ELEC 342: Electromechanical Energy Conversion and Transmission

Homework 3 (Chapters 2 and 3 from the Textbook)

✓ Examples (questions with full answers)
✓ **Assignments** (questions with no answers)

You need to submit your answers (procedure + final answer) for "Assignments" only by 11:59 pm on due date.

Summary:

- **Chapter 2:** Follow through **Examples 2.2, 2.6, 2.7, 2.8** from the textbook.
- **Assignments**: Solve Problems **P2.3, P2.9, P2.26, P2.32**.
- **Chapter 3:** Follow through **Examples 3.2, 3.3, 3.4** from the textbook.
- **Assignments**: Solve Problems **P3.5**, **P3.7**.

Example 2.2:

Tests are performed on a 1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer and the following results are obtained.

	Open-Circuit Test (high-voltage side open)	Short-Circuit Test (low-voltage side shorted)
Voltmeter	220 V	150 V
Ammeter	2.5 A	4.55 A
Wattmeter	100 W	215 W

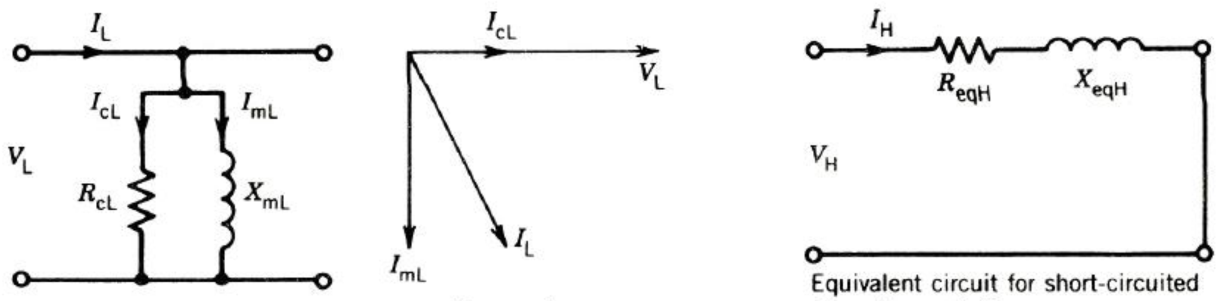
(a) Derive the parameters for the approximate equivalent circuits referred to the low-voltage side and the high-voltage side.

Solution

Note that for the no-load test the supply voltage (full-rated voltage of 220 V) is applied to the low-voltage winding, and for the short-circuit test the supply voltage is applied to the high-voltage winding with the low-voltage winding shorted. The subscripts H and L will be used to represent quantities for the high-voltage and low-voltage windings, respectively.

The ratings of the windings are as follows:

$$V_{H(rated)} = 2200 \text{ V}$$
$$V_{L(rated)} = 220 \text{ V}$$
$$I_{H(rated)} = \frac{10,000}{2200} = 4.55 \text{ A}$$
$$I_{L(rated)} = \frac{10,000}{220} = 45.5 \text{ A}$$
$$V_H I_H|_{rated} = V_L I_L|_{rated} = 10 \text{ kVA}$$



Equivalent circuit at no load — Phasor diagram

(a)

Equivalent circuit for short-circuited low-voltage winding

(b)

(a) The equivalent circuit and the phasor diagram for the open-circuit test are shown in Fig. E2.2*a*.

$$\text{Power, } P_{oc} = \frac{V_L^2}{R_{cL}}$$

$$R_{cL} = \frac{220^2}{100} = 484\ \Omega$$

$$I_{cL} = \frac{220}{484} = 0.45\ \text{A}$$

$$I_{mL} = (I_L^2 - I_{cL}^2)^{1/2} = (2.5^2 - 0.45^2)^{1/2} = 2.46\ \text{A}$$

$$X_{mL} = \frac{V_L}{I_{mL}} = \frac{220}{2.46} = 89.4\ \Omega$$

The corresponding parameters for the high-voltage side are obtained as follows:

$$\text{Turns ratio } a = \frac{2200}{220} = 10$$

$$R_{cH} = a^2 R_{cL} = 10^2 \times 484 = 48{,}400\ \Omega$$

$$X_{mH} = 10^2 \times 89.4 = 8940\ \Omega$$

The equivalent circuit with the low-voltage winding shorted is shown in Fig. E2.2*b*.

$$\text{Power } P_{sc} = I_H^2\, R_{eqH}$$

$$R_{eqH} = \frac{215}{4.55^2} = 10.4\ \Omega$$

$$Z_{eqH} = \frac{V_H}{I_H} = \frac{150}{4.55} = 32.97\ \Omega$$

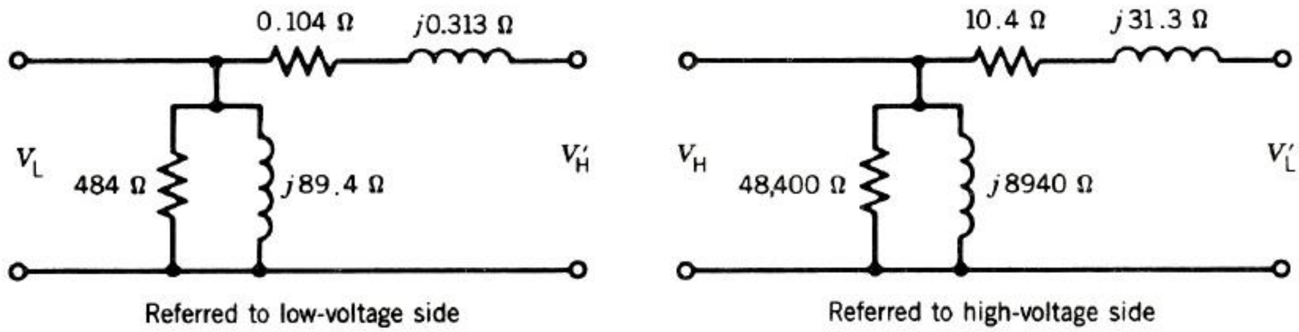
$$X_{eqH} = (Z_{eqH}^2 - R_{eqH}^2)^{1/2} = (32.97^2 - 10.4^2)^{1/2} = 31.3\ \Omega$$

The corresponding parameters for the low-voltage side are as follows:

$$R_{eqL} = \frac{R_{eqH}}{a^2} = \frac{10.4}{10^2} = 0.104\ \Omega$$

$$X_{eqL} = \frac{31.3}{10^2} = 0.313\ \Omega$$

The approximate equivalent circuits referred to the low-voltage side and the high-voltage side are shown in Fig. E2.2*c*. Note that the impedance of the shunt branch is much larger than that of the series branch.



(*c*) Equivalent circuits

FIGURE E2.2

Note: Here, the Rc||Xm branch is moved to the left to simplify the circuit for further calculation, and this is a very reasonable approximation that is made in practice. Otherwise, we can put the Rc||Xm branch in the middle: for that, you need to split the series impedance (0.104+j0.313 or 10.4+j31.3) and put half of it to the left of the Rc||Xm branch and half of it to its right.

Example 2.6:

A 1ϕ, 100 kVA, 2000/200 V two-winding transformer is connected as an autotransformer as shown in Fig. E2.6 such that more than 2000 V is obtained at the secondary. The portion ab is the 200 V winding, and the portion bc is the 2000 V winding. Compute the kVA rating as an autotransformer.

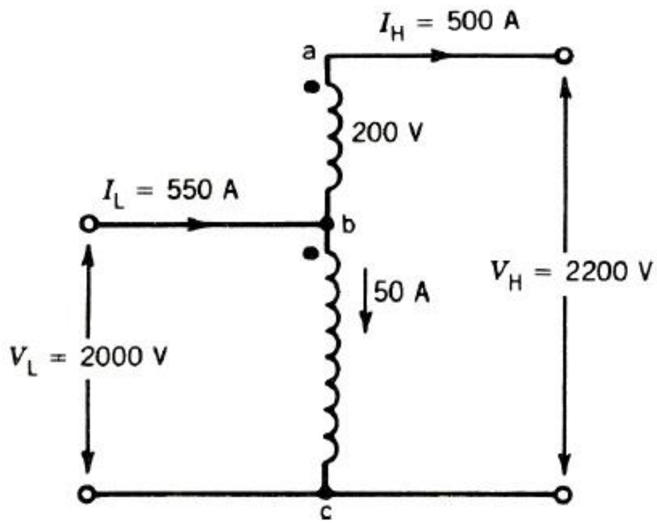


FIGURE E2.6

Solution

The current ratings of the windings are

$$I_{ab} = \frac{100{,}000}{200}\,\text{A} = 500\text{ A}$$

$$I_{bc} = \frac{100{,}000}{2000} = 50\text{ A}$$

Therefore, for full-load operation of the autotransformer, the terminal currents are

$$I_H = 500\text{ A}$$
$$I_L = 500 + 50 = 550\text{ A}$$

Now, $V_L = 2000$ V and

$$V_H = 2000 + 200 = 2200\text{ V}$$

Therefore,

$$\text{kVA}|_L = \frac{2000 \times 550}{1000} = 1100$$

$$\text{kVA}|_H = \frac{2200 \times 500}{1000} = 1100$$

A single-phase, 100 kVA, two-winding transformer when connected as an autotransformer can deliver 1100 kVA. Note that this higher rating of an autotransformer results from the conductive connection. Not all of the 1100 kVA is transformed by electromagnetic induction. Also note that the 200-volt winding must have sufficient insulation to withstand a voltage of 2200 V to ground. ■

Example 2.7:

Three 1ϕ, 50 kVA, 2300/230 V, 60 Hz transformers are connected to form a 3ϕ, 4000/230 V transformer bank. The equivalent impedance of each transformer referred to low voltage is $0.012 + j0.016\ \Omega$. The 3ϕ transformer supplies a 3ϕ, 120 kVA, 230 V, 0.85 PF (lag) load.

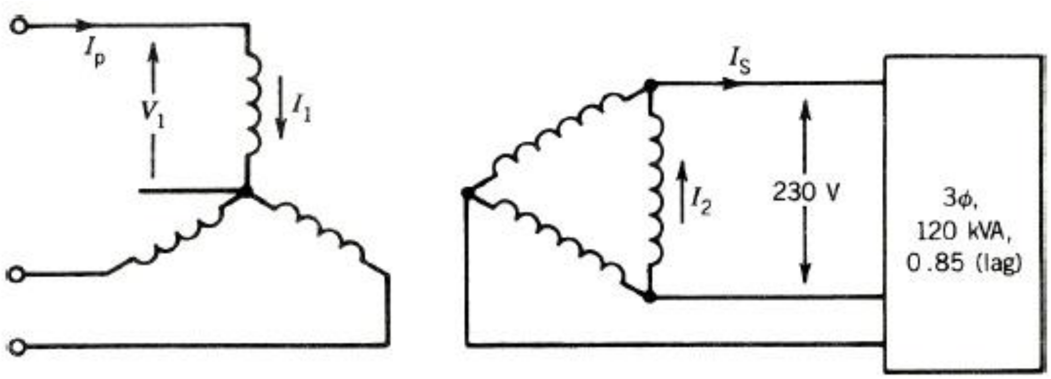
(a) Draw a schematic diagram showing the transformer connection.

(b) Determine the transformer winding currents.

(c) Determine the primary voltage (line-to-line) required.

Solution

(a) The connection diagram is shown in Fig. E2.7*a*. The high-voltage windings are to be connected in wye so that the primary can be connected to the 4000 V supply. The low-voltage winding is connected in delta to form a 230 V system for the load.



(*a*)

FIGURE E2.7

(b)

$$I_s = \frac{120{,}000}{\sqrt{3}\times 230} = 301.24\ \text{A}$$

$$I_2 = \frac{301.24}{\sqrt{3}} = 173.92\ \text{A}$$

$$a = \frac{2300}{230} = 10$$

$$I_1 = \frac{173.92}{10} = 17.39\ \text{A}$$

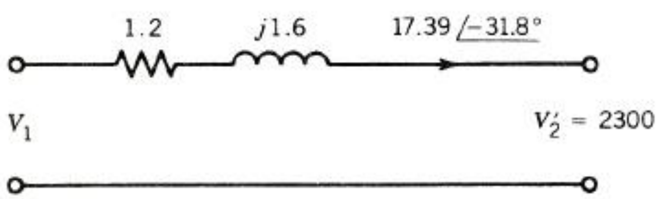
(c) Computation can be carried out on a per-phase basis.

$$Z_{eq1} = (0.012 + j0.016)10^2$$
$$= 1.2 + j1.6\,\Omega$$
$$\phi = \cos^{-1}0.85 = 31.8^\circ$$

The primary equivalent circuit is shown in Fig. E2.7*b*.

$$V_1 = 2300\underline{/0^\circ} + 17.39\underline{/-31.8^\circ}(1.2 + j1.6)$$
$$|V_1| = 2332.4\text{ V}$$

$$\text{Primary line-to-line voltage} = \sqrt{3}V_1 = 4039.8\text{ V}$$



(b)

FIGURE E2.7

Example 2.8:

EXAMPLE 2.8

A 3ϕ, 230 V, 27 kVA, 0.9 PF (lag) load is supplied by three 10 kVA, 1330/230 V, 60 Hz transformers connected in Y–Δ by means of a common 3ϕ feeder whose impedance is $0.003 + j0.015\,\Omega$ per phase. The transformers are supplied from a 3ϕ source through a 3ϕ feeder whose impedance is $0.8 + j5.0\,\Omega$ per phase. The equivalent impedance of one transformer referred to the low-voltage side is $0.12 + j0.25\,\Omega$. Determine the required supply voltage if the load voltage is 230 V.

Solution

The circuit is shown in Fig. E2.8*a*.

The equivalent circuit of the individual transformer referred to the high-voltage side is

$$R_{\text{eqH}} + jX_{\text{eqH}} = \left(\frac{1330}{230}\right)^2 (0.12 + j0.25)$$

$$= 4.01 + j8.36$$

The turns ratio of the equivalent Y–Y bank is

$$a' = \frac{\sqrt{3} \times 1330}{230} = 10$$

The single-phase equivalent circuit of the system is shown Fig. E2.8*b*. All the impedances from the primary side can be transferred to the secondary side and combined with the feeder impedance on the secondary side.

$$R = (0.80 + 4.01)\frac{1}{10^2} + 0.003 = 0.051\ \Omega$$

$$X = (5 + 8.36)\frac{1}{10^2} + 0.015 = 0.149\ \Omega$$

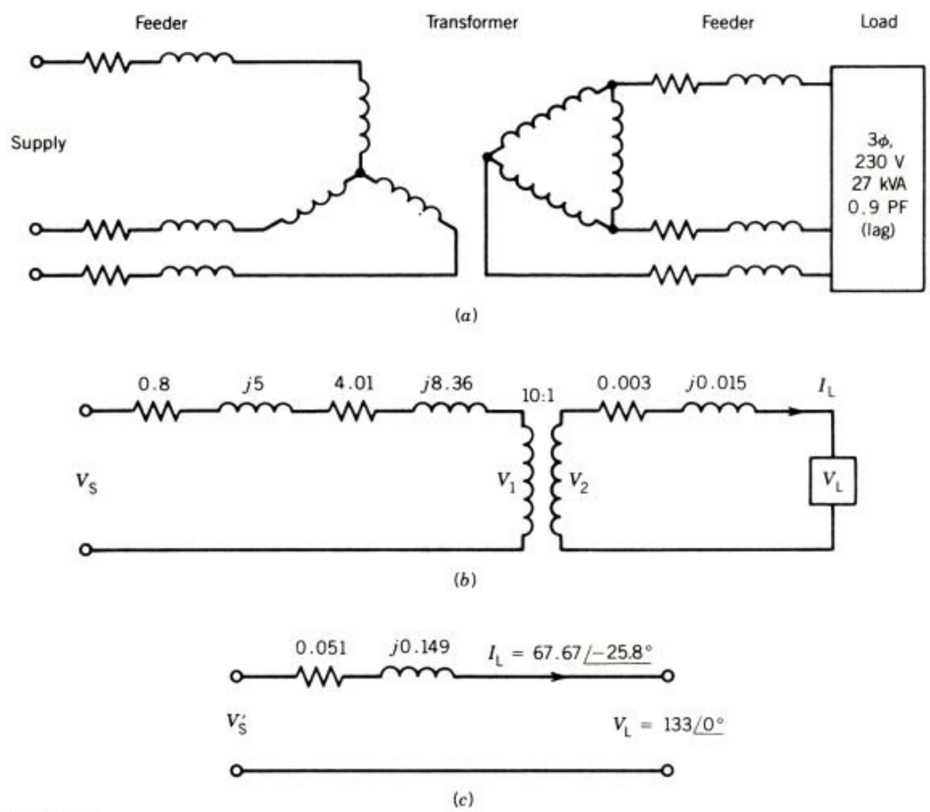


FIGURE E2.8

The circuit is shown in Fig. E2.8*c*.

$$V_L = \frac{230}{\sqrt{3}}\underline{/0^\circ} = 133\underline{/0^\circ}\text{ V}$$

$$I_L = \frac{27 \times 10^3}{3 \times 133} = 67.67\text{ A}$$

$$\phi_L = -\cos^{-1}0.9 = -25.8^\circ$$

$$V'_s = 133\underline{/0^\circ} + 67.67\underline{/-25.8^\circ}(0.051 + j0.149)$$

$$= 133\underline{/0^\circ} + 10.6571\underline{/45.3^\circ}$$

$$= 140.7\underline{/3.1^\circ}\text{ V}$$

$$V_s = 140.7 \times 10 = 1407\text{ V}$$

The line-to-line supply voltage is

$$1407\sqrt{3} = 2437\text{ V} \quad \blacksquare$$

Chapter 2 Assignments – Problems to Solve

P 2.3):

A single-phase transformer has 800 turns on the primary winding and 400 turns on the secondary winding. The cross-sectional core area is 50 cm^2. If the primary winding is connected to a 1 φ, 1000 V, 60 Hz supply, calculate

(a) The maximum value of the flux density in the core, B_{max}.

(b) The induced voltage in the secondary winding.

P 2.9):

A 1ϕ, two-winding transformer has 1000 turns on the primary and 500 turns on the secondary. The primary winding is connected to a 220 V supply and the secondary winding is connected to a 5 kVA load. The transformer can be considered ideal.

(a) Determine the load voltage.

(b) Determine the load impedance.

(c) Determine the load impedance referred to the primary.

P 2.26):

2.26 A 1ϕ, 10 kVA, 460/120 V, 60 Hz transformer has an efficiency of 96% when it delivers 9 kW at 0.9 power factor. This transformer is connected as an autotransformer to supply load to a 460 V circuit from a 580 V source.

(a) Show the autotransformer connection.

(b) Determine the maximum kVA the autotransformer can supply to the 460 V circuit.

(c) Determine the efficiency of the autotransformer for full load at 0.9 power factor.

P 2.32):

2.32 A 1ϕ, 200 kVA, 2100/210 V, 60 Hz transformer has the following characteristics. The impedance of the high-voltage winding is $0.25 + j1.5\,\Omega$ with the low-voltage winding short-circuited. The admittance (i.e., inverse of impedance) of the low-voltage winding is $0.025 - j0.075$ mhos with the high-voltage winding open-circuited.

(a) Taking the transformer rating as base, determine the base values of power, voltage, current, and impedance for both the high-voltage and low-voltage sides of the transformer.

(b) Determine the per-unit value of the equivalent resistance and leakage reactance of the transformer.

(c) Determine the per-unit value of the excitation current at rated voltage.

(d) Determine the per-unit value of the total power loss in the transformer at full-load output condition.

Chapter 3: Examples & Problems

EXAMPLE 3.2

The $\lambda - i$ relationship for an electromagnetic system is given by

$$i = \left(\frac{\lambda g}{0.09}\right)^2$$

which is valid for the limits $0 < i < 4\,\text{A}$ and $3 < g < 10\,\text{cm}$. For current $i = 3\,\text{A}$ and air gap length $g = 5\,\text{cm}$, find the mechanical force on the moving part, using energy and coenergy of the field.

Solution

The λ–i relationship is nonlinear. From the $\lambda - i$ relationship

$$\lambda = \frac{0.09 i^{1/2}}{g}$$

The coenergy of the system is

$$W'_f = \int_0^i \lambda\, di = \int_0^i \frac{0.09 i^{1/2}}{g} di$$

$$= \frac{0.09}{g}\frac{2}{3} i^{3/2} \text{ joules}$$

From Eq. 3.26

$$f_m = \left.\frac{\partial W'_f(i,g)}{\partial g}\right|_{i = \text{constant}}$$

$$= -0.09 \times \left.\frac{2}{3} i^{3/2} \frac{1}{g^2}\right|_{i = \text{constant}}$$

For $g = 0.05\,\text{m}$ and $i = 3\,\text{A}$,

$$f_m = -0.09 \times \frac{2}{3} \times 3^{3/2} \times \frac{1}{0.05^2}\ \text{N} \cdot \text{m}$$

$$= -124.7\ \text{N} \cdot \text{m}$$

The energy of the system is

$$W_f = \int_0^\lambda i\,d\lambda = \int_0^\lambda \left(\frac{\lambda g}{0.09}\right)^2 d\lambda$$
$$= \frac{g^2}{0.09^2}\frac{\lambda^3}{3}$$

From Eq. 3.27

$$f_m = -\left.\frac{\partial W_f(\lambda, g)}{\partial g}\right|_{\lambda = \text{constant}}$$
$$= -\frac{\lambda^3 2g}{3 \times 0.09^2}$$

For $g = 0.05\,\text{m}$ and $i = 3\,\text{A}$,

$$\lambda = \frac{0.09 \times 3^{1/2}}{0.05} = 3.12\,\text{Wb-turn}$$

and

$$f_m = -\frac{3.12^3 \times 2 \times 0.05}{3 \times 0.09^2}$$
$$= -124.7\,\text{N} \cdot \text{m}$$

The forces calculated on the basis of energy and coenergy functions are the same, as they should be. The selection of the energy or coenergy function as the basis for calculation is a matter of convenience, depending on the variables given. The negative sign for the force indicates that the force acts in such a direction as to decrease the air gap length. ■

EXAMPLE 3.3

The magnetic system shown in Fig. E3.3 has the following parameters:

$N = 500$
$i = 2$ A
Width of air gap = 2.0 cm
Depth of air gap = 2.0 cm
Length of air gap = 1 mm

Neglect the reluctance of the core, the leakage flux, and the fringing flux.

(a) Determine the force of attraction between both sides of the air gap.
(b) Determine the energy stored in the air gap.

Solution

(a)

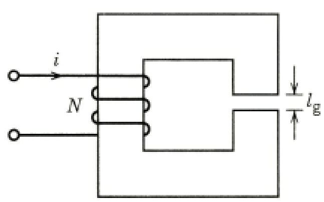


FIGURE E3.3

$$B_g = \frac{\mu_0 N i}{l_g}$$

$$f_m = \frac{B_g^2}{2\mu_0} \times A_g$$

$$= \mu_0 \frac{N^2 i^2}{2 l_g^2} A_g$$

$$= \frac{4\pi 10^{-7} (500 \times 2)^2}{2 \times 1 \times 1 \times 10^{-6}} \times 2.0 \times 2.0 \times 10^{-4} \quad = 251.33 \text{ N}$$

(b) $$W_f = \frac{B_g^2}{2\mu_0} \times V_g \quad = \frac{B_g^2}{2\mu_0} \times A_g \times l_g \quad = 251.33 \times 10^{-3} \text{ J} \quad = 0.25133 \text{ joules} \quad \blacksquare$$

EXAMPLE 3.4

The lifting magnetic system shown in Fig. E3.4 has a square cross section $6 \times 6\ \text{cm}^2$. The coil has 300 turns and a resistance of 6 ohms. Neglect reluctance of the magnetic core and field fringing in the air gap.

(a) The air gap is initially held at 5 mm and a dc source of 120 V is connected to the coil. Determine

(i) The stored field energy.

(ii) The lifting force.

(b) The air gap is again held at 5 mm and an ac source of 120 V (rms) at 60 Hz is connected to the coil. Determine the average value of the lift force.

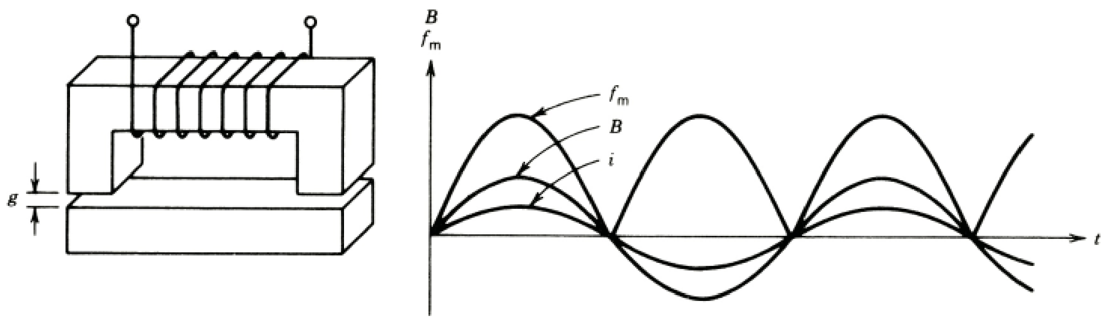


FIGURE E3.4

Solution

(a) Current in the coil is $i = \dfrac{120}{6} = 20\ \text{A}$

Because the reluctance of the magnetic core is neglected, field energy in the magnetic core is negligible. All field energy is in the air gaps.

$$Ni = H_g l_g = \frac{B_g}{\mu_0} l_g$$

$$B_g = \frac{\mu_0 Ni}{2g} = \frac{4\pi 10^{-7} \times 300 \times 20}{2 \times 5 \times 10^{-3}} = 0.754\ \text{tesla}$$

Field energy is $W_f = \dfrac{B_g^2}{2\mu_0} \times \text{volume of air gap}$

$$= \frac{0.754^2}{2 \times 4\pi 10^{-7}} \times 2 \times 6 \times 6 \times 5 \times 10^{-7}\ \text{J} = 8.1434\ \text{J}$$

From Eq. 3.37, the lift force is $f_m = \dfrac{B_g^2}{2\mu_0} \times \text{air gap area}$

$$= \frac{0.754^2}{2 \times 4\pi 10^{-7}} \times 2 \times 6 \times 6 \times 10^{-4}\ \text{N} = 1628.7\ \text{N}$$

(b) For ac excitation, the impedance of the coil is $Z = R + j\omega L$

Inductance of the coil is

$$L = \frac{N^2}{R_g} = \frac{N^2 \mu_0 A_g}{l_g} = \frac{300^2 \times 4\pi 10^{-7} \times 6 \times 6 \times 10^{-4}}{2 \times 5 \times 10^{-3}} = 40.7 \times 10^{-3}\ \text{H}$$

$$Z = 6 + j377 \times 40.7 \times 10^{-3}\ \Omega \quad = 6 + j15.34\ \Omega$$

Current in the coil is $I_{\mathrm{rms}} = \dfrac{120}{\sqrt{(6^2 + 15.34^2)}} = 7.29\ \mathrm{A}$

The flux density is $B_g = \dfrac{\mu_0 N i}{2g}$

The flux density is proportional to the current and therefore changes sinusoidally with time as shown in Fig. E3.4. The rms value of the flux density is

$$B_{\mathrm{rms}} = \frac{\mu_0 N I_{\mathrm{rms}}}{2g} \quad = \frac{4\pi 10^{-7} \times 300 \times 7.29}{2 \times 5 \times 10^{-3}}\ \mathrm{T} \quad = 0.2748\ \mathrm{T} \tag{3.38a}$$

The lift force is $f_{\mathrm{m}} = \dfrac{B_g^2}{2\mu_0} \times 2A_g \quad \propto B_g^2$ (3.38b)

The force varies as the square of the flux density, as shown in Fig. E3.4.

$$f_{\mathrm{m|avg}} = \left.\frac{B_g^2}{2\mu_0}\right|_{\mathrm{avg}} \times 2A_g \quad = \frac{B_{\mathrm{rms}}^2}{2\mu_0} \times 2A_g \quad = \frac{B_{\mathrm{rms}}^2}{2\mu_0} \times \text{air gap area} \tag{3.38c}$$

$$= \frac{0.2748^2 \times 2 \times 6 \times 6 \times 10^{-4}}{2 \times 4\pi 10^{-7}} \quad = 216.3\ \mathrm{N}$$

which is almost one-eighth of the lift force obtained with a dc supply voltage. Lifting magnets are normally operated from dc sources. ■

Chapter 3 Assignments – Problems to Solve

P 3.5):

3.5 An electromagnet lift system is shown in Fig. P3.5. The coil has 2500 turns. The flux density in the air gap is 1.25 T. Assume that the core material is ideal.

(a) For an air gap, $g = 10\,\text{mm}$:

- **(i)** Determine the coil current.
- **(ii)** Determine the energy stored in the magnetic system.
- **(iii)** Determine the force on the load (sheet of steel).
- **(iv)** Determine the mass of the load (acceleration due to gravity $= 9.81\ \text{m/sec}^2$).

(b) If the air gap is 2 mm, determine the coil current required to lift the load.

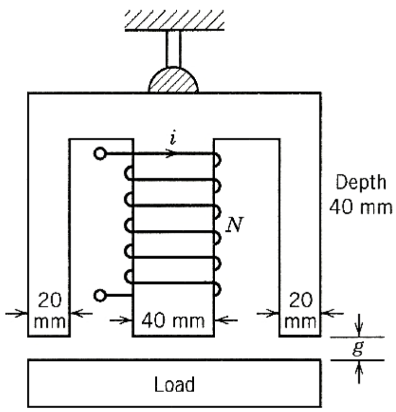


FIGURE P3.5

P 3.7):

3.7 The electromagnet shown in Fig. P3.7 can be used to lift a sheet of steel. The coil has 400 turns and a resistance of 5 ohms. The reluctance of the magnetic material is negligible. The magnetic core has a square cross section of 5 cm by 5 cm. When the sheet of steel is fitted to the electromagnet, air gaps, each of length $g = 1$ mm, separate them. An average force of 550 newtons is required to lift the sheet of steel.

(a) For dc supply,

- **(i)** Determine the dc source voltage.
- **(ii)** Determine the energy stored in the magnetic field.

(b) For ac supply at 60 Hz, determine the ac source voltage.

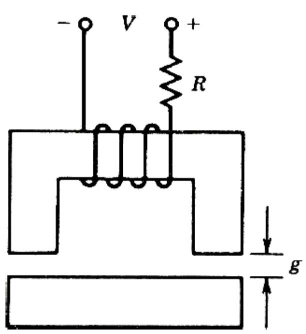


FIGURE P3.7