ELEC 342: Electromechanical Energy Conversion and Transmission

Homework 3 (Chapters 2 and 3 from the Textbook)

- ✓ **Solutions only to Assignment Problems**

Summary:

- **Chapter 2:** Problems **P2.3, P2.9, P2.26, P2.32** from the textbook.
- **Chapter 3:** Problems **P3.5**, **P3.7** from the textbook.

Chapter 2: Problems

P 2.3):

A single-phase transformer has 800 turns on the primary winding and 400 turns on the secondary winding. The cross-sectional core area is 50 cm^2. If the primary winding is connected to a 1 φ, 1000 V, 60 Hz supply, calculate

(a) The maximum value of the flux density in the core, B_{max}.

(b) The induced voltage in the secondary winding.

Solution:

(a) $$V_1 = E_1 = 4.44\, N_1 f \Phi_{max}$$

$$\Phi_{max} = \frac{1000}{4.44 \times 800 \times 60} = 4.67 \times 10^{-3}\ \text{Wb}$$

$$B_{max} = \frac{4.67 \times 10^{-3}}{50 \times 10^{-4}} = 0.934\ \text{T}$$

(b) $$E_1 = V_2 = \frac{N_2}{N_1} V_1 = \frac{400}{800} \times 1000 = 500\ \text{V}$$

P 2.9):

A 1ϕ, two-winding transformer has 1000 turns on the primary and 500 turns on the secondary. The primary winding is connected to a 220 V supply and the secondary winding is connected to a 5 kVA load. The transformer can be considered ideal.

(a) Determine the load voltage.

(b) Determine the load impedance.

(c) Determine the load impedance referred to the primary.

Solution:

(a) $$V_2 = \frac{N_2}{N_1} V_1 = \frac{500}{1000} \times 220 = 110\ \text{V}$$

(b) $$I_2 = \frac{5000}{110} = 45.4545\ \text{A}$$

$$Z_2 = \frac{110}{45.4545} = 2.42\ \Omega$$

(c) $$Z_2' = \left(\frac{1000}{500}\right)^2 \times 2.42 = 9.68\ \Omega$$

P 2.26):

2.26 A 1ϕ, 10 kVA, 460/120 V, 60 Hz transformer has an efficiency of 96% when it delivers 9 kW at 0.9 power factor. This transformer is connected as an autotransformer to supply load to a 460 V circuit from a 580 V source.

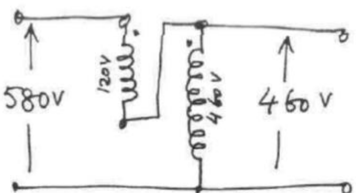
(a) Show the autotransformer connection.

(b) Determine the maximum kVA the autotransformer can supply to the 460 V circuit.

(c) Determine the efficiency of the autotransformer for full load at 0.9 power factor.

Solution:

2.26 (a)



(b) $I_{120V} = \dfrac{10 \times 10^3}{120} = 83.33\,A$

$KVA\big|_{max} = 580 \times 83.33 = 48.33\ KVA$

(c) Transformer losses when rated current flows.

$P_{loss} = 9 \times 10^3 (1 - 0.96) = 360\,W$

$Eff = \dfrac{48330 \times 0.9}{48330 \times 0.9 + 360} \times 100\% = 99.2\%$

P 2.32):

2.32 A 1ϕ, 200 kVA, 2100/210 V, 60 Hz transformer has the following characteristics. The impedance of the high-voltage winding is $0.25 + j1.5\,\Omega$ with the low-voltage winding short-circuited. The admittance (i.e., inverse of impedance) of the low-voltage winding is $0.025 - j0.075$ mhos with the high-voltage winding open-circuited.

(a) Taking the transformer rating as base, determine the base values of power, voltage, current, and impedance for both the high-voltage and low-voltage sides of the transformer.

(b) Determine the per-unit value of the equivalent resistance and leakage reactance of the transformer.

(c) Determine the per-unit value of the excitation current at rated voltage.

(d) Determine the per-unit value of the total power loss in the transformer at full-load output condition.

Solution:

$\boxed{2.32}$

(a)

	High voltage	Low voltage
Base Voltage V_b	2100 V	210 V
Base Volt-ampere S_b	200 KVA	200 KVA
Base current $I_b = S_b / V_b$	95.24 A	952.4 A
Base impedance $Z_b = V_b / I_b$	22.05	0.2205 Ω

(b) $Z_{eq} = (0.25 + j1.5)/22.05 = 0.01134 + j0.068$

(c) $I_m = \left|(0.025 - j0.075)\frac{20}{952.4}\right| = 0.01743$ pu.

(d) $P_{core} = 0.025 \times 210^2 = 1102.5$ W

$= \frac{1102.5}{200{,}000} = 0.00551$ pu.

$P_{cu} = 0.01134$ pu.

$P_{Loss} = P_{core} + P_{cu} = 0.01685$ pu.

Chapter 3: Problems

3.5 An electromagnet lift system is shown in Fig. P3.5. The coil has 2500 turns. The flux density in the air gap is 1.25 T. Assume that the core material is ideal.

(a) For an air gap, $g = 10$ mm:

(i) Determine the coil current.

(ii) Determine the energy stored in the magnetic system.

(iii) Determine the force on the load (sheet of steel).

(iv) Determine the mass of the load (acceleration due to gravity = 9.81 m/sec^2).

(b) If the air gap is 2 mm, determine the coil current required to lift the load.

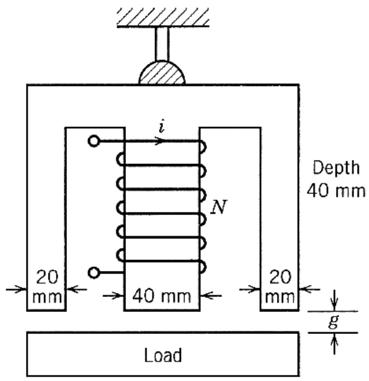


FIGURE P3.5

Solution:

$\boxed{3.5}$ (a) $g = 10\ mm = 10 \times 10^{-3}\ m$

(i) $Ni = H_g \cdot 2g = \frac{B_g}{\mu_0} \times 2g$

$$i = \frac{1.25 \times 2 \times 10 \times 10^{-3}}{2500 \times 4\pi\ 10^{-7}} = 7.9577\ A$$

(ii) $$W_f = W_{fg} = \frac{B_g^2}{2\mu_0} \times V_g = \frac{1.25^2}{2 \times 4\pi \times 10^{-7}} \times 80 \times 40 \times 10^{-6} \times 10 \times 10^{-3}$$

$= 19.8\ J$

(iii) $$f_m = \frac{B_g^2}{2\mu_0} \times Area = \frac{1.25^2}{2 \times 4\pi \times 10^{-7}} \times 80 \times 40 \times 10^{-6} = 1980 N.$$

(iv) $$Mass = \frac{1980}{9.81} = 201.64\ kg.$$

(b) $g = 5\ mm = 5 \times 10^{-3}\ m.$

$$f_m = \frac{B_g^2}{2\mu_0} \times Area \propto B_g^2 \propto \left(\frac{Ni}{2g}\right)^2 \propto \left(\frac{i}{g}\right)^2$$

Force remains same for $g = 5\ mm$ and $g = 10\ mm$.

$$\left(\frac{i_2}{g_2}\right)^2 = \left(\frac{i_1}{g_1}\right)^2 \rightarrow \left(\frac{i_2}{5}\right)^2 = \left(\frac{7.9577}{10}\right)^2$$

$i_2 = 3.9789\ A.$

3.7 The electromagnet shown in Fig. P3.7 can be used to lift a sheet of steel. The coil has 400 turns and a resistance of 5 ohms. The reluctance of the magnetic material is negligible. The magnetic core has a square cross section of 5 cm by 5 cm. When the sheet of steel is fitted to the electromagnet, air gaps, each of length $g = 1$ mm, separate them. An average force of 550 newtons is required to lift the sheet of steel.

(a) For dc supply,

(i) Determine the dc source voltage.

(ii) Determine the energy stored in the magnetic field.

(b) For ac supply at 60 Hz, determine the ac source voltage.

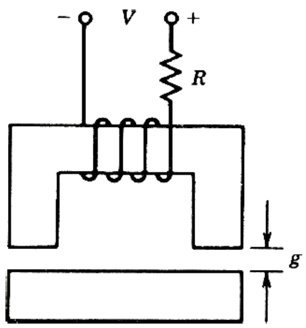


FIGURE P3.7

Solution:

$\boxed{3.7}$ (a)(i) $f_m = 550 = \frac{B^2}{2\mu_0} \times$ pole area.

$$B = \sqrt{\frac{550 \times 2 \times 4\pi 10^{-7}}{2 \times 5 \times 5 \times 10^{-4}}} = 0.5258\,T$$

$$Ni = H_g l_g = \frac{B}{\mu_0} 2g$$

$$i = \frac{0.5258 \times 2 \times 1 \times 10^{-3}}{4\pi\, 10^{-7} \times 400} = 2.09\,A$$

$$V = 5 \times 2.09 = 10.45\ V\ (dc)$$

(ii) $$W_f = \frac{B^2}{2\mu_0} \times \text{volume of airgap}$$

$$= \frac{0.5258^2}{2 \times 4\pi 10^{-7}} \times 2 \times 5 \times 5 \times 10^{-4} \times 10^{-3}$$

$$= 0.55\ J.$$

or $W_f = \frac{1}{2}\lambda i$ (linear system)

$$= \frac{1}{2} NABi = \frac{1}{2} \times 400 \times 5 \times 5 \times 10^{-4} \times .5258 \times 2.09$$

$$= 0.55\ J$$

(b) $$L = \frac{N^2}{R_g} = \frac{N^2}{2g/\mu_0 A_g} = \frac{N^2 \mu_0 A_g}{2g}$$

$$= \frac{400^2 \times 4\pi 10^{-7} \times 5 \times 5 \times 10^{-4}}{2 \times 10^{-3}} = 0.2513\ H$$

$$Z = \sqrt{5^2 + (2\pi 60 \times 0.2513)^2} = 94.87\,\Omega$$

$f_m \propto B_{rms}^2$, $B_{rms} \propto I_{rms}$.

Hence, same rms current is required to produce the same average torque.

$$I_{rms} = 2.09\,A.$$

$$V_{rms} = 2.09 \times 94.87 = 198.3\ V.$$