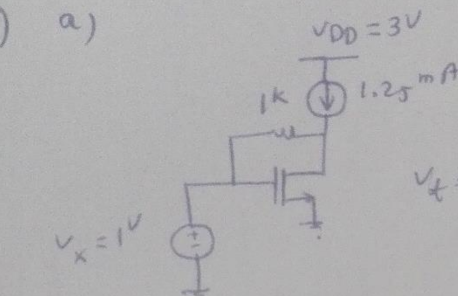
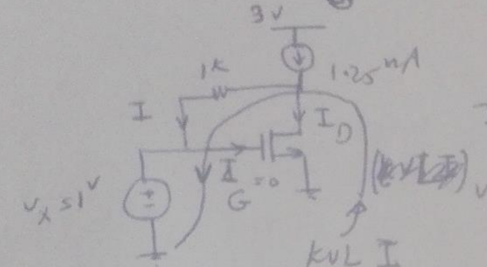
Assignment #1

1) a)



$V_t = .5^V \quad \lambda_n = 0 \quad \mu_n C_{ox} = 200 \,\mu A/V^2$

Drain Current: $I_G = 0$



$I + I_D = 1.25^{mA}$

$V_X = V_{GS} = 1^V$

Assume the transistor is in Sat. : $\Rightarrow$

$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right)(V_{GS} - V_t)^2 \Rightarrow I_D = \frac{1}{2} \times (.2) \times \left(\frac{5^{\mu}}{.5^{\mu}}\right)(1 - .5)^2$

$\Rightarrow \boxed{I_D = 0.25^{mA}} \Rightarrow$

Now, we should check if our assumption is right:

The Conditions for Sat. region is

$V_{GS} > V_t$ and $V_{DS} \geq (V_{GS} - V_t)$

$\Rightarrow V_{DS} = V_{GS} + 1^k \times (1.25^{mA} - .25^{mA}) \longleftarrow$ KVL I

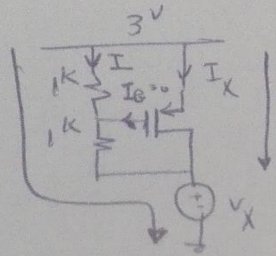
$\boxed{V_{DS} = 1 + 1 = 2^V}$ ✓ , $V_{GS} - V_t = 1 - .5 = .5^V \Rightarrow$

$V_{DS} > V_{GS} - V_t$, $V_{GS} > V_t \Rightarrow$ The trans. is in Sat. region ✓

1

1) b)

KVL I



KVL II $V_t = -.5^V$ $\lambda = 0$, $k'_p = 0.1\ ^{mA}/_{V^2}$

$I_G = 0$

KVL I: $3^V = 1^{k\Omega} \times I^{mA} + 1^{k\Omega} \times I^{mA} + 1^V \Rightarrow I = 1^{mA}$

$\Rightarrow V_{SG} = 1^{k\Omega} \times 1^{mA} = 1^V$

Assume the trans. is in Saturation. $\Rightarrow$

$I_X = \frac{1}{2} \times 0.1^{mA/V^2} \times (\frac{W}{L})(V_{SG} - |V_t|)^2 \Rightarrow \boxed{I_X = \frac{1}{2} \times .1 \times (\frac{5}{.5})(1-.5)^2 = .125^{mA}}$

Now, we should check our assumption.

The Condition for tran to be in Sat. is

$|V_{GS}| > |V_t|$, $|V_{DS}| > (|V_{GS}| - |V_t|)$

$|V_{GS}| = 1^V > .5$ ✓

to find V_{DS}: KVL II: $3 = V_{SD} + 1 \Rightarrow \boxed{V_{SD} = 2^V}$

$\Rightarrow 2 > 1 - .5 = .5^V$ ✓

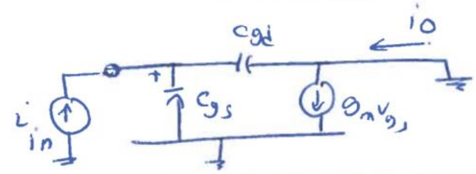
$\boxed{\text{So, the trans. is in Sat. ✓}}$

$I_D = 0.125^{mA}$

$|V_{DS}| = 2^V$

Region: Saturation

3) a) $f_T = \dfrac{g_m}{2\pi(C_{gs}+C_{gd})}$



$$\frac{i_o}{i_{in}} = \frac{g_m}{2\pi f(C_{gd}+C_{gs})} \Rightarrow A_i = 1 \Rightarrow \boxed{f_T = \frac{g_m}{2\pi(C_{gd}+C_{gs})}}$$

b) $g_m = k'\left(\frac{W}{L}\right)(V_{ov})$

$C_{gd} = C_{ox}W$

$C_{gs} = \frac{2}{3}C_{ox}W(L_p - 2L_D) + C_{ov}W \approx \frac{2}{3}C_{ox}WL$, $C_{gs} >> C_{gd}$

$$\Rightarrow f_T = \frac{\mu_n \cancel{C_{ox}}\left(\frac{\cancel{W}}{L}\right)V_{eff}}{2\pi\frac{2}{3}\cancel{C_{ox}}\cancel{W}L} = \frac{3\mu_n V_{eff}}{4\pi L^2}$$

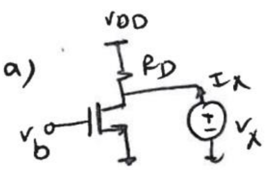
c) $I_D = I_0 e^{V_{gs}/nV_T} \Rightarrow g_m = \frac{\partial I_D}{\partial V_{gs}} = \boxed{\frac{I_D}{nV_T}}$

$$\Rightarrow f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} = \frac{I_D}{2\pi n V_T (C_{gs}+C_{gd})}$$

$C_{gs} = C_{gd} = C_{ox}W \Rightarrow f_T = \frac{I_D}{2\pi n V_T C_{ox} W}$

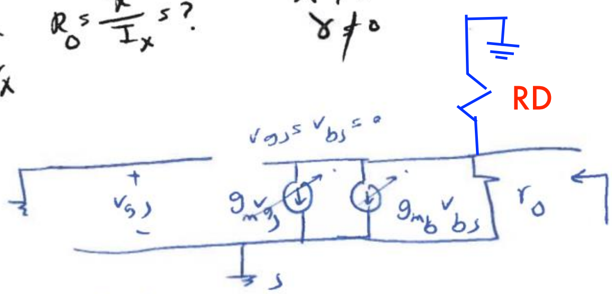
d) Although the denuminator of f_T in part c is smaller than that of part (a), its I_D is also (much) smaller than I_D in part a. So, f_T of trans. in <u>sub-threshold</u> is comparable or smaller than ~~that~~ f_T in strong inversion.

4) a)

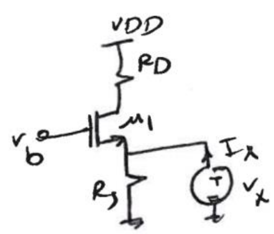


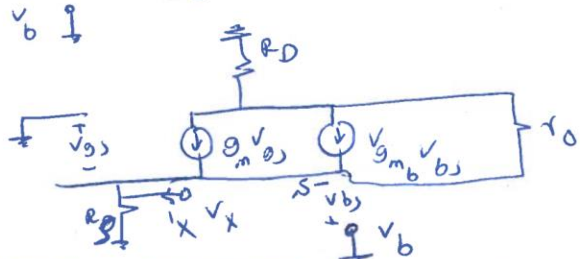
$R_0 = \frac{V_x}{I_x} = ?$

$\lambda \neq 0$
$\gamma \neq 0$



$\frac{V_x}{I_x} = r_0 || RD$

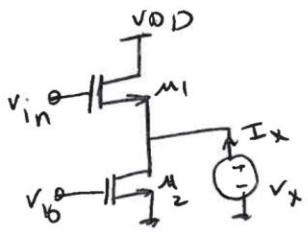


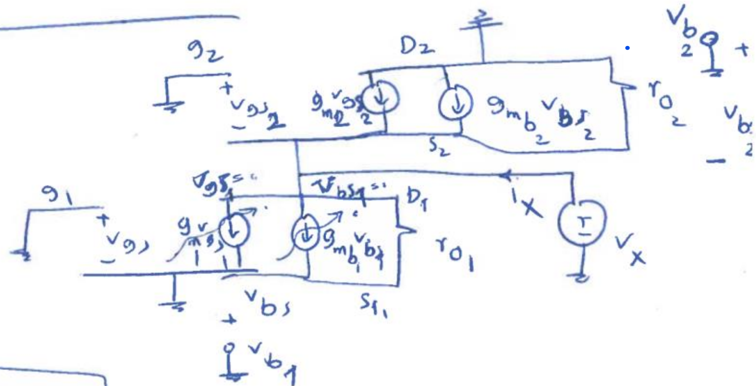


$$\boxed{R_{out} = \frac{V_x}{i_x} = \left\{\frac{R_D}{1+(g_{mb}+g_m)r_0} + \frac{1}{g_{mb}+g_m} || r_0\right\} || R_S}$$

$$R_{out} = R_S \,||\, \left\{ \frac{R_D}{1+(g_{mb}+g_m)r_o} + \left(r_o \,||\, \frac{1}{g_{mb}+g_m}\right) \right\} =$$

$$\boxed{R_S \,||\, \left\{ \frac{R_D + r_o}{1+(g_{mb}+g_m)r_o} \right\}}$$

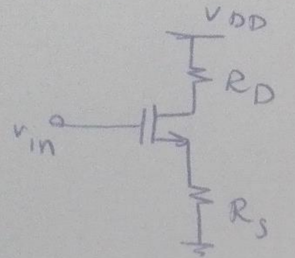




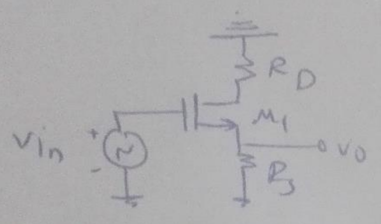
$$\boxed{\frac{v_x}{i_x} = r_{o_1} \,||\, r_{o_2} \,||\, \frac{1}{g_{m_2} + g_{mb_2}}}$$

5) a) Finding the small signal gain:

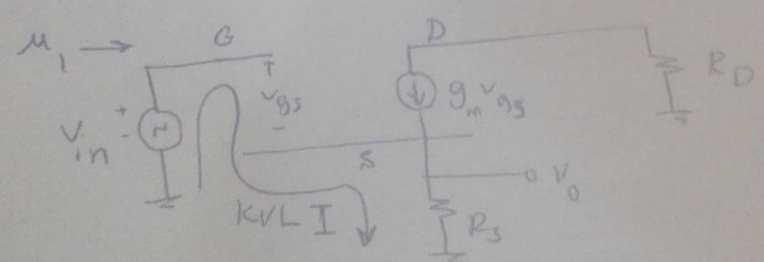
$\lambda = \gamma = 0$



To find gain, first all DC independent sources are required to be set to ZERO $\Rightarrow$



Using small signal model for the transistor:

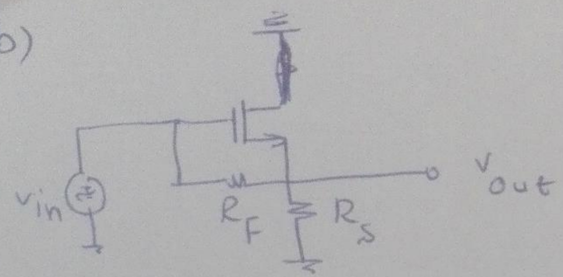


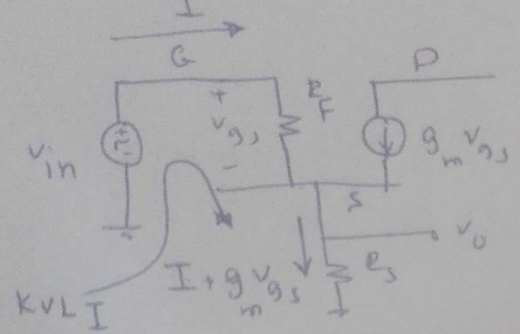
KVL I : $V_{in} = V_{gs} + R_S g_m V_{gs} \Rightarrow V_{gs} = \dfrac{V_{in}}{1 + g_m R_S}$

and $V_o = R_S \times g_m V_{gs} \Rightarrow V_o = g_m R_S \times \dfrac{1}{1 + g_m R_S}$

$$\Rightarrow \boxed{A_v = \frac{V_o}{V_{in}} = \frac{g_m R_S}{1 + g_m R_S}}$$

5) b)





KVL I: $v_{in} = R_F I + R_S (g_m v_{gs} + I)$ (I)

and

$v_{gs} = R_F I$ (II)

(II) → (I) $\Rightarrow v_{in} = R_F I + R_S I + g_m R_S \times (R_F I)$

$$\Rightarrow I = \frac{v_{in}}{R_F + R_S + R_F R_S g_m}$$

$$v_o = R_S \times (I + g_m v_{gs}) = \frac{R_S v_{in}}{R_F + R_S + R_F R_S g_m} + \frac{g_m R_S R_F v_{in}}{R_F + R_S + R_F R_S g_m}$$

$$\Rightarrow \boxed{A_v = \frac{R_S + g_m R_S R_F}{R_F + R_S + R_F R_S g_m}}$$