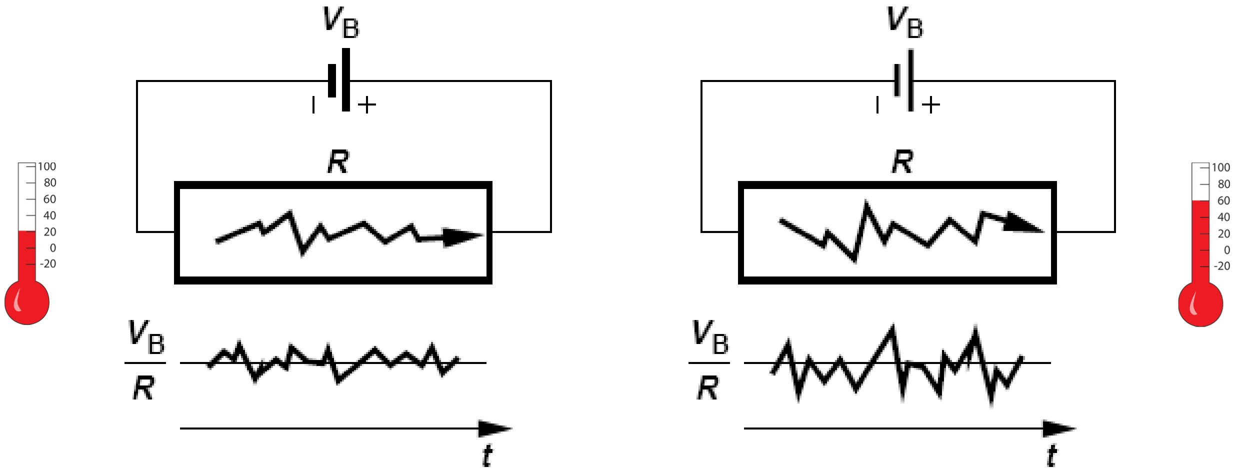
Integrated Circuits for Phase-Locked Loops

Lecture :

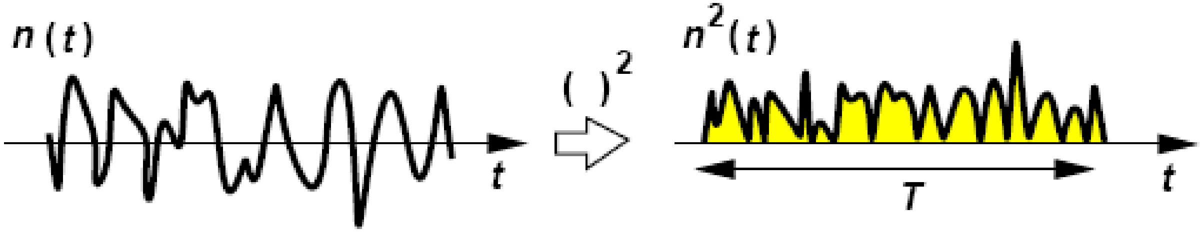
Device Noise

Adapted from *The Design of CMOS Radio-Frequency Integrated Circuits.* T. Lee. Copyright 2004 Cambridge.
Adapted from *RF Microelectronics*. B. Razavi. Copyright 2012 Prentice Hall

Noise – Random Process



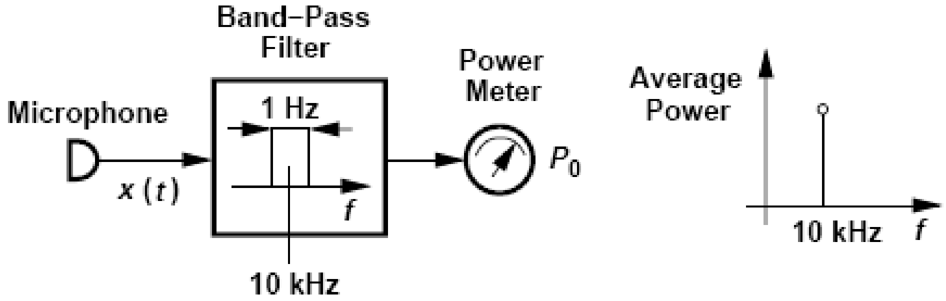
- **Avg. current = V_B/R but the instantaneous current displays random values**
- **Avg. Power of noise:**



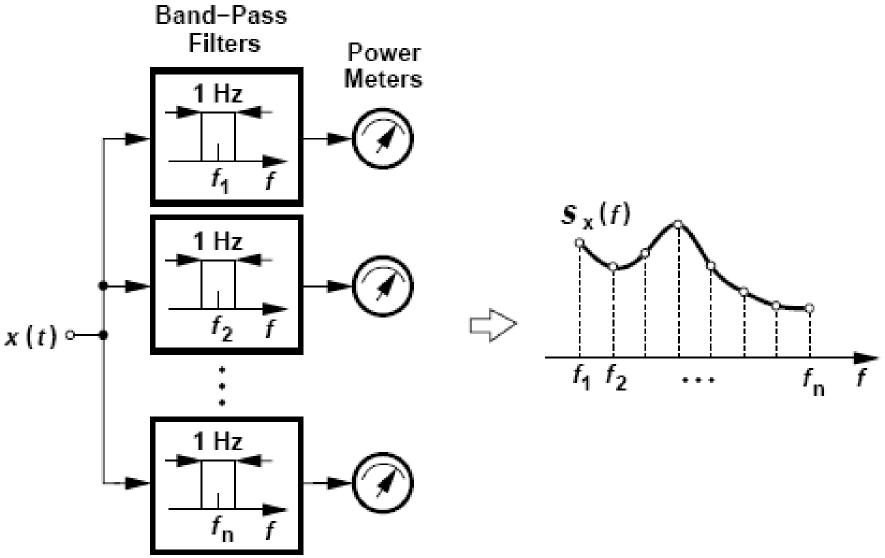
$$P_n = \lim_{T\to\infty} \frac{1}{T} \int_0^T n^2(t)\,dt$$

- ***T* must be long enough to accommodate several cycles of the lowest frequency.**

Measurement of Spectrum & PSD



- To measure frequency content at 10kHz, filter out the remainder of the spectrum and measure the average power of the 10kHz component.

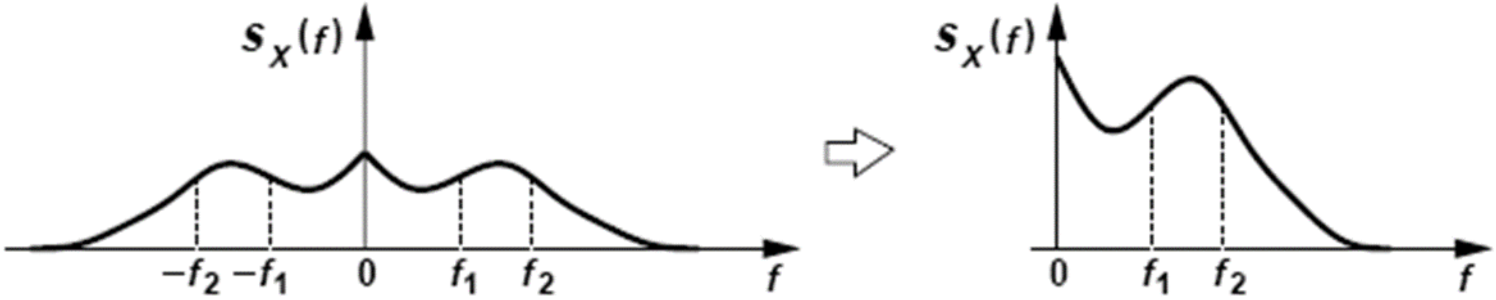


$$\int_0^{\infty} S_x(f)df = \lim_{T\to\infty} \frac{1}{T}\int_0^{T} x^2(t)dt$$

- Total area under $S_x(f)$ represents the average power carried by $x(t)$
- PSD is defined as the Fourier Transform of the autocorrelation of a signal

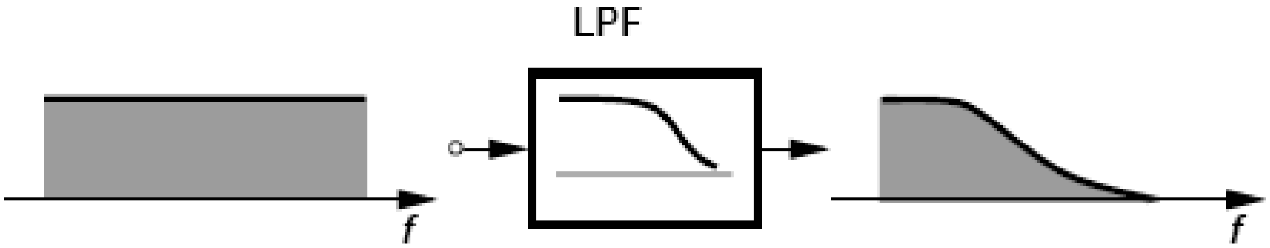
Two-sided/One-sided PSD

Two-Sided **One-Sided**



- Scale the single-sided spectrum by ½ to get the double-sided spectrum

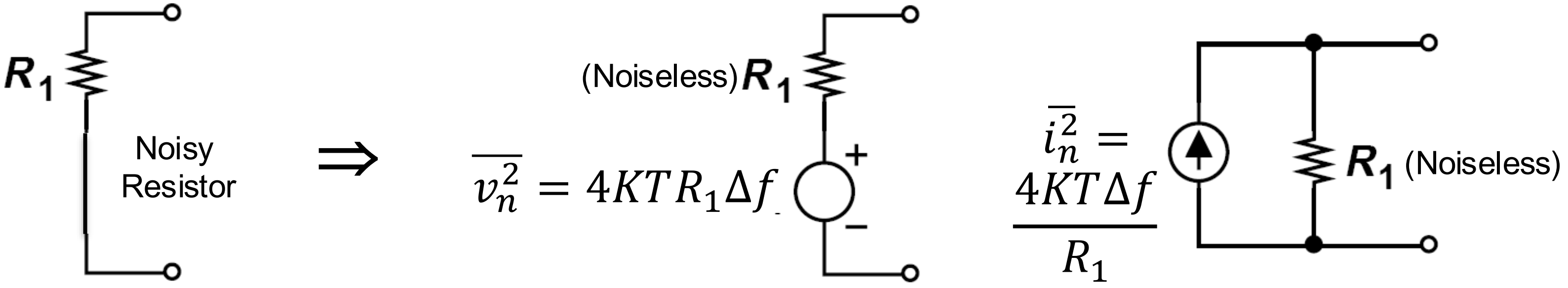
Noise and Transfer Function



$$S_y(f) = S_x(f)|H(f)|^2$$

- **$|H(f)|$ for the LTI system must be squared because $S_x(f)$ is a squared quantity**

Thermal Noise of Resistors



$$\overline{v_n^2} = 4KTR_1\Delta f$$

$$\overline{i_n^2} = \frac{4KT\Delta f}{R_1}$$

Δf= measurement BW

- **Mean-square noise voltage density**

$$\frac{\overline{v_n^2}}{\Delta f} = 4KTR_1 = 4\frac{KT}{q}qR_1 = 4(25mV)(1.6\times10^{-19})R_1 = (1.6\times10^{-20})R_1\,V^2/Hz$$

@ Room Temp.

- **RMS noise voltage density**

$$\frac{\overline{v_n}}{\sqrt{\Delta f}} = \sqrt{4KTR_1} = 0.9\,nV/\sqrt{Hz}$$

R_1 = 50Ω

- **Mean-square noise current density**

$$\frac{\overline{i_n^2}}{\Delta f} = \frac{\overline{v_n^2}}{R_1^2\Delta f} = 4\frac{KT}{R_1}A^2/Hz$$

- **RMS noise current density**

$$\frac{\overline{i_n}}{\sqrt{\Delta f}} = \sqrt{\frac{4KT}{R_1}}$$

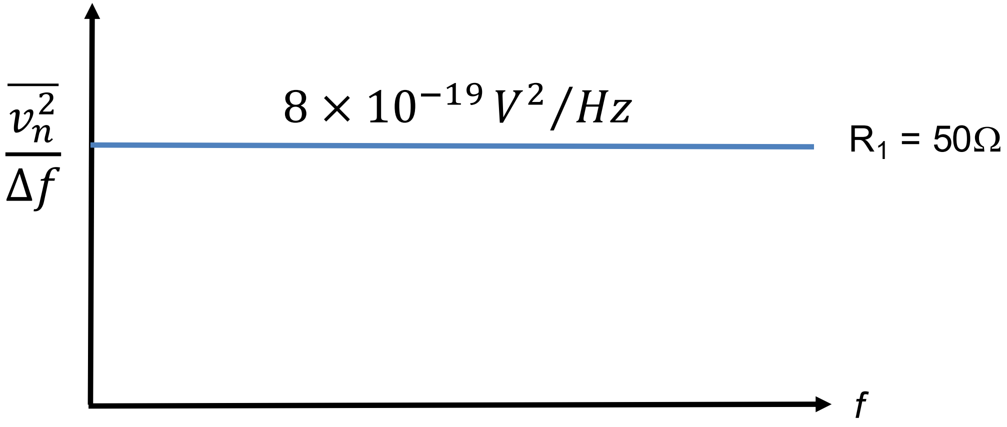
- **Source Polarity kept same throughout the calculations**

Spot Noise and White Noise

- Spot noise → Mean-square noise voltage density for 1Hz BW @ frequency of interest

$$\overline{v_n^2}(\Delta f = 1Hz) = 4KTR_1 = (1.6 \times 10^{-20})R_1V^2$$

- White noise → Mean-square spectral density is constant with frequency → spot noise is independent of frequency

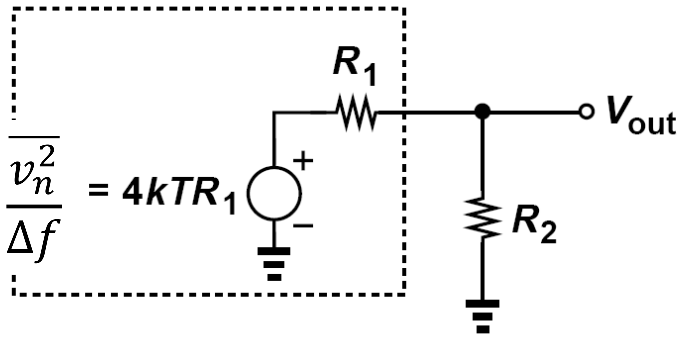


- Falls at f > 1THz → finite total energy.

Noise Power

- **Noise power delivered to the load resistor under conjugate match condition**

Suppose R_2 is held at T = 0 K



$$P_{R2} = \frac{\overline{V_{out}^2}}{R_2} = \overline{V_n^2}\left(\frac{R_2}{R_1 + R_2}\right)^2 \frac{1}{R_2}$$

$$\frac{P_{R2}}{\Delta f} = 4kT\frac{R_1R_2}{(R_1 + R_2)^2}.$$

This quantity reaches a maximum if $R_2 = R_1$:

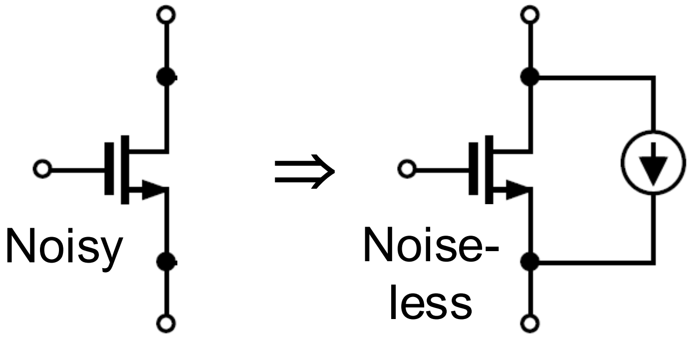
$P_{R2,max} = P_{avn} = kTB$

- **Proportional to absolute temperature and BW → keep them minimum possible for low-noise**

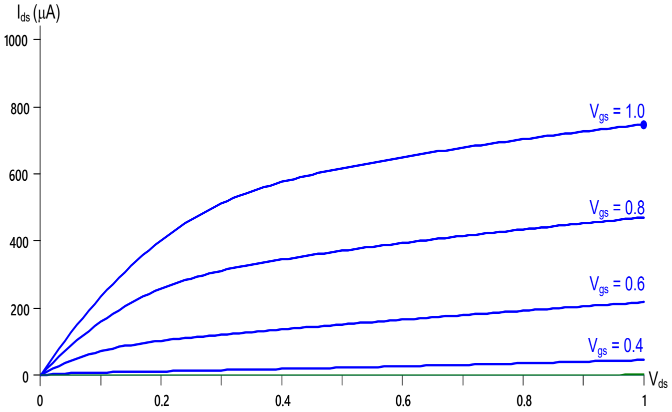
P_{avn}/Hz = 4.143 x 10^{-21} W/Hz = -174 dBm/Hz

Thermal Noise: MOSFET in Triode

- **Operating as a resistor**



$$\overline{i_{nd}^2} = 4KT\gamma g_{d0}\Delta f$$



g_{d0}=Slope at Origin

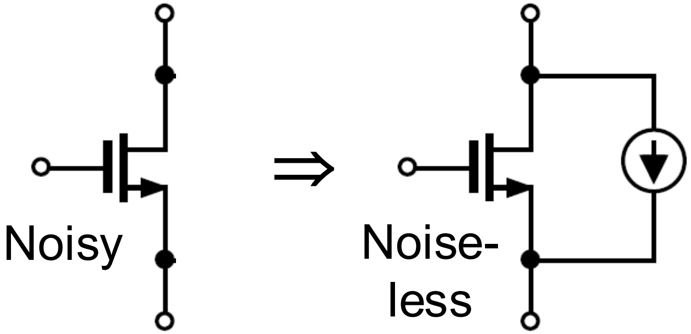
$$I_d = \beta\left[(V_{gs} - V_T)V_{ds} - V_{ds}^2/2\right]$$

$$g_d = \frac{\partial I_d}{\partial V_{ds}} = \beta\left[V_{gs} - V_T - V_{ds}\right]$$

$$g_{d0}|_{V_{ds}=0} = \beta\left[V_{gs} - V_T\right]$$

- **γ = Empirical drain current noise parameter ≈ 1**

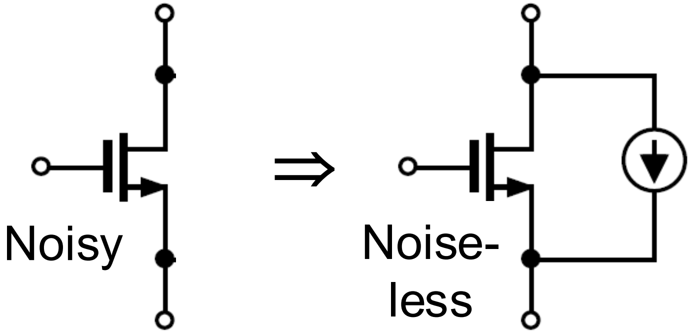
Thermal Noise: MOSFET in Saturation



$$\overline{i_{nd}^2} = 4KT\gamma g_{d0}\Delta f$$

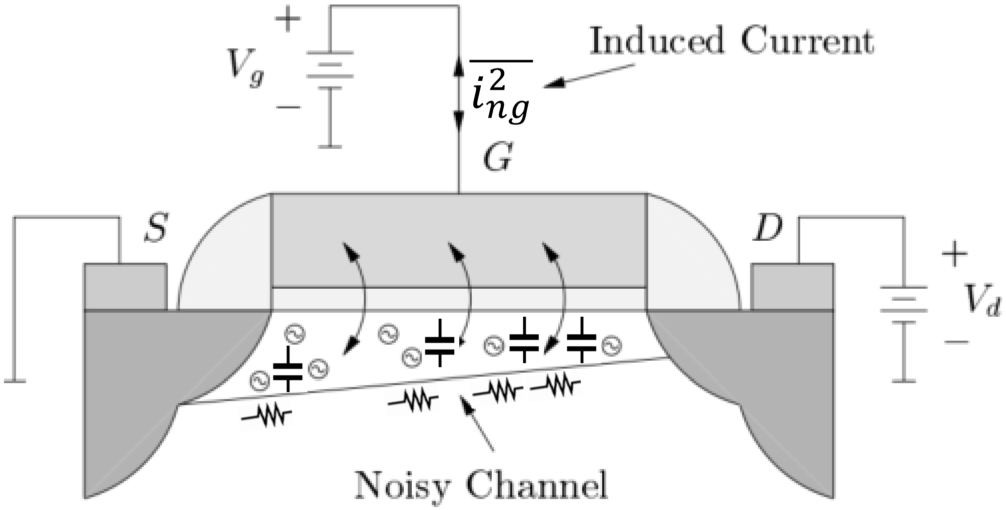
- $\gamma \approx$ **2/3 (long channel)**
- $\gamma \approx$ **2-3 (short channel) [Hot-carrier and substrate effects]**

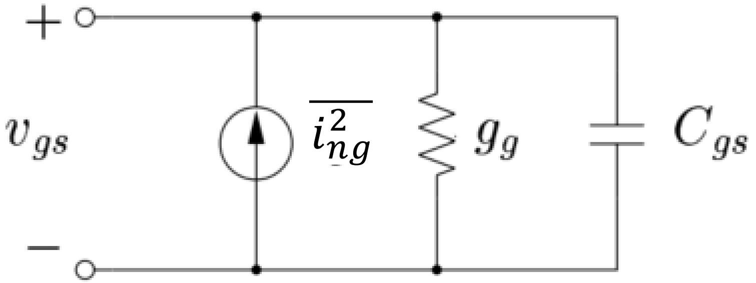
$$g_m = \frac{\partial I_d}{\partial V_{gs}} = \beta\left[V_{gs} - V_T\right] = g_{d0}$$



$$\overline{i_{nd}^2} = 4KT\gamma g_m\Delta f$$

Induced Gate-noise





$$\overline{i_{ng}^2} = 4KT\delta g_g \Delta f$$

$$g_g = \frac{\omega^2 C_{gs}^2}{5 g_{d0}}$$

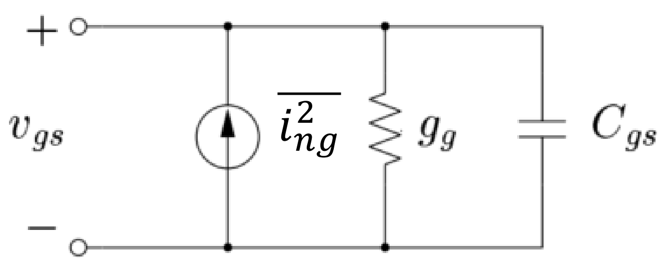
$$\delta = 2\gamma$$

[D. Shaeffer]

- **Channel charge variations cause local potential variations in the channel. These couple through the gate oxide capacitance to produce gate noise (displacement current).**
- **Not white**
- **Important at high frequency**

D. Shaeffer & T. Lee, " A 1.5V, 1.5GHz CMOS low noise amplifier," *JSSC* 1997.

Induced Gate-noise: Transformation

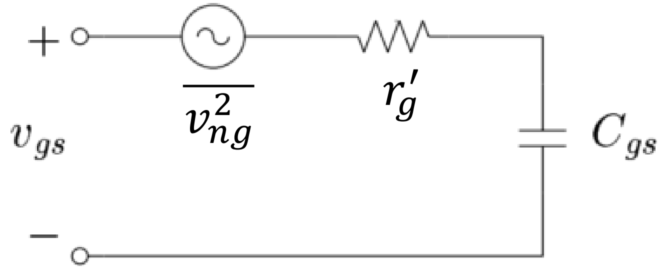


$$\overline{i_{ng}^2} = 4KT\delta g_g \Delta f$$

$$g_g = \frac{\omega^2 C_{gs}^2}{5 g_{d0}}$$

$$\delta = 2\gamma$$

[D. Shaeffer]



$$\overline{v_{ng}^2} = 4KT\delta r_g' \Delta f$$

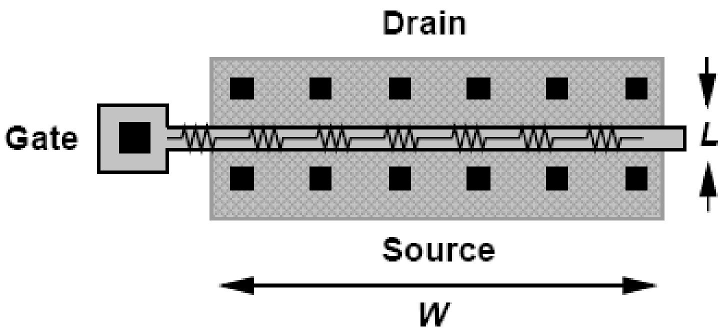
$$r_g' = \frac{1}{5 g_{d0}}$$

$$\delta = 2\gamma$$

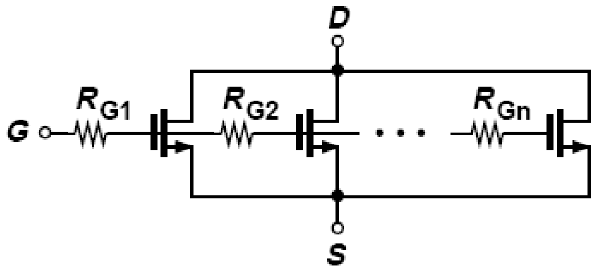
- **Transformation results in white noise!**
- **Narrowband transformation, Q>>1**
- **Important at high frequency**

D. Shaeffer & T. Lee, " A 1.5V, 1.5GHz CMOS low noise amplifier," *JSSC* 1997.

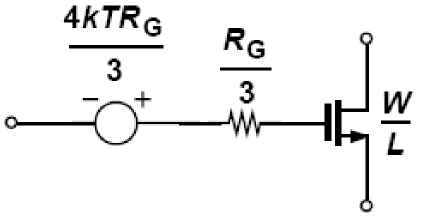
Gate-noise



$$R_G = \frac{W}{L} R_\square$$



$R_{G1} + R_{G2} + \cdots + R_{Gn} = R_G$



[B. Razavi]

$$\overline{e_{nG}^2} = 4KT\frac{R_G}{3}\Delta f$$

- Noisy resistive gate material (polysilicon) also produces gate noise.
- Important at high frequency, and for short-channel devices.
- Gate and drain terminal exhibit physical resistance too.

B. Razavi, " Impact of distributed gate resistance on the performance of MOS devices," *TCAS-I* 1994.

Correlation between Drain/Gate-Noise

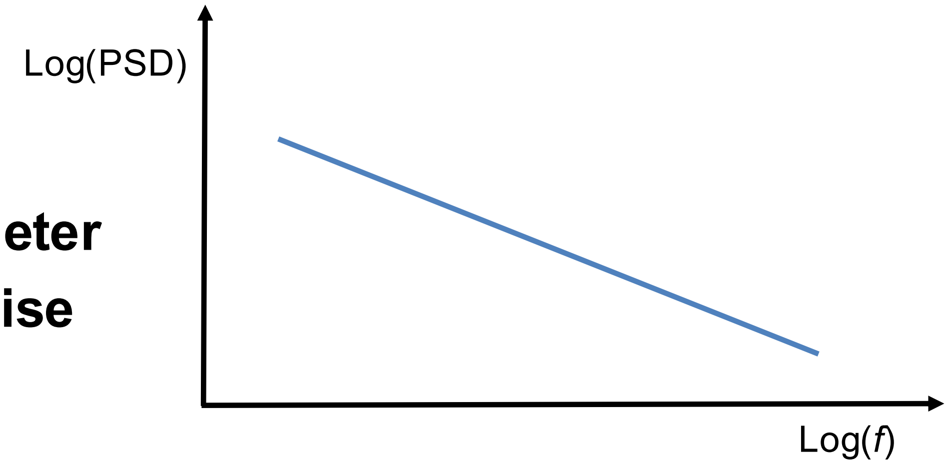
- Drain noise creates gate noise → they are partially correlated.
- Correlation coefficient $c = \frac{\overline{i_{ng} i_{nd}^*}}{\sqrt{\overline{i_{ng}^2 i_{nd}^2}}} \approx 0.395j$
- Notice the reference polarity of the correlated component

D. Shaeffer & T. Lee, " A 1.5V, 1.5GHz CMOS low noise amplifier," JSSC 1997.

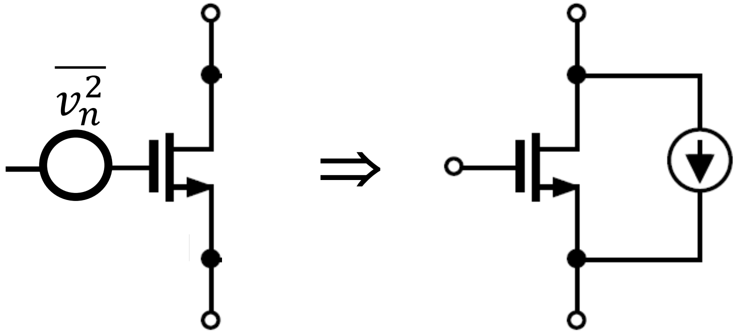
Flicker Noise

- Flow of DC current in resistors and transistors result in “1/f” noise
- In MOSFETs, random trapping and release of charge carriers is given as one reason

$$\overline{v_n^2} = \frac{k_{1/f}}{C_{ox}WL}\frac{\Delta f}{f}$$

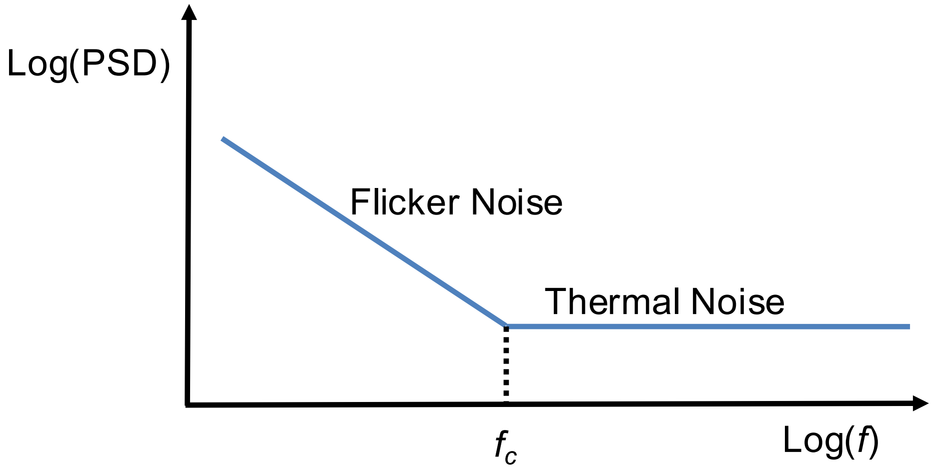


- $k_{1/f}$ is process dependent parameter
- Increase area to decrease 1/f noise



$$\overline{i_{n1/f}^2} = g_m^2 \frac{k_{1/f}}{C_{ox}WL}\frac{\Delta f}{f}\frac{\mathrm{A}^2}{\mathrm{Hz}}$$

Flicker Noise Corner Frequency



$$4KT\gamma g_m = \frac{k_{1/f}}{C_{ox}WL}\frac{g_m^2}{f_c}$$

$$f_c = \frac{k_{1/f}}{4KT\gamma}\frac{g_m}{C_{ox}WL}$$

$$\boxed{f_c \approx \frac{2\pi k_{1/f}}{4KT\gamma} f_\mathrm{T}}$$

$$\boxed{f_\mathrm{T} \approx \frac{g_m}{2\pi(C_{gs} + C_{gd})}}$$

- **f_T = unity current gain [i_d/i_{in}] frequency**
- **Faster process → Larger corner frequency for same area**

Overall MOSFET Noise Model