ELEC 342: Electromechanical Energy Conversion and Transmission

Homework 5

Solution of the Assignments

Assignments

P 5.2):

A three-phase, 5 hp, 208 V, 60 Hz induction motor runs at 1746 rpm when it delivers rated output power.

(a) Determine the number of poles of the machine.

(b) Determine the slip at full load.

(c) Determine the frequency of the rotor current.

(d) Determine the speed of the rotor field with respect to the

(i) Stator.

(ii) Stator rotating field.

Solution:

(a) $\frac{120 f}{P} \simeq 1746$

$$P = \frac{120 \times 60}{1746} \rightarrow 4$$

(b) $$n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$s = \frac{1800 - 1746}{1800} = 0.03$$

(c) $$f_2 = 0.03 \times 60 = 1.8 \text{ Hz}$$

(d) (i) 1800 rpm

(ii) 0 rpm

P 5.3):

A 3ϕ, 460 V, 100 hp, 60 Hz, six-pole induction machine operates at 3% slip (positive) at full load.

(a) Determine the speeds of the motor and its direction relative to the rotating field.

(b) Determine the rotor frequency.

(c) Determine the speed of the stator field.

(d) Determine the speed of the air gap field.

(e) Determine the speed of the rotor field relative to

- **(i)** the rotor structure.
- **(ii)** the stator structure.
- **(iii)** the stator rotating field.

Solution:

(a) $0.03 = \frac{1200 - n}{1200}$, $n_s = \frac{120 \times 60}{6} = 1200$ rpm.

$n = 1164$ rpm $\rightarrow$ direction same as the rotating field.

(b) $f_2 = 0.03 \times 60 = 1.8$ Hz

(c) 1200 rpm

(d) 1200 rpm

(e) (i) $n_{sl} = 0.03 \times 1200 = 36$ rpm $\rightarrow$ direction same as the rotor motion.

(ii) 1200 rpm

(iii) 0 rpm

P 5.14):

The following test results are obtained for a 3ϕ, 280 V, 60 Hz, 6.5 A, 500 W induction machine.

Blocked-rotor test:	44 V, 60 Hz, 25 A, 1250 W
No-load test:	208 V, 60 Hz, 6.5 A, 500 W

The average resistance measured by a dc bridge between two stator terminals is 0.54 Ω.

(a) Determine the no-load rotational loss.

(b) Determine the parameters of the equivalent circuit.

(c) What type of induction motor is this?

(d) Determine the output horsepower at $s = 0.1$.

Solution:

(a) $P_{rot} = 500 - 3 \times 6.5^2 \times \frac{0.54}{2} = 465.78\ W$

(b) From no-load test $R_1 = \frac{0.54}{2} = 0.27\ \Omega$

$$V_1 = \frac{208}{\sqrt{3}} = 120.1\ V$$

$$Z_{NL} = \frac{120.1}{6.5} = 18.48\ \Omega$$

$$R_{NL} = \frac{500}{3 \times 6.5^2} = 3.94\ \Omega$$

$$X_{NL} = \sqrt{18.48^2 - 3.94^2} = 18.05\ \Omega$$

$$X_1 + X_m = X_{NL} = 18.05\ \Omega$$

From blocked-rotor test

$$R_{BL} = \frac{1250}{3 \times 25^2} = 0.6667\ \Omega$$

$$R_2' = 0.6667 - 0.27 = 0.3967\ \Omega$$

$$Z_{BL} = \frac{44/\sqrt{3}}{25} = 1.0162\ \Omega$$

$$X_{BL} = \sqrt{1.0162^2 - 0.6667^2} = 0.7669\ \Omega$$

$$X_1 = X_2' = 0.3834\ \Omega$$

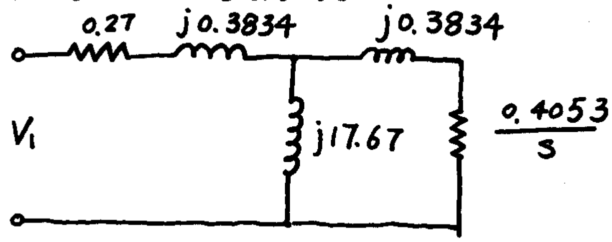
$$X_m = 18.05 - 0.3834 = 17.6666\ \Omega$$

Precise value for R_2'

$$R = R_{BL} - R_1 = 0.6667 - 0.27 = 0.3967\ \Omega$$

$$R_2' = \frac{0.3834 + 17.6667}{17.6667} \times 0.3967 = 0.4053\ \Omega$$

Equivalent Circuit is



(c) Neglecting X_m, $S_b \approx \dfrac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \dfrac{0.4053}{\sqrt{0.27^2 + 0.7668^2}}$

$= 0.4985$

Maximum torque occurs at 49.85% of synchrouns speed.

It is class-D type motor

(d) $\dfrac{R_2'}{s} = \dfrac{0.4053}{0.1} = 4.053\,\Omega$

$Z_1 = 0.27 + j0.3834 + \dfrac{j17.67(4.053 + j0.3834)}{4.053 + j18.05} = 4.273\angle 21.83^\circ$

$I_1 = \dfrac{208/\sqrt{3}}{4.273\angle 21.83^\circ} = 28.104\angle -21.83^\circ$

$P_s = 3 \times 120.1 \times 28.104 \times \cos(-21.83^\circ) = 9396.809\,W$

$P_{ag} = 9396.809 - 3 \times 28.104^2 \times 0.27 = 8757.045\,W$

$P_{mech} = (1 - 0.1) \times 8757.045 = 7881.341\,W$

$P_{out} = P_{shaft} = P_{mech} - P_{rot} = 7881.341 - 465.78 = 7415.561\,W$

$HP = \dfrac{7415.561}{746} = 9.94\ (HP)$

P 5.15):

A 3ϕ, 280 V, 60 Hz, 20 hp, four-pole induction motor has the following equivalent circuit parameters.

$$R_1 = 0.12\,\Omega \qquad R_2' = 0.1\,\Omega$$
$$X_1 = X_2' = 0.25\,\Omega$$
$$X_m = 10.0\,\Omega$$

The rotational loss is 400 W. For 5% slip, determine

(a) The motor speed in rpm and radians per sec.
(b) The motor current.
(c) The stator cu-loss.
(d) The air gap power.
(e) The rotor cu-loss.
(f) The shaft power.
(g) The developed torque and the shaft torque.
(h) The efficiency.

Use the IEEE-recommended equivalent circuit (Fig. 5.15).

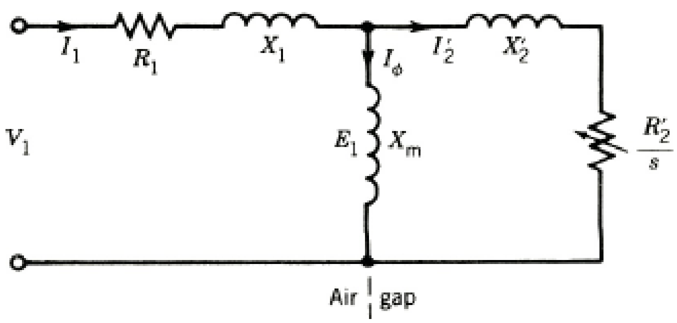


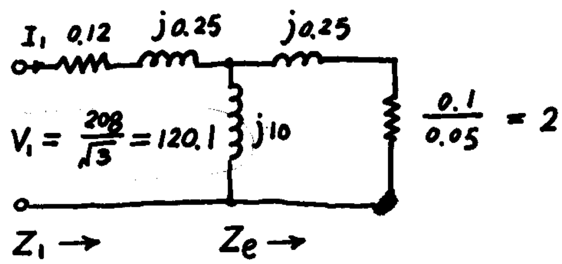
FIGURE 5.15 IEEE-recommended equivalent circuit.

Solution:

(a) $n_s = \frac{120 \times 60}{4} = 1800$ rpm $\quad \omega_s = \frac{1800}{60} \times 2\pi = 188.5$ rad/sec

$n=(1-s)n_s=(1-0.05)*1800=1710$ rpm $\quad w_r=(1-0.05)*188.5=179.1$ rad/s

(b)



$$Z_1 = 0.12 + j0.25 + R_e + jX_e = 0.12 + j0.25 + \frac{j10 \times (2 + j0.25)}{2 + j10.25}$$

$$= 0.12 + j0.25 + 1.9538 + j0.8517 = 2.1314 \angle 23.5533^\circ \Omega$$

$R_e = 1.9538$, $X_e = 0.8517$

$$I_1 = \frac{120.1}{2.1314 \angle 23.5533^\circ} = 56.3479 \angle -23.5533^\circ$$

(c) $P_1 = 3 \times (56.3479)^2 \times 0.12 = 1143.0309$ W

(d) $P_s = 3 \times 120.1 \times 56.3479 \times \cos(-23.5533) = 18610.9794$ W

$P_{ag} = P_s - P_1 = 17467.9485$ W

(e) $P_2 = s P_{ag} = 0.05 \times 17467.9485 = 873.3974$ W

(f) $P_{mech} = (1 - s) P_{ag} = 16594.5511$ W

$P_{shaft} = P_{mech} - P_{rotational} = 16194.5511$ W

(g) $T = \frac{P_{ag}}{188.5} = \frac{17467.9485}{188.5} = 92.6682$ N.m

$T_{shaft} = \frac{P_{shaft}}{188.5} = \frac{16194.5511}{188.5} = 85.9127$ N.m

(h) $Eff = \frac{P_{shaft}}{P_s} = 0.8702 \times 100\% = 87.02\%$

P 5.17):

A 3ϕ, 460 V, 60 Hz, six-pole induction motor has the following single-phase equivalent circuit parameters:

$$R_1 = 0.2\,\Omega \qquad X_1 = 1.055\,\Omega$$
$$R_2' = 0.28\,\Omega \qquad X_2' = 1.055\,\Omega$$
$$X_m = 33.9\,\Omega$$

The induction motor is connected to a 3ϕ, 460 V, 60 Hz supply.

(a) Determine the starting torque.

(b) Determine the breakdown torque and the speed at which it occurs.

(c) The motor drives a load for which $T_L = 1.8$ N · m. Determine the speed at which the motor will drive the load. Assume that near the synchronous speed the motor torque is proportional to slip. Neglect rotational losses.

Use the approximate equivalent circuit of Fig. 5.14*b*.

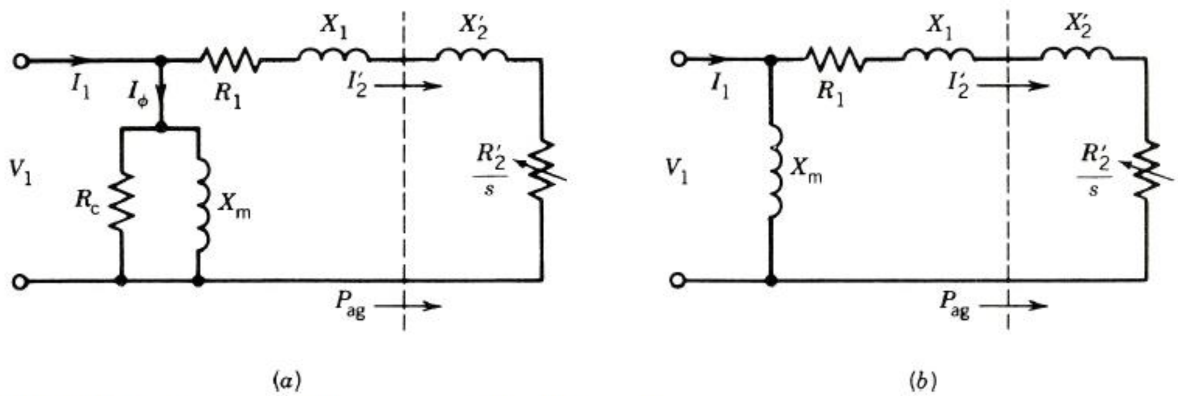
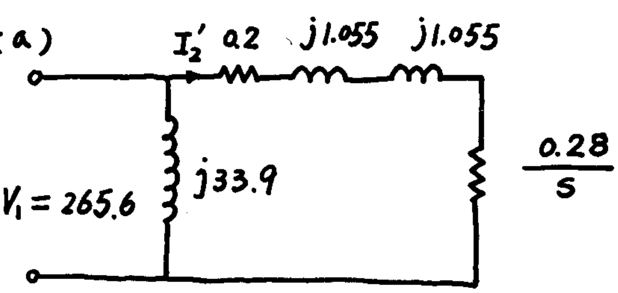


FIGURE 5.14 Approximate equivalent circuit.

Solution:

(a)



$$n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm} \qquad \omega_{syn} = \frac{1200}{60} \times 2\pi = 125.66 \text{ rad/sec}$$

$$T_{st} = \frac{3}{125.66} \times \frac{265.6^2}{(0.48)^2 + (2.11)^2} \times 0.28 = 100.7 \text{ N.m.}$$

$Z_{th} = R_{th} + jX_{th} = R_1 + jX_1 \Rightarrow R_{th} = R_1 = 0.2\,\Omega \quad X_{th} = X_1 = 1.055\,\Omega. \quad V_{th} = \frac{V_s}{\sqrt{3}} = \frac{460\,V}{\sqrt{3}} = 265.581\,V$

b) $$\boxed{T_{e,m}} = \frac{1}{2W_{syn}} \frac{3V_{th}^2}{R_{th} + \left[R_{th}^2 + (X_{th} + X_2')^2\right]^{1/2}} = \frac{1}{2\times 120\pi\times \frac{2}{6}} \frac{3\times\left(\frac{460}{\sqrt{3}}\right)^2}{0.2 + \left[0.2^2 + (1.055+1.055)^2\right]^{1/2}} = \boxed{362.9856 \text{ Nm}}$$

$$\boxed{S_m} = \frac{R_2'}{\left(R_{th}^2 + (X_{th} + X_2')^2\right)^{1/2}} = \frac{0.28}{\left(0.2^2 + (1.055+1.055)^2\right)^{1/2}} = \boxed{0.1321}$$

c) Approximate Solution

5.17 (c) <u>Approximate Solution</u>

$$T_e = 3\cdot\frac{V_1^2}{\omega_{syn}}\cdot\frac{R_2/s}{(R_1 + R_2/s)^2 + (X_1+X_2)^2}$$

Note: $T = 1.8$ N.m is a very small torque!

$\Rightarrow$ The slip s will also be very small

$\Rightarrow$ $\frac{R_2}{s}$ dominates R_1, X_1 and X_2.

$$T_e \simeq 3\frac{V_1^2}{\omega_{syn}}\cdot\frac{R_2/s}{(R_2/s)^2} = 3\cdot\frac{V_1^2}{\omega_{syn}\cdot R_2}\cdot S = 6013.8\cdot S$$

So, at very small torques (slips), the torque-slip relation can be approximated as;

$$T = 6013.8\cdot S \Rightarrow S = \frac{1.8}{6013.8} = 2.99\times 10^{-4} = 0.000299$$

For 6-pole machine $n_{syn} = 1200$ rpm.

$$n = n_{syn}(1-s) = 1200\cdot(1-2.99e{-4}) = \underline{1199.6 \text{ rpm}}$$

c) Accurate (Correct) Solution

5.17 (c) $T_L = 1.8\ N \cdot m$

In steady state & no rotational losses $\Rightarrow T_e = T_L = 1.8\ N \cdot m$

Recall $T_e = 3 \cdot \frac{V_1^2}{\omega_{syn}} \cdot \frac{R_2/s}{(R_1 + R_2/s)^2 + (X_1 + X_2)^2}$

$$\frac{T \cdot \omega_{syn}}{3 V_1^2} = \frac{R_2/s}{(R_1 + R_2/s)^2 + (X_1 + X_2)^2}$$ Define $\alpha = R_2/s$

$$(R_1 + \alpha)^2 + (X_1 + X_2)^2 = \alpha \cdot \frac{3 V_1^2}{T \omega_{syn}}$$

$$\alpha^2 + 2 R_1 \alpha + R_1^2 + (X_1 + X_2)^2 - \alpha \frac{3 V_1^2}{T \omega_{syn}} = 0$$

$$\alpha^2 + \underbrace{\left[2 R_1 - \frac{3 V_1^2}{T \omega_{syn}}\right]}_{b} \alpha + \underbrace{R_1^2 + (X_1 + X_2)^2}_{c} = 0$$

$\alpha^2 + b\alpha + c = 0$ - quadratic equation in α!

$$b = 2 R_1 - \frac{3 V_1^2}{T \cdot \omega_{syn}} = 2 \cdot 0.2 - \frac{3 \cdot (265.58)^2}{1.8 \cdot 125.66} = -935.077$$

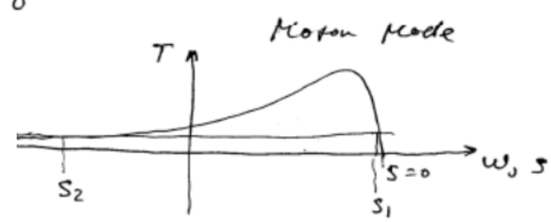
$$c = R_1^2 + (X_1 + X_2)^2 = 0.2^2 + (1.055 + 1.055)^2 = 4.492$$

$$\alpha_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{1}{2} b \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

$\alpha_1 = 935.0726$ $\qquad \alpha_2 = 0.0048$

$$S_1 = \frac{R_2}{\alpha_1} = \frac{0.2}{935.0726} = 0.000299$$

$$S_2 = \frac{R_2}{\alpha_2} = \frac{0.2}{0.0048} = 58.2846$$



Take $S = S_1 = 0.000299$

Speed $n_{syn} = 1200$ rpm. $\quad n = (1 - s) \cdot n_{syn} = 1199.6$ rpm

P 5.20):

A 3ϕ, 25 hp, 460 V, 60 Hz, 1760 rpm, wound-rotor induction motor has the following equivalent circuit parameters:

$$R_1 = 0.25\,\Omega, \quad X_1 = 1.2\,\Omega$$
$$R_2' = 0.2\,\Omega, \quad X_2' = 1.1\,\Omega$$
$$X_m = 35.0\,\Omega$$

The motor is connected to a 3ϕ, 460 V, 60 Hz, supply.

(a) Determine the number of poles of the machine.

(b) Determine the starting torque.

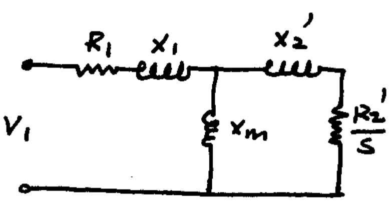
(c) Determine the value of the external resistance required in each phase of the rotor circuit such that the maximum torque occurs at starting. Use Thevenin's equivalent circuit.

Solution:

(a) $n = 1760 \simeq \dfrac{120 \times 60}{P}$

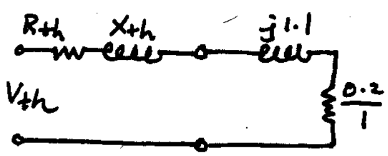
$P = 4.09 \longrightarrow 4$

(b) $V_1 = \dfrac{460}{\sqrt{3}} = 265.59$



$V_{th} = \dfrac{35}{1.2 + 35} V_1$

$= 0.967 \times 265.59 = 256.79\,V \Rightarrow$



$R_{th} \approx 0.967^2 \times 0.25 = 0.234\,\Omega$

$X_{th} = 1.2\,\Omega = X_1$

$\omega_{syn} = \dfrac{1800}{60} \times 2\pi = 188.496$ rad./sec.

$$T_{st} = \frac{3}{188.496} \; \frac{256.79^2}{(0.234 + 0.2)^2 + (1.2 + 1.1)^2} \times 0.2$$

$= 38.31$ N.m

(c) $$S_{T\max} = 1 = \frac{R_2' + R_{ext}'}{\sqrt{0.234^2 + (1.2 + 1.1)^2}} = \frac{R_2' + R_{ext}'}{2.31}$$

$R_{ext}' = 2.31 - 0.2 = 2.11\,\Omega = R_{ext}.$

$$V_{th} = \frac{X_m}{[R_1^2 + (X_1 + X_m)^2]^{1/2}} V_1 \tag{5.45}$$

If $R_1^2 \ll (X_1 + X_m)^2$, as is usually the case,

$$V_{th} \approx \frac{X_m}{X_1 + X_m} \tag{5.45a}$$

$$= K_{th} V_1 \tag{5.45b}$$

The Thevenin impedance is

$$Z_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

$$= R_{th} + jX_{th}$$

If $R_1^2 \ll (X_1 + X_m)^2$,

$$R_{th} \simeq \left(\frac{X_m}{X_1 + X_m}\right)^2 R_1 \tag{5.46}$$

$$= K_{th}^2 R_1 \tag{5.46a}$$

and since $X_1 \ll X_m$,

$$X_{th} \simeq X_1 \tag{5.47}$$

$$T_{mech} = \frac{1}{\omega_{syn}} \frac{V_{th}^2}{(R_{th} + R_2'/s)^2 + (X_{th} + X_2')^2} \frac{R_2'}{s} \tag{5.54}$$

$$s_{T_{max}} = \frac{R_2'}{[R_{th}^2 + (X_{th} + X_2')^2]^{1/2}} \tag{5.58}$$

$$T_{max} = \frac{1}{2\omega_{syn}} \frac{V_{th}^2}{R_{th} + [R_{th}^2 + (X_{th} + X_2')^2]^{1/2}} \tag{5.59}$$