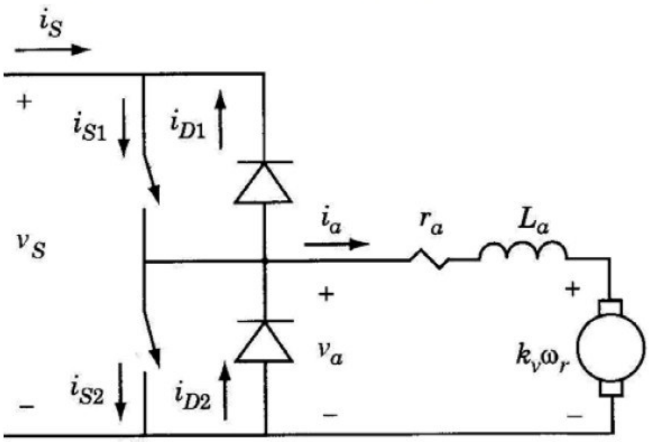
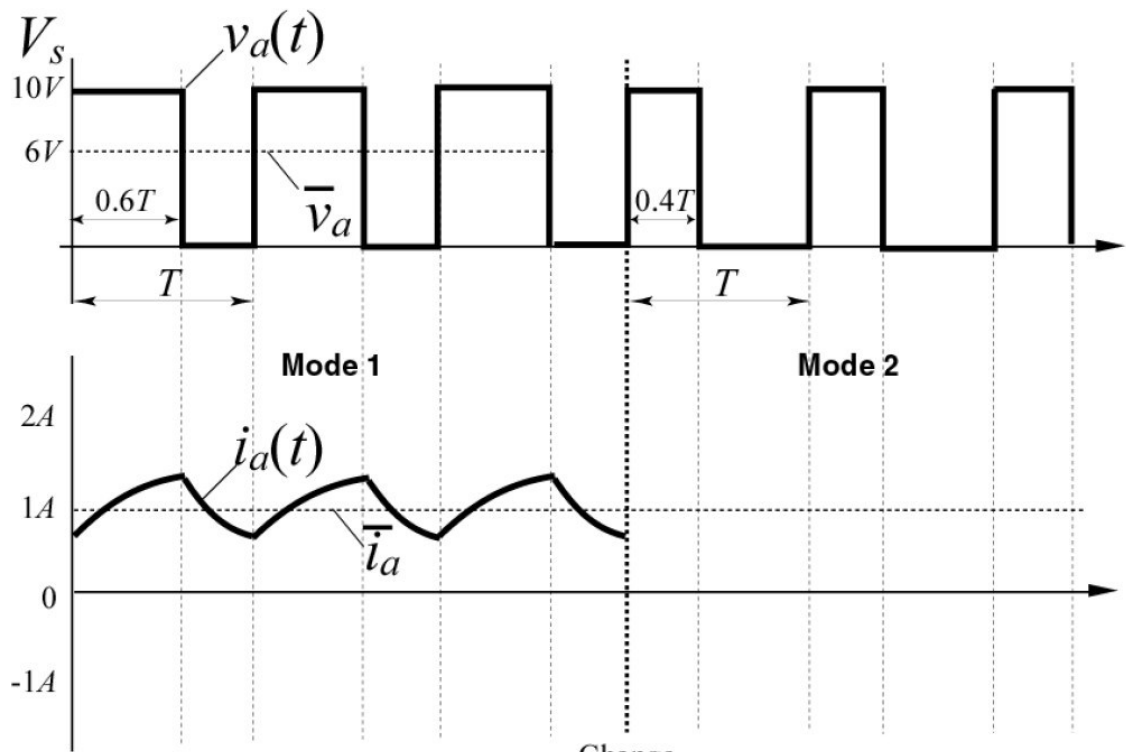
Assume a PM DC motor is driven by a PWM converter shown below. The motor shaft is connected to a constant speed mechanical system (load) with $\omega_r = 10\, r/s$. The motor parameters are $k_v = 0.5\, Vs/r$ and $r_a = 1\Omega$. The converter is supplied form a 10 V dc source. Initially, in **Mode 1**, the duty cycle $d = 0.6$ (60% on and 40% off). The corresponding voltage and current waveforms are depicted on the figure below (first interval), wherein the average values of the voltage $\bar{v}_a = 6V$ and current $\bar{i}_a = 1A$ are also shown. Assume that the duty cycle has changed to 0.4 (40% on and 60 off), as shown on the figure. This is a new steady-state operation in **Mode 2**:



(a) (5pts): Determine whether machine is **Motoring** or **Generating** in each **Mode?** and circle appropriate

(b) (5pts): Find the value of the average voltage $\bar{v}_a$ for the **Mode 2** and show it on the figure

(c) (5pts): Sketch the steady-state waveform of the current $\bar{i}_a$ in Mode 2 and find/show its average value



Motoring
Generaging

Change
in duty
cycle *d*

Motoring
Generaging

Given,

$W_m = 10\,r/s$, $K_v = 0.5\,Vs/r$, $R_a = 1\Omega$, $V_s = 10V$

Solution:

For Mode 1:

duty cycle, $d = 60\% = 0.6$

Average voltage, $\bar{V_a} = d.V_s = 0.6 \times 10$
$= 6V$

For motoring / Generating, we check V_a & E_a,
if $\bar{V_a} > E_a \rightarrow$ motoring ; $\bar{V_a} < E_a \rightarrow$ Generating

Back emf, $E_a = K_v . W_m = 0.5 \times 10 = 5V$

$\because \bar{V_a} > E_a \rightarrow \boxed{\text{motoring mode}} \rightarrow$ mode 1

Mode 2:

$\bar{V_a} = d.V_s = 0.4 \times 10 = 4V$ [$\because d = 40\%$ for mode 2]

And, $E_a = 5V$ [From previous part]

$\therefore \bar{V_a} < E_a \rightarrow \boxed{\text{Generating mode}} \rightarrow$ mode 2

Average current for mode 1 and 2:

For mode 1:

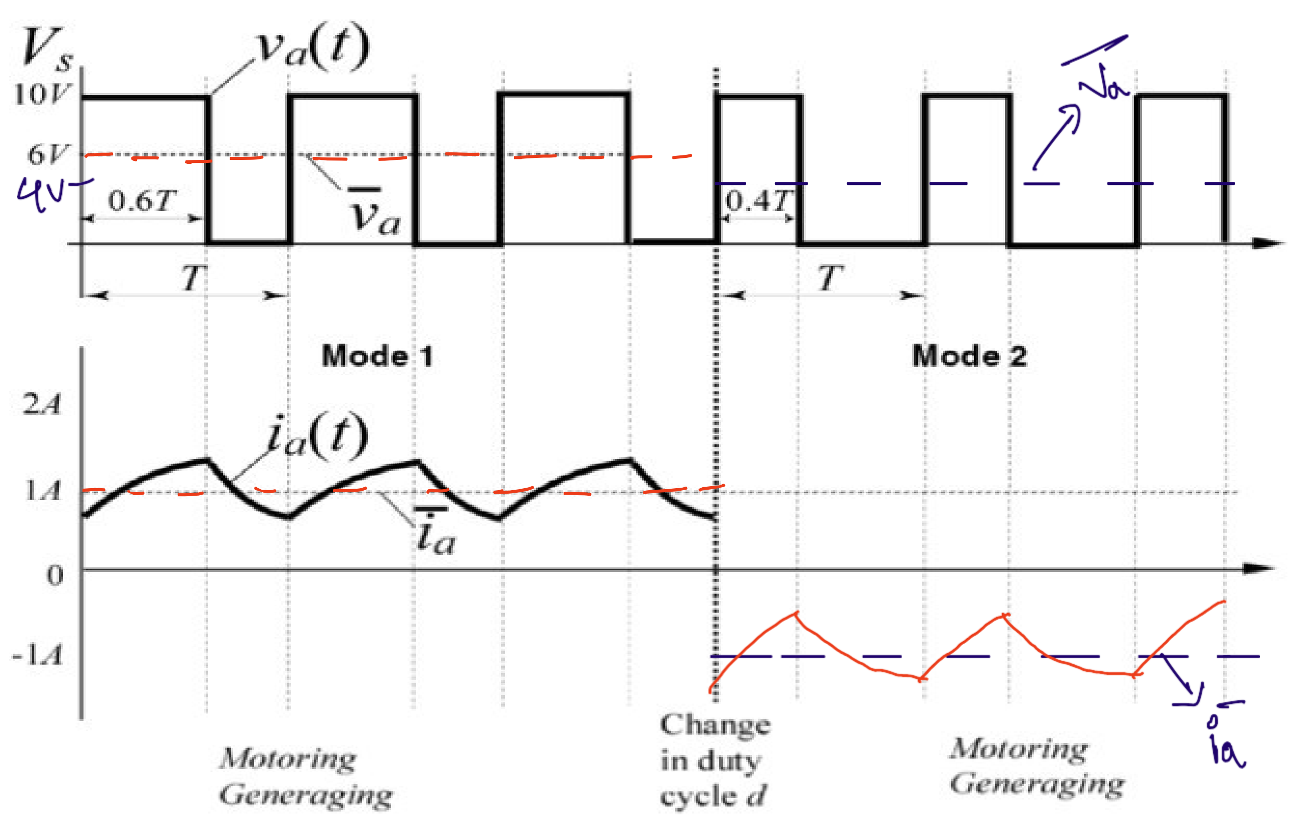
$$\bar{i}_a = \frac{V_a - E_a}{R_a} = \frac{6-5}{1} = 1A$$

For mode 2:

$$\bar{i}_a = \frac{V_a - E_a}{R_a} = \frac{4-5}{1} = -1A$$

[$\bar{V}_a$ values are calculated in the previous part]

[∵ steady-state, L_a values are not accounted]



A dc shunt machine (10 kW, 250 V, 1200 rpm) has $R_a = 0.25\ \Omega$. The machine is connected to a 250 V dc supply, draws rated armature current, and rotates at 1200 rpm.

(a) Determine the generated voltage, the electromagnetic power developed, and the torque developed.

(b) The mechanical load on the motor shaft is thrown off, and the motor draws 4 A armature current.

- **(i)** Determine the rotational loss.
- **(ii)** Determine speed

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* For this problem, since we are not given any information about the field resistance, flux or current, we can ignore the field winding circuit and proceed to solve the problem as a PMDC motor.

Solution 2(a): Given, $R_a = 0.25\,\Omega$, $V_s = 250\,V$, $P = 10\,kW$

$n_m = 1200\,rpm$

$$\omega = \frac{n}{60} \times 2\pi = \frac{1200}{30} \times \pi = 125.7\ rad/s$$

$$I_a = \frac{P}{V_s} = \frac{10000}{250} = 40\,A$$

PMDC equivalent ckt

Generated voltage, Ea

$$E_a = V_s - I_a \times R_a$$

$$= 250 - (0.25 \times 40)$$

$$\boxed{E_a = 240\,V}$$

Electromagnetic powe, $P_e = E_a I_a$

$= 240 \times 40$

$\boxed{P_e = 9600\ W}$

Torque developed, $T_e = \frac{P_e}{\omega} = \frac{9600}{125.7}$

$\boxed{T_e = 76.4\ Nm}$

2(b): mechanical load removed → Ia becomes 4A

So, new Ia becomes $I_a' = 4A$

Changed Ea due to I_a', $E_a' = V_s - R_a \times I_a'$

$= 250 - (0.25 \times 4)$

$= 249\ V$

i) Rotational loss, $P_{rot} = E_a' \times I_a'$

$= 249 \times 4$

$= \boxed{996\ W}$

ii) We know, $E_a = k_a \phi \omega$

From here, if ϕ & k_a are constant, $E_a \propto \omega \Rightarrow E_a \propto n$

proportional

speeds

$\because E_1 \propto n_1$ & $E_2 \propto n_2$; we can say,

$$\frac{E_1}{E_2} = \frac{n_1}{n_2}$$ [$\because$ both quantities are proportional]

So, if the new speed is n_2 and changed E_a' is E_2 the previous E_a & I_a are represented as E_1 & n_1

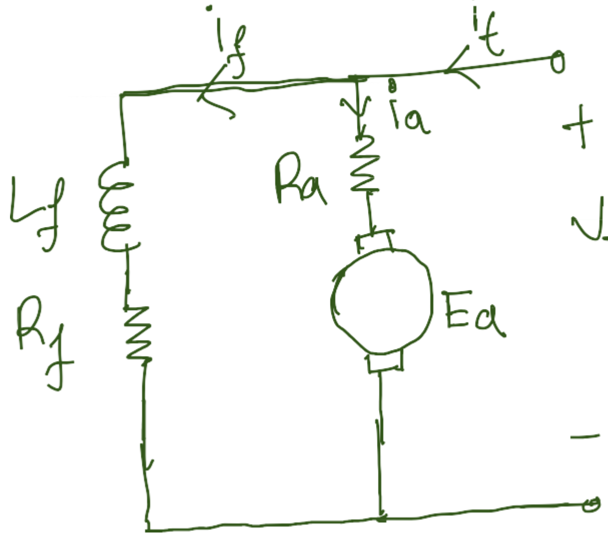
We can say, $\frac{E_2}{E_1} = \frac{n_2}{n_1} \Rightarrow n_2 = \frac{E_2}{E_1} \times n_1$

$$\Rightarrow n_2 = \frac{249}{240} \times 1200$$

$$= 1245 \text{ rpm}$$

$$\boxed{n_2 = 1245 \text{ rpm}}$$

* If you are given field winding information, you can use the DC shunt machine equivalent circuit:



→ DC shunt machine equivalent ckt.

P 4.38):

A dc motor is mechanically connected to a constant-torque load. When the armature is connected to a 120 V dc supply, it draws an armature current of value 8 A and runs at 1800 rpm. The armature resistance is $R_a = 0.08\ \Omega$. Accidentally, the field circuit breaks and the flux drops to the residual flux, which is only 5% of the original flux.

(a) Determine the value of the armature current immediately after the field circuit breaks (i.e., before the speed has had time to change from 1800 rpm).

(b) Determine the theoretical final speed of the motor after the field circuit breaks.

Solution 3(a):

Given data, $V_s = 120V$, $I_a = 8A$, $n = 1800 rpm$, $R_a = 0.08\Omega$, flux reduced to 5% of original

$\phi_1 \to$ original, $\phi_2 = 5\%$ of ϕ_1 --- (1)

Immediately after field ckt breaks, I_a becomes $\to I_a'$

Initial back emf, $E_a = V_s - I_a R_a$
$= 120 - (8 \times 0.08) = 119.36V$ } before field ckt broke

We know, $E_a = k_a \phi \omega$; So at constant speed, $E_a \propto \phi$ and after field ckt. broke, $E_a \to E_a'$

So, $E_a \propto \phi_1$ and $E_a' \propto \phi_2$ ---- (11)

$E_a' = 5\%$ of E_a [comparing (11) & (1)]

$= 0.05 \times 119.36 = 5.968\ V$

$$\therefore I_a' = \frac{V_s - E_a'}{R_a} = \frac{120 - 5.968}{0.08} = \boxed{1425.4\ A}$$

3(b): We know,

$$E_a = k\phi\omega \Rightarrow \omega = \frac{E_a}{k\phi} \quad \text{--- (I)}$$

and, $$T = k\phi I_a \Rightarrow I_a = \frac{T}{k\phi} \quad \text{--- (II)}$$

If E_a, T and k are constants in (I) & (II), we can say, $\omega \propto \frac{1}{\phi}$, $I_a \propto \frac{1}{\phi}$ --- (III)

After field ckt. breaks, E_a becomes E_a', I_a becomes I_a' and ϕ becomes ϕ'

$$I_a' = \frac{I_a}{5\%} = \frac{8}{0.05} = 160A \quad \text{[From (III)]}$$

So, $E_a' = V_s - R_a I_a' = 120 - (0.08 \times 160) = 107.2\,V$

$$\frac{E_a'}{E_a} = \frac{\cancel{ka}\,\phi'\omega'}{\cancel{ka}\,\phi\,\omega} = \frac{0.05 \times \cancel{\phi} \times \omega'}{\cancel{\phi} \times \omega} \quad [\because \phi' = 0.05 \times \phi]$$

$$\Rightarrow \frac{E_a'}{E_a} = 0.05 \times \frac{\omega'}{\omega}$$

$$\Rightarrow \omega' = \frac{E_a'}{E_a} \times \frac{\omega}{0.05}$$

[ω and n are both speeds just in different units]

So, $n' = \dfrac{E_a'}{E_a} \times \dfrac{n}{0.05}$

$$= \frac{107.2}{120} \times \frac{1800}{0.05}$$

$$\therefore \boxed{n' = 32160 \text{ rpm}}$$

↳ theoretical final speed.