ELEC 342: Electromechanical Energy Conversion and Transmission

Homework 5

- ✓ Examples (questions with full answers)
- ✓ **Assignments** (questions with no answers)

You need to submit your answers (procedure + final answer) for "Assignments" only on Canvas by 11:59 pm on the due date.

Summary:

- Follow through **Examples 5.2 to 5.6** below from P. C. Sen textbook.
- **Assignments**: Solve Problems **P5.2**, **P5.3, P5.14, P5.15, P5.17, P5.20**.

Examples (from Chapter 5 of P. C. Sen textbook):

EXAMPLE 5.2

A 3ϕ, 15 hp, 460 V, four-pole, 60 Hz, 1728 rpm induction motor delivers full output power to a load connected to its shaft. The windage and friction loss of the motor is 750 W. Determine the

(a) Mechanical power developed.
(b) Air gap power.
(c) Rotor copper loss.

Solution

(a) Full-load shaft power $= 15 \times 746 = 11{,}190$ W

$$\text{Mechanical power developed} = \text{shaft power} + \text{windage and friction loss}$$
$$= 11{,}190 + 750 = 11{,}940 \text{ W}$$

(b)

$$\text{Synchronous speed} \quad n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$\text{Slip} \quad s = \frac{1800 - 1728}{1800} = 0.04$$

$$\text{Air gap power} \quad P_{ag} = \frac{11{,}940}{1 - 0.04} = 12{,}437.5 \text{ W}$$

(c)

$$\text{Rotor copper loss} \quad P_2 = 0.04 \times 12{,}437.5$$
$$= 497.5\ W \quad \blacksquare$$

EXAMPLE 5.3

The following test results are obtained from a 3ϕ, 60 hp, 2200 V, six-pole, 60 Hz squirrel-cage induction motor.

1. No-load test:

$$\text{Supply frequency} = 60 \text{ Hz}$$
$$\text{Line voltage} = 2200 \text{ V}$$
$$\text{Line current} = 4.5 \text{ A}$$
$$\text{Input power} = 1600 \text{ W}$$

2. Blocked-rotor test:

$$\text{Frequency} = 15 \text{ Hz}$$
$$\text{Line voltage} = 270 \text{ V}$$
$$\text{Line current} = 25 \text{ A}$$
$$\text{Input power} = 9000 \text{ W}$$

3. Average dc resistance per stator phase:

$$R_1 = 2.8\,\Omega$$

(a) Determine the no-load rotational loss.
(b) Determine the parameters of the IEEE-recommended equivalent circuit of Fig. 5.15.
(c) Determine the parameters (V_{th}, R_{th}, X_{th}) for the Thevenin equivalent circuit of Fig. 5.16.

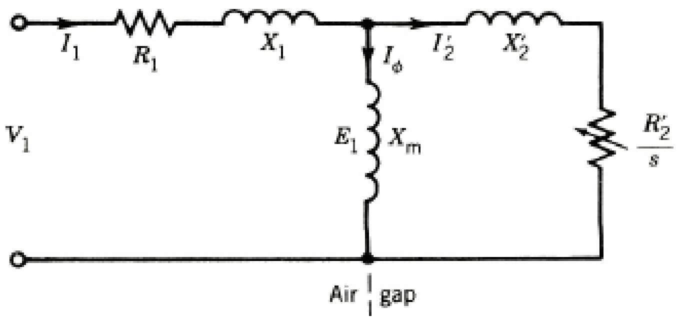

FIGURE 5.15 IEEE-recommended equivalent circuit.

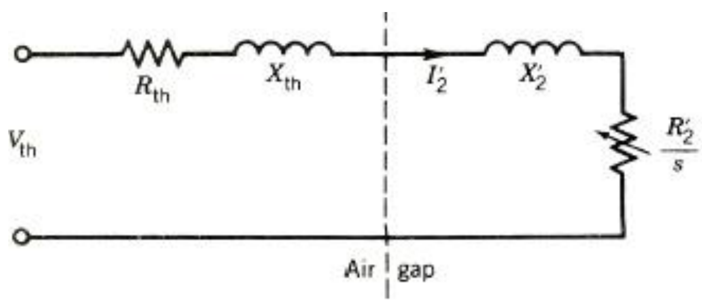

FIGURE 5.16 Thevenin equivalent circuit.

Solution

(a) From the no-load test, the no-load power is

$$P_{NL} = 1600\text{ W}$$

The no-load rotational loss is

$$\begin{aligned} P_{Rot} &= P_{NL} - 3I_1^2R_1 \\ &= 1600 - 3 \times 4.5^2 \times 2.8 \\ &= 1429.9\text{ W} \end{aligned}$$

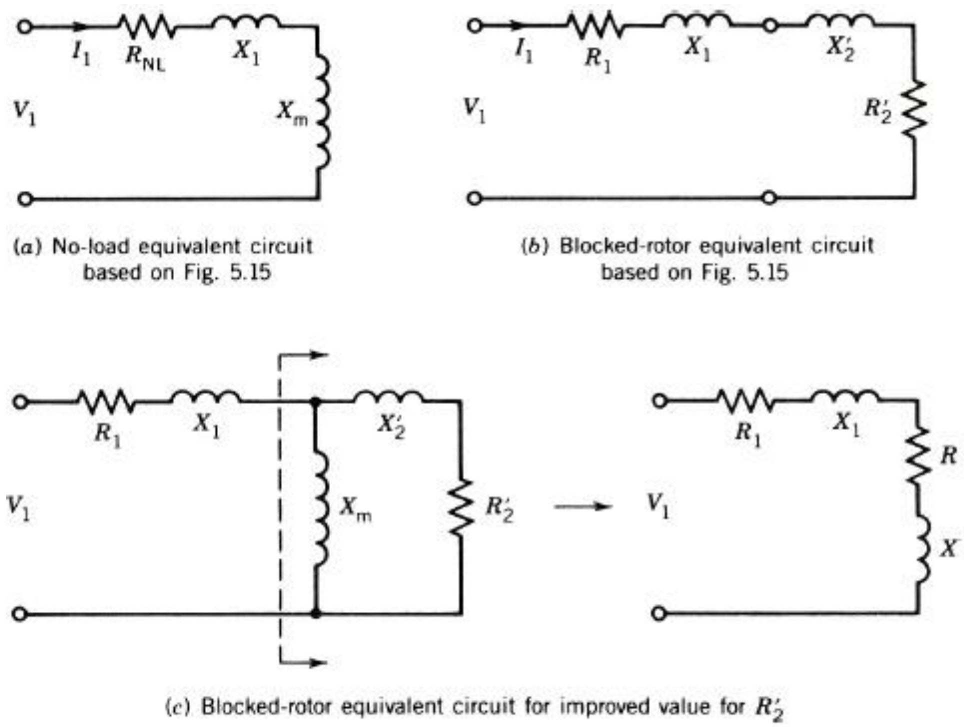

FIGURE E5.3

(b) *IEEE-recommended equivalent circuit*. For the no-load condition, R_2'/s is very high. Therefore, in the equivalent circuit of Fig. 5.15, the magnetizing reactance X_m is shunted by a very high resistive branch representing the rotor circuit. The reactance of this parallel combination is almost the same as X_m. Therefore, the total reactance X_{NL}, measured at no load at the stator terminals, is essentially $X_1 + X_m$. The equivalent circuit at no load is shown in Fig. E5.3*a*.

$$V_1 = \frac{2200}{\sqrt{3}} = 1270.2\ \mathrm{V/phase}$$

The no-load impedance is

$$Z_{NL} = \frac{V_1}{I_1} = \frac{1270.2}{4.5} = 282.27\ \Omega$$

The no-load resistance is

$$R_{NL} = \frac{P_{NL}}{3I_1^2} = \frac{1600}{3 \times 4.5^2} = 26.34\ \Omega$$

The no-load reactance is

$$\begin{aligned} X_{NL} &= (Z_{NL}^2 - R_{NL}^2)^{1/2} \\ &= (282.27^2 - 26.34^2)^{1/2} \\ &= 281.0\ \Omega \end{aligned}$$

Thus, $X_1 + X_m = X_{NL} = 281.0\ \Omega$.

For the blocked-rotor test the slip is 1. In the equivalent circuit of Fig. 5.15, the magnetizing reactance X_m is shunted by the low-impedance branch $jX_2' + R_2'$. Because $|X_m| \gg |R_2' + jX_2'|$, the impedance X_m can be neglected and the equivalent circuit for the blocked-rotor test reduces to the form shown in Fig. E5.3*b*. From the blocked-rotor test, the blocked-rotor resistance is

$$R_{BL} = \frac{P_{BL}}{3I_1^2}$$
$$= \frac{9000}{3 \times 25^2}$$
$$= 4.8\,\Omega$$

Therefore, $R_2' = R_{BL} - R_1 = 4.8 - 2.8 = 2\,\Omega$. The blocked-rotor impedance at 15 Hz is

$$Z_{BL} = \frac{V_1}{I_1} = \frac{270}{\sqrt{3} \times 25} = 6.24\,\Omega$$

The blocked-rotor reactance at 15 Hz is

$$X_{BL} = (6.24^2 - 4.8^2)^{1/2}$$
$$= 3.98\,\Omega$$

Its value at 60 Hz is

$$X_{BL} = 3.98 \times \frac{60}{15} = 15.92\,\Omega$$
$$X_{BL} = X_1 + X_2'$$

Hence,

$$X_1 = X_2' = \frac{15.92}{2} = 7.96\,\Omega \quad \text{(at 60 Hz)}$$

The magnetizing reactance is therefore

$$X_m = 281.0 - 7.96 = 273.04\,\Omega$$

Comments: The rotor equivalent resistance R_2' plays an important role in the performance of the induction machine. A more accurate determination of R_2' is recommended by the IEEE as follows: The blocked resistance R_{BL} is the sum of R_1 and an equivalent resistance, say R, which is the resistance of $R_2' + jX_2'$ in parallel with X_m as shown in Fig. E5.3*c*; therefore,

$$R = \frac{X_m^2}{R_2'^2 + (X_2' + X_m)^2} R_2'$$

If $X_2' + X_m \gg R_2'$, as is usually the case,

$$R \simeq \left(\frac{X_m}{X_2' + X_m}\right)^2 R_2'$$

or

$$R_2' = \left(\frac{X_2' + X_m}{X_m}\right)^2 R$$

Now $R = R_{BL} - R_1 = 4.8 - 2.8 = 2\ \Omega$. So,

$$R_2' = \left(\frac{7.96 + 273.04}{273.04}\right)^2 \times 2 = 2.12\,\Omega$$

(c) From Eq. 5.45,

$$V_{th} \simeq \frac{273.04}{7.96 + 273.04} V_1$$

$$= 0.97 V_1$$

From Eq. 5.46a,

$$R_{th} \simeq 0.97^2 R_1 = 0.97^2 \times 2.8 = 2.63\ \Omega$$

From Eq. 5.47,

$$X_{th} \simeq X_1 = 7.96\ \Omega \quad \blacksquare$$

$$V_{th} = \frac{X_m}{[R_1^2 + (X_1 + X_m)^2]^{1/2}} V_1 \tag{5.45}$$

If $R_1^2 \ll (X_1 + X_m)^2$, as is usually the case,

$$V_{th} \approx \frac{X_m}{X_1 + X_m} \tag{5.45a}$$

$$= K_{th} V_1 \tag{5.45b}$$

The Thevenin impedance is

$$Z_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

$$= R_{th} + jX_{th}$$

If $R_1^2 \ll (X_1 + X_m)^2$,

$$R_{th} \simeq \left(\frac{X_m}{X_1 + X_m}\right)^2 R_1 \tag{5.46}$$

$$= K_{th}^2 R_1 \tag{5.46a}$$

and since $X_1 \ll X_m$,

$$X_{th} \simeq X_1 \tag{5.47}$$

EXAMPLE 5.4

A three-phase, 460 V, 1740 rpm, 60 Hz, four-pole wound-rotor induction motor has the following parameters per phase:

$$R_1 = 0.25 \text{ ohms}, \qquad R_2' = 0.2 \text{ ohms}$$
$$X_1 = X_2' = 0.5 \text{ ohms}, \qquad X_m = 30 \text{ ohms}$$

The rotational losses are 1700 watts. With the rotor terminals short-circuited, find

(a) (i) Starting current when started direct on full voltage.
(ii) Starting torque.
(b) (i) Full-load slip.
(ii) Full-load current.
(iii) Ratio of starting current to full-load current.
(iv) Full-load power factor.
(v) Full-load torque.
(vi) Internal efficiency and motor efficiency at full load.
(c) (i) Slip at which maximum torque is developed.
(ii) Maximum torque developed.
(d) How much external resistance per phase should be connected in the rotor circuit so that maximum torque occurs at start?

Solution

(a)

$$V_1 = \frac{460}{\sqrt{3}} = 265.6 \text{ volts/phase}$$

(i) At start $s = 1$. The input impedance is

$$Z_1 = 0.25 + j0.5 + \frac{j30(0.2 + j0.5)}{0.2 + j30.5}$$
$$= 1.08\angle 66^\circ\ \Omega$$

$$I_{st} = \frac{265.6}{1.08\angle 66^\circ} = 245.9\angle -66^\circ \text{ A}$$

$\frac{120 \times 60}{4}$

(ii)

$$\omega_{syn} = \frac{1800}{60} \times 2\pi = 188.5 \text{ rad/sec}$$

$$V_{th} = \frac{265.6(j30.0)}{(0.25 + j30.5)} \approx 261.3 \text{ V}$$

$$Z_{th} = \frac{j30(0.25 + j0.5)}{0.25 + j30.5} = 0.55\angle 63.9^\circ$$
$$= 0.24 + j0.49$$

$$R_{th} = 0.24\ \Omega$$
$$X_{th} = 0.49 \simeq X_1$$
$$T_{st} = \frac{P_{ag}}{\omega_{syn}} = \frac{I_2'^2 R_2'/s}{\omega_{syn}}$$
$$= \frac{3}{188.5}\frac{261.3^2}{(0.24+0.2)^2 + (0.49+0.5)^2} \times \frac{0.2}{1}$$

from eq. (5-54)

$$= \frac{3}{188.5} \times (241.2)^2 \times \frac{0.2}{1}$$
$$T_{st} = 185.2\ \text{N}\cdot\text{m}$$

(b) **(i)** $$s = \frac{1800 - 1740}{1800} = 0.0333$$

(ii) $$\frac{R_2'}{s} = \frac{0.2}{0.0333} = 6.01\ \Omega$$

$$Z_1 = (0.25 + j0.5) + \frac{(j30)(6.01 + j0.5)}{6.01 + j30.5}$$
$$= 0.25 + j0.5 + 5.598 + j1.596$$
$$= 6.2123\underline{/19.7^\circ}\ \Omega$$
$$I_{FL} = \frac{265.6}{6.2123\underline{/19.7}}$$
$$= 42.754\underline{/-19.7^\circ}$$

(iii) $$\boxed{\frac{I_{st}}{I_{FL}} = \frac{245.9}{42.754} = 5.75}$$

(iv) $$\text{PF} = \cos(19.7^\circ) = 0.94\ \text{(lagging)}$$

(v) $$T = \frac{3}{188.5}\frac{(261.3)^2}{(0.24+6.01)^2 + (0.49+0.5)^2} \times 6.01$$
$$= \frac{3}{188.5} \times 41.29^2 \times 6.01$$
$$= 163.11\ \text{N}\cdot\text{m}$$

(vi) Air gap power:

$$P_{ag} = T\omega_{syn} = 163.11 \times 188.5 = 30{,}746.2\ \text{W}$$

Rotor copper loss:

$$P_2 = sP_{ag} = 0.0333 \times 30{,}746.2 = 1023.9\ \text{W}$$

$$P_{mech} = (1 - 0.0333)30{,}746.2 = 29{,}722.3\ \text{W}$$

$$P_{out} = P_{mech} - P_{rot} = 29{,}722.3 - 1700 = 28{,}022.3\ \text{W}$$

$$P_{input} = 3V_1 I_1 \cos\theta$$

$$= 3 \times 265.6 \times 42.754 \times 0.94 = 32{,}022.4\ \text{W}$$

$$\text{Eff}_{motor} = \frac{28{,}022.3}{32{,}022.4} \times 100 = 87.5\%$$

$$\text{Eff}_{internal} = (1 - s) = 1 - 0.0333 = 0.967 \rightarrow 96.7\%$$

(c) **(i)** From Eq. 5.58,

$$s_{T_{max}} = \frac{0.2}{[0.24^2 + (0.49 + 0.5)^2]^{1/2}} = \frac{0.2}{1.0187} = 0.1963$$

(ii) From Eq. 5.59,

$$T_{max} = \frac{3}{2 \times 188.5}\left[\frac{261.3^2}{0.24 + [0.24^2 + (0.49 + 0.5)^2]^{1/2}}\right]$$

$$= 431.68\ \text{N}\cdot\text{m}$$

$$\frac{T_{max}}{T_{FL}} = \frac{431.68}{163.11} = 2.65$$

(d)

$$s_{T_{max}} = \frac{R_2' + R_{ext}'}{[0.24^2 + (0.49 + 0.5)^2]^{1/2}} = \frac{R_2' + R_{ext}'}{1.0186}$$

$$R_{ext}' = 1.0186 - 0.2 = 0.8186\ \Omega/\text{phase}$$

Note that for parts (a) and (b), it is not necessary to use Thevenin's equivalent circuit. Calculation can be based on the equivalent circuit of Fig. 5.15 as follows:

$$Z_1 = R_1 + jX_1 + R_e + jX_e$$

$$= 0.25 + j0.5 + 5.598 + j1.596$$

$$T = \frac{3}{\omega_{syn}} I_1^2 R_e$$

$$= \frac{3}{188.5} \times 42.754^2 \times 5.598$$

$$= 163\ \text{N}\cdot\text{m} \quad \blacksquare$$

$$T_{\text{mech}} = \frac{1}{\omega_{\text{syn}}} \frac{V_{\text{th}}^2}{(R_{\text{th}} + R_2'/s)^2 + (X_{\text{th}} + X_2')^2} \frac{R_2'}{s} \quad (5.54)$$

$$s_{T_{\max}} = \frac{R_2'}{[R_{\text{th}}^2 + (X_{\text{th}} + X_2')^2]^{1/2}} \quad (5.58)$$

$$T_{\max} = \frac{1}{2\omega_{\text{syn}}} \frac{V_{\text{th}}^2}{R_{\text{th}} + [R_{\text{th}}^2 + (X_{\text{th}} + X_2')^2]^{1/2}} \quad (5.59)$$

EXAMPLE 5.5

A three-phase, 460 V, 60 Hz, six-pole wound-rotor induction motor drives a constant load of 100 N · m at a speed of 1140 rpm when the rotor terminals are short-circuited. It is required to reduce the speed of the motor to 1000 rpm by inserting resistances in the rotor circuit. Determine the value of the resistance if the rotor winding resistance per phase is 0.2 ohms. Neglect rotational losses. The stator-to-rotor turns ratio is unity.

Solution

The synchronous speed is

$$n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm}$$

Slip at 1140 rpm:

$$s_1 = \frac{1200 - 1140}{1200} = 0.05$$

Slip at 1000 rpm:

$$s_2 = \frac{1200 - 1000}{1200} = 0.167$$

From the equivalent circuits, it is obvious that if the value of R_2'/s remains the same, the rotor current I_2 and the stator current I_1 will remain the same, and the machine will develop the same torque (Eq. 5.54). Also, if the rotational losses are neglected, the developed torque is the same as the load torque. Therefore, for unity turns ratio,

$$\frac{R_2}{s_1} = \frac{R_2 + R_{\text{ext}}}{s_2}$$

$$\frac{0.2}{0.05} = \frac{0.2 + R_{\text{ext}}}{0.167}$$

$$R_{\text{ext}} = 0.468\ \Omega/\text{phase} \quad \blacksquare$$

EXAMPLE 5.6

The rotor current at start of a three-phase, 460 volt, 1710 rpm, 60 Hz, four-pole, squirrel-cage induction motor is six times the rotor current at full load.

(a) Determine the starting torque as percent of full-load torque.
(b) Determine the slip and speed at which the motor develops maximum torque.
(c) Determine the maximum torque developed by the motor as percent of full-load torque.

Solution

Note that the equivalent circuit parameters are not given. Therefore, the equivalent circuit parameters cannot be used directly for computation.

(a) The synchronous speed is

$$n_s = \frac{120 \times 60}{4} = 1800\,\text{rpm}$$

The full-load slip is

$$s_{\text{FL}} = \frac{1800 - 1710}{1800} = 0.05$$

From Eq. 5.52a,

$$T = \frac{I_2^2 R_2}{\omega_{\text{syn}}} \propto \frac{I_2^2 R_2}{s}$$

Thus,

$$\frac{T_{\text{st}}}{T_{\text{FL}}} = \left|\frac{I_{2(\text{st})}}{I_{2(\text{FL})}}\right|^2 s_{\text{FL}}$$

$$T_{\text{st}} = 6^2 \times 0.05 \times T_{\text{FL}} = 1.8T_{\text{FL}}$$

$$= 180\%\, T_{\text{FL}}$$

(b) From Eq. 5.64,

$$\frac{T_{\text{st}}}{T_{\max}} = \frac{2s_{T_{\max}}}{1 + s_{T_{\max}}^2}$$

$$\frac{T_{\text{FL}}}{T_{\max}} = \frac{2s_{T_{\max}} s_{\text{FL}}}{s_{T_{\max}}^2 + s_{\text{FL}}^2}$$

From these two expressions,

$$\frac{T_{\text{st}}}{T_{\text{FL}}} = \frac{s_{T_{\max}}^2 + s_{\text{FL}}^2}{s_{\text{FL}} + s_{\text{FL}} \times s_{T_{\max}}^2}$$

$$1.8 = \frac{s_{T_{\max}}^2 + 0.0025}{0.05 + 0.05 \times s_{T_{\max}}^2}$$

$$s_{T_{\max}}^2 + 0.0025 = 0.09 + 0.09 s_{T_{\max}}^2$$

$$s_{T_{\max}} = \left(\frac{0.0875}{0.91}\right)^{1/2} = 0.31$$

$$\text{Speed at maximum torque} = (1 - 0.31) \times 1800$$

$$= 1242\,\text{rpm}$$

(c) From Eq. 5.64,

$$T_{\max} = \left| \frac{1 + s_{T_{\max}}^2}{2 s_{T_{\max}}} \right| T_{\text{st}}$$

$$= \frac{1 + 0.31^2}{2 \times 0.31} \times 1.8 T_{\text{FL}}$$

$$= 3.18 T_{\text{FL}} = 318\% \, T_{\text{FL}} \quad \blacksquare$$

$$T_{\text{mech}} = \frac{1}{\omega_{\text{syn}}} P_{\text{ag}} \tag{5.52}$$

$$= \frac{1}{\omega_{\text{syn}}} I_2^2 \frac{R_2}{s} \tag{5.52a}$$

$$= \frac{1}{\omega_{\text{syn}}} I_2'^2 \frac{R_2'}{s} \tag{5.53}$$

$$\frac{T_{\max}}{T} \simeq \frac{s_{T_{\max}}^2 + s^2}{2 s_{T_{\max}} s} \tag{5.64}$$

Assignments

P 5.2):

A three-phase, 5 hp, 208 V, 60 Hz induction motor runs at 1746 rpm when it delivers rated output power.

(a) Determine the number of poles of the machine.

(b) Determine the slip at full load.

(c) Determine the frequency of the rotor current.

(d) Determine the speed of the rotor field with respect to the

- **(i)** Stator.
- **(ii)** Stator rotating field.

P 5.3):

A 3ϕ, 460 V, 100 hp, 60 Hz, six-pole induction machine operates at 3% slip (positive) at full load.

(a) Determine the speeds of the motor and its direction relative to the rotating field.

(b) Determine the rotor frequency.

(c) Determine the speed of the stator field.

(d) Determine the speed of the air gap field.

(e) Determine the speed of the rotor field relative to

- **(i)** the rotor structure.
- **(ii)** the stator structure.
- **(iii)** the stator rotating field.

P 5.14):

The following test results are obtained for a 3ϕ, 280 V, 60 Hz, 6.5 A, 500 W induction machine.

Blocked-rotor test:	44 V, 60 Hz, 25 A, 1250 W
No-load test:	208 V, 60 Hz, 6.5 A, 500 W

The average resistance measured by a dc bridge between two stator terminals is 0.54 Ω.

(a) Determine the no-load rotational loss.

(b) Determine the parameters of the equivalent circuit.

(c) What type of induction motor is this?

(d) Determine the output horsepower at $s = 0.1$.

P 5.15):

A 3ϕ, 280 V, 60 Hz, 20 hp, four-pole induction motor has the following equivalent circuit parameters.

$$R_1 = 0.12\,\Omega \qquad R_2' = 0.1\,\Omega$$
$$X_1 = X_2' = 0.25\,\Omega$$
$$X_m = 10.0\,\Omega$$

The rotational loss is 400 W. For 5% slip, determine

(a) The motor speed in rpm and radians per sec.

(b) The motor current.

(c) The stator cu-loss.

(d) The air gap power.

(e) The rotor cu-loss.

(f) The shaft power.

(g) The developed torque and the shaft torque.

(h) The efficiency.

Use the IEEE-recommended equivalent circuit (Fig. 5.15).

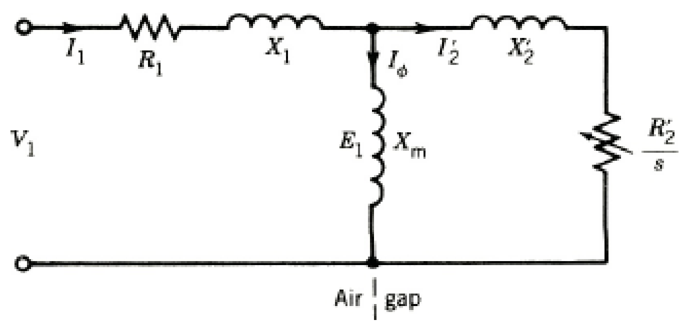

FIGURE 5.15 IEEE-recommended equivalent circuit.

P 5.17):

A 3ϕ, 460 V, 60 Hz, six-pole induction motor has the following single-phase equivalent circuit parameters:

$$R_1 = 0.2\,\Omega \qquad X_1 = 1.055\,\Omega$$
$$R_2' = 0.28\,\Omega \qquad X_2' = 1.055\,\Omega$$
$$X_m = 33.9\,\Omega$$

The induction motor is connected to a 3ϕ, 460 V, 60 Hz supply.

(a) Determine the starting torque.

(b) Determine the breakdown torque and the speed at which it occurs.

(c) The motor drives a load for which $T_L = 1.8\ \text{N}\cdot\text{m}$. Determine the speed at which the motor will drive the load. Assume that near the synchronous speed the motor torque is proportional to slip. Neglect rotational losses.

Use the approximate equivalent circuit of Fig. 5.14*b*.

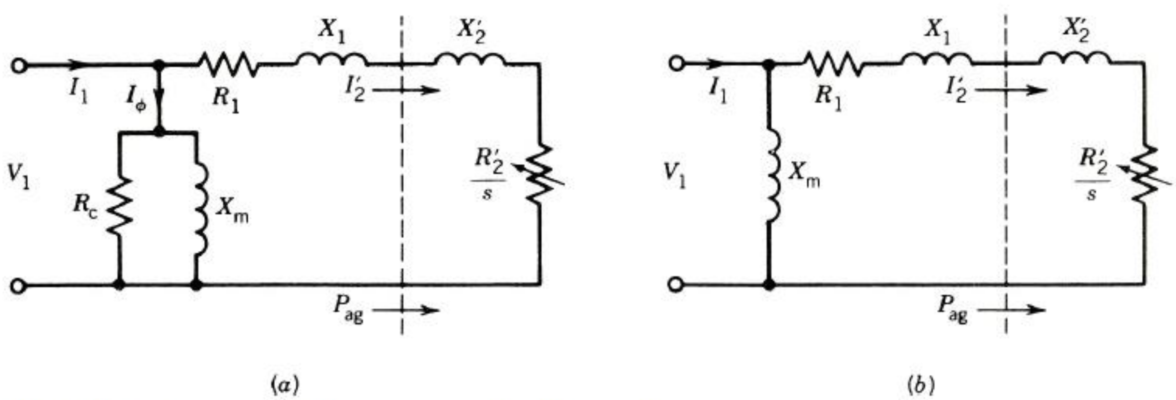

FIGURE 5.14 Approximate equivalent circuit.

P 5.20):

A 3ϕ, 25 hp, 460 V, 60 Hz, 1760 rpm, wound-rotor induction motor has the following equivalent circuit parameters:

$$R_1 = 0.25\,\Omega, \qquad X_1 = 1.2\,\Omega$$
$$R_2' = 0.2\,\Omega, \qquad X_2' = 1.1\,\Omega$$
$$X_m = 35.0\,\Omega$$

The motor is connected to a 3ϕ, 460 V, 60 Hz, supply.

(a) Determine the number of poles of the machine.

(b) Determine the starting torque.

(c) Determine the value of the external resistance required in each phase of the rotor circuit such that the maximum torque occurs at starting. Use Thevenin's equivalent circuit.