** (questions with no answers)

You need to submit your answers (procedure + final answer) for "Assignments" only by 11:59 pm on the posted due date.

Summary:

- Follow through **Examples B.1, B.2** from the textbook.
- Follow through with **Additional Examples 1, 2, 3**.
- **Assignments**: Solve **Q1**, **Q2, Q3**, **Q4**.

Example B.1:

For the circuit shown in Fig. EB.1*a*:

(a) Determine the rms value of the supply current.

(b) Express the supply voltage v and supply current i in the time domain.

(c) Determine the power from the supply and the supply power factor.

(d) Draw the phasor diagram for V, I, V_r, V_L, and V_C.

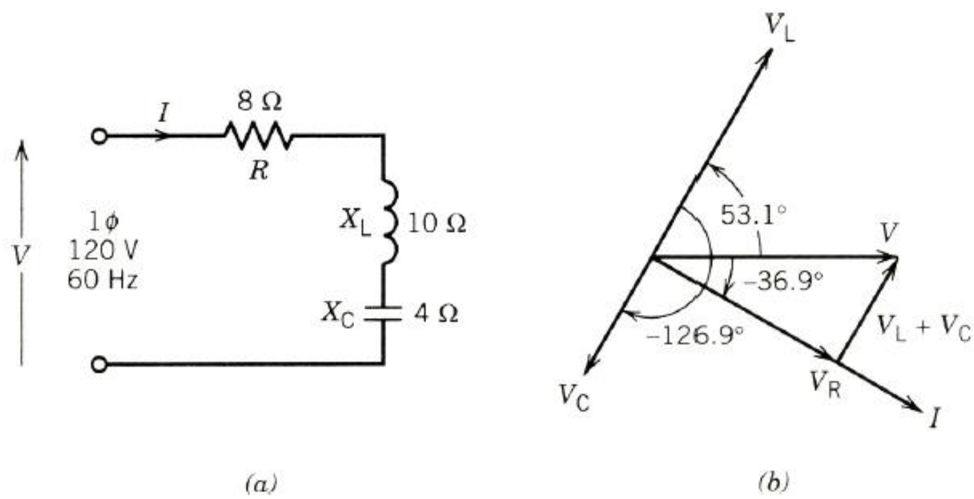


FIGURE EB.1

Solution

(a)

$$V = 120\ \underline{/0^\circ}\ \text{V}$$

$$I = \frac{120\ \underline{/0^\circ}}{8 + j10 - j4}$$

$$= \frac{120\ \underline{/0^\circ}}{10\ \underline{/36.9^\circ}}$$

$$= 12\ \underline{/-36.9^\circ}\ \text{A}$$

(b)

$$v = \sqrt{2} \times 120 \sin \omega t$$

$$i = \sqrt{2} \times 12 \sin(\omega t - 36.9^\circ)$$

Note: They could also be written as cos (…) instead of sin (…)

(c)

$$P = 120 \times 12 \times \cos 36.9^\circ = 1152 \text{ W}$$

or

$$P = I^2 R = 12^2 \times 8 = 1152 \text{ W}$$

(No real power absorbed by L or C)

$$\text{PF} = \cos 36.9^\circ = 0.8$$

(d)

$$V_R = IR = 12 \underline{/-36.9^\circ} \times 8 \underline{/0^\circ} = 96 \underline{/-36.9^\circ} \text{ V}$$

$$V_L = IjX_L = 12 \underline{/-36.9^\circ} \times 10 \underline{/90^\circ}$$

$$= 120 \underline{/-36.9^\circ + 90^\circ}$$

$$= 120 \underline{/53.1^\circ} \text{ V}$$

$$V_C = I(-jX_c) = 12 \underline{/-36.9^\circ} \times 4 \underline{/-90^\circ}$$

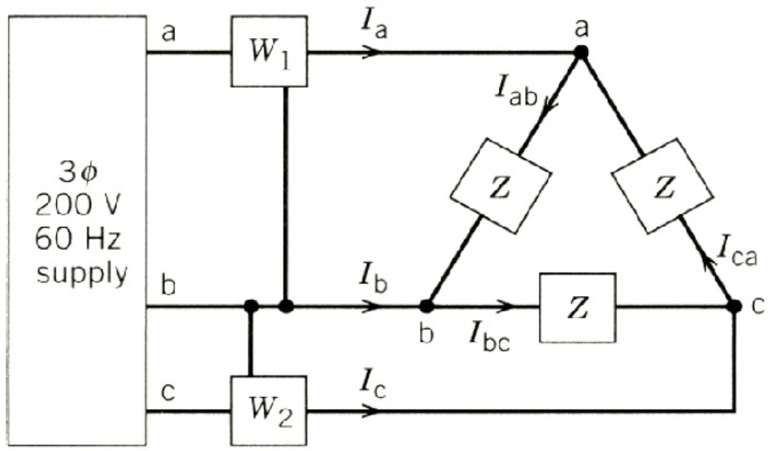
$$= 48 \underline{/-36.9^\circ - 90^\circ}$$

$$= 48 \underline{/-126.9^\circ} \text{ V}$$

The phasor diagram is shown in Fig. EB.1*b*.
Note: $V = V_R + V_L + V_C$ ■

Example B.2:

In the three-phase balanced system shown below, the load is $Z = 8 + j6\,\Omega$ and consider $V_{ab} = 200\,\angle 0^\circ$ V (rms voltage) and phase sequence a, b, and c.



(a) Determine the phase currents in phasor (I_{ab}, I_{bc}, and I_{ca}).

(b) Determine the line currents in phasor (I_a, I_b, and I_c).

(c) Determine the total three-phase power.

(d) Determine the wattmeter readings W_1 and W_2.

Solution

(a)

$$Z = 8 + j6 = 10\,\angle 36.9^\circ\ \Omega$$

$$I_{ab} = \frac{200\,\angle 0^\circ}{10\,\angle 36.9^\circ} = 20\,\angle -36.9^\circ \text{ A}$$

$$I_{bc} = \frac{200\,\angle -120^\circ}{10\,\angle 36.9^\circ} = 20\,\angle -36.9^\circ - 120^\circ = 20\,\angle -156.9^\circ \text{ A}$$

$$I_{ca} = 20\,\angle -36.9^\circ + 120^\circ = 20\,\angle 83.1^\circ \text{ A}$$

(b)

$$I_a = I_{ab} - I_{ca} = 20\,\underline{/-36.9^\circ} - 20\,\underline{/83.1^\circ}$$

$$= \sqrt{3} \times 20\,\underline{/-36.9^\circ - 30^\circ} = 34.641\,\underline{/-66.9^\circ}\text{ A}$$

$$I_b = 34.641\,\underline{/-66.9^\circ - 120^\circ} = 34.641\,\underline{/-186.9^\circ}\text{ A}$$

$$I_c = 34.641\,\underline{/-66.9^\circ + 120^\circ} = 34.641\,\underline{/53.1^\circ}\text{ A}$$

(c)

$$P_{3\phi} = 3V_p I_p \cos\theta = 3 \times 200 \times 20 \times \cos 36.9^\circ = 9600\text{ W}$$

or

$$= \sqrt{3}\; V_l I_l \cos\theta = \sqrt{3} \times 200 \times 34.641 \cos 36.9^\circ = 9600\text{ W}$$

(d)

$$W_1 = V_{ab} I_a \cos\; \underline{/V_{ab}, I_a} = 200 \times 34.641 \times \cos(36.9^\circ + 30^\circ)$$

$$= 2721.45\text{ W}$$

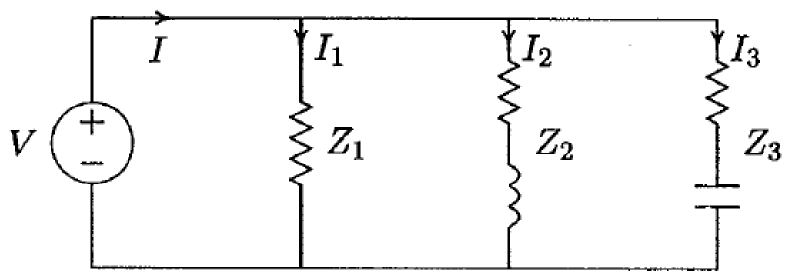
$$W_2 = V_{cb} I_c \cos\; \underline{/V_{cb}, I_C} = 200 \times 34.641 \times \cos(36.9^\circ - 30^\circ)$$

$$= 6878.26\text{ W}$$

Check $W_1 + W_2 = 2721.45 + 6878.26 = 9600\text{ W} = P_{3\phi}$.

Additional Example 1:

In the circuit $V = 1200\angle 0°$ V, $Z_1 = 60 + j0\ \Omega$, $Z_2 = 6 + j12\ \Omega$ and $Z_3 = 30 - j30\ \Omega$. Find the power absorbed by each load and the total complex power.



Solution:

$$I_1 = \frac{1200\angle 0°}{60\angle 0} = 20 + j0\ \text{A}$$

$$I_2 = \frac{1200\angle 0°}{6 + j12} = 40 - j80\ \text{A}$$

$$I_3 = \frac{1200\angle 0°}{30 - j30} = 20 + j20\ \text{A}$$

$$S_1 = VI_1^* = 1200\angle 0°(20 - j0) = 24,000 \text{ W} + j0 \text{ var}$$
$$S_2 = VI_2^* = 1200\angle 0°(40 + j80) = 48,000 \text{ W} + j96,000 \text{ var}$$
$$S_3 = VI_3^* = 1200\angle 0°(20 - j20) = 24,000 \text{ W} - j24,000 \text{ var}$$

The total load complex power adds up to

$$S = S_1 + S_2 + S_3 = 96,000 \text{ W} + j72,000 \text{ var}$$

Alternatively, the sum of complex power delivered to the load can be obtained by first finding the total current.

$$I = I_1 + I_2 + I_3 = (20 + j0) + (40 - j80) + (20 + j20)$$
$$= 80 - j60 = 100\angle -36.87° \text{ A}$$

and

$$S = VI^* = (1200\angle 0°)(100\angle 36.87°) = 120,000\angle 36.87° \text{ VA}$$
$$= 96,000 \text{ W} + j72,000 \text{ var}$$

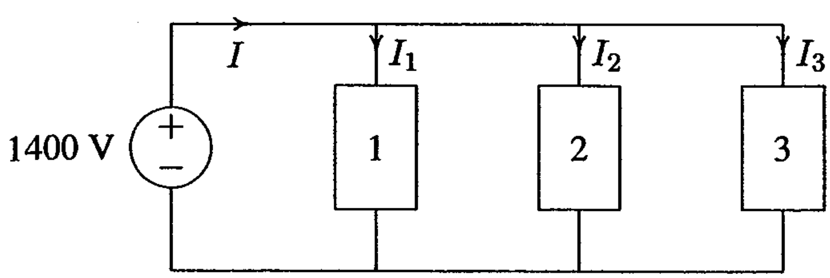
Additional Example 2:

Three loads are connected in parallel across a 1400-V rms, 60-Hz single-phase supply as shown in Figure .

Load 1: Inductive load, 125 kVA at 0.28 power factor.

Load 2: Capacitive load, 10 kW and 40 kvar.

Load 3: Resistive load of 15 kW.



(a) Find the total kW, kvar, kVA, and the supply power factor.

(b) A capacitor of negligible resistance is connected in parallel with the above loads to improve the power factor to 0.8 lagging. Determine the kvar rating of this capacitor and the capacitance in μF.

Solution (a):

An inductive load has a lagging power factor, the capacitive load has a leading power factor, and the resistive load has a unity power factor.

For Load 1:

$$\theta_1 = \cos^{-1}(0.28) = 73.74^\circ \text{ lagging}$$

The load complex powers are

$$\begin{aligned} S_1 &= 125\angle 73.74 \text{ kVA} = 35 \text{ kW} + j120 \text{ kvar} \\ S_2 &= 10 \text{ kW} - j40 \text{ kvar} \\ S_3 &= 15 \text{ kW} + j0 \text{ kvar} \end{aligned}$$

The total apparent power is

$$\begin{aligned} S = P + jQ &= S_1 + S_2 + S_3 \\ &= (35 + j120) + (10 - j40) + (15 + j0) \\ &= 60 \text{ kW} + j80 \text{ kvar} = 100\angle 53.13 \text{ kVA} \end{aligned}$$

The total current is

$$I = \frac{S^*}{V^*} = \frac{100,000\angle -53.13^\circ}{1400\angle 0^\circ} = 71.43\angle -53.13^\circ \text{ A}$$

The supply power factor is

$$PF = \cos(53.13) = 0.6 \text{ lagging}$$

Solution (b):

Total real power $P = 60$ kW at the new power factor of 0.8 lagging results in the new reactive power Q'.

$$\theta' = \cos^{-1}(0.8) = 36.87^\circ$$
$$Q' = 60\tan(36.87^\circ) = 45 \text{ kvar}$$

Therefore, the required capacitor kvar is

$$Q_c = 80 - 45 = 35 \text{ kvar}$$

and

$$X_c = \frac{|V|^2}{S_c^*} = \frac{1400^2}{j35,000} = -j56\ \Omega$$
$$C = \frac{10^6}{2\pi(60)(56)} = 47.37\ \mu\text{F}$$

and the new current is

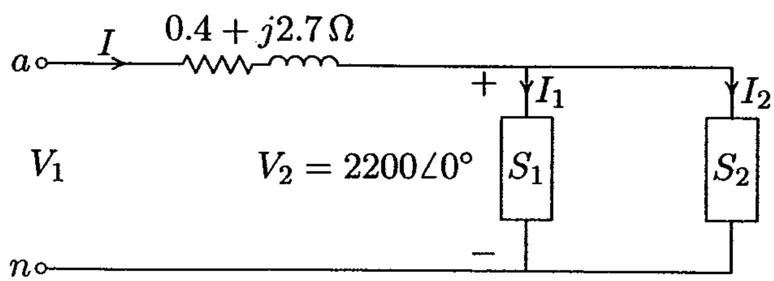
$$I' = \frac{S'^*}{V^*} = \frac{60,000 - j45,000}{1400\angle 0^\circ} = 53.57\angle -36.87^\circ \text{ A}$$

Note the reduction in the supply current from 71.43 A to 53.57 A.

Additional Example 3:

A three-phase line has an impedance of $0.4 + j2.7\ \Omega$ per phase. The line feeds two balanced three-phase loads that are connected in parallel. The first load is absorbing 560.1 kVA at 0.707 power factor lagging. The second load absorbs 132 kW at unity power factor. The line-to-line voltage at the load end of the line is 3810.5 V. Determine:

(a) The magnitude of the line voltage at the source end of the line.
(b) Total real and reactive power loss in the line.
(c) Real power and reactive power supplied at the sending end of the line.



Single-phase equivalent diagram

Solution:

(a) The phase voltage at the load terminals is

$$V_2 = \frac{3810.5}{\sqrt{3}} = 2200 \text{ V}$$

The total complex power is

$$S_{R(3\phi)} = 560.1(0.707 + j0.707) + 132 = 528 + j396$$
$$= 660\angle 36.87^\circ \text{ kVA}$$

With the phase voltage V_2 as reference, the current in the line is

$$I = \frac{S^*_{R(3\phi)}}{3V_2^*} = \frac{660,000\angle{-36.87^\circ}}{3(2200\angle 0^\circ)} = 100\angle{-36.87^\circ} \text{ A}$$

The phase voltage at the sending end is

$$V_1 = 2200\angle 0^\circ + (0.4 + j2.7)100\angle{-36.87^\circ} = 2401.7\angle 4.58^\circ \text{ V}$$

The magnitude of the line voltage at the sending end of the line is

$$|V_{1L}| = \sqrt{3}|V_1| = \sqrt{3}(2401.7) = 4160 \text{ V}$$

(b) The three-phase power loss in the line is

$$S_{L(3\phi)} = 3R|I|^2 + j3X|I|^2 = 3(0.4)(100)^2 + j3(2.7)(100)^2$$
$$= 12 \text{ kW} + j81 \text{ kvar}$$

(c) The three-phase sending power is

$$S_{S(3\phi)} = 3V_1 I^* = 3(2401.7\angle 4.58°)(100\angle 36.87°) = 540 \text{ kW} + j477 \text{ kvar}$$

It is clear that the sum of load powers and the line losses is equal to the power delivered from the supply, i.e.,

$$S_{S(3\phi)} = S_{R(3\phi)} + S_{L(3\phi)} = (528 + j396) + (12 + j81) = 540 \text{ kW} + j477 \text{ kvar}$$

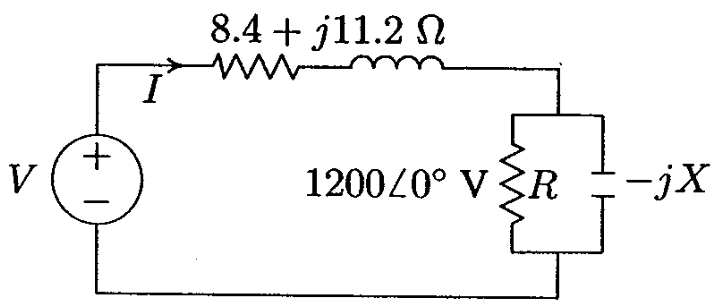
Assignments

Q1):

An inductive load consisting of R and X in series feeding from a 2400-V rms supply absorbs 288 kW at a lagging power factor of 0.8. Determine R and X.

Q2):

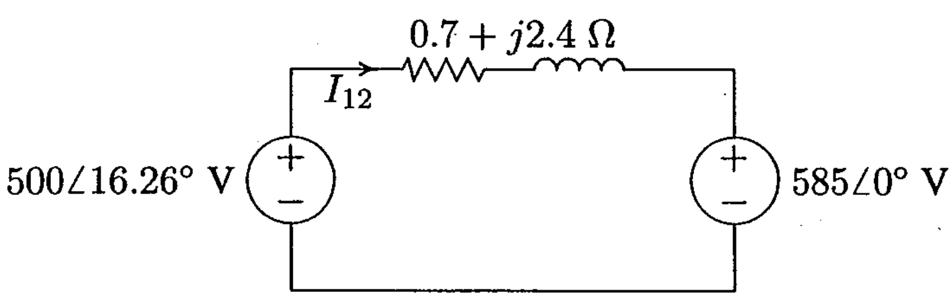
The load shown in Figure consists of a resistance R in parallel with a capacitor of reactance X. The load is fed from a single-phase supply through a line of impedance $8.4 + j11.2\ \Omega$. The rms voltage at the load terminal is $1200\angle 0°$ V rms, and the load is taking 30 kVA at 0.8 power factor leading.
(a) Find the values of R and X.
(b) Determine the supply voltage V.



Q3):

Two single-phase ideal voltage sources are connected by a line of impedance of $0.7 + j2.4\ \Omega$ as shown in Figure . $V_1 = 500\angle 16.26°$ V and $V_2 = 585\angle 0°$ V.

Find the complex power for each machine and determine whether they are delivering or receiving real and reactive power. Also, find the real and the reactive power loss in the line.



Q4):

A balanced delta connected load of $15 + j18\ \Omega$ per phase is connected at the end of a three-phase line as shown in Figure . The line impedance is $1 + j2\ \Omega$ per phase. The line is supplied from a three-phase source with a line-to-line voltage of 207.85 V rms. Taking V_{an} as reference, determine the following:

(a) Current in phase a.

(b) Total complex power supplied from the source.

(c) Magnitude of the line-to-line voltage at the load terminal.