ELEC 342: Electromechanical Energy Conversion and Transmission

Homework 2

- ✓ Examples (questions with full answers)
- ✓ **Assignments** (questions with no answers)

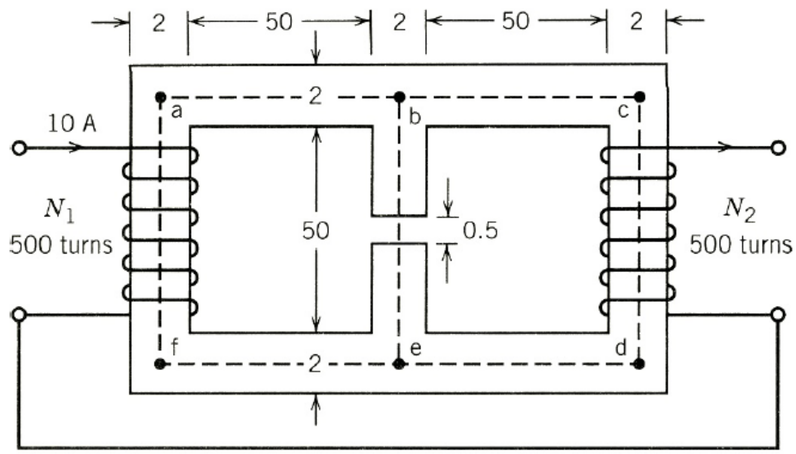
You need to submit your answers (procedure + final answer) for "Assignments" only by 11:59 pm on due date.

Summary:

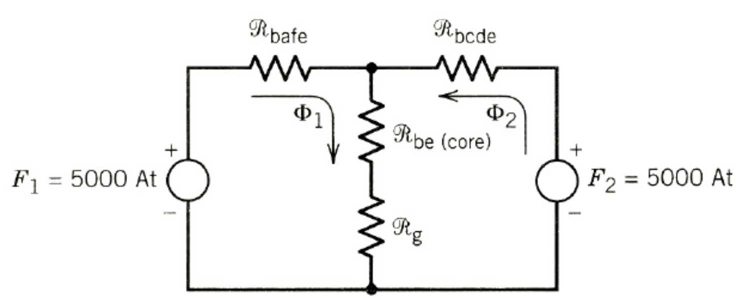
- Follow through **Examples 1.3, 1.4, 1.5, 1.6** from the textbook.
- **Assignments**: Solve Problems **P1.1**, **P1.4, P1.5, P1.11**.

Example 1.3:

In the magnetic circuit of Fig. El.3*a*, the relative permeability of the ferromagnetic material is 1200. Neglect magnetic leakage and fringing. All dimensions are in centimeters, and the magnetic material has a square cross-sectional area. Determine the air gap flux, the air gap flux density, and the magnetic field intensity in the air gap.



(*a*)



(*b*)

FIGURE E1.3

Solution

The mean magnetic paths of the fluxes are shown by dashed lines in Fig. El.3*a*. The equivalent magnetic circuit is shown in Fig. El.3*b*.

$$F_1 = N_1 I_1 = 500 \times 10 = 5000 \text{ At}$$

$$F_2 = N_2 I_2 = 500 \times 10 = 5000 \text{ At}$$

$$\mu_c = 1200\mu_0 = 1200 \times 4\pi 10^{-7}$$

$$\mathcal{R}_{bafe} = \frac{l_{bafe}}{\mu_c A_c}$$

$$= \frac{3 \times 52 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}}$$

$$= 2.58 \times 10^6 \text{ At/Wb}$$

From symmetry

$$\mathscr{R}_{\text{bcde}} = \mathscr{R}_{\text{bafe}}$$

$$\mathscr{R}\text{g} = \frac{l_g}{\mu_0 A_g}$$

$$= \frac{5 \times 10^{-3}}{4\pi 10^{-7} \times 2 \times 2 \times 10^{-4}}$$

$$= 9.94 \times 10^6 \text{ At/Wb}$$

$$\mathscr{R}_{\text{be(core)}} = \frac{l_{\text{be(core)}}}{\mu_c A_c}$$

$$= \frac{51.5 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}}$$

$$= 0.82 \times 10^6 \text{ At/Wb}$$

The loop equations are

$$\Phi_1(\mathscr{R}_{\text{bafe}} + \mathscr{R}_{\text{be}} + \mathscr{R}_{\text{g}}) + \Phi_2(\mathscr{R}_{\text{be}} + \mathscr{R}_{\text{g}}) = F_1$$

$$\Phi_1(\mathscr{R}_{\text{be}} + \mathscr{R}_{\text{g}}) + \Phi_2(\mathscr{R}_{\text{bcde}} + \mathscr{R}_{\text{be}} + \mathscr{R}_{\text{g}}) = F_2$$

or

$$\Phi_1(13.34 \times 10^6) + \Phi_2(10.76 \times 10^6) = 5000$$

$$\Phi_1(10.76 \times 10^6) + \Phi_2(13.34 \times 10^6) = 5000$$

or

$$\Phi_1 = \Phi_2 = 2.067 \times 10^{-4} \text{ Wb}$$

The air gap flux is

$$\Phi_g = \Phi_1 + \Phi_2 = 4.134 \times 10^{-4} \text{ Wb}$$

The air gap flux density is

$$B_g = \frac{\Phi_g}{A_g} = \frac{4.134 \times 10^{-4}}{4 \times 10^{-4}} = 1.034 \text{ T}$$

The magnetic intensity in the air gap is

$$H_g = \frac{B_g}{\mu_0} = \frac{1.034}{4\pi 10^{-7}} = 0.822 \times 10^6 \text{ At/m} \qquad \blacksquare$$

Example 1.4:

For the magnetic circuit of Fig. 1.9, $N = 400$ turns.
Mean core length $l_c = 50$ cm.
Air gap length $l_g = 1.0$ mm.
Cross-sectional area $A_c = A_g = 15\,\text{cm}^2$.
Relative permeability of core $\mu_r = 3000$.
$i = 1.0$ A.
Find

(a) Flux and flux density in the air gap.
(b) Inductance of the coil.

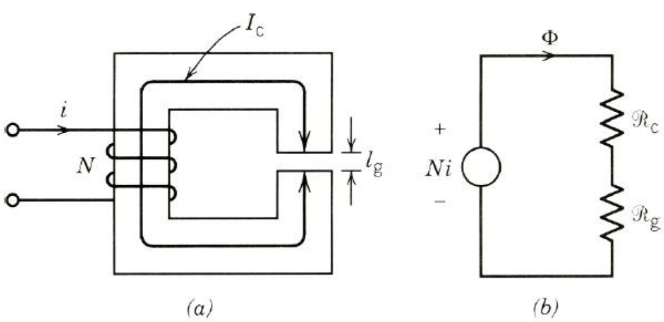


FIGURE 1.9 Composite structure. (a) Magnetic core with air gap. (b) Magnetic equivalent circuit.

Solution

(a)

$$\mathscr{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{50 \times 10^{-2}}{3000 \times 4\pi 10^{-7} \times 15 \times 10^{-4}}$$

$$= 88.42 \times 10^3 \text{ AT/Wb}$$

$$\mathscr{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{1 \times 10^{-3}}{4\pi 10^{-7} \times 15 \times 10^{-4}}$$

$$= 530.515 \times 10^3 \text{ At/Wb}$$

$$\Phi = \frac{Ni}{R_c + R_g}$$

$$= \frac{400 \times 1.0}{(88.42 + 530.515)10^3}$$

$$B = \frac{\Phi}{A_g} = \frac{0.6463 \times 10^{-4}}{15 \times 10^{-4}} = 0.4309 \text{ T}$$

(b)

$$L = \frac{N^2}{\mathscr{R}_c + \mathscr{R}_g} = \frac{400^2}{(88.42 + 530.515)10^3}$$

$$= 258.52 \times 10^{-3}\ \mathrm{H}$$

or

$$L = \frac{\lambda}{i} = \frac{N\Phi}{i} = \frac{400 \times 0.6463 \times 10^{-3}}{1.0}$$

$$= 258.52 \times 10^{-3}\ \mathrm{H} \quad \blacksquare$$

Example 1.5:

The coil in Fig. 1.4 has 250 turns and is wound on a silicon sheet steel. The inner and outer radii are 20 and 25 cm, respectively, and the toroidal core has a circular cross section. For a coil current of 2.5 A, find

(a) The magnetic flux density at the mean radius of the toroid.

(b) The inductance of the coil, assuming that the flux density within the core is uniform and equal to that at the mean radius.

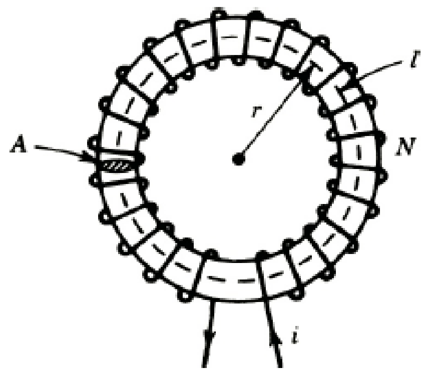


FIGURE 1.4 Toroid magnetic circuit.

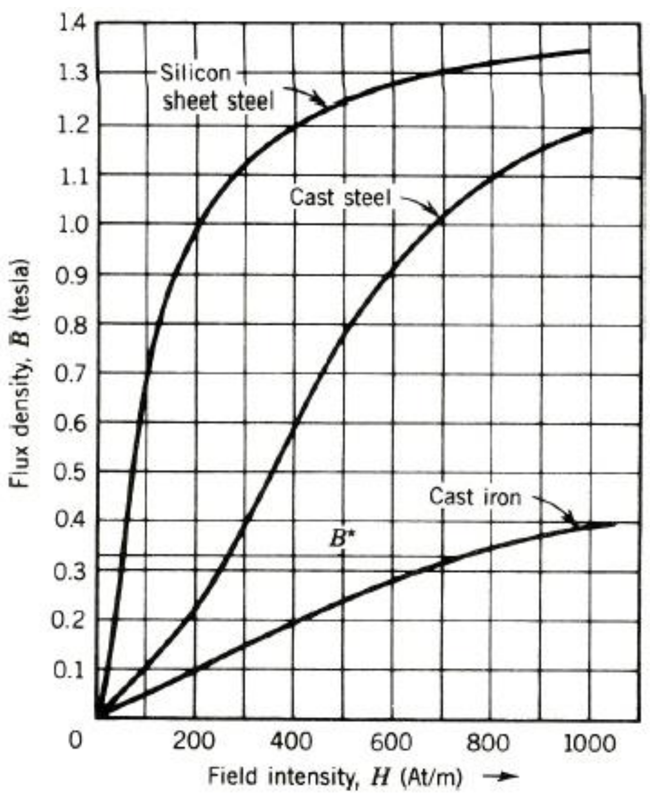


FIGURE 1.7 Magnetization curves.

Solution

(a) Mean radius is $\frac{1}{2}(25+20) = 22.5$ cm

$$H = \frac{Ni}{l} = \frac{250 \times 2.5}{2\pi 22.5 \times 10^{-2}} = 442.3 \text{ At/m}$$

From the $B-H$ curve for silicon sheet steel (Fig. 1.7),

$$B = 1.225 \text{ T}$$

(b) The cross-sectional area is

$$A = \pi(\text{radius of core})^2$$

$$= \pi\left(\frac{25-20}{2}\right)^2 \times 10^{-4}$$

$$= \pi 6.25 \times 10^{-4} \text{ m}^2$$

$$\Phi = BA$$

$$= 1.225 \times \pi 6.25 \times 10^{-4}$$

$$= 24.04 \times 10^{-4} \text{ Wb}$$

$$\lambda = 250 \times 24.04 \times 10^{-4}$$

$$= 0.601 \text{ Wb} \cdot \text{turn}$$

$$L = \frac{\lambda}{i} = \frac{0.601}{2.5} = 0.2404 \text{ H}$$

$$= 240.4 \text{ mH}$$

Inductance can also be calculated using Eq. 1.25:

Flux linkage $\lambda = N\Phi$ (1.22)

Inductance $L = \frac{\lambda}{i}$ (1.23)

$$L = \frac{N\Phi}{i} = \frac{NBA}{i} = \frac{N\mu HA}{i}$$

$$= \frac{N\mu HA}{Hl/N} = \frac{N^2}{l/\mu A} \tag{1.24}$$

$$L = \frac{N^2}{\mathscr{R}} \tag{1.25}$$

$$\mu \text{ of core} = \frac{B}{H} = \frac{1.225}{442.3}$$

$$\mathscr{R}_{\text{core}} = \frac{l}{\mu A} = \frac{2\pi 22.5 \times 10^{-2}}{(1.225/442.3) \times \pi 6.25 \times 10^{-4}}$$

$$= 2599.64 \times 10^2 \text{ At/Wb}$$

$$= \frac{N^2}{\mathscr{R}} L = \frac{250^2}{2599.64 \times 10^2} = 0.2404 \text{ H}$$

$$= 240.4 \text{ mM} \quad \blacksquare$$

Example 1.6:

A 1Φ, 120 V, 60 Hz supply is connected to the coil of Fig. The coil has 200 turns. The parameters of the core are as follows:

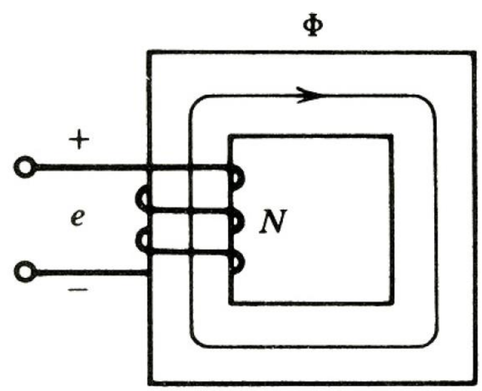
Length of core = 100 cm

Cross-sectional area of core = 20 cm^2

Relative permeability of core = 2500

(a) Obtain an expression for the flux density in the core.

(b) Obtain an expression for the current in the coil.



Solution

(a)

$$e(t) = E_{\max}\cos\omega t = \sqrt{2}\,E_{\mathrm{rms}}\cos\omega t$$

From Faraday's law, the voltage induced in the N-turn coil is

$$e(t) = N\frac{d\Phi}{dt} \quad \Rightarrow \quad \Phi(t) = \int \frac{e(t)}{N}\,dt = \Phi_{\max}\sin\omega t$$

where $\Phi_{\max}$ is the amplitude of the core flux

$$\Phi_{\max} = \frac{\sqrt{2}\,E_{\mathrm{rms}}}{N\omega} = \frac{E_{\mathrm{rms}}}{4.44Nf}$$

From Eq. of $\Phi_{\max}$

$$\Phi_{\max} = \frac{120}{4.44 \times 200 \times 60}$$

$$= 0.002253 \text{ Wb}$$

$$B_{\max} = \frac{0.002253}{20 \times 10^{-4}} = 1.1265 \text{ T}$$

$$B = 1.1265 \sin 2\pi 60t$$

(b)

$$H_{\max} = \frac{1.1265}{2500 \times 4\pi 10^{-7}}$$

$$= 358.575 \text{ At/m}$$

$$i_{\max} = \frac{Hl}{N}$$

$$= \frac{358.575 \times 100 \times 10^{-2}}{200}$$

$$= 1.79328 \text{ A}$$

$$i = 1.7928 \sin 2\pi 60t \quad \blacksquare$$

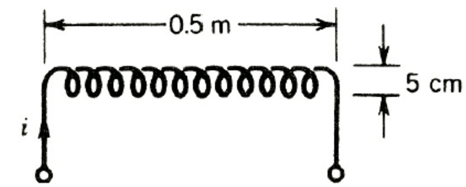
Assignments

P 1.1):

The long solenoid coil shown in Fig. has 250 turns. As its length is much greater than its diameter, the field inside the coil may be considered uniform. Neglect the field outside.

(a) Determine the field intensity (H) and flux density (B) inside the solenoid ($i = 100$ A).

(b) Determine the inductance of the solenoid coil.



P 1.4):

Two coils are wound on a toroidal core as shown in Fig. P1.4. The core is made of silicon sheet steel and has a square cross section. The coil currents are $i_1 = 0.28$ A and $i_2 = 0.56$ A.

(a) Determine the flux density at the mean radius of the core.

(b) Assuming constant flux density (same as at the mean radius) over the cross section of the core, determine the flux in the core.

(c) Determine the relative permeability, μ_r, of the core.

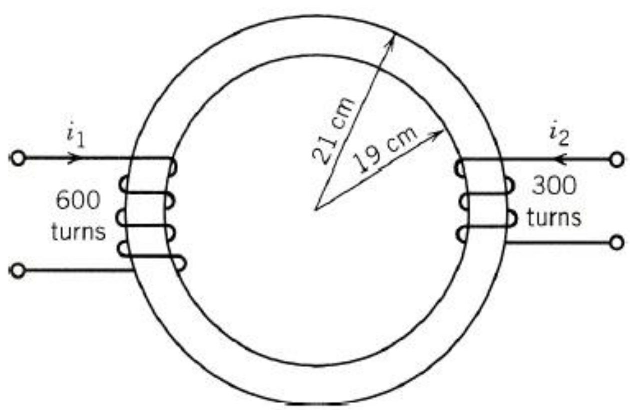


FIGURE P1.4

P 1.5):

The magnetic circuit of Fig. P1.5 provides flux in the two air gaps. The coils ($N_1 = 700$, $N_2 = 200$) are connected in series and carry a current of 0.5 ampere. Neglect leakage flux, reluctance of the iron (i.e., infinite permeability), and fringing at the air gaps. Determine the flux and flux density in the air gaps.

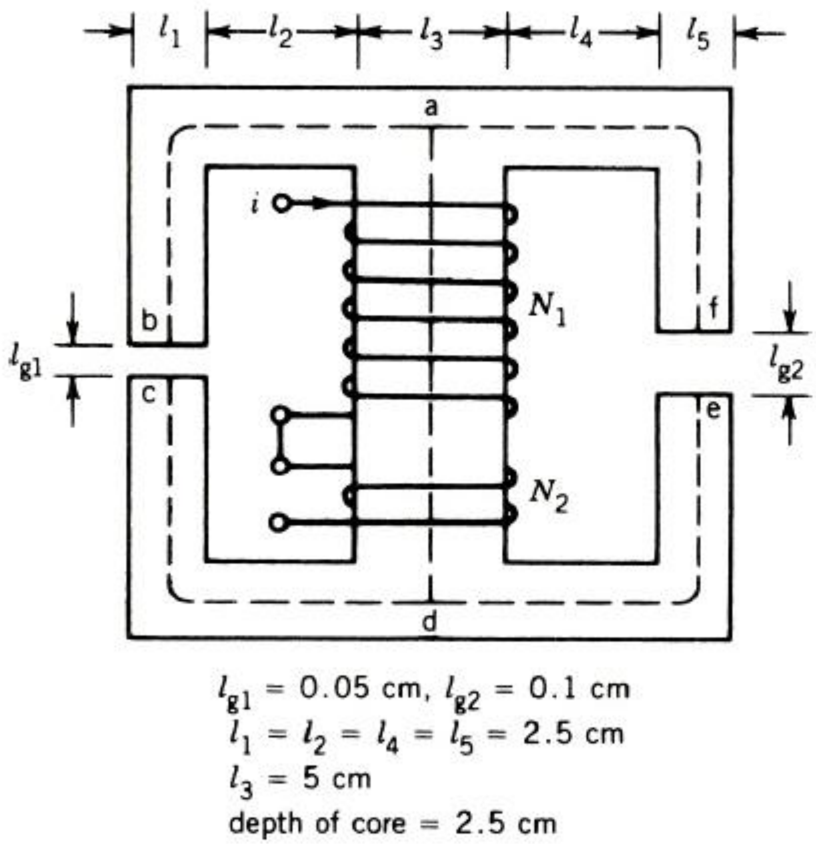


FIGURE P1.5

P 1.11):

The magnetic circuit of Fig. P1.11 has a core of relative permeability $\mu_r = 2000$. The depth of the core is 5 cm. The coil has 400 turns and carries a current of 1.5 A.

(a) Draw the magnetic equivalent circuit.

(b) Find the flux and the flux density in the core.

(c) Determine the inductance of the coil.

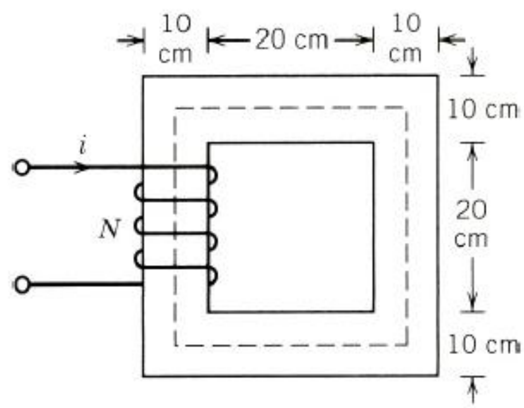


FIGURE P1.11