Lecture 2:
Passive RLC Networks

ELEC 404

Acknowledgement: *RF Microelectronics*. B. Razavi

Passive RLC Networks

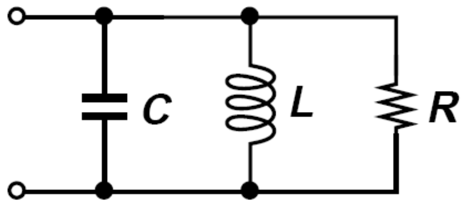
Applications of Resonant Networks

- Tuning
- Matching and Impedance Transformation

Examples

- Low Noise Amplifiers (LNAs)
- Voltage Controlled Oscillators (VCOs)
- Mixers
- Power Amplifiers (PAs)
- Band Pass Filters (BPFs)
- Duplexers
- Baluns

Parallel RLC Network



@low freq. @high freq. @ω_0

At resonance,

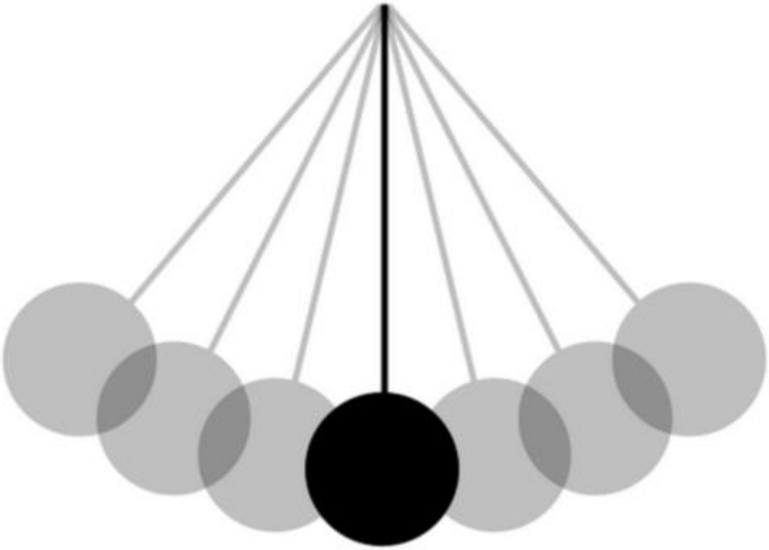
- **1pF & 1nH → 5GHz**
- **Remember the 2π!**

Quality Factor, Q

$$Q = \omega \frac{Energy\ Stored}{Average\ Power\ Dissipated}$$

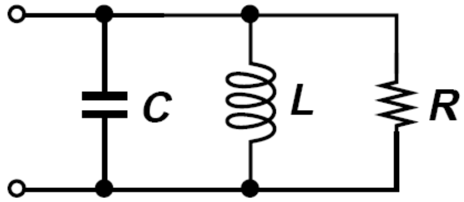
At resonance,

- Dimensionless
- Defined at both resonance and non-resonance



Q for a Parallel RLC Network

$$E_{Total} = E_{PK,L} = E_{PK,C} = E_{inst,L} + E_{inst,C}$$

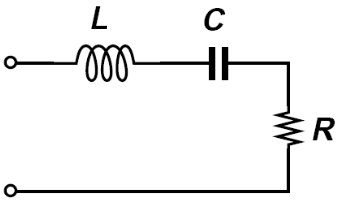


Current in L,C for a || RLC Network

At resonance,

- Large currents (Q↑) may flow
- Functionality and Reliability Concerns

Series RLC Network



@low freq. **@high freq.** **@ω_0**

$$Z(\omega) = R + j\omega L + \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C})$$

At resonance,

$$\omega_0 = \frac{1}{\sqrt{LC}}.\ |Z_L| = |Z_C| = \sqrt{\frac{L}{C}}$$

Expressions for *Q*:

Voltage across L,C for a Series RLC

At resonance,

- Large voltages (Q↑) may appear
- High Field → Breakdown → Functionality and Reliability Concerns
- Utilize smartly for voltage amplification – eg. in LNA

Maximum Power Transfer Theorem

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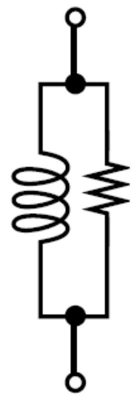
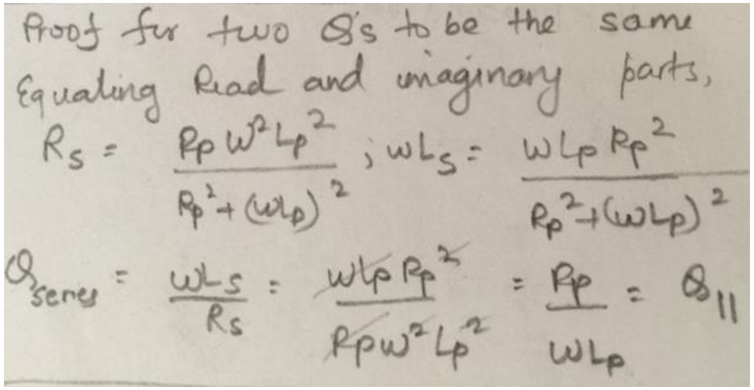
- If Z_L is constrained or specified already (say, 50Ω) insert an impedance transformer or a matching-network to match with Z_s.

Impedance Transformation





Proof for two Q's to be the same

Equating Real and imaginary parts,

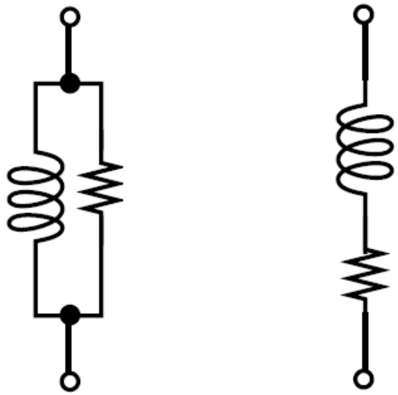
$R_s = \frac{R_p \omega^2 L_p^2}{R_p^2 + (\omega L_p)^2}$; $\omega L_s = \frac{\omega L_p R_p^2}{R_p^2 + (\omega L_p)^2}$

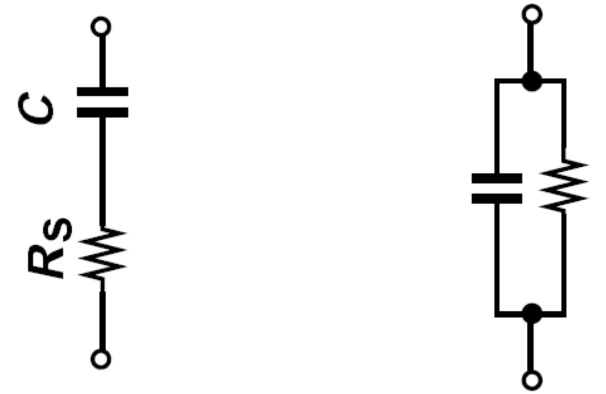
$Q_{series} = \frac{\omega L_s}{R_s} = \frac{\omega L_p R_p^2}{R_p \omega^2 L_p^2} = \frac{R_p}{\omega L_p} = Q_{||}$

- **Generic formula for narrowband transformation:**

$$R_p = R_s(1 + Q^2)$$
$$X_p = X_s(1 + Q^{-2})$$

Series/Parallel Transformation





$$R_p = R_s(1 + Q^2)$$
$$X_p = X_s(1 + Q^{-2})$$

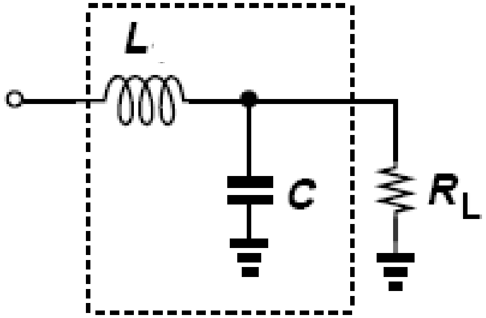
Impedance Transformation: L-Match

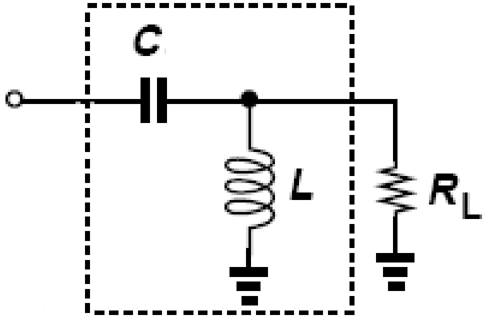
- **Upconvert 50Ω to 200Ω @ 5GHz:**

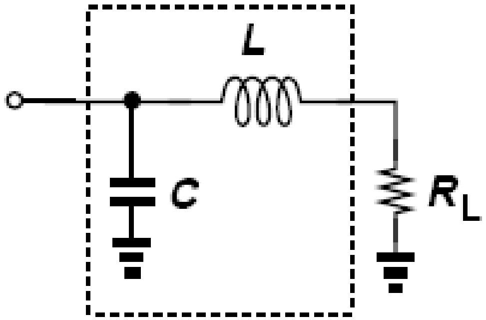
$$R_p = R_s(1 + Q^2)$$
$$X_p = X_s(1 + Q^{-2})$$

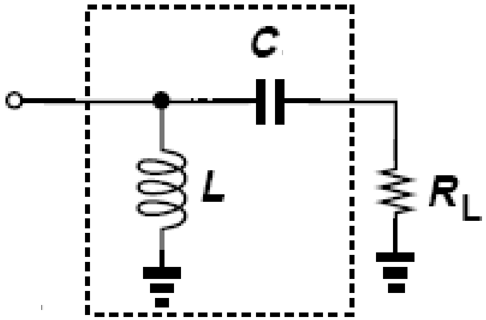
Choice of Matching Network

- Degrees of Freedom
- High-pass or low-pass
- Smallest area
- Parasitics
- Quality of the match (S_{11})









Tapped-Capacitor Match - I

- **Used in Colpitts Oscillator, LNA O/P Match**
- **3 degrees of freedom (η, ω_0, *Q*)**

$$Z_{2s} = \frac{R_2 \dfrac{1}{sC_2}}{R_2 + \dfrac{1}{sC_2}} = \frac{R_2}{1 + sC_2R_2}$$

$$Z_{in} = \frac{1}{sC_1} + \frac{R_2}{1 + sC_2R_2} = \frac{1 + sC_2R_2 + sC_1R_2}{sC_1 + s^2C_2C_1R_2} = \frac{1 + j\omega R_2(C_1 + C_2)}{j\omega C_1 - \omega^2 C_2C_1R_2}$$

$$Y_{in} = \frac{\omega^2 R_2 C_1^2}{1 + \omega^2 R_2^2 (C_1 + C_2)^2} + \frac{j\omega C_1[1 + \omega^2 C_2 R_2^2 (C_1 + C_2)]}{1 + \omega^2 R_2^2 (C_1 + C_2)^2}$$

- **At high frequency,**

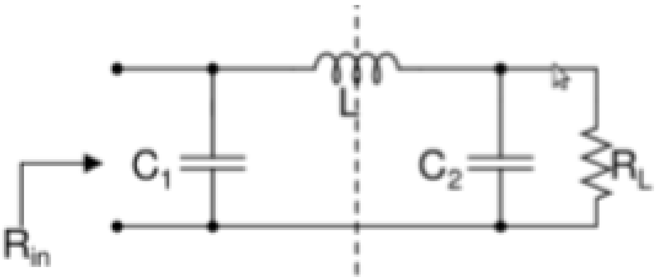
$$Y_{in} = \frac{1}{R_2}\left(\frac{C_1}{C_1 + C_2}\right)^2 + j\omega\frac{C_1C_2}{C_1 + C_2} = \frac{\eta^2}{R_2} + j\omega C_{series}$$

Tapped-Capacitor Match - II

Tapped-Inductor Match

- Used in Hartley Oscillator
- Uses two inductors

π-Match



- Analyze by splitting L into L_1 and L_2.
- Consider the network as two L-match networks, both transforming R_L and R_{in} to an intermediate resistance R_I
- $Q_{overall} = Q_{left} + Q_{right}$