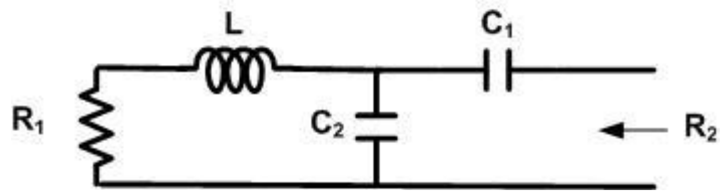
ELEC404/571F, Winter 2018: Assignment #1(1)
Due Thursday February 15th, 11am (Hand it to the TA in class)

Instructions: Please do not write your name on the assignment sheet. Just write your student ID #. If you worked with or asked help from another student in this class to do this homework, acknowledge him!

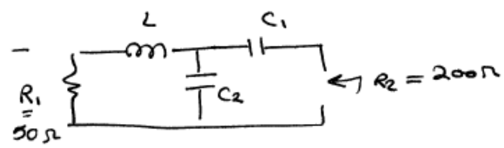
This assignment is 15% of your final grade. It is advised that you carry out the calculations using parameters (L, C, R) and plug the numbers only in the end. This will ensure that any calculation error does not percolate, and you do not lose extra points.

Q1. Consider the matching circuit shown in the figure below, used to up-convert R_1 = 50Ohm to R_2 = 200Ohm, with Q=10 @ f_0=1.9GHz. [Total 8]

(a) Determine the required LC values using transformation calculations. [6]
(b) Use *SPECTRE-RF* to simulate the magnitude (dB) and phase of the input impedance versus frequency from 100MHz-4GHz. [2]
(Hint: You can also use the 50Ohm component "port" from analogLib to do an S-parameter response, then plot the Z-parameter.).

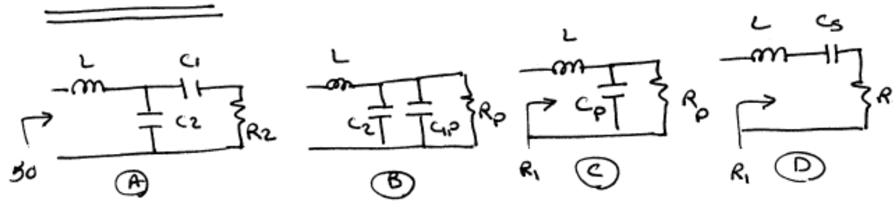


Q3 –



Q = 10
f = 1.9 GHz
R_2 = 200 Ω
R_1 = 50 Ω
L, C_1, C_2 = ?

Method 1 :



In D $\quad Q = \frac{\omega_0 L}{R_1} \Rightarrow L = \frac{QR_1}{\omega_0} = \frac{10\times 50}{\omega_0} = \boxed{41.88nH}$

$$Q = \frac{1}{\omega_0 R_1 C_S} \Rightarrow C_S = \frac{1}{\omega_0 Q R_1} = \frac{1}{2\pi \times 1.9G \times 10 \times 50} = 167.53fF$$

from C to D $\quad R_P = R_1(1+Q^2) \quad$ (i)

$$C_P = C_S \frac{Q^2}{1+Q^2} \quad \text{(ii)}$$

from A to B $\quad R_P = R_2(1+Q_1^2) \quad$ (iii)

(i)&(iii) $\Rightarrow R_1(1+Q^2) = R_2(1+Q_1^2) \quad$ (iv)

$$Q_1 = \sqrt{\frac{R_1}{R_2}(1+Q^2)-1} = \sqrt{\frac{50}{200}(1+100)-1} = 4.92$$

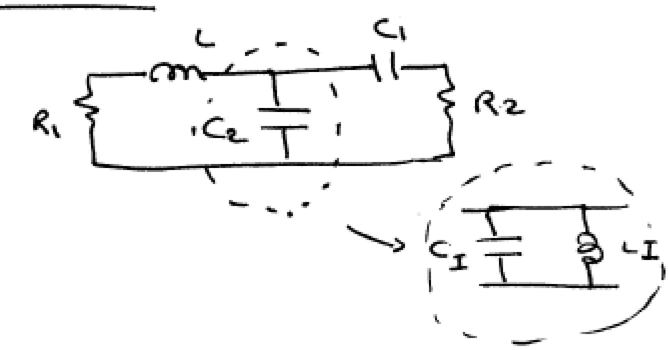
Also $Q_1 = \frac{1}{R_2 C_1 \omega_0} \Rightarrow C_1 = \frac{1}{\omega_0 R_2 Q_1} = \boxed{85.12\ fF}$

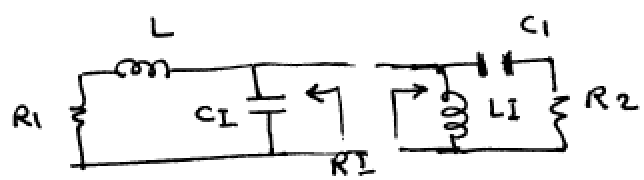
$$C_2 = C_p - C_{1P}$$

$$= C_s\left(\frac{Q^2}{1+Q^2}\right) - C_1\left(\frac{Q_1^2}{1+Q_1^2}\right)$$

$$= \boxed{84.13\ fF}$$

<u>Method 2</u> 8





Note: Method 2 is **not** that accurate. If we replace a capacitor with an inductor and capacitor, it is only accurate for one frequency, somewhat accurate in the vicinity, but usually not accurate over the -3dB BW. Hence, the calculations may not be accurate!

$$Q_{total} = Q_L + Q_R$$

$$\Rightarrow \quad 10 = \sqrt{\frac{R_I}{50} - 1} + \sqrt{\frac{R_I}{200} - 1} \quad \Rightarrow \quad \boxed{R_I = 2322\ \Omega}$$

left hand side:

$$Q_L = \sqrt{\frac{R_I}{R_1} - 1} = 6.74$$

$$Q_L = \frac{\omega_0 L}{R_1} \Rightarrow L = \frac{Q_L \cdot R_1}{\omega_0} = \boxed{2.82 \times 10^{-8}\ H}$$

$$Q_L = R_I C_I \omega_0 \Longrightarrow C_I = \frac{Q_L}{R_I \omega_0} = \boxed{0.243\ pF}$$

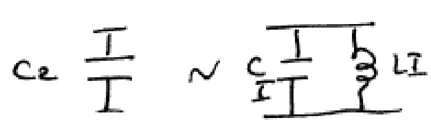
Right hand side:

$$Q_R = \sqrt{\frac{R_I}{R_2} - 1} = 3.26$$

$$Q_R = \frac{1}{R_2 C_1 \omega_0} \Rightarrow C_1 = \frac{1}{Q_R R_2 \omega_0} = \boxed{128.4\ fF}$$

$$Q_R = \frac{R_I}{L_I \omega_0} \Rightarrow L_I = \frac{R_I}{Q_R \omega_0} = \boxed{59.6\ nH}$$

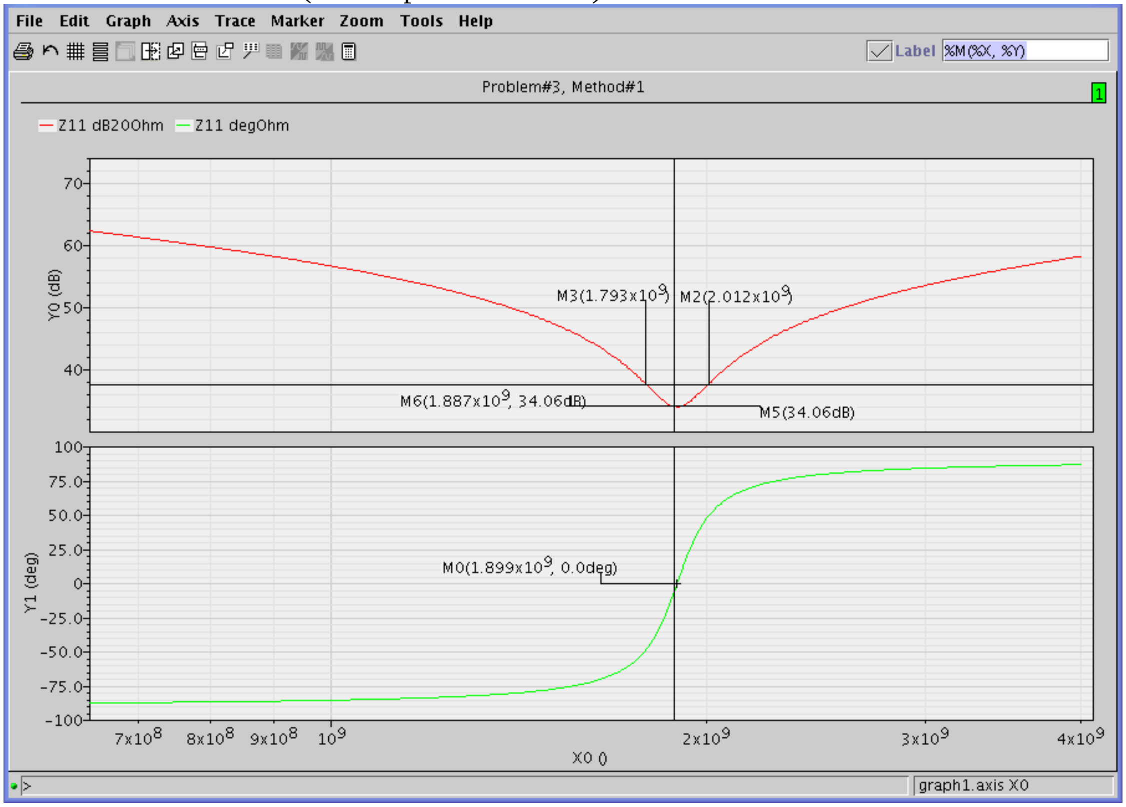
To find C_2:



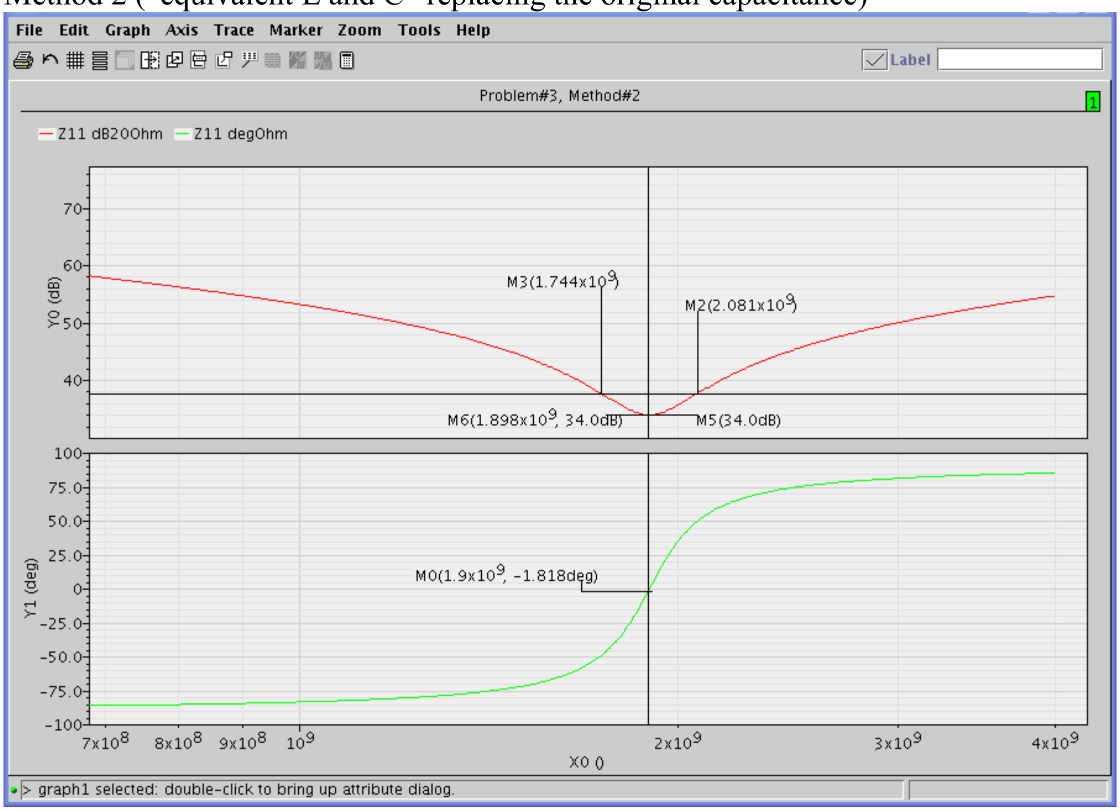
$$C_2 (j\omega_0) = C_I (j\omega_0) - \frac{j}{L\omega_0}$$

$$C_2 = C_I - \frac{1}{L\omega_0^2} = \boxed{125.2\ fF}$$

Method 1 or Method 2 (with separate L and C)



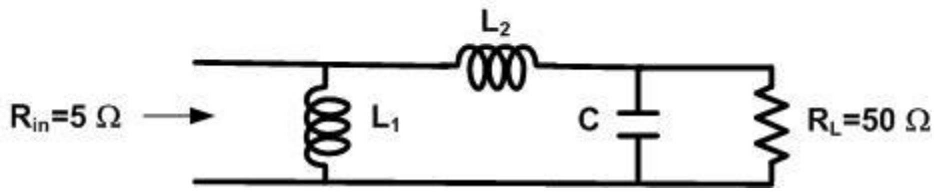
Method 2 ("equivalent L and C" replacing the original capacitance)



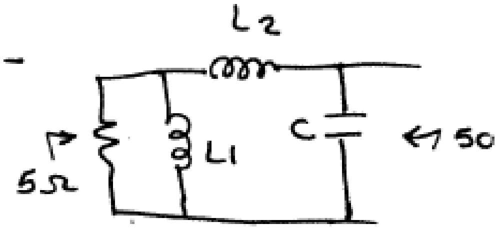
Note that final Q is only about 6.5!

Q2. Jeff is designing a power-amplifier at 2.4GHz with an output impedance of 5 Ohm, and wants to interface to a 50 Ohm antenna through a passive *transformer* (matching network). The process limits the maximum zero-peak voltage swing to 1.2 V. [10]

a. What is the maximum power (in dBm) he can deliver to the antenna? Assume perfect matching; Jeff is a good engineer! [2]
b. As a first pass design, he decides to use the matching network below for the impedance transformation, with a *Q* of 10. Calculate the LC values? Verify using *SPECTRE*, submit plot showing the bandwidth. [6+2]



Q4 -



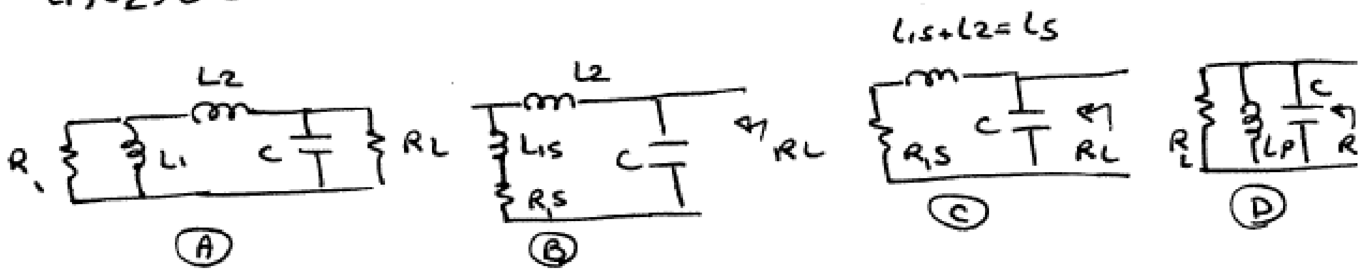
$Q = 10$
$f = 2.4$ GH
max power = ?
$L_1, L_2, \& C = ?$

max power :

$$P_{max} = \frac{V_{peak}^2}{8R_S} = \frac{1.2^2}{8 \times 5} = 0.036\,W = 36\,mW$$

$$P_{max} \text{ in dBm} = 10\log\left(\frac{P_{mW}}{1\,mW}\right) = 10\log\frac{36}{1} = \boxed{15.56\,dB_m}$$

$L_1, L_2, \& C$:



from (D) $\quad Q = \omega_0 R_L C \Rightarrow C = \frac{Q}{\omega_0 R_L} = \boxed{13.26\,pF}$

$$Q = \frac{R_L}{\omega_0 L_p} \Rightarrow L_p = \frac{R_L}{\omega_0 Q} = 331.5\,pH$$

(D)→(C) $\quad R_{1S} = \frac{R_L}{Q^2+1}$, (i)

$$L_S = L_p \cdot \frac{Q^2}{Q^2+1}$$

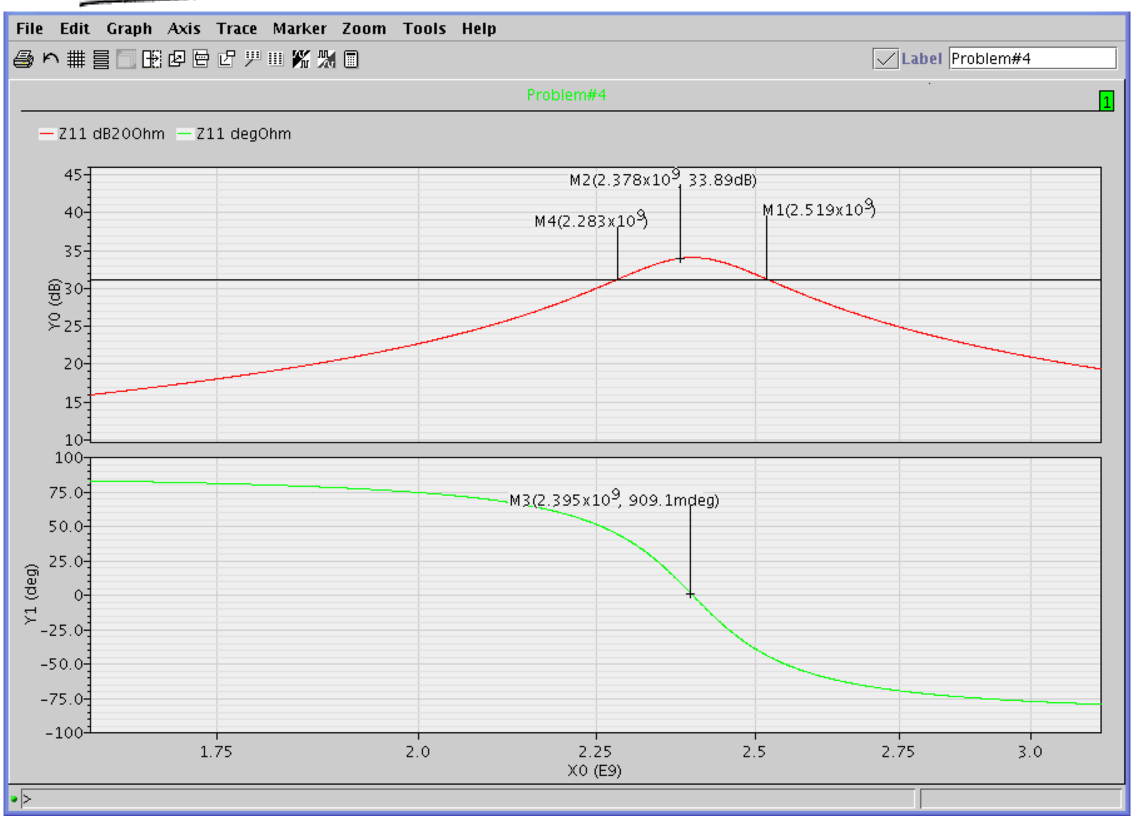
(A)→(B) $\quad R_{1S} = \frac{R_1}{Q_1^2+1}$ (ii) from (i), (ii) $\Rightarrow$ $Q_1 = \sqrt{\frac{R_1}{R_L}(Q^2+1) - 1} = 3.01$

(A) $Q_1 = \frac{R_1}{\omega_0 L_1} \Rightarrow L_1 = \frac{R_1}{\omega_0 Q_1} = \boxed{110\,pH}$

$L_2 = L_S - L_{1S}$

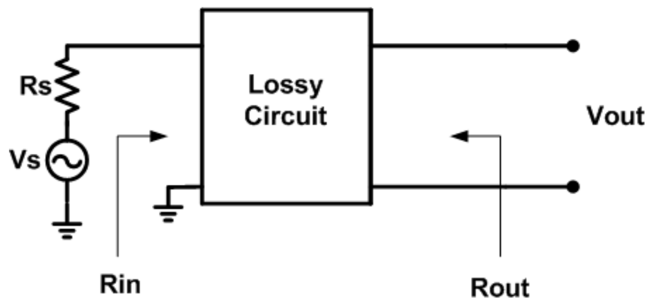
$= L_P \frac{Q^2}{1+Q^2} - L_1 \frac{Q_1^2}{1+Q_1^2}$

$\boxed{L_2 = 235\,pH}$



Q3. *Noise Figure of Lossy Circuits:*
Consider the passive reciprocal lossy circuit shown below. Like the concept of available power gain discussed in the class, the power loss of this circuit can be defined as $L = P_{s,av}/P_{out,av}$, where $P_{s,av}$ is available source power and $P_{out,av}$ is the available power at the output. $P_{s,av}$ is given by $V_s^2/4R_s$, and $P_{out,av}$ is given by $V_{TH}^2/4R_{out}$, V_{TH} and R_{out} being the Thevenin voltage and resistance at the output port, respectively. It can be shown that the noise factor of this passive reciprocal network is same as L.

$F_{lossy} = L = (V_s^2/V_{TH}^2)*(R_{out}/R_s)$

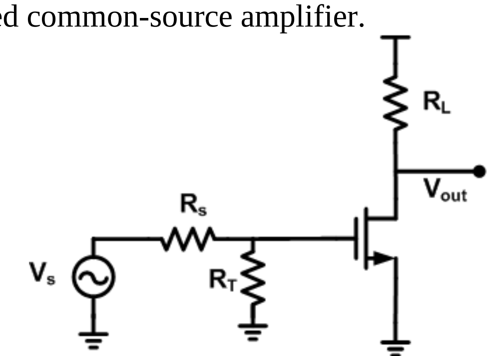


An example of such a lossy network is the bandpass filter that sits in between the antenna and the LNA. Modeling the antenna as $\{V_s, R_s\}$ and the lossy filter with a power loss of L, and connecting an LNA to the output of this filter, what will be the total noise factor of the system? Assume that the noise factor of the LNA, calculated with respect to the R_{out} of the filter, is F_{LNA}. [2]

$\boldsymbol{F_{total} = F_{filter} + (F_{LNA}\text{-}1)/L^{-1} = L + (F_{LNA}\text{-}1)L = L*F_{LNA}}$
Thus, overall noise figure, $\boldsymbol{NF_{total} = L_{(dB)} + NF_{LNA}}$

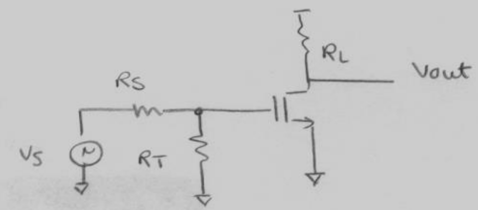
Q4. *Noise Factor calculation for amplifiers:*
For the amplifiers shown below, ignore channel-length modulation, any parasitic capacitance, and gate-induced noise. All the transistors are operating in the saturated region. (Biasing details are not shown). Z_{in} is the input resistance, defined looking into the core-amplifier (excluding R_s and V_s).

(a) Resistive-terminated common-source amplifier.

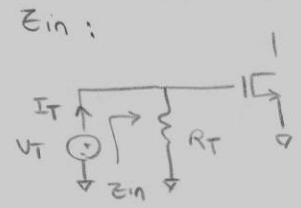


Calculate the noise factor (F) and Z_{in}. [4+2]
Further simplify this expression assuming that $Z_{in} = R_s$ for matching purposes. [1]

Q3) part a)

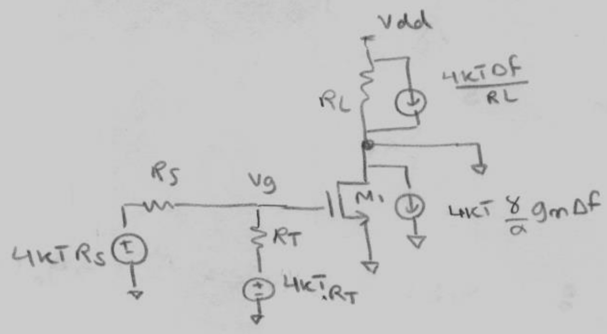


Zin :



$$\boxed{Z_{in} = \frac{V_T}{I_T} = R_T}$$

NF :



① R_S : $\left(\frac{R_T}{R_S+R_T}\right)^2 \overline{V_{RS}^2}\, g_m^2 = \left(\frac{R_T}{R_S+R_T}\right)^2 \times (4kTR_S\Delta f)\, g_m^2$

② R_T : $\left(\frac{R_S}{R_S+R_T}\right)^2 \overline{V_{RT}^2}\, g_m^2 = \left(\frac{R_S}{R_S+R_T}\right)^2 \times (4kTR_T\Delta f)\, g_m^2$

③ M_1 : $4kT\frac{\gamma}{\alpha} g_m \Delta f$

④ R_L : $\frac{4kT\Delta f}{R_L}$

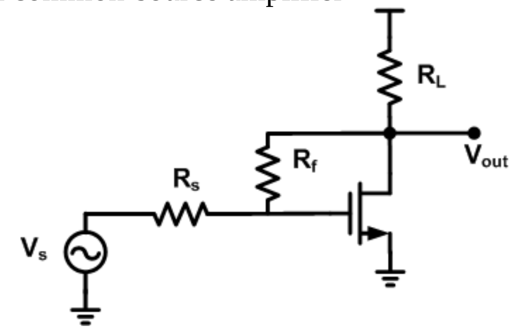
$$NF = \frac{① + ② + ③ + ④}{①} = 1 + \frac{\left(\frac{R_S}{R_S+R_T}\right)^2 (4kTR_T\Delta f g_m^2) + \frac{4kT\Delta f}{R_L} + 4kT\frac{\gamma}{\alpha} g_m \Delta f}{\left(\frac{R_T}{R_S+R_T}\right)^2 \times (4kTR_S\Delta f g_m^2)}$$

$$NF = 1 + \frac{R_S}{R_T} + \frac{(R_S+R_T)^2}{R_T^2} \times \frac{1}{R_S R_L g_m^2} + \frac{(R_S+R_T)^2}{R_T^2} \times \frac{1}{R_S g_m} \times \frac{\gamma}{\alpha}$$

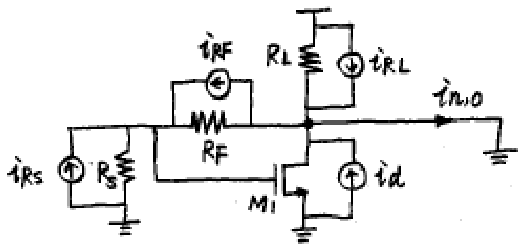
Therefore, for $R_s = R_T$, $F = 2+4/(g_m*R_s)[\,1/R_L+\gamma/\alpha]$

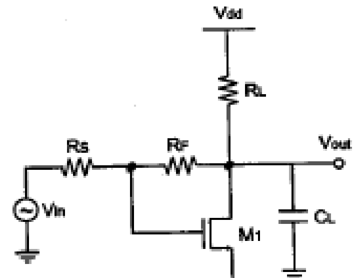
Note that $g_m = g_{d0}$ for a long-channel NMOS only. For a short channel device, another factor α is defined as the ratio of g_m and g_{d0}. Hence, the drain current noise expression incorporates α for accuracy.

(b) Resistive-feedback common-source amplifier



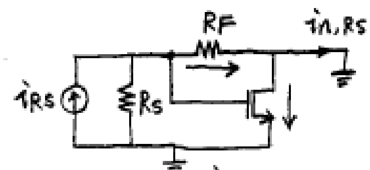
Calculate the noise factor (F) and Z_{in}. [5+2]



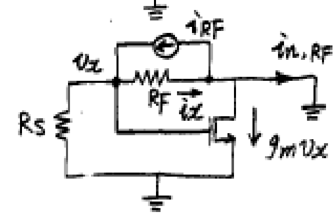


There are four noise sources:

$$\overline{i_{RS}^2} = \frac{4kT}{R_S}\Delta f,\quad \overline{i_{RF}^2} = \frac{4kT}{R_F}\Delta f,\quad \overline{i_{RL}^2} = \frac{4kT}{R_L}\Delta f,\quad \overline{i_d^2} = 4kT\gamma g_{do}\Delta f$$



- $$\overline{i_{n,RS}^2} = \overline{i_{RS}^2}\cdot\left(\frac{R_F R_S}{R_F+R_S}\right)^2\cdot\left(\frac{1}{R_F}-g_m\right)^2$$



- $\overline{i_{n,RF}^2}$:

$$v_x = i_{RF}\cdot\left(\frac{R_F R_S}{R_F+R_S}\right)$$

$$i_x = i_{RF}\cdot\left(\frac{R_S}{R_S+R_F}\right)$$

$$\begin{aligned} i_{n,RF} &= i_x - i_{RF} - g_m v_x \\ &= i_{RF}\cdot\frac{R_S}{R_S+R_F} - i_{RF} - g_m\cdot i_{RF}\cdot\frac{R_F R_S}{R_F+R_S} \\ &= -i_{RF}\cdot\frac{R_F}{R_F+R_S}\cdot(1+g_m R_S) \end{aligned}$$

$$\overline{i_{n,RF}^2} = \overline{i_{RF}^2}\cdot\left(\frac{R_F}{R_F+R_S}\right)^2(1+g_m R_S)^2$$

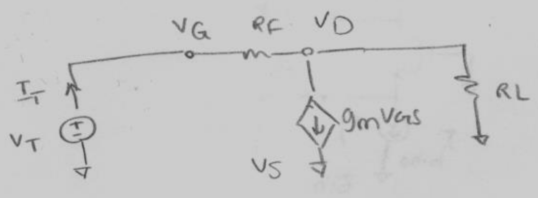
- $$\overline{i_{n,RL}^2} = \frac{4kT}{R_L}\Delta f$$

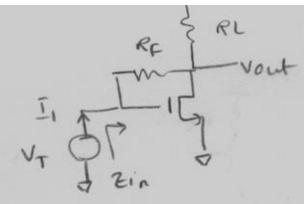
- $$\overline{i_{n,d}^2} = 4kT\gamma g_{do}\Delta f$$

$$\begin{aligned} F &= 1 + \frac{\overline{i_{n,RF}^2} + \overline{i_{n,RL}^2} + \overline{i_{n,d}^2}}{\overline{i_{n,RS}^2}} \\ &= 1 + \frac{\frac{4kT}{R_F}\Delta f\left(\frac{R_F}{R_F+R_S}\right)^2(1+g_m R_S)^2 + \frac{4kT}{R_L}\Delta f + 4kT\gamma g_{do}\Delta f}{\frac{4kT}{R_S}\Delta f\cdot\left(\frac{R_F R_S}{R_F+R_S}\right)^2\cdot\left(g_m-\frac{1}{R_F}\right)^2} \end{aligned}$$

$$\Rightarrow \boxed{F = 1 + \frac{R_F}{R_S}\cdot\frac{(1+g_m R_S)^2}{(1-g_m R_F)^2} + \frac{1}{R_S}\left(\frac{1}{R_L}+\gamma g_{do}\right)\cdot\frac{(R_F+R_S)^2}{(1-g_m R_F)^2}}$$

Q3) b) Zin:





at Drain: (KCL)

$$I_T = g_m V_{GS} + \frac{V_D}{R_L} \Rightarrow I_T = g_m V_T + \frac{V_D}{R_L} \quad (i)$$

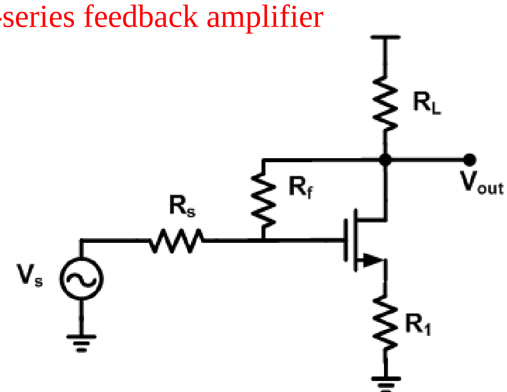
KVL: $V_T = R_F I_T + V_D$ (ii)

(i) in (ii) $\Rightarrow$ $V_T = R_F I_T + R_L I_T - g_m R_L V_T$

$$\Rightarrow (1+g_m R_L) V_T = (R_F + R_L) I_T$$

$$\boxed{\frac{V_T}{I_T} = Z_{in} = \frac{R_F + R_L}{1 + g_m R_L}}$$

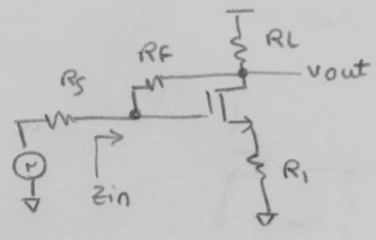
(c) [571F only] Shunt-series feedback amplifier



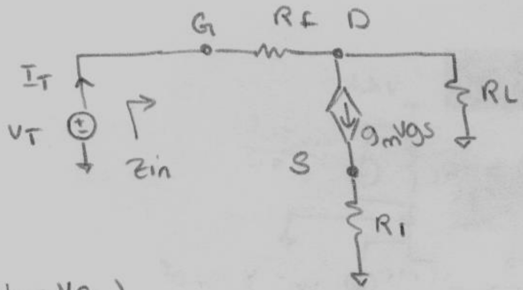
Calculate Zin. [3]

Comment on the noise factor of this amplifier compared to that of (b), if $R_1 >> 1/g_m$. Do not bother to solve for F. [6]

Q3) Part C :



Zin :



$(v_T = V_G)$

at source : kcl: $g_m v_{gs} = \frac{v_S}{R_1} \Rightarrow g_m(v_T - v_S) = \frac{v_S}{R_1}$

$$v_S = \frac{g_m R_1}{1 + g_m R_1} v_T \quad (i)$$

at Drain : kcl :

$$I_T = \frac{v_D}{R_L} + g_m v_{gs} \quad (ii)$$

kvl . $v_T = R_f I_T + v_D \Rightarrow v_D = v_T - R_f I_T \quad (iii)$

(i) & (iii) in (ii) $\Rightarrow$

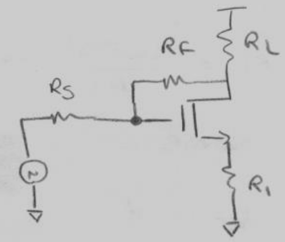
$$I_T = \frac{v_T - R_f I_T}{R_L} + g_m \left(v_T - \frac{g_m R_1 v_T}{1 + g_m R_1}\right)$$

$$\left(1 + \frac{R_f}{R_L}\right) I_T = \left(\frac{1}{R_L} + \frac{g_m}{1 + g_m R_1}\right) v_T$$

$$Z_{in} = \frac{v_T}{I_T} = \frac{\frac{(R_L + R_f)}{R_L}}{1 + g_m R_1 + g_m R_L} = \boxed{\frac{(1 + g_m R_1)(R_L + R_F)}{1 + g_m(R_L + R_1)}}$$

Q3) Test C)

NF: (You didn't need to solve NF)

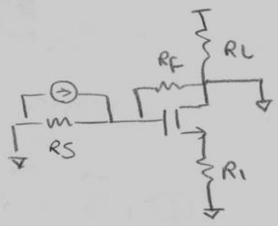


$$\left(G_m = \frac{g_m}{1+g_m R_1} \right)$$

① RS:

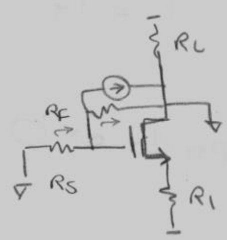
$$\left(\frac{R_F R_S}{R_F+R_S}\right)^2 \bar{I}^2_{RS,n} \left(\frac{1}{R_F} - G_m\right)^2$$

$$= \bar{I}^2_{RS,n} \left(\frac{R_S}{R_F+R_S}\right)^2 (1 - G_m R_F)^2$$



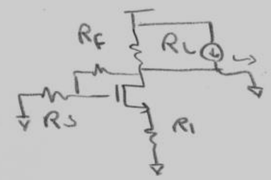
② RF:

$$\left(\frac{R_F}{R_S+R_F}\right)^2 (1 + G_m R_S)^2$$



③ RL:

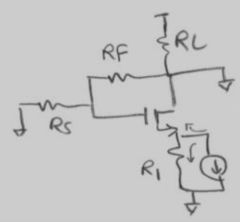
$$\frac{4kT}{R_L} \Delta f$$



④ R1:

$$\left(\frac{R_1}{R_1+\frac{1}{g_m}}\right)^2 \bar{I}^2_{R_1,n} = \left(\frac{g_m}{R_1 g_m+1}\right)^2 I^2_{R_1,n}$$

$$= (G_m R_1)^2 I^2_{R_1,n}$$



⑤ M1:

$$\left(\frac{1/g_m}{1/g_m+R_1}\right)^2 \bar{I}^2_{d,n} = \left(\frac{1}{1+g_m R_1}\right)^2 I_d^2$$

$$NF = \frac{①+②+③+④+⑤}{①}$$

Compared to 3(b), for $g_m R_1 \gg 1$

(5) has decreased.

(4) $\approx \overline{i^2_{R_1}} = \frac{4kT\Delta f}{R_1}$ is small.

(3) is same.

(2) is decreased.

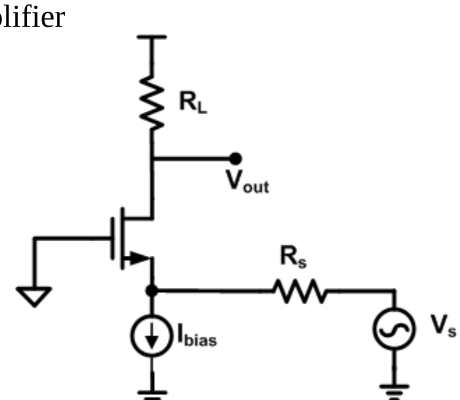
(1) depends on the product of $G_m R_F$ – how big/small/compa

it is with respect to 1.

$$F = 1 + \frac{\overset{\downarrow}{(2)} + \overset{\leftrightarrow}{(3)} + \overset{\approx 0}{(4)} + \overset{\downarrow}{(5)}}{(1)} \; ?$$

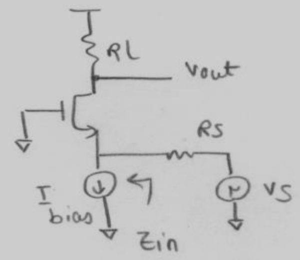
Depends on (1), hard to say without knowing R_F & R_s.

(d) Common-gate amplifier

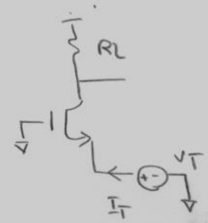


Calculate the noise factor (F) and Z_{in}. [4+1]
Further simplify this expression assuming that $Z_{in} = R_s$ for matching purposes.

Q3) part d:

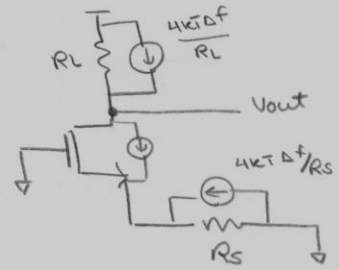


Zin:



$$Z_{in} = \frac{V_T}{I_T} = \boxed{\frac{1}{g_m}}$$

NF:



① R_S:

$$\left(\frac{R_S}{R_S+\frac{1}{g_m}}\right)^2 \overline{I^2_{n R_S}} = \left(\frac{R_S}{R_S+\frac{1}{g_m}}\right)^2 \frac{4kT}{R_S}\Delta f$$

② R_L:

$$\overline{I^2_{n R_L}} = \frac{4kT\Delta f}{R_L}$$

③ M_1:

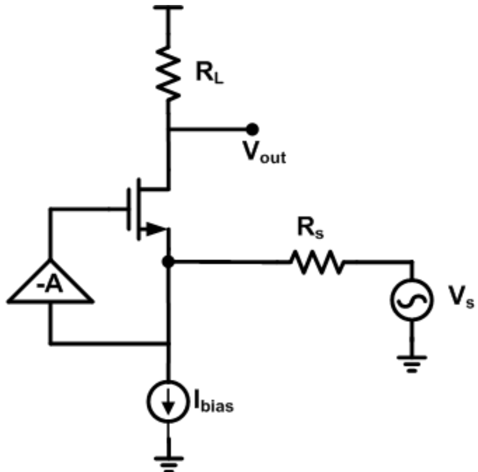
$$\left(\frac{1/g_m}{1/g_m+R_S}\right)^2 \overline{I^2_{nd}} = \left(\frac{1/g_m}{1/g_m+R_S}\right)^2 \times 4kT\frac{8}{\alpha}g_m\Delta f$$

$$NF = \frac{① + ② + ③}{①} = \boxed{1 + \frac{8}{\alpha}\left(\frac{1}{g_m R_S}\right) + \frac{\left(R_S+\frac{1}{g_m}\right)^2}{R_S R_L}}$$

if $R_S = \frac{1}{g_m}$

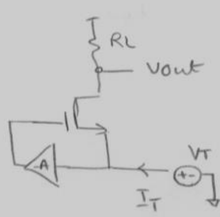
$$\boxed{\Rightarrow NF = 1 + \frac{8}{\alpha} + \frac{4}{g_m R_L}}$$

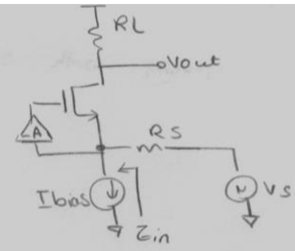
(e) [571F only] Common-gate amplifier with g_m-boosting



Calculate the noise factor (F) and Z_{in}. A is an ideal voltage gain block. [4+2]
Further simplify this expression assuming that $Z_{in} = R_s$ for matching purposes. [1]

Zin :

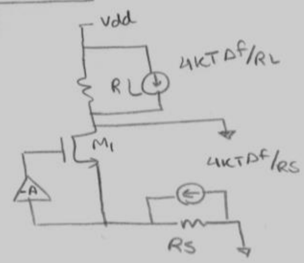




$$\left.\begin{array}{l} I_T = -g_m v_{gs} \\ V_g = -A v_S \\ v_S = V_T \end{array}\right\} \Rightarrow I_T = -g_m(-A-1)V_T$$

$$\Rightarrow \boxed{Z_{in} = \frac{V_T}{I_T} = \frac{1}{g_m(A+1)}}$$

NF :



① RS : $\left(\frac{R_S}{R_S + \frac{1}{g_m(A+1)}}\right)^2 \cdot \overline{i^2_{n,RS}}$

② RL : $\overline{i^2_{n,RL}}$

③ M1 : $\left(\frac{\frac{1}{g_m(A+1)}}{\frac{1}{g_m(A+1)} + R_S}\right)^2 \cdot \overline{I^2_{nd}} = \frac{\left(\frac{1}{g_m(1+A)}\right)^2}{\left(R_S + \frac{1}{g_m(1+A)}\right)^2} \times 4KT\frac{8}{\alpha}\Delta f\, g_m$

$$NF = \frac{① + ② + ③}{①} = \boxed{1 + \frac{\left(R_S + \frac{1}{g_m(A+1)}\right)^2}{R_L R_S} + \frac{1}{R_S \cdot g_m(1+A)^2}\,\frac{8}{\alpha}}$$

if $R_S = \frac{1}{g_m(1+A)} = Z_{in}$,

$$\boxed{NF = 1 + \frac{8}{\alpha(1+A)} + \frac{4}{R_L \cdot g_m(1+A)}}$$

Q5. Hand-calculations for transistor parameters using the square law I-V equations are good for intution, but are not very accurate. Given a CMOS process and spice simulation, one can use current density as an important parameter for hand-calculation/graphical design. g_m and g_{ds} of a transistor is primarily dependent on I_o (drain bias current) and W (the width of the transistor) → g_m (I_o, W), gds (I_o, W). Length of the transitor (L) is considerd fixed at $L_{minimum}$. Current density is then defined as $I_{den} = I_o/W$.

For the given process in this course, plot g_m (I_o, W), gds (I_o, W) and Vgs(I_o, W) of an NMOS transistor as a function of the current density. [6]

One easy way to generate a set of the above curves in Cadence is to bias a diode-connected NMOS device with an ideal drain current source of I_o. Next, set W = 1um, L=Lmin and sweep I_o to generate the curves. Use nmos_1v transistor in the library.