P 2.2):

The flux in the core of a 1 φ transformer varies with time as shown in Fig. P2.2. The primary coil has 400 turns and the secondary coil has 100 turns. Sketch the waveform of the induced voltage e_1 in the primary winding.

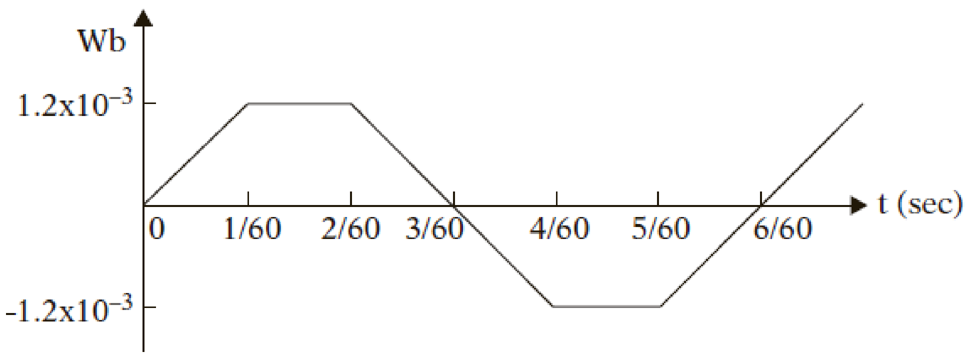


FIGURE P2.2

Solution:

$$0 < t < \frac{1}{60}\ \text{sec}$$

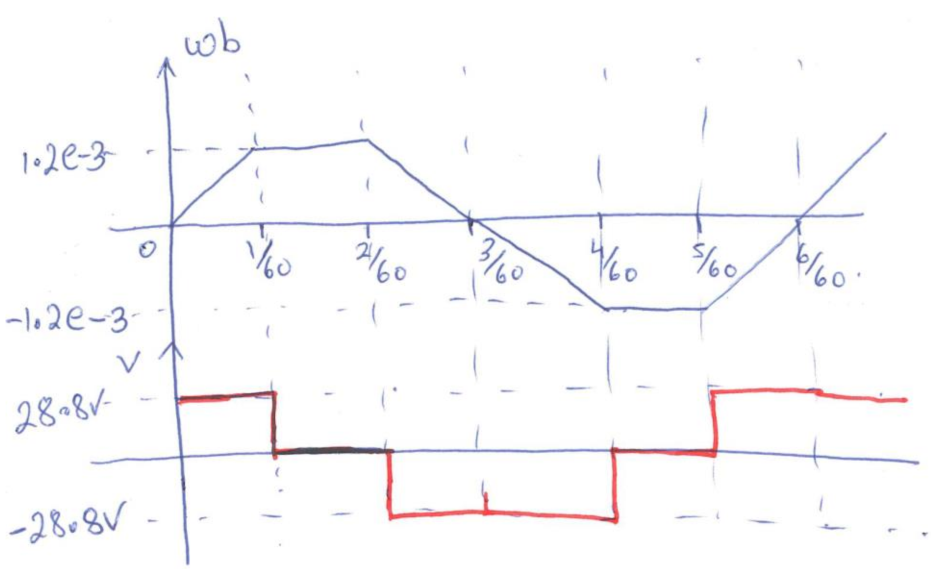
$$e = N\frac{d\Phi}{dt} = 400 \times \frac{1.2 \times 10^{-3}}{\frac{1}{60}} = 28.8\ \text{V}$$

$$\frac{1}{60} < t < \frac{2}{60}\ \text{sec}$$

$$e = N \times 0 = 0$$

$$\frac{2}{60} < t < \frac{4}{60}\ \text{sec}$$

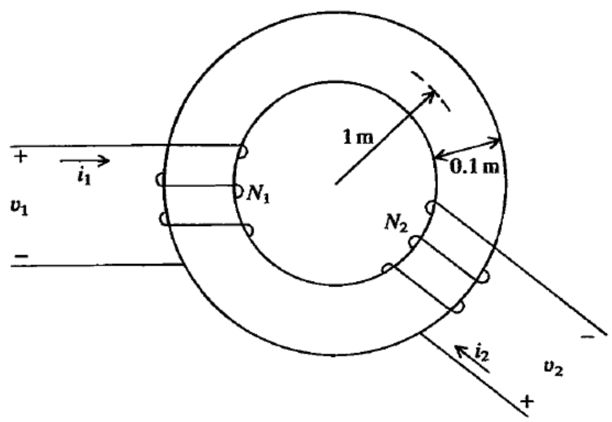
$$e = -400 \times \frac{2 \times 1.2 \times 10^{-3}}{\frac{2}{60}} = -28.8\ \text{V}$$



Q2):

An iron "doughnut" (toroid) with two coils is shown in $N_1 = 100$ turns and $N_2 = 200$ turns, $\mu_r = 10^4/4\pi$. Calculate L_{12}.

Also calculate L_{21}, L_{11}, L_{22}.



Remember:

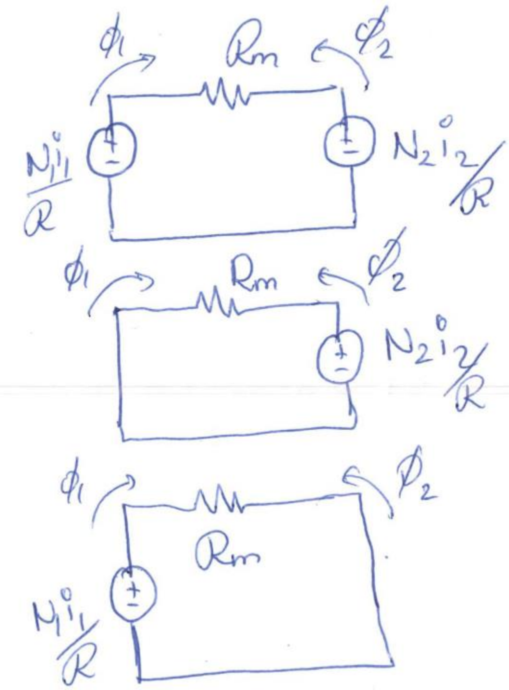
$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$
$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$

Solution:

$$\mathcal{R}_m = \frac{l}{\mu A} = \frac{(2\pi)(1)}{(4\pi \times 10^{-7})(\frac{1}{4\pi} \times 10^4)\,\pi\,(0.05)^2} = 8 \times 10^5 \text{ H}^{-1}$$

$$L_{12} = -\frac{N_1 N_2}{\mathcal{R}_m} = -\frac{(100)(200)}{8 \times 10^5} = -25 \text{ mH}$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$



$$\lambda_1 \big|_{i_1=0} = L_{12} i_2$$

$$L_{12} = \frac{\lambda_1}{i_2} = \frac{N_1 \phi_1}{i_2} = -\frac{N_1 N_2 \cancel{i_2}}{\cancel{i_2} R}$$

$$\lambda_1 \big|_{i_2=0} = L_{11} i_1$$

$$L_{11} = \frac{\lambda_1}{i_1} = \frac{N_1 \phi_1}{i_1} = \frac{N_1 \cdot N_1 \cancel{i_1}}{\cancel{i_1} R}$$

$$\lambda_2 = L_{22} i_2 + L_{21} i_1$$

$$\lambda_2 \big|_{i_1=0} = L_{22} i_2$$

$$L_{22} = \frac{\lambda_2}{i_2} = \frac{N_2 \phi_2}{i_2} = \frac{N_2 \cdot N_2 \cancel{i_2}}{\cancel{i_2} \cdot R}$$

$$\lambda_2 \big|_{i_2=0} = L_{21} i_1$$

$$L_{21} = \frac{\lambda_2}{i_1} = \frac{N_2 \phi_2}{i_1} = -\frac{N_2 N_1 \cancel{i_1}}{\cancel{i_1} R}$$

$$L_{11} = \frac{N_1^2}{\mathcal{R}} = \frac{100^2}{8 \times 10^5} = 12.5\,\text{mH}$$

$$L_{12} = -\frac{N_1 N_2}{\mathcal{R}} = -\frac{100 \times 200}{8 \times 10^5} = -25\,\text{mH}$$

$$L_{21} = -\frac{N_1 N_2}{\mathcal{R}} = \frac{-2 \times 10^4}{8 \times 10^5} = -25\,\text{mH}$$

$$L_{22} = \frac{N_2^2}{\mathcal{R}} = \frac{4 \times 10^4}{8 \times 10^5} = 50\,\text{mH}$$

Q3):

Two coupled coils have the following parameters: $r_1 = 10\,\Omega$, $L_{l1} = 0.1\,L_{11}$, $r_2 = 2.5\,\Omega$, $L_{l2} = 0.1\,L_{22}$, $L_{11} = 100\,\text{mH}$, $N_1 = 100$ turns, $L_{22} = 25\,\text{mH}$, $N_2 = 50$ turns. Develop an equivalent T circuit with (a) winding 1 as the reference winding and (b) winding 2 as reference winding.

Solution:

(a) $L_{m1} = L_{11} - L_{l1} = 90\ \text{mH}$

$$L'_{l2} = \left(\frac{N_1}{N_2}\right)^2 L_{l2} = \left(\frac{100}{50}\right)^2 (2.5 \times 10^{-3}) = 10\ \text{mH}$$

$$r'_2 = \left(\frac{N_1}{N_2}\right)^2 r_2 = \left(\frac{100}{50}\right)^2 (2.5) = 10\ \Omega$$

(b) $L_{m2} = L_{22} - L_{l2} = 22.5\ \text{mH}$

$$L'_{l1} = \left(\frac{N_2}{N_1}\right)^2 L_{l1} = \left(\frac{50}{100}\right)^2 (10 \times 10^{-3}) = 2.5\ \text{mH}$$

$$r'_1 = \left(\frac{N_2}{N_1}\right)^2 r_1 = \left(\frac{50}{100}\right)^2 (10) = 2.5\ \Omega$$

P 2.16):

A 1ϕ, 100 kVA, 11000/2200 V, 60 Hz transformer has the following parameters.

$R_{HV} = 6.0\ \Omega$ $\qquad R_{LV} = 0.28\ \Omega$

$L_{HV} = 0.08$ H $\qquad L_{LV} = 0.0032$ H

$L_{m(HV)} = 160$ H

$R_{c(HV)} = 125\ k\Omega$

Obtain an equivalent circuit of the transformer:

(a) Referred to the high-voltage side.

(b) Referred to the low-voltage side.

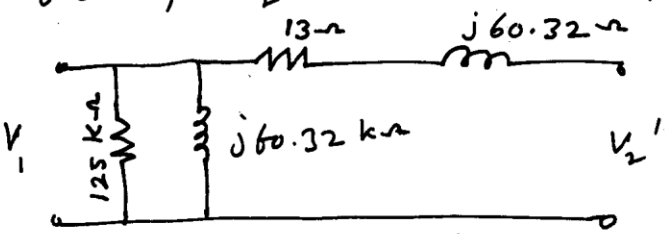
Solution:

(a) $n = \dfrac{11000}{2200} = 5$

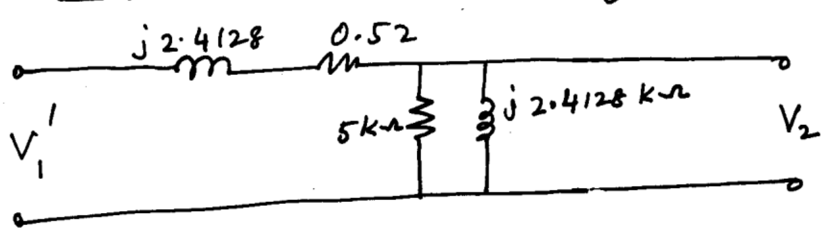
$X_{L_1} = 377 \times 0.08 = 30.16\ \Omega$, $X'_{L2} = 5^2 \times 377 \times 0.0032 = 30.16\ \Omega$

$X_{m_1} = 377 \times 160 = 60.32\ k\Omega$, $R_{c_1} = 125\ k\Omega$.

$R_1 = 6.0\ \Omega$, $R_2' = 5^2 \times 0.28 = 7.0\ \Omega$



b)



Q5):

A 1ϕ, 100 kVA, 1000/100 V transformer gave the following test results:

open-circuit test (HV side open)

100 V, 6.0 A, 400 W

short-circuit test

50 V, 100 A, 1800 W

(a) Determine the rated voltage and rated current for the high-voltage and low-voltage sides.

(b) Derive an approximate equivalent circuit referred to the HV side.

Solution:

(a) $V_{H(rated)} = 1000V$, $I_{H(rated)} = \dfrac{100\times10^3}{1000} = 100A.$

$V_{L(rated)} = 100V$, $I_{L(rated)} = \dfrac{100\times10^3}{100} = 1000A$

(b) From open circuit test,

$$R_{cL} = \frac{100^2}{400} = 25\Omega.$$

$$I_{cL} = \frac{100}{25} = 4A.$$

$$I_{mL} = \sqrt{6^2-4^2} = 4.47A$$

$$X_{mL} = \frac{100}{4.47} = 22.37\Omega$$

Turns ratio $a = \dfrac{1000}{100} = 10$

Refer to high voltage side,

$R_{cH} = 25\times10^2 = 2500\Omega$, $X_{mH} = 2237\Omega$

From short circuit test,

$$R_{eq\,H} = \frac{1800}{100^2} = 0.18\Omega.$$

$$Z_{eq\,H} = \frac{50}{100} = 0.5\,\Omega$$

$$X_{eq\,H} = \sqrt{0.5^2 - 0.18^2} = 0.4665\,\Omega$$

Equivalent circuit referred to H.V. side