ELEC 342: Electromechanical Energy Conversion and Transmission

Homework 6

- ✓ Examples (questions with full answers)
- ✓ **Assignments** (questions with no answers)

You need to submit your answers (procedure + final answer) for "Assignments" only on Canvas by 11:59 pm on the due date.

Summary:

- Follow through **Examples 6.2, 6.3, 6.5, 6.7** below from P. C. Sen textbook.
- **Assignments**: Solve Problems **P6.3 only parts (a) and (b), P6.9, P6.12, P6.13, P6.30**.

Examples (from Chapter 6 of P. C. Sen textbook):

EXAMPLE 6.2

The following data are obtained for a 3ϕ, 10 MVA, 14 kV, star-connected synchronous machine.

I_f(A)	Open-Circuit Voltage (kV) (line-to-line)	Air Gap Line Voltage (kV) (line-to-line)	Short-Circuit Current (A)
100	9.0		
150	12.0		
200	14.0	18	490
250	15.3		
300	15.9		
350	16.4		

The armature resistance is 0.07 Ω/phase.

(a) Find the unsaturated and saturated values of the synchronous reactance in ohms and also in pu.

(b) Find the field current required if the synchronous generator is connected to an infinite bus and delivers rated MVA at 0.8 lagging power factor.

(c) If the generator, operating as in part (b), is disconnected from the infinite bus without changing the field current, find the terminal voltage.

Solution

The data are plotted in Fig. E6.2.

$$\text{Base voltage} \quad V_b = \frac{14{,}000}{\sqrt{3}} = 8083 \text{ V/phase}$$

$$\text{Base current} \quad I_b = \frac{10 \times 10^6}{\sqrt{3} \times 14{,}000} = 412.41 \text{ A}$$

$$\text{Base impedance} \quad Z_b = \frac{8083}{412.41} = 19.6\ \Omega$$

(a) From Eq. (6.8) and Fig. E6.2,

$$Z_s|_{\text{unsat}} = \frac{18{,}000/\sqrt{3}}{490} = 21.21\ \Omega$$

$$X_s|_{\text{unsat}} = \sqrt{21.21^2 - 0.07^2} = 21.2\ \Omega$$

$$= \frac{21.2}{19.6} \text{pu} = 1.08 \text{ pu}$$

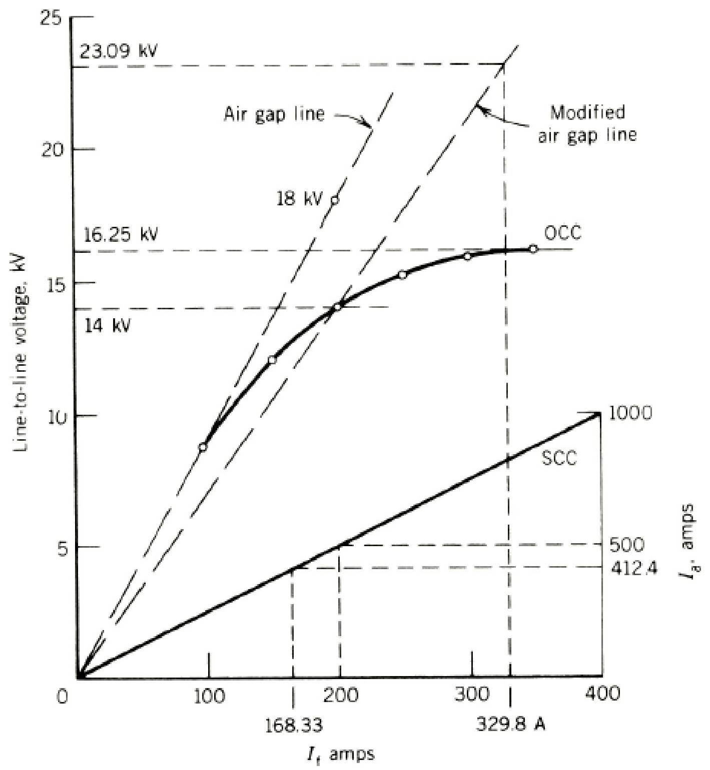

FIGURE E6.2

From Eq. (6.11) and Fig. E6.2, and because R_a is very small,

$$Z_{s(sat)} = \frac{14{,}000/\sqrt{3}}{490} = 16.5\ \Omega = (X_{s(sat)}^2 + 0.07^2)^{1/2}$$

$$X_{s(sat)} \simeq 16.5\ \Omega = \frac{16.5}{19.6} = 0.84\ \text{pu}$$

(b) It will be convenient to carry out the calculation in pu values rather than in actual values.

$$V_t = 1\underline{/0^\circ}\ \text{pu}$$

$$\text{PF} = 0.8 = \cos 36.9^\circ$$

$$I_a = 1\ \underline{/-36.9}\ \text{pu}$$

$$Z_s = 0.84\underline{/\tan^{-1}\frac{16.5}{0.07}} = 0.84\underline{/89.8^\circ}\ \text{pu}$$

Now

$$\begin{aligned} E_f &= V_t + I_a R_a + I_a jX_s \\ &= V_t + I_a Z_s \\ &= 1\underline{/0^\circ} + 1\underline{/-36.9^\circ} \cdot 0.85\underline{/89.8^\circ} \end{aligned}$$

$$
\begin{aligned}
&= 1\underline{/0^\circ} + 0.84\underline{/52.9^\circ} \\
&= 1.5067 + j0.67 \\
&= 1.649\underline{/24^\circ}\ \text{pu} \\
&= 1.649 \times 14\underline{/24^\circ}\ \text{kV}, \qquad \text{line-to-line} \\
&= 23.09\underline{/24^\circ}\ \text{kV}, \qquad \text{line-to-line}
\end{aligned}
$$

Note that $\delta = 24^\circ$ and is positive, as it should be for generator operation.
The required field current from the modified air gap line (Fig. E6.2) is

$$I_f = 1.649 \times 200 = 329.8\ \text{A}$$

(c) From the open-circuit data (Fig. E6.2) at $I_f = 329.8$ A, the terminal voltage is

$$V_t = 16.25\ \text{kV} \quad \text{(line-to-line)}$$

EXAMPLE 6.3

A 3ϕ, 5 kVA, 208 V, four-pole, 60 Hz, star-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 ohms per phase at rated terminal voltage.

The machine is first operated as a generator in parallel with a 3ϕ, 208 V, 60 Hz power supply.

(a) Determine the excitation voltage and the power angle when the machine is delivering rated kVA at 0.8 PF lagging. Draw the phasor diagram for this condition.

(b) If the field excitation current is now increased by 20 percent (without changing the prime mover power), find the stator current, power factor, and reactive kVA supplied by the machine.

(c) With the field current as in (a), the prime mover power is slowly increased. What is the steady-state (or static) stability limit? What are the corresponding values of the stator (or armature) current, power factor, and reactive power at this maximum power transfer condition?

Solution

The per-phase equivalent circuit for the synchronous generator is shown in Fig. E6.3*a*.

(a) $V_t = \dfrac{208}{\sqrt{3}} = 120 \text{ V/phase}$

Stator current at rated kVA;

$$I_a = \frac{5000}{\sqrt{3} \times 208} = 13.9 \text{ A}$$

$$\phi = -36.9^\circ \quad \text{for lagging pf of } 0.8$$

From Fig. E6.3*a*,

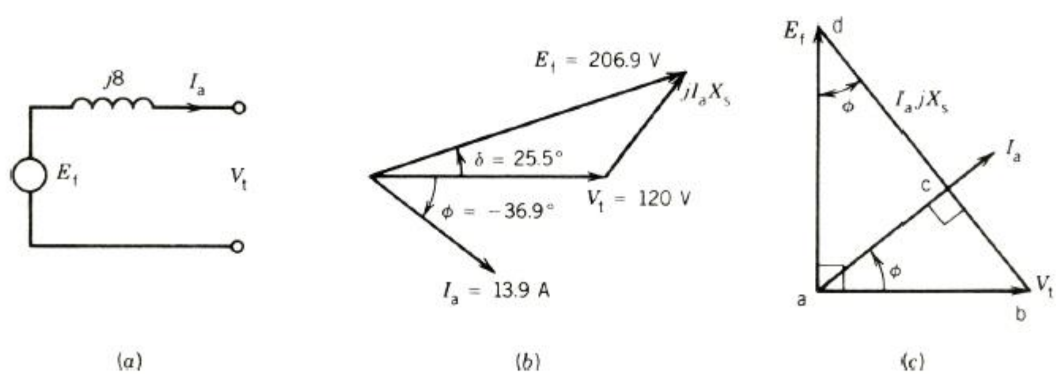

FIGURE E6.3

$$
\begin{aligned}
E_{\mathrm{f}} &= V_{\mathrm{t}}\underline{/0^\circ} + I_{\mathrm{a}} jX_{\mathrm{s}} \\
&= 120\underline{/0^\circ} + 13.9\underline{/-36.9^\circ} \cdot 8\underline{/90^\circ} \\
&= 206.9\underline{/25.5^\circ}
\end{aligned}
$$

$$
\text{Excitation voltage} \quad E_{\mathrm{f}} = 206.9 \text{ V/phase}
$$

$$
\text{Power angle} \quad \delta = +25.5^\circ
$$

Note that because of generator action, the power angle is positive.
The phasor diagram is shown in Fig. E6.3*b*.

(b) The new excitation voltage $E'_{\mathrm{f}} = 1.2 \times 206.9 = 248.28$ V. Because power transfer remains same,

$$
\frac{V_{\mathrm{t}} E_{\mathrm{f}}}{X_{\mathrm{s}}} \sin \delta = \frac{V_{\mathrm{t}} E'_{\mathrm{f}}}{X_{\mathrm{s}}} \sin \delta'
$$

or

$$
E_{\mathrm{f}} \sin \delta = E'_{\mathrm{f}} \sin \delta'
$$

or

$$
\sin \delta' = \frac{E_{\mathrm{f}}}{E'_{\mathrm{f}}} \sin \delta = \frac{\sin 25.5^\circ}{1.2}
$$

$$
\delta' = 21^\circ
$$

The stator current is

$$
\begin{aligned}
I_{\mathrm{a}} &= \frac{E_{\mathrm{f}} - V_{\mathrm{t}}}{jX_{\mathrm{s}}} \\
&= \frac{248.28\underline{/21^\circ} - 120\underline{/0^\circ}}{8\underline{/90^\circ}} \\
&= \frac{142.87\underline{/38.52^\circ}}{8\underline{/90^\circ}} \\
&= 17.86\underline{/-51.5^\circ} \text{ A}
\end{aligned}
$$

$$
\text{Power factor} = \cos 51.5^\circ = 0.62 \text{ lag}
$$

$$
\begin{aligned}
\text{Reactive kVA} &= 3|V_{\mathrm{t}}||I_{\mathrm{a}}| \sin 51.5^\circ \\
&= 3 \times 120 \times 17.86 \times 0.78 \times 10^{-3} \\
&= 5.03
\end{aligned}
$$

or, from Eq. 6.26,

$$Q = 3\left(\frac{120 \times 248.28}{8}\cos 21^\circ - \frac{120^2}{8}\right) \times 10^{-3}$$

$$= 3(3476.86 - 1800)$$

$$= 5.03$$

(c) From Eq. 6.25, the maximum power transfer occurs at $\delta = 90°$.

$$P_{\max} = \frac{3E_f V_t}{X_s} = \frac{3 \times 206.9 \times 120}{8} = 9.32 \text{ kW}$$

$$I_a = \frac{E_f - V_t}{jX_s} = \frac{206.9\underline{/+90^\circ} - 120\underline{/0^\circ}}{8\underline{/90^\circ}}$$

$$= 29.9\underline{/30.1^\circ} \text{ A}$$

$$\text{Stator current} \qquad I_a = 29.9 \text{ A}$$

$$\text{Power factor} = \cos 30.1^\circ = 0.865 \quad \text{leading}$$

The stator current and power factor can also be obtained by drawing the phasor diagram for the maximum power transfer condition. The phasor diagram is shown in Fig. E6.3*c*.

Because $\delta = +90°$, E_f leads V_t by 90°. The distance *bd* between phasors V_t and E_f is the voltage drop $I_a X_s$, and the current phasor I_a is in quadrature with $I_a X_s$.

From the phasor diagram,

$$|I_a X_s|^2 = |E_f|^2 + |V_t|^2$$

$$I_a = \left(\frac{206.9^2 + 120^2}{8^2}\right)^{1/2} = 29.9 \text{ A}$$

From the two triangles *abc* and *abd*,

$$\underline{/bac} = \underline{/adb} = \phi$$

$$\tan\phi = \frac{ab}{ad} = \frac{120}{206.9} = 0.58$$

$$\phi = 30.1^\circ$$

$$\text{PF} = \cos 30.1^\circ = 0.865 \quad \text{lead}$$

EXAMPLE 6.5

A 3ϕ, 460 V, 60 Hz, 1200 rpm, 125 hp synchronous motor has the following equivalent circuit parameters:

$$R_a = 0.078\ \Omega, \quad X_{al} = 0.05\ \Omega, \quad X_{ar} = 1.85\ \Omega$$

$$N_{re}/N_{se} = 28.2$$

For rated conditions the field current is adjusted to make the motor power factor unity. Neglect all rotational losses and power lost in the field winding.

(a) For rated operating conditions, determine the motor current I_a, field current I_f, and power angle δ.

(b) Draw the phasor diagram.

Solution

(a) For rated conditions, from Fig. 6.17*c*,

$$P_{in} = \sqrt{3} \times 460 \times I_a = 3 \times 0.078 I_a^2 + 125 \times 746$$

$$I_a = 121.4\ A$$

Let

$$V_t = \frac{460}{\sqrt{3}} \angle 0^\circ = 265.6 \angle 0^\circ \text{ V}$$

$$I_a = 121.4\ \angle 0^\circ \text{ A}$$

$$E_a = V_t - I_a R_a$$

$$= 265.6 \angle 0^\circ - 121.4 \angle 0^\circ \times 0.078$$

$$= 256.13 \angle 0^\circ \text{ V}$$

$$X_s = 0.05 + 1.85 = 1.9\ \Omega$$

$$I_m = \frac{256.13 \angle 0^\circ}{1.9 \angle 90^\circ} = 134.74 \angle -90^\circ \text{ A}$$

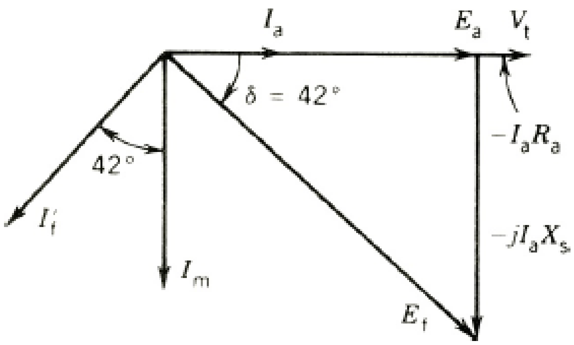

FIGURE E6.5

$$
\begin{aligned}
I_f' &= I_m - I_a \\
&= 134.74\underline{/-90°} - 121.4\underline{/0°} \\
&= 181.4\underline{/-132°\ \text{A}} \\
\beta &= -132°
\end{aligned}
$$

From Eq. 6.7c,

$$n = \frac{\sqrt{2}}{3} \times 28.2 = 13.29$$

From Eq. 6.7b,

$$I_f = \frac{181.4}{13.29} \times \frac{1.9}{1.85} = 14.02\ \text{A}$$

$$\delta = -132° + 90° = -42°$$

(b) The phasor diagram is drawn in Fig. E6.5.

EXAMPLE 6.7

A three-phase, 50 MVA, 11 kV, 60 Hz, salient pole, synchronous machine has reactances $X_d = 0.8$ pu and $X_q = 0.4$ pu. The synchronous motor is loaded to draw rated current at a

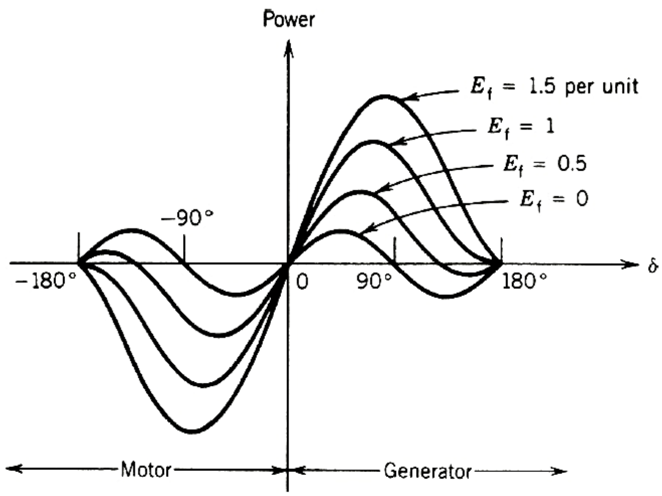

FIGURE 6.29 Power–angle characteristics for various field currents.

supply power factor of 0.8 lagging. Rotational losses are 0.15 pu. Neglect armature resistance losses.

(a) Determine the excitation voltage E_f in pu.

(b) Determine the power due to field excitation and that due to saliency of the machine.

(c) If the field current is reduced to zero, will the machine stay in synchronism?

(d) If the shaft load is removed before the field current is reduced to zero, determine the resultant supply current in pu and the supply power factor. Draw the phasor diagram for the machine in this condition.

Solution

(a) $V_t = 1\underline{/0^\circ}$ pu, $I_a = 1\underline{/-36.9^\circ}$ pu.
From the phasor diagram (Fig. 6.27*d*), $\psi = \phi - \delta$
From Eq. 6.46,

$$\tan\delta = \frac{1\times 0.4\times 0.8}{1-1\times 0.4\times 0.6} = 0.42$$

$$\delta = 22.8^\circ$$

From Fig. 6.27*d*,

$$\psi = 36.9 - 22.8 = 14.1^\circ$$

From Eqs. 6.43 and 6.44,

$$I_d = 1\ \sin 14.1^\circ = 0.24\ \text{pu}$$

$$I_q = 1\ \cos 14.1^\circ = 0.97\ \text{pu}$$

From Eq. 6.47,

$$E_f = 1\ \cos 22.8^\circ - 0.24\times 0.8 = 0.73\ \text{pu}$$

(b) From Eq. 6.52, power due to field excitation

$$P_f = \frac{1\times 0.73}{0.8}\sin 22.8^\circ = 0.35\ \text{pu}$$

and power due to saliency of the machine

$$P_r = \frac{1^2\times(0.8-0.4)}{2\times 0.8\times 0.4}\sin 45.6^\circ = 0.45\ \text{pu}$$

(c) Power output is

$$P_{out} = V_t I_a \cos\phi = 1\times 1\times 0.8 = 0.8\ \text{pu}$$

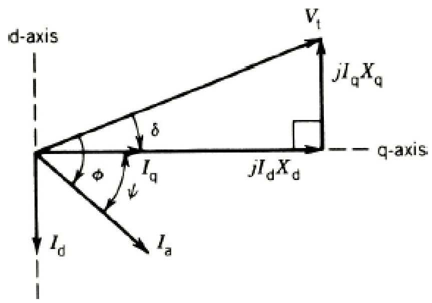

FIGURE E6.7

From Eq. 6.52, the maximum power due to saliency of the machine is

$$P_r|_{max} = \frac{1^2 \times (0.8 - 0.4)}{2 \times 0.8 \times 0.4} = 0.63 \text{ pu}$$

The output power is more than the power the machine can develop. Therefore, the machine will lose synchronism.

(d) No-load power is 0.15 pu.

$$0.15 = 0.63 \sin 2\delta$$

or

$$\delta = 6.89°$$

The phasor diagram is shown in Fig. E6.7. With V_t as reference, the q-axis is $\delta = 6.89°$ behind it. The right-angle triangle is formed by V_t, I_dX_d, and I_qX_q.

$$I_dX_d = V_t \cos\delta = 1 \cos 6.89° = 0.99 \text{ pu}$$

$$I_d = \frac{0.99}{0.8} = 1.24 \text{ pu}$$

$$I_qX_q = V_t \sin\delta = 1 \sin 6.89° = 0.12 \text{ pu}$$

$$I_q = \frac{0.12}{0.4} = 0.3 \text{ pu}$$

$$I_a = (1.24^2 + 0.3^2)^{1/2} = 1.276 \text{ pu}$$

$$\psi = \tan^{-1}\frac{1.24}{0.3} = 76.4°$$

$$\phi = 76.4° + 6.89° = 83.3°$$

$$\text{Power factor} = \cos 83.3° = 0.117 \quad \text{lagging}$$

Assignments

6.3 The following test data are obtained for a 3ϕ, 195 MVA, 15 kV, 60 Hz star-connected synchronous machine.

Open-circuit test:

I_f(A)	150	300	450	600	750	900	1200
V_{LL}(kV)	3.75	7.5	11.2	13.6	15	15.8	16.5

Short-circuit test:

$I_f = 750$ A, $I_a = 7000$ A

The armature resistance is very small.

(a) Draw the open-circuit characteristic, the short-circuit characteristic, the air gap line, and the modified air gap line.

(b) Determine the unsaturated and saturated values of the synchronous reactance in ohms and also in pu.

6.9 A 3 φ, 11 kV, 60 Hz, 25 MVA, Y-connected cylindrical-rotor synchronous machine generator has $R_a = 0.45\ \Omega$ per phase and $X_s = 4.5\ \Omega$ per phase. The generator delivers the rated load at 11 kV and 0.85 lagging power factor. Determine the excitation voltage, E_f, for this operating condition.

6.12 A 3 φ, 10 kVA, 220 V, Y-connected synchronous generator has $R_a = 0.25\ \Omega$ per phase and $X_s = 5.0\ \Omega$ per phase. Determine the excitation voltage, E_f, when the generator is delivering full load at power factor of

(a) 0.85 lagging.

(b) 1.0 unity.

(c) 0.8 leading.

6.13 The synchronous generator of Problem 6.12 has a field resistance of $R_f = 5.0\ \Omega$, core loss of 250 W and rotational loss of 200 W.

Determine the efficiency of the generator in all cases of operating conditions of Problem 6.4. The excitation current is 8 A at unity power factor and assume that excitation voltage E_f varies linearly with the excitation current.

6.30 A salient pole synchronous machine delivers rated power to a load of unity power factor. The d-axis and q-axis reactances are

$$X_d = 0.95 \text{ pu}, \quad X_q = 0.45 \text{ pu}$$

The power angle is $\delta = 25°$.

The stator winding resistance is negligible.

(a) Determine the excitation voltage (E_f) and the terminal voltage (V_t).

(b) Determine I_a, I_d, and I_q, and draw the phasor diagram.