Radio-Frequency IC Design

Lecture 8: Non-Linearity

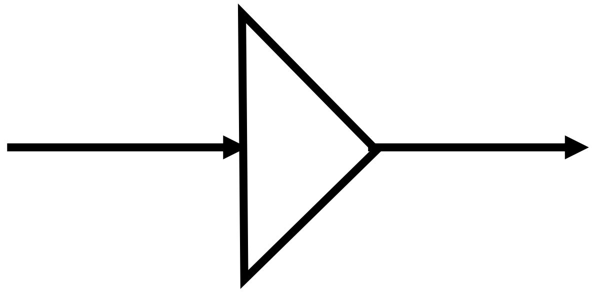
EECE 404

Acknowledgement: *RF Microelectronics*. B. Razavi

A linear system

Follows the principle of superposition

$x(t)$



$y(t) = \alpha x(t)$

$x_1(t) \rightarrow y_1(t) = \alpha x_1(t)$

$x_2(t) \rightarrow y_2(t) = \alpha x_2(t)$

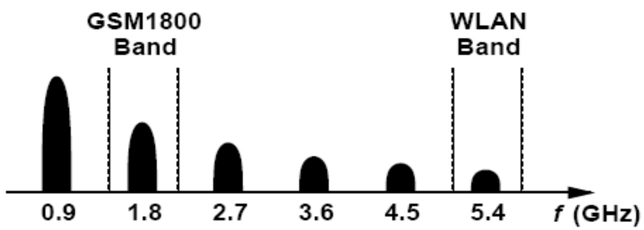
$$a x_1(t) + b x_2(t) \rightarrow y(t) = \alpha[a x_1(t) + b x_2(t)]$$
$$= a\alpha x_1(t) + b\alpha x_2(t)$$
$$= a y_1(t) + b y_2(t)$$

Memoryless non-linearity $y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$

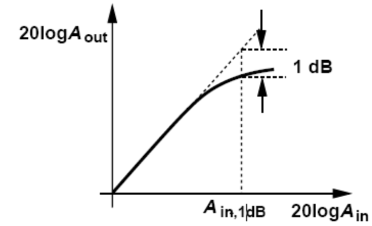
Non-linearity effects

- **Signal**

Harmonic Distortion

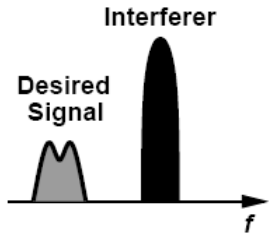


Gain Compression

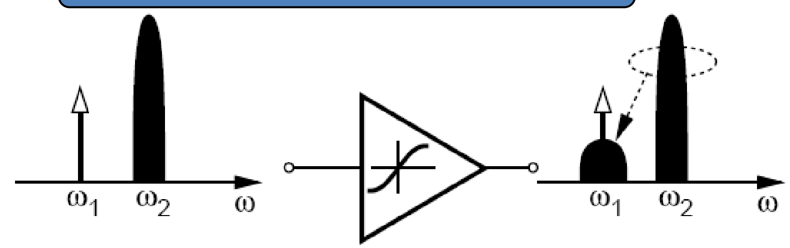


- **Signal + one large interferer**

Desensitization

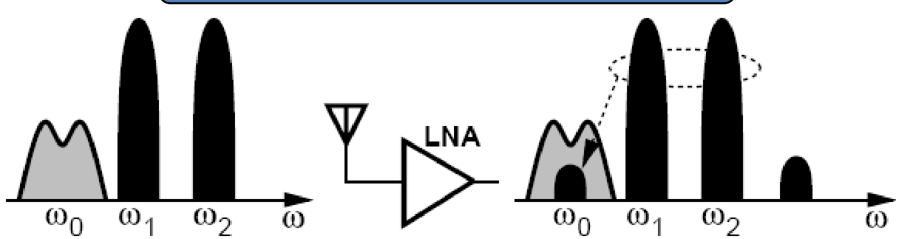


Cross Modulation



- **Signal + two large interferers**

Intermodulation



Harmonic Distortion (Single Tone I/P)

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$x(t) = A\cos\omega t$$

$$\begin{aligned} y(t) &= \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t \\ &= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t) \\ &= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t. \end{aligned}$$

DC offset

Fundamental

Harmonics

Desired

Undesired

What if $\alpha_1\alpha_3$ is negative (compressive circuits)?

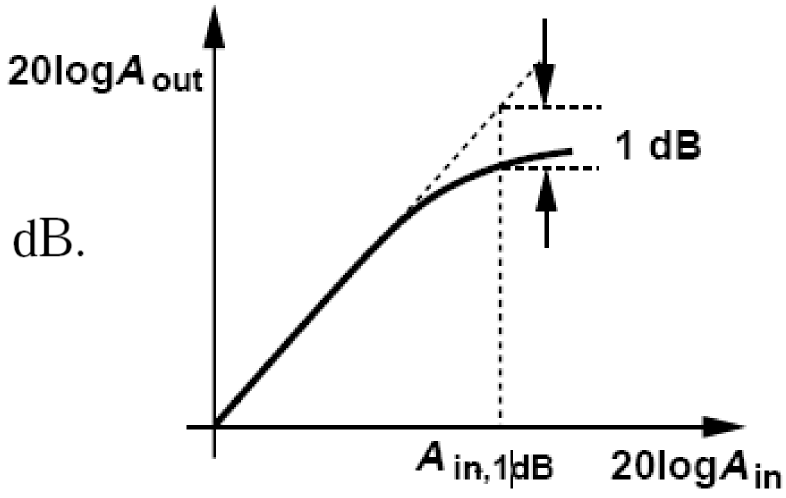
Gain Compression: -1dB Compression Pt.

What if $\alpha_1\alpha_3$ is negative (compressive circuits)?

$$y(t) \approx \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) cos\omega t$$

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A^2_{in,1dB}\right| = 20\log|\alpha_1| - 1\ \text{dB}.$$

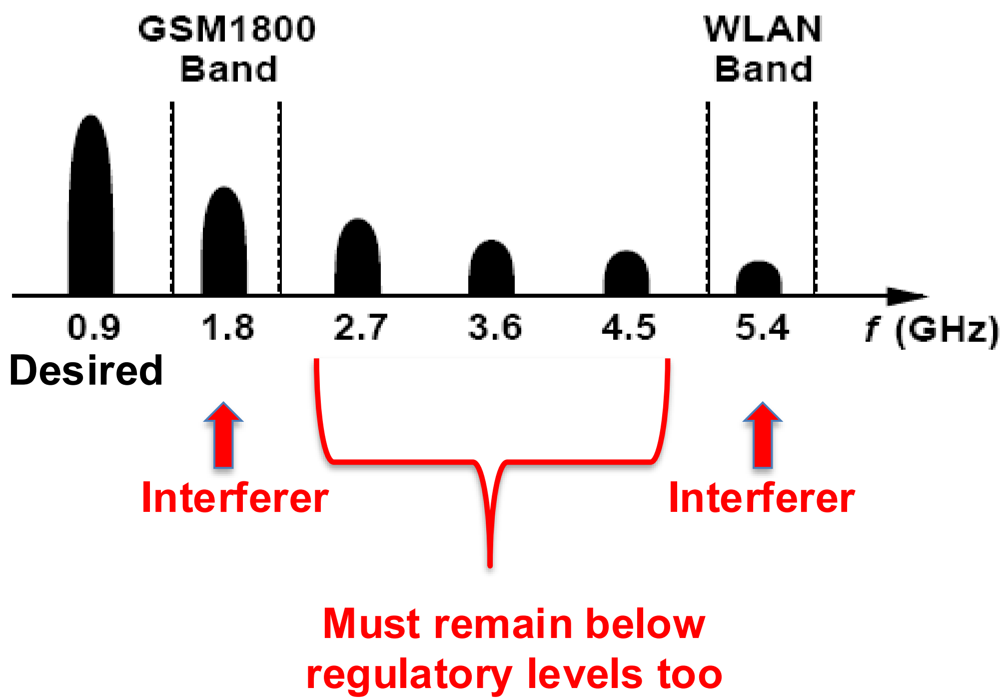
$$A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}.$$



- **Output falls below its ideal value by 1 dB at the 1-dB compression point**
- **Peak value instead of peak-to-peak value**

Example of Harmonics on GSM Signal

A 900-MHz GSM cellphone TX delivers 1 W of power to the antenna. Effect of the harmonics?

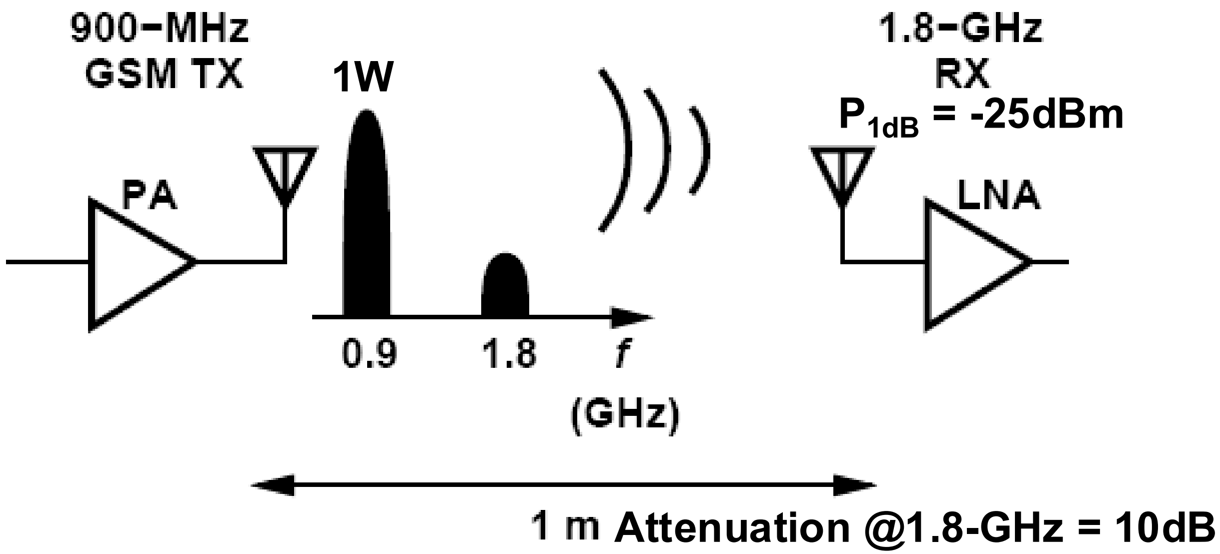


Example of Gain Compression

By how much must the 2nd harmonic of the GSM TX be suppressed (filtered) to avoid 1.8-GHz RX desensitization?

@ TX output

- 2nd harmonic < -25dBm + 10dB = -15dBm for it to be below P_{1dB} of the RX.
- Fundamental 1W → +30dBm.
- 2nd harmonic < 45dB below the fundamental
- In practice, several dB lower to ensure the RX does not compress.



Non-Linearity (Two-Tone I/P)

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

$$y(t) \approx \alpha_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + \alpha_3(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

Different terms at p*ω_1+q*ω_2.

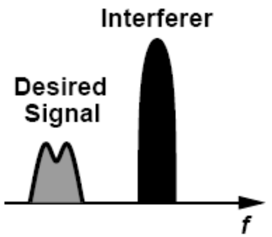
Fundamental

$$\omega = \omega_1, \omega_2 \colon \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos \omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos \omega_2 t$$

Desensitization & Blocking

$$\omega = \omega_1, \omega_2: \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos\omega_2 t$$

- Suppose ω_1 is desired and ω_2 is interferer
- Also, $A_1 \ll A_2$ (Strong interferer)

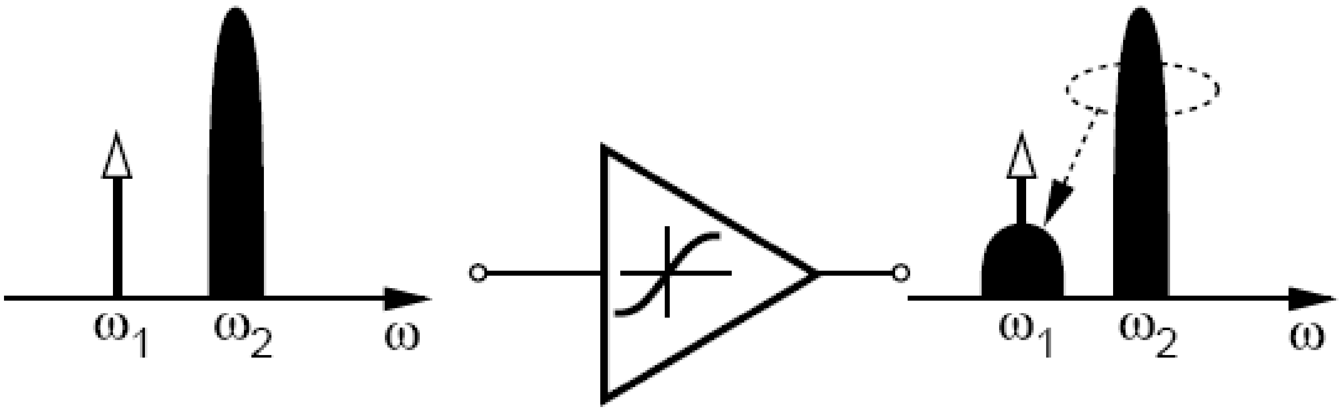


$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$

0

- For compressive circuits, strong interferers desensitize and can completely Block the signal!

Cross Modulation



Suppose that the interferer is an amplitude-modulated (AM) signal

$$A_2(1 + m \cos \omega_m t) \cos \omega_2 t$$

Thus

$$y(t) = \left[\alpha_1 + \frac{3}{2}\alpha_3 A_2^2 \left(1 + \frac{m^2}{2} + \frac{m^2}{2} \cos 2\omega_m t + 2m \cos \omega_m t\right)\right] A_1 \cos \omega_1 t + \cdots .$$

- **Desired signal at output suffers from amplitude modulation**
- **If interferer is a phase-modulated (only) signal, no cross-modulation for desired signal**

Non-linearity (two tone i/p)

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

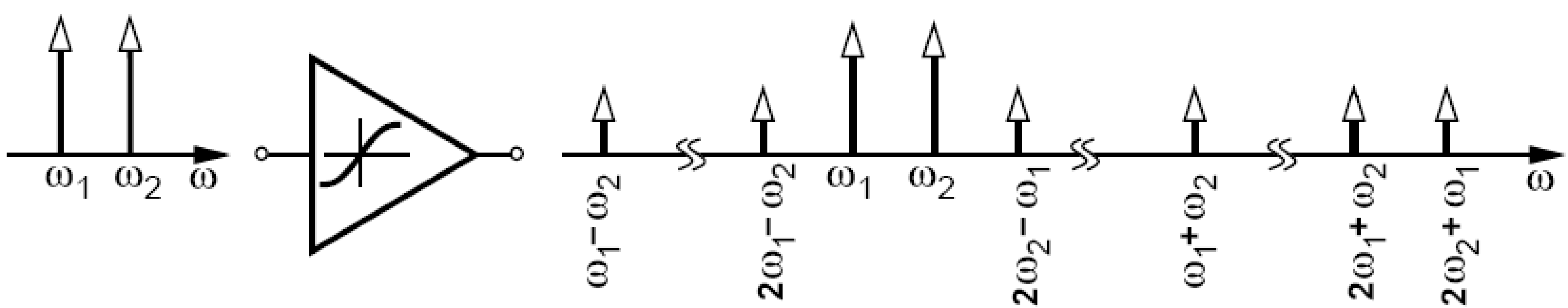
$$y(t) = \alpha_1(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t) + \alpha_2(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^2 + \alpha_3(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^3$$

Intermodulation (IM) products:

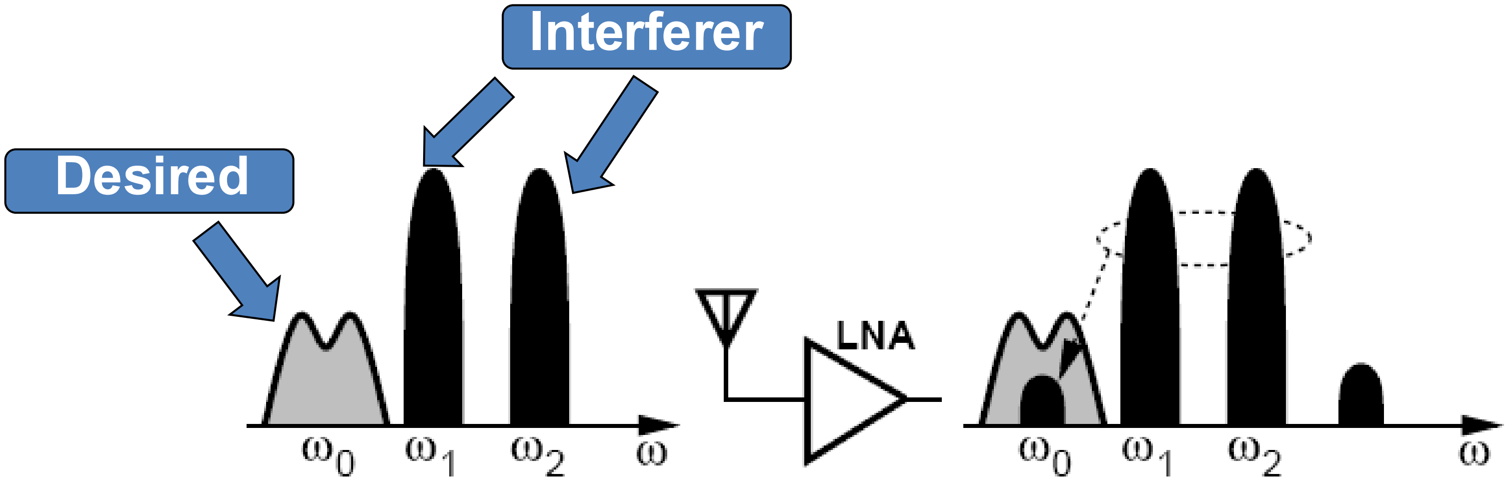
$$\omega = 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 - \omega_2)t$$

$$\omega = 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2 - \omega_1)t$$

Can be troublesome



IM3 Falling on Desired Channel

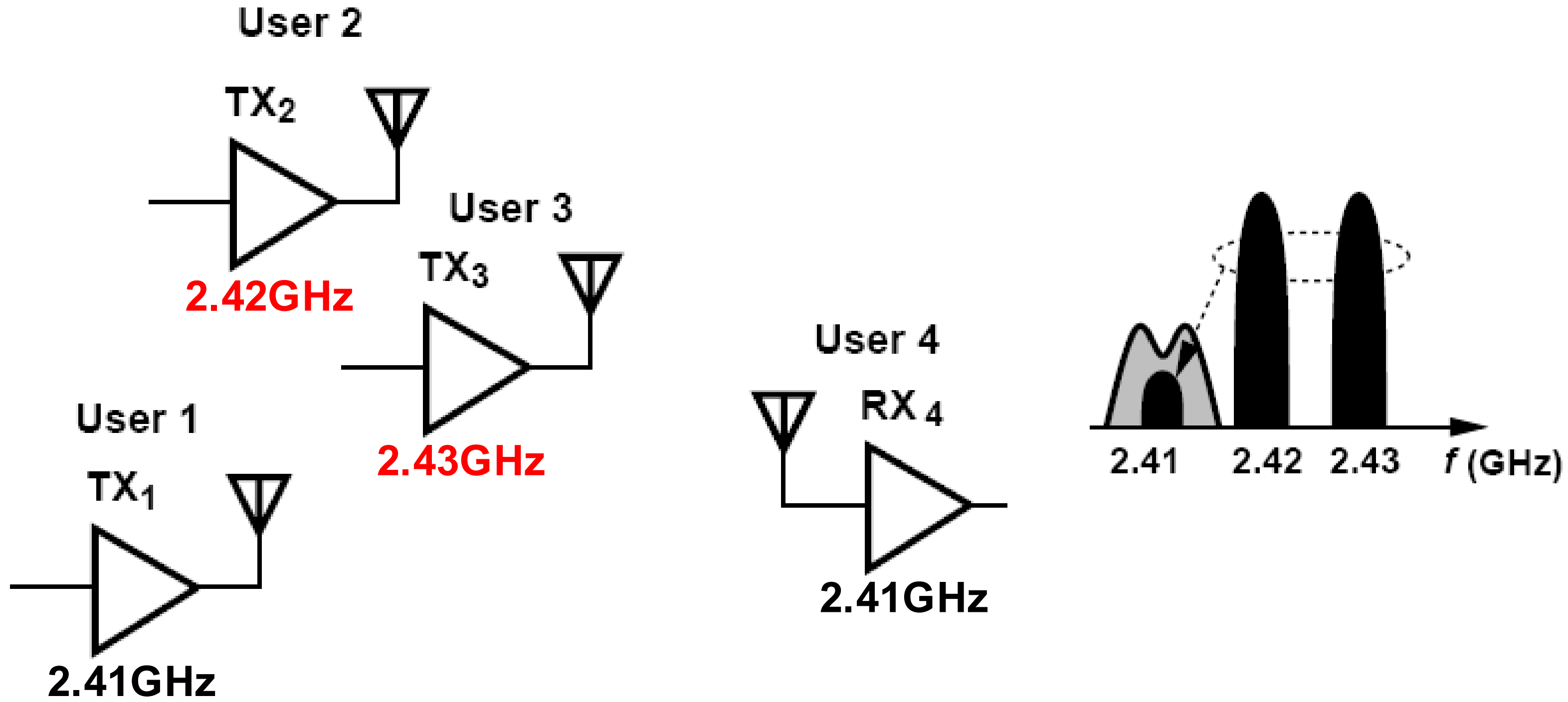


$$2\omega_1 - \omega_2 = \omega_0$$

- **A received small desired signal along with two large interferers**
- **Intermodulation product falls onto the desired channel, corrupts signal.**

Example of IM

Four Bluetooth users. User 4 RX attempts to sense a weak signal by User 1 TX. What happens?



- **Equally spaced frequencies from TX 1, 2, and 3**
- **IM3 in the LNA of R_{X4} corrupts the desired signal @ 2.410 GHz**

Input Third-order Intercept Point

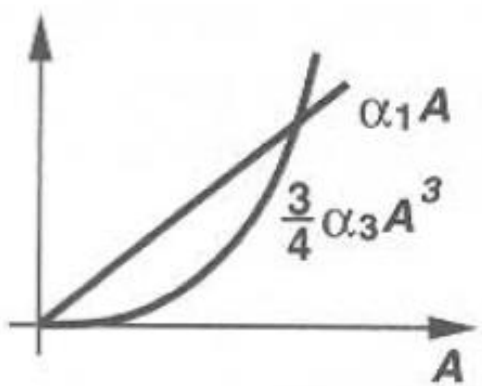
$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

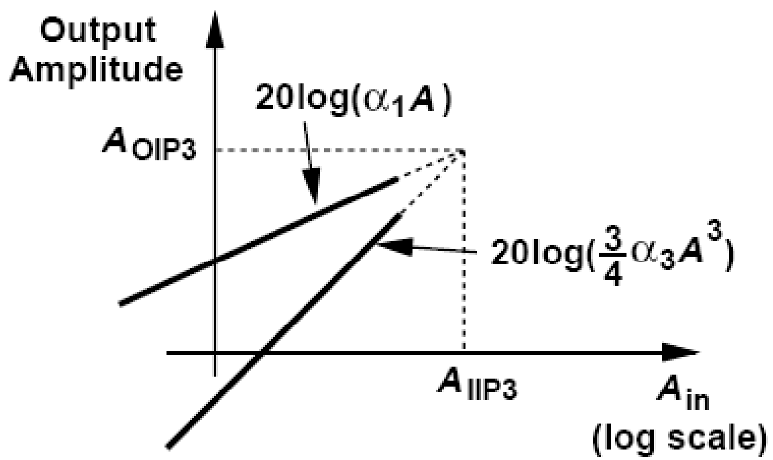
$$x(t) = A\cos\omega_1 t + A\cos\omega_2 t$$

$$y(t) = \left(\alpha_1 + \frac{9}{4}\alpha_3 A^2\right) A\cos\omega_1 t + \left(\alpha_1 + \frac{9}{4}\alpha_3 A^2\right) A\cos\omega_2 t + \frac{3}{4}\alpha_3 A^3 \cos(2\omega_1 - \omega_2)t + \frac{3}{4}\alpha_3 A^3 \cos(2\omega_2 - \omega_1)t + \cdots.$$

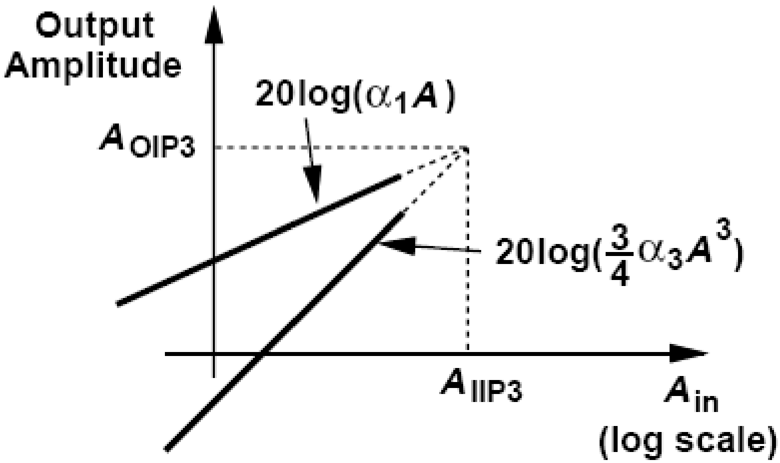
If $\alpha_1 \gg 9\alpha_3 A^2/4$ $\quad |\alpha_1| A_{IP3} = \frac{3}{4}|\alpha_3| A_{IP3}^3 \Rightarrow A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$

Crude approx.





IIP3

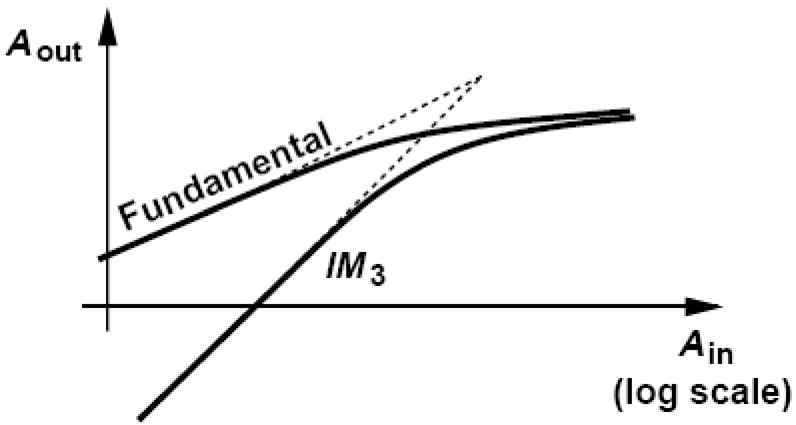


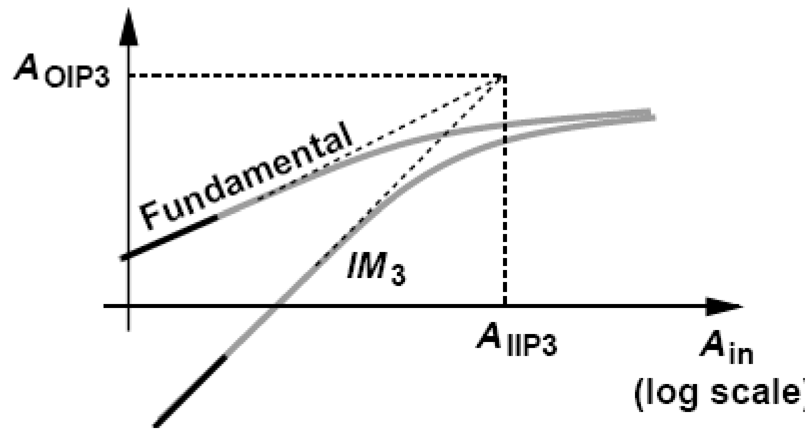
$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A_{IIP3}^3\right|$$

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}}$$

$$\approx 9.6 \text{ dB}.$$

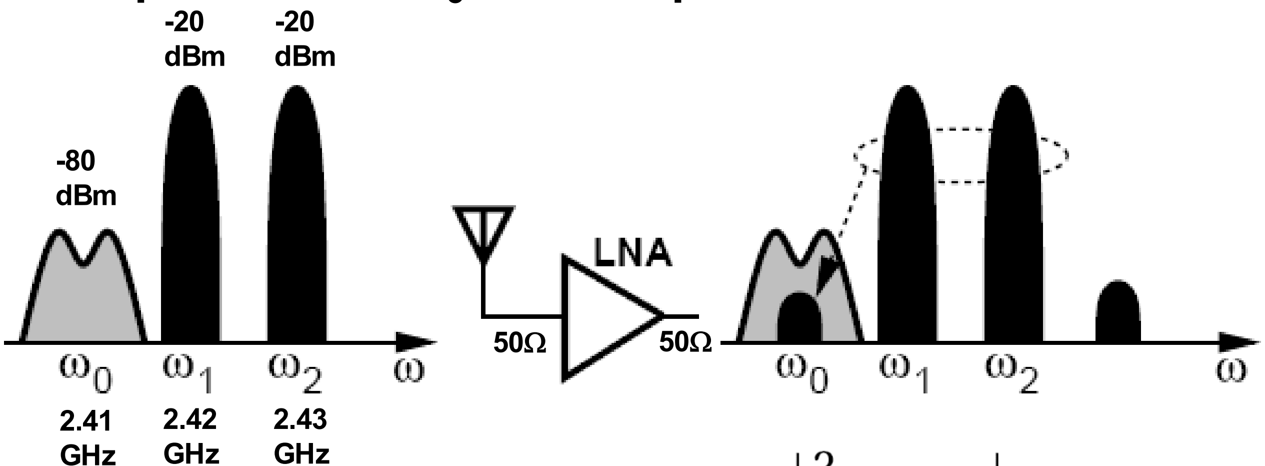




- **IP3 is not a directly measureable quantity, but a point obtained by extrapolation**

Example of IIP3

What is the required LNA IIP$_3$ if the IM products must remain 20 dB below signal?



@ LNA O/P: $$20\log|\alpha_1 A_{sig}| - 20\text{ dB} = 20\log\left|\frac{3}{4}\alpha_3 A_{int}^3\right|$$

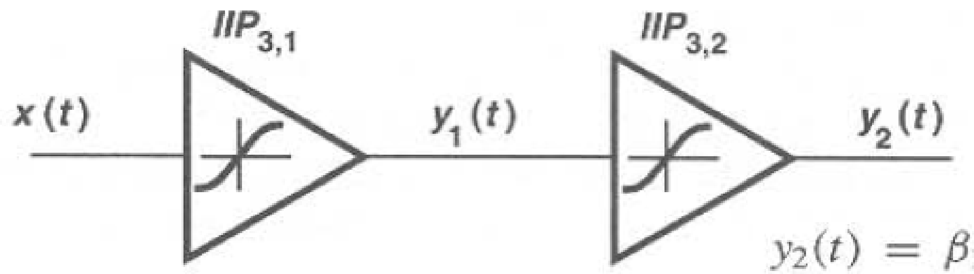
$$|\alpha_1 A_{sig}| = \left|\frac{30}{4}\alpha_3 A_{int}^3\right|$$

@ LNA I/P: -80dBm signal → A_{sig} = 31.6µV$_p$, and -20dBm interferer → A_{int} = 31.6mV$_p$.

$$\text{IIP}_3 = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$= 3.16V_p = +20\text{dBm}$$

IIP3 for Cascaded Systems



$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 + \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3$$

$$y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + \alpha_1^3\beta_3)x^3(t) + \cdots$$

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + \alpha_1^3\beta_3}\right|}$$

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots$$

$$\frac{1}{IIP3} \approx \frac{1}{IIP3_1} + \frac{G_1}{IIP3_2} + \frac{G_1G_2}{IIP3_2} + ...$$

- **Thus, if each G >1, the nonlinearity of the latter stages becomes increasingly more critical because IP3 of each stage is equivalently scaled down by the total gain preceding that stage.**