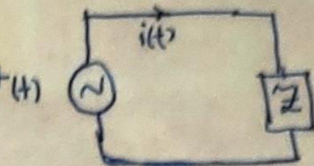
* The voltage source is connected to a load and draws current.

Let
$$\begin{cases} v(t) = \sqrt{2} \times 20 \times \cos(\omega t) \\ i(t) = \sqrt{2} \times 4 \times \cos(\omega t - 60^\circ) \\ \omega = 2\pi \times 60 \text{ Rad/s} \end{cases}$$



Calculate:

1) Phasor of voltage: $V = |V| \angle \theta_v$ $\begin{cases} |V| = ? & 20\ V \\ \theta_v = ? & 0^\circ \end{cases}$

2) Phasor of current: $I = |I| \angle \theta_i$ $\begin{cases} |I| = ? & 4\ A \\ \theta_i = ? & -60^\circ \end{cases}$

3) Active power: $|V||I|\cos\phi = 20 \times 4 \times \cos(60^\circ) = 40\ W$

4) Reactive power: $|V||I|\sin\phi = 20 \times 4 \times \sin(60^\circ) = 40 \times \sqrt{3} = 69.3$ VAr

5) Apparent power: $|V||I| = 20 \times 4 = 80$ V.A

6) Complex power: $S = P + jQ = 40 + j\,40\sqrt{3}$

7) power factor: $\cos(60^\circ) = 40/80 = 0.5$

8) power factor angle: $\phi = \theta_v - \theta_i = 60^\circ$

9) Impedance Z: $Z = |Z| \angle \theta_z$ $\begin{cases} |Z| = ? & |V|/|I| = 20/4 = 5\ \Omega \\ \theta_z = ? & \theta_v - \theta_i = 60^\circ \end{cases}$

$Z = 5 \angle 60^\circ$

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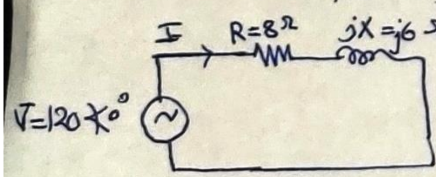
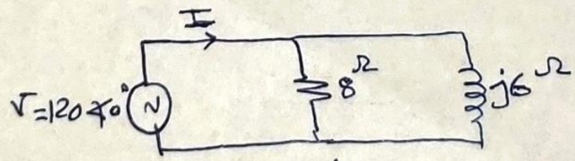
* Consider the following systems:



(a)



(b)

For each case, calculate:

1) Phasor of current I.

2) Active power

3) Reactive power

4) Apparent power

(a)

$I = \dfrac{120\angle 0^\circ}{8 + j6} = \dfrac{120\angle 0^\circ}{10\angle 36.87^\circ}$

$I = 12 \angle -36.87^\circ$

$|V||I|\cos\phi = 120 \times 12 \times \cos(36.87^\circ) = 1152$ W

$|V||I|\sin\phi = 120 \times 12 \times \sin(36.87^\circ) = 864$ Var

$|V||I| = 120 \times 12 = 1440$ V.A

$\begin{cases} \cos(36.87^\circ) = 0.8 \\ \sin(36.87^\circ) = 0.6 \end{cases}$

(b)

$I = \dfrac{120}{8} + \dfrac{120}{6j} \Rightarrow$

$I = 15 - j20 = 25 \angle -53.13^\circ$

$120 \times 25 \times \cos(53.13^\circ) = 1800$ W

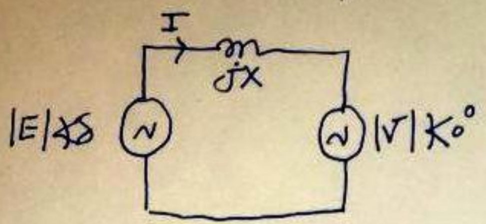
$120 \times 25 \times \sin(53.13^\circ) = 2400$ Var

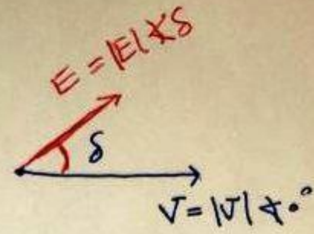
$120 \times 25 = 3000$ V.A

$\begin{cases} \cos(53.13^\circ) = 0.6 \\ \sin(53.13^\circ) = 0.8 \end{cases}$

Solution

Consider one phase:





complex power from left to right: $S = E \cdot I^*$

$$I = \frac{E - V}{jX} = \frac{|E|\angle\delta - |V|\angle 0^\circ}{jX} = \frac{|E|}{X}\angle(\delta - 90^\circ) - \frac{|V|}{X}\angle(-90^\circ)$$

$$I^* = \frac{|E|}{X}\angle(90^\circ - \delta) - \frac{|V|}{X}\angle 90^\circ$$

$$S = EI^* = \left(|E|\angle\delta\right) \times \left(\frac{|E|}{X}\angle(90 - \delta) - \frac{|V|}{X}\angle 90^\circ\right) =$$

$$\frac{|E|^2}{X}\angle 90^\circ - \frac{|E|\cdot|V|}{X}\angle(\delta + 90^\circ)$$

$$S = j\frac{|E|^2}{X} - \frac{|E||V|}{X}\cos(\delta + 90^\circ) - j\frac{|E||V|}{X}\sin(\delta + 90^\circ)$$

$$= \frac{|E||V|}{X}\sin(\delta) + j\left(\frac{|E|^2 - |E|\cdot|V|\cos\delta}{X}\right)$$

$S = P + jQ$

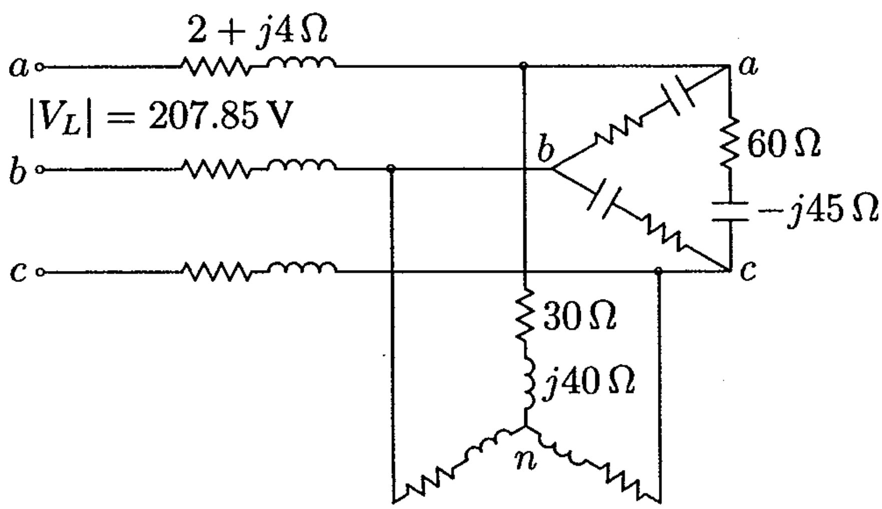
$$\Rightarrow \begin{cases} P = \dfrac{|E|\cdot|V|}{X}\sin\delta \\ Q = \dfrac{|E|^2 - |E||V|\cdot\cos\delta}{X} \end{cases}$$

for the three-phase system:

$$\begin{cases} P = 3\left(\dfrac{|E||V|}{X}\sin\delta\right) \\ Q = 3\left(\dfrac{|E|^2 - |E|\cdot|V|\cos\delta}{X}\right) \end{cases}$$

Problem:

A three-phase line has an impedance of $2 + j4\ \Omega$ as shown in Figure ·



The line feeds two balanced three-phase loads that are connected in parallel. The first load is Y-connected and has an impedance of $30+j40\ \Omega$ per phase. The second load is Δ-connected and has an impedance of $60 - j45\ \Omega$. The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85 V. Taking the phase voltage V_a as reference, determine:

(a) The current, real power, and reactive power drawn from the supply.
(b) The line voltage at the combined loads.
(c) The current per phase in each load.
(d) The total real and reactive powers in each load and the line.

Solution:

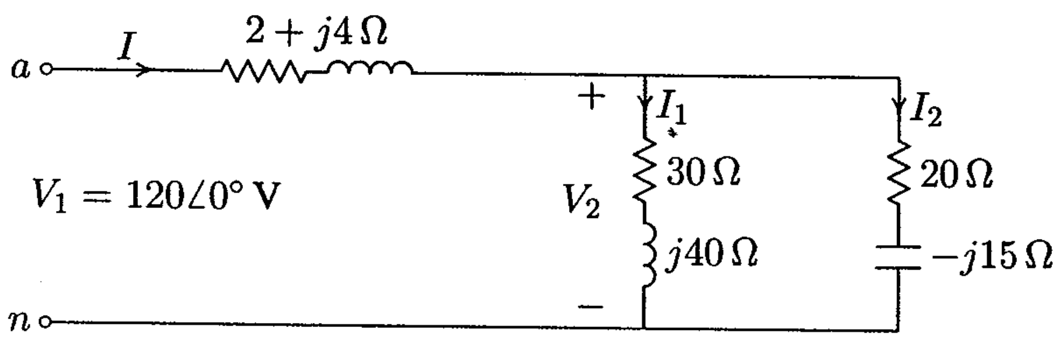
(a) The Δ-connected load is transformed into an equivalent Y. The impedance per phase of the equivalent Y is

$$Z_2 = \frac{60 - j45}{3} = 20 - j15 \ \Omega$$

The phase voltage is

$$V_1 = \frac{207.85}{\sqrt{3}} = 120 \text{ V}$$

The single-phase equivalent circuit is shown in Figure



The total impedance is

$$Z = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)}$$
$$= 2 + j4 + 22 - j4 = 24 \ \Omega$$

With the phase voltage V_{an} as reference, the current in phase a is

$$I = \frac{V_1}{Z} = \frac{120\angle 0°}{24} = 5 \text{ A}$$

The three-phase power supplied is

$$S = 3V_1I^* = 3(120\angle 0°)(5\angle 0°) = 1800 \text{ W}$$

(b) The phase voltage at the load terminal is

$$V_2 = 120\angle 0^\circ - (2 + j4)(5\angle 0^\circ) = 110 - j20$$
$$= 111.8\angle{-10.3^\circ}\ \text{V}$$

The line voltage at the load terminal is

$$V_{2ab} = \sqrt{3}\,\angle 30^\circ\ V_2 = \sqrt{3}\ (111.8)\angle 19.7^\circ = 193.64\angle 19.7^\circ\ \text{V}$$

(c) The current per phase in the Y-connected load and in the equivalent Y of the Δ load is

$$I_1 = \frac{V_2}{Z_1} = \frac{110 - j20}{30 + j40} = 1 - j2 = 2.236\angle - 63.4^\circ\ \text{A}$$
$$I_2 = \frac{V_2}{Z_2} = \frac{110 - j20}{20 - j15} = 4 + j2 = 4.472\angle 26.56^\circ\ \text{A}$$

The phase current in the original Δ-connected load, i.e., I_{ab} is given by

$$I_{ab} = \frac{I_2}{\sqrt{3}\angle{-30^\circ}} = \frac{4.472\angle 26.56^\circ}{\sqrt{3}\angle{-30^\circ}} = 2.582\angle 56.56^\circ\ \text{A}$$

(d) The three-phase power absorbed by each load is

$$S_1 = 3V_2 I_1^* = 3(111.8\angle - 10.3^\circ)(2.236\angle 63.4^\circ) = 450\ \text{W} + j600\,\text{var}$$
$$S_2 = 3V_2 I_2^* = 3(111.8\angle - 10.3^\circ)(4.472\angle{-26.56^\circ}) = 1200\ \text{W} - j900\,\text{var}$$

The three-phase power absorbed by the line is

$$S_L = 3(R_L + jX_L)|I|^2 = 3(2 + j4)(5)^2 = 150\ \text{W} + j300\ \text{var}$$

It is clear that the sum of load powers and line losses is equal to the power delivered from the supply, i.e.,

$$S_1 + S_2 + S_L = (450 + j600) + (1200 - j900) + (150 + j300)$$
$$= 1800\ \text{W} + j0\ \text{var}$$