# ELEC 404 RFIC Design
**University of British Columbia**
**April 2018**
Final Examination
Thursday, April 19th, 2018

***Honor Code:***

*The student agrees to not give or receive aid in the exam. This includes – copying from another's exam paper or allowing another to copy from one's own paper, unpermitted collaboration and plagiarism. The student also agrees to take an active part in seeing to it that other students as well uphold the spirit and letter of this honor code.*
*Breach of the honor code will lead to disciplinary measures as dictated by UBC rules.*
*The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring this exam and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above.*

Square Wave Fourier Series : $f(t) = \frac{4}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin(n\omega t)$

Trignometric Identities :

$$\cos(A)\cos)B) = \frac{1}{2}(\cos(A+B)+\cos(A-B))$$

$$\sin(A)\cos(B) = \frac{1}{2}(\sin(A+B)+\sin(A-B))$$

$$\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B)-\cos(A+B))$$

$$\int\frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

**Instructions:**

Each questions carries 5 marks.

| Problem | Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

Student ID #: ______________________________

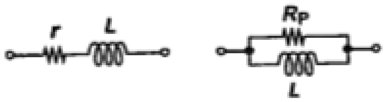
Signature:

1. For each of the question below, choose the most appropriate answer. You do not need to show how you reached the correct answer. [1+1+1+1+1 marks]

I. The loss of an inductor $L$ is modeled with a series resistance of $r$. If this series $r$-$L$ circuit is transformed to an equivalent parallel circuit of $R_p||L$ at a given frequency $\omega$, and assuming $Q>>1$,

(a) $R_p \alpha r$
[✓ (b) $R_p \alpha r^{-1}$]
(c) $R_p \alpha r^{-2}$
(d) $R_p$ is independent of r.

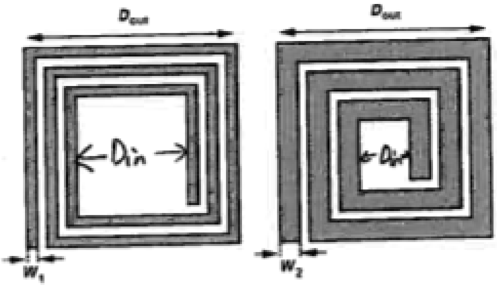
$$R_p = (1+Q^2)r$$
$$\simeq Q^2 r = \left(\frac{\omega L}{r}\right)^2 r$$
$$= \frac{\omega^2 L^2}{r}$$



II. Consider the spiral inductor shown below. If the line width is doubled to reduce its resistance, while the other parameters (outer dimension, line spacing, and number of turns) stay the same, what happens to the total inductance, $L_{tot}$? Recall that the total inductance $L_{tot}$ is the sum of individual turn-inductances and mutual-inductances between different turns.

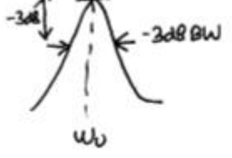
[✓ (a)] $L_{tot}$ decreases because some individual turn-inductance decreases as well as mutual-inductance decreases.
(b) $L_{tot}$ increases because some individual turn-inductance increases as well as mutual-inductance increases.
(c) $L_{tot}$ decreases because some individual turn-inductance decreases, even though mutual-inductance increases.
(d) $L_{tot}$ increases because some individual turn-inductance increases, even though mutual-inductance decreases.

Din decreases
⇒ inductance of inner turns decrease.
Spacing between legs increases
⇒ mutual inductance decreases



III. An 802.11a LNA must achieve a -3dB BW from 5GHz to 6GHz. If the LNA incorporates a parallel LC tank as its load, what is the maximum allowable tank $Q$?

(a) 11
[✓ (b) 5.5]
(c) 3 ~ 5
(d) ∞



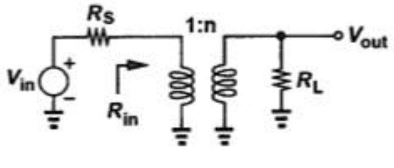
$$Q = \frac{\omega_0}{3dB\,BW} = \frac{5.5\,GHz}{1\,GHz} = 5.5$$

IV. An ideal step-up transformer shown below has a turns ratio (primary:secondary) of 1:n, and "amplifies" the voltage across its primary by a factor of n (secondary voltage = n x primary voltage). Assuming that an ideal transformer does not result in any power loss,

(a) $R_{in} = n^2R_L$
[✓ (b) $R_{in} = R_L/n^2$]
(c) $R_{in} = nR_L$
(d) $R_{in}$ is independent of n.

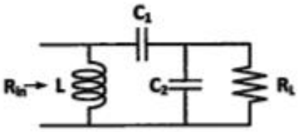
$$\frac{v_p^2}{R_{in}} = \frac{v_s^2}{R_L} = \frac{(n v_p)^2}{R_L}$$

$$\Rightarrow \quad R_{in} = \frac{R_L}{n^2}.$$



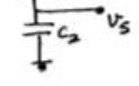
V. The figure below shows a tapped capacitor matching network which transforms a resistance $R_L$ to $R_{in}$ at a frequency $\omega_0$. At $\omega_0$, the transformed capacitance resonates completely with the inductor $L$. Assuming $\omega_0$ is sufficiently high, and $n = 1 + C_2/C_1$,

[✓ (a) $R_{in} \cong n^2R_L$]
(b) $R_{in} \cong R_L/n^2$
(c) $R_{in} \cong nR_L$
(d) $R_{in}$ is independent of n.



Approx. Method

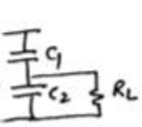
Although not perfectly lossless, the tapped capacitor is transforming the voltage



$$\frac{v_s}{v_p} \simeq \left(\frac{1/sC_2}{1/sC_1 + 1/sC_2}\right) = \frac{C_1}{C_1+C_2} = 1/n \Rightarrow v_s = \frac{v_p}{n}$$

$$\Rightarrow n:1 \text{ xfmr action } \Rightarrow R_{in} = n^2 R_L$$

Method 2 (Shown in the class)

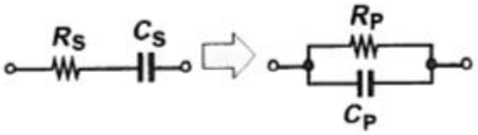


$$Z_{in} = \frac{1}{sC_1} + \frac{R_L}{1+sC_2R_L}$$

$$Re\{Y_{in}\} = \frac{\omega^2 R_L C_1^2}{1+\omega^2 R_L^2 (C_1+C_2)^2} \simeq \frac{1}{R_L}\left(\frac{C_1}{C_1+C_2}\right. = \frac{1}{n^2 R_L}$$

2. (a) The series network shown below is transformed to an equivalent parallel network at a given frequency, $\omega$. Prove that the quality factor of the series network $Q_s$ is the same as that of the parallel network, $Q_p$, i.e., $Q_s = Q_p$. [3 marks]

Equating the impedances @ $\omega$ (s)



$$R_s + \frac{1}{sC_s} = \frac{R_p \cdot \frac{1}{sC_p}}{R_p + \frac{1}{sC_p}}$$

$$\Rightarrow \quad \frac{1 + sR_sC_s}{sC_s} = \frac{R_p}{1 + sR_pC_p}$$

$$\Rightarrow \quad 1 + s\,\overline{R_sC_s + R_pC_p} + s^2 R_sC_sR_pC_p = sR_pC_s$$

$$\Rightarrow \quad 1 - w^2 R_sC_sR_pC_p + jw\,\overline{R_sC_s + R_pC_p - R_pC_s} = 0$$

Equating <u>real</u> and imaginary parts,

$$1 - w^2 R_sC_sR_pC_p = 0 \quad \Rightarrow \quad w^2 R_sC_sR_pC_p = 1$$

$$\Rightarrow \quad wR_pC_p = \frac{1}{wR_sC_s}$$

$$\Rightarrow \quad \frac{R_p}{1/wC_p} = \frac{1/wC_s}{R_s}$$

$$\Rightarrow \quad Q_p = Q_s .$$

Q.E.D

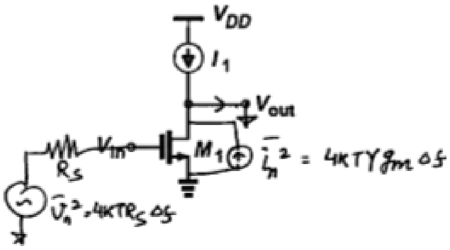
**(b) Why are S-parameters more popular compared to conventional Z or Y parameters in RFIC and microwave design? [2 marks]**

At high frequency → difficult to provide open-circuit or short-circuit, especially over a broad frequency range

→ active circuits may not work accurately if terminated into open/short.

Thus Z/Y parameters not so useful.

3. Determine the noise factor of the common-source amplifier shown below with respect to the source impedance $R_S$. Neglect the capacitances, flicker noise and gate-induced noise, channel-length modulation and body effect. Assume $I_1$ is an ideal-current source.
[3 marks]



Both $r_o$ of M1 and I1 $\rightarrow \infty$

Output noise current due to

$i_n \rightarrow i_n$ ①

$v_n \rightarrow g_m v_n$. ②

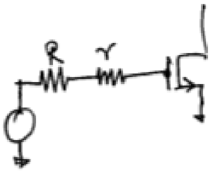
$\overline{i_{n,out\,1}^2} = \overline{i_n^2} = 4KT\gamma g_m \Delta f$ (I)

$\overline{i_{n,out\,2}^2} = g_m^2\, 4KTR_s \Delta f$. (II)

$$F = \frac{(I)+(II)}{(II)} = \frac{\gamma g_m + g_m^2 R_s}{g_m^2 R_s} = 1 + \frac{\gamma}{g_m R_s} \;//$$

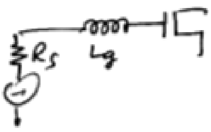
**Now assume that there is a parasitic resistance *r* due to improper layout of M₁, routing loss, etc. that appears in series with the source impedance *Rₛ*. What is the new noise factor with respect to the source impedance *Rₛ*? [2 marks]**

$\overline{i_{n\,out3}^2} = g_m^2\, 4KT r \Delta f$ (III)



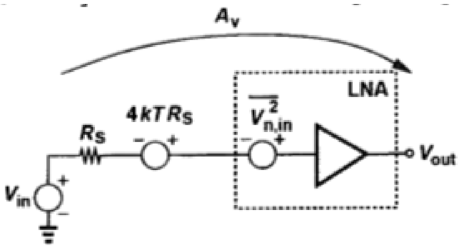
$$F = \frac{(I)+(II)+(III)}{(II)} = 1 + \frac{\gamma}{g_m R_s} + \frac{r}{R_s}.$$

Note: In an inductive-degenerated CSLNA with $L_g$, any series resistance of $L_g$ degrades NF.



4. (a) An LNA circuit exhibits an NF = 3dB. What % of the output noise power is due to the source resistance, $R_s$? [$\log_{10}2 = 0.3$] [3 marks]

$$F = 1 + \frac{\overline{v_{n,in}^2}}{4KTR_s}$$



$NF = 10\log_{10}F$

$\Rightarrow 3dB = 10\log_{10}2 = 10\log_{10}F$

$\Rightarrow F = 2$

$\Rightarrow \overline{v_{n,in}^2} = 4KTR_s.$

50%.

(b) Several optimizations of the LNA reduce the NF = 1dB. Repeat the above problem. Does it make sense to further optimize the LNA with respect to its noise contribution? [$\log_{10}1.25 \approx 0.1$] [2 marks]

$1dB = 10\log_{10}(1.25) = 10\log_{10}F$

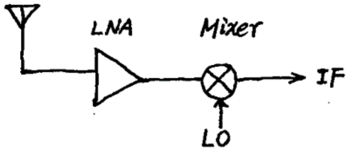
$\Rightarrow F = 1.25 = \frac{5}{4} = 1 + \frac{\overline{v_{n,in}^2}}{4KTR_s}$

$\Rightarrow \overline{v_{n,in}^2} = \frac{1}{4} \cdot 4KTR_s$

% noise power due to $R_s$ = $\frac{1}{1+\frac{1}{4}} \times 100 = 80\%$

Most of the noise from $R_s$ $\Rightarrow$ further optimizing the LNA will bring diminishing returns

5. Calculate the overall noise figure and IIP3 of the RX front-end shown below, given the parameters for the LNA and the Mixer. List any assumptions.



| | | |
|---|---|---|
| $A_p =$ | 18dB | 20dB |
| $NF =$ | 2.5dB | 12dB |
| $IIP3 =$ | −15dBm | +5dBm |

Assuming impedances are matched,

$F_1 = 1.78$ ; $F_2 = 15.85$ ; $A_{p_1} = 63.1$ ;

$$F_{tot.} = F_1 + \frac{F_2 - 1}{A_{p_1}} = 1.78 + \frac{14.85}{63.1} = 2.02$$

$$NF_{tot.} = 10 \log F_{tot.} = 3.04 \text{ dB}$$

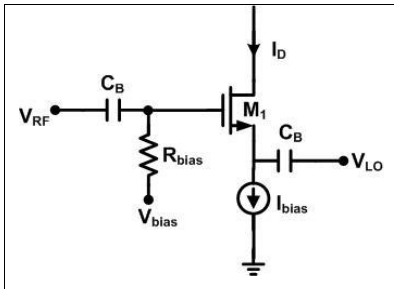
$$\frac{1}{IIP_{3_{tot}}^2} = \frac{1}{IIP_{3_1}^2} + \frac{A_p}{IIP_{3,2}^2}$$ ; $IIP_{3_1}^2 = 3.16 \times 10^{-5}$

$IIP_{3_2}^2 = 3.16 \times 10^{-3}$

$$\frac{1}{IIP_{3,tot}^2} = \frac{1}{3.16 \times 10^{-5}} + \frac{63.1}{3.16 \times 10^{-3}} = 51613.92$$

$$IIP_{3,tot}^2 = 1.94 \times 10^{-5} = \underline{-17.12 \text{ dBm}}$$

6. The circuit shown below acts as a mixer, with the RF signal, $V_{RF}*\cos(\omega_{RF}t)$ and a DC bias voltage, $V_{bias}$, applied to the gate and the LO signal, $V_{LO}*\cos(\omega_{LO}t)$ applied to the source of the transistor M1. Assume M1 is a square-law device, $C_B$ is a DC blocking capacitor, $R_{bias}$ is a biasing resistor, and ignore the parasitics of M1.



(a) Calculate the amplitude of the upconverted & downconverted signal in $I_D$.

(b) Calculate the amplitude of LO feedthrough & RF feedthrough.

(c) Calculate the amplitude of the 2nd harmonic of LO & RF.

(d) Calculate the conversion gain of the Mixer

(e) Is this a good mixer? How will this mixer behave if $M_1$ is a short-channel device?

$$I_d = \frac{\beta}{2}\left(V_{gs}-V_T\right)^2 = \frac{\beta}{2}\left[\left(V_{bias}-V_T\right)+\left(V_{RF}\cos\omega_{RF}t - V_{LO}\cos\omega_{LO}t\right)\right]^2$$

$$= \frac{\beta}{2}\left[\left(V_{bias}-V_T\right)^2 + 2\left(V_{bias}-V_T\right)\left(V_{RF}\cos\omega_{RF}t - V_{LO}\cos\omega_{LO}t\right) + \left(V_{RF}\cos\omega_{RF}t - V_{LO}\cos\omega_{LO}t\right)^2\right]$$

$$= \frac{\beta}{2}\Big[\underbrace{\left(V_{bias}-V_T\right)^2}_{\text{DC bias current}} + 2\left(V_{bias}-V_T\right)\Big(\overset{\text{RF feedthrough}}{V_{RF}\cos\omega_{RF}t} - \overset{\text{lo feedthrough}}{V_{LO}\cos\omega_{LO}t}\Big)$$

$$+ \frac{V_{RF}^2}{2}\left(1+\overset{\text{RF harmonic}}{\cos 2\omega_{RF}t}\right) + \frac{V_{LO}^2}{2}\left(1+\overset{\text{LO harmonic}}{\cos 2\omega_{LO}t}\right)$$

$$- \underset{\text{downconverted}}{\underbrace{V_{RF}V_{LO}\cos\left(\omega_{RF}-\omega_{LO}\right)t}} - \underset{\text{upconverted}}{\underbrace{V_{RF}V_{LO}\cos\left(\omega_{RF}+\omega_{LO}\right)t}}\Big]$$

(a) Downconverted/Upconverted signal: $\frac{\beta}{2} V_{RF} V_{LO}$

(b) LO feedthrough: $- \beta (V_{bias} - V_T) V_{LO}$

RF feedthrough: $\beta (V_{bias} - V_T) V_{RF}$

(c) LO 2nd harmonic: $\frac{\beta}{4} V_{LO}^2$

RF 2nd harmonic: $\beta/4 \, V_{RF}^2$

(d) $G_C$ = $\underline{\text{Conversion gain}}$ = $\frac{\text{IF o/p amplitude}}{\text{RF i/p amplitude}}$ (assume down-conversion)

$$= \frac{\frac{\beta}{2} V_{RF} V_{LO}}{V_{RF}} = \frac{\beta}{2} V_{LO}$$

(e) No! LO/RF feedthroughs, harmonics, dc offsets.

A short-channel device is no longer a square law device, hence the desired mixing term ($\cos\omega_{RF}t \cdot \cos\omega_{LO}t$) has smaller magnitude.